Analog Electronic Circuits
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Lecture - 06
Common source amplifier, small signal analysis

Welcome, back to Analog Electronic Circuits, this is lecture 6 and today, we are going to talk about developing the small signal model for the MOSFET. So, that is the plan for the lecture. As we were discussing yesterday we worked out how the biasing is done, right.

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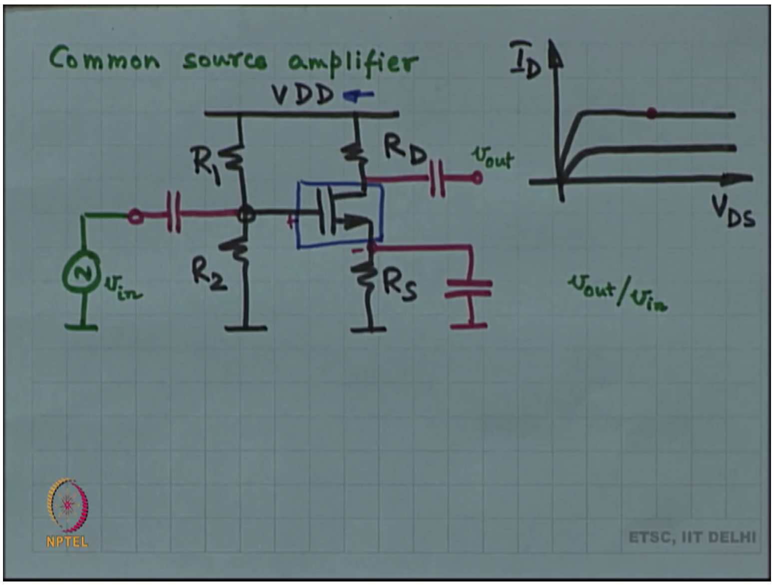

First of all we worked out that to set the DC operating point of the MOSFET this kind of a circuit is a good idea, ok. So, something that we worked out in the last class, we also worked out how to precisely compute the DC operating point that is; what is V DS what is id given the values of these resistors and given some details about the MOSFET, ok.

The objective of the biasing or the DC operating point exercise is to make sure that V DS is greater than V GS minus V T that is an important objective of this entire exercise. Once you have established the DC operating point in a way that V DS is indeed greater than V GS minus V T, Now, you know that the MOSFET in the MOSFET characteristics I have V DS and I have ID.

So, now we know that in the MOSFET characteristics, I have managed to place the device in the flat region of operation. So, somewhere over here maybe because V DS is more than V GS minus V t, ok. So, this is one job that we have done in the last class. Then we said that all right this has been biased, the MOSFET has been set at it is correct operating point. Now, how do I make an amplifier out of it and the answer was that we use these big fat capacitors to couple in the signal or to couple out the signal, all right.

So, this is going to be a big fat capacitor these capacitors are going to be a big fat capacitors, right. So, this big fat capacitor is going to work in a way that at DC it is an open circuit all capacitors are open circuit at DC and for the signal which is changing with time this capacitor is going to behave like a short circuit. So, that is the plan and likewise we had another problem and that problem was that to do good amplification it turns out that R S is not required R S is harmful, ok. Because we want all of the signal to come across V GS and none of it to appear across R S.

So, again a capacitor is going to come to the rescue and we are going to place that capacitor in shunt with R S to make sure that the source terminal is static with respect to time, ok. So, these are the jobs that we did in the last class. Now, today our plan is going to be to analyze the circuit or to work out how to analyze the circuit, ok. So, that is the job today and to do this job we have to understand how the MOSFET responds to small signals.

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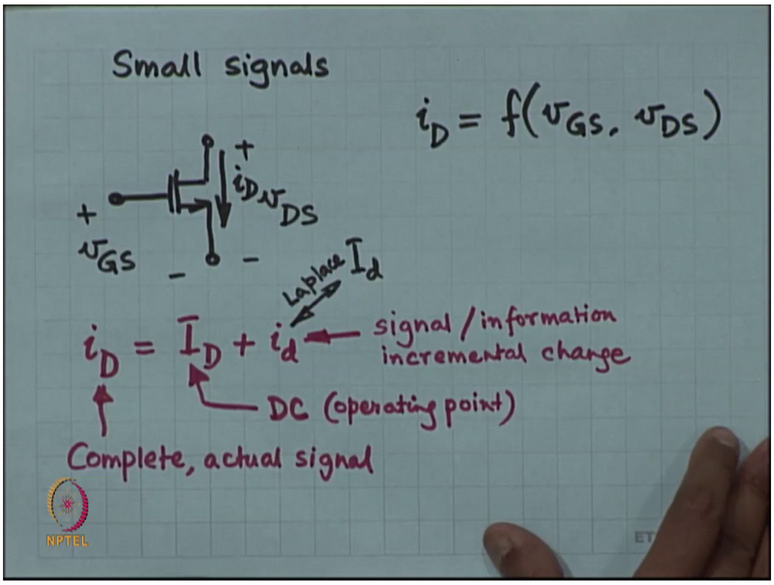

Small signals

$i_D = f(v_{GS}, v_{DS})$

$i_D = I_D + i_d$

i_d: signal / information, incremental change; Laplace I_d

I_D: DC (operating point)

i_D: Complete, actual signal

Now these small signals are so small that something's happen, all right, let us let us do it nicely. So, the MOSFET behaves in this fashion, if I have a V GS and I have a V DS then I will get an i D, where i D is some function of V GS and V DS and really I do not care what that function is right this is the characteristics of the MOSFET is rather difficult some people think that it is flat over here it need not be flat lot of Phd's are being awarded just to figure out what this function f is, I mean there are hundreds of peoples doing their Phd's to figure out what that f should be, ok.

So, it is very difficult to say what it is. In fact, it all it depends on so many other parameters that it is not worth in this course, we are not going to work out what is f, right that is not the objective, you do not worry about f, all right. Now, what you are going to do is something called Taylor series you have learnt this in maths and we are going to say that all these small letters of capital letter like i D can be broken up into a capital sub capital and a small sub small, all right.

So, this is going to be our terminology. The small sub capital is going to be the complete signal actual, signal, ok. The capital sub capital is going to be the DC part of it, that is the DC operating point, part of it and the small sub small is just the signal part of it or rather the information or you could call this an incremental change, ok. So, this is going to be our terminology. So, I have a DC operating point, on top of the DC operating point riding on top of the DC operating point I will incrementally change that parameter either the voltage or the current, that incremental change part that is where the information is right that is going to be small sub small, then capital sub capital is going to be the DC operating point the total thing is small sub capital. So, this is going to be a notation all right.

So, we are going to follow this notation throughout the course. In fact, you will find that this is the notation that is widely adopted. There is one more combination that is left out that is capital sub small and we will talk about it later on, right, capital sub small is nothing much it is a Fourier transform of small sub small, it is the Laplace transform of the frequency domain representation of small sub small, that is going to be a convention ok. So, so far what I have done is I have said that i d is a function of V GS and V DS this is all that I have done and then I am going to say that V GS this is small sub capital this is the actual thing this is the actual quantity this can be broken up into capital and small small, like wise this. So, let us do that.

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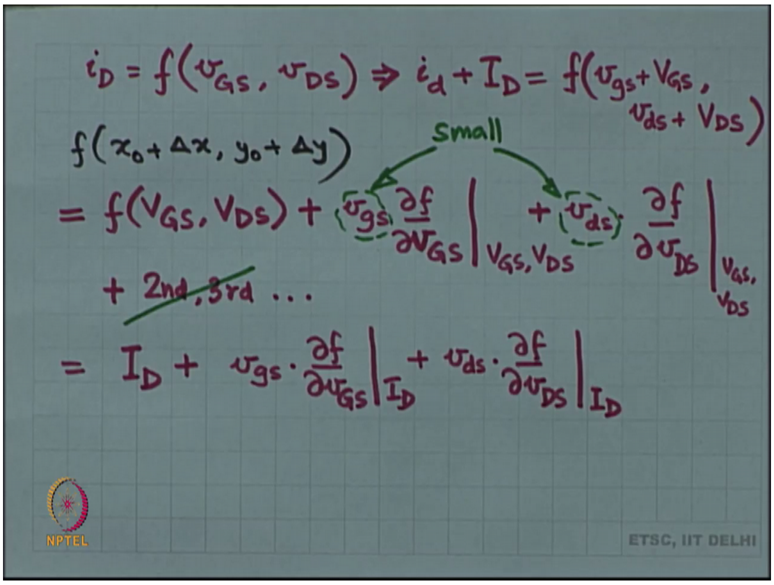

$$i_D = f(v_{GS}, v_{DS}) \Rightarrow i_d + I_D = f(v_{gs}+V_{GS},\ v_{ds}+V_{DS})$$

$$f(x_0+\Delta x,\ y_0+\Delta y)$$

small

$$= f(V_{GS}, V_{DS}) + v_{gs}\frac{\partial f}{\partial v_{GS}}\Big|_{V_{GS},V_{DS}} + v_{ds}\frac{\partial f}{\partial v_{DS}}\Big|_{V_{GS},V_{DS}}$$

$$+ \cancel{2nd, 3rd} \ldots$$

$$= I_D + v_{gs}\cdot\frac{\partial f}{\partial v_{GS}}\Big|_{I_D} + v_{ds}\cdot\frac{\partial f}{\partial v_{DS}}\Big|_{I_D}$$

So far so good, so I have broken up all the voltages and the current, ok. Now, I know one thing I know Taylor series, right, this function of x naught plus delta x, right. So, this is like this function of x naught plus delta x comma y naught plus delta y, ok. So, you this is something that you have done in your mathematics and this can be broken up as a Taylor series. So, I get back to our particular one. So, this is equal to f of V GS comma V DS plus v gs times partial derivative of f with respect to V g workout at V GS comma V DS plus v ds times partial derivative of f were with respect to v DS worked out at V GS comma V DS plus second order third order etcetera curves infinite terms that is the Taylor series.

Now, the assumption if we make an assumption that this small v gs and small v ds, right. These two quantities are small. If we make an assumption that these are very small, how small? Infinitesimally small, if you are not happy with how small it is, make it smaller than that, ok. If these two are small then all the second third and the higher order terms do not matter at all and this is all you are left with.

Now, this can be rewritten as f of V GS comma V DS this is nothing, but I D plus v gs times partial derivative of worked out at the operating point plus v ds times partial derivative of f worked out at the operating point, ok. So, this is the idea. Now, if you think about it effectively what does it transpire to? So, I am going to keep this over here and then this is a circuits course right, now the maths course.

So, in a circuit course what we try to do is to represent this mathematical equations as a circuit, ok, that is our job.

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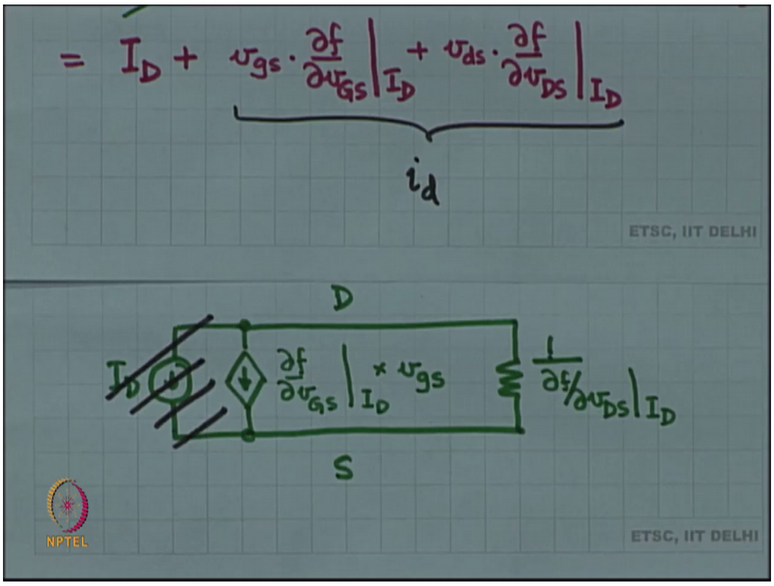

$$= I_D + \underbrace{v_{gs} \cdot \frac{\partial f}{\partial v_{GS}}\Big|_{I_D} + v_{ds} \cdot \frac{\partial f}{\partial v_{DS}}\Big|_{I_D}}_{i_d}$$

So, I D and then in addition to I D, I need some more current which is the function of v gs and then in addition to that I need some more current that is proportional to v ds. Now, if you think about it this is the drain terminal this is the source terminal something that is proportional to v ds would look like a resistor ok, not in this case. In this case it will look like a resistor of value 1 by dou f by dou v DS computed at I D, all right. So, this is what this equation looks like.

Now, the incremental part of this equation; so this is all of this entire picture is I sub capital D, but if I ask you what is i sub small d only this portion is i sub small d. So, as far as I sub small d is concerned you have only these two elements, between drain and source. One is a voltage controlled current source proportional to v gs and the second is a resistor, and what are these two values? These two values are the slopes of the curve computed at the operating point. So, let us simplify this. So, the simplified version looks like this.

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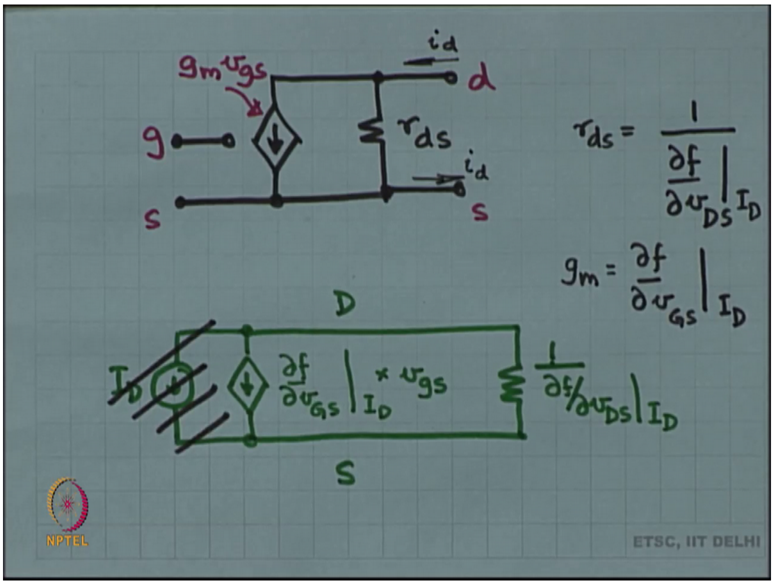

So, this is going to be the model. So, it is looks like a two port network; one port is between gate and source the other port is between drain and source, these are the two ports, ok. Between drain and source I have got r ds and the value of r ds is 1 by dou f by dou v DS computed at the operating point, that is the value of r ds, right. Some people write it as a conductance in which case they will write it as g ds, that is ok, right r and g, you have to do that one by on the fly, ok.

This one is popularly known as g m right and all the technical literature this is called a transconductance and the symbol for a transconductance is g m this is denoted as a small g sub small m and the reason why it is small is because this is the small signal transconductance, and the definition of g m over here is dou f by dou v GS computed at the operating point, all right. So, this is going to be the model for the MOSFET.

So, if I apply some gate to source voltage and incremental gate to source voltage according to this MOSFET model, I will get an incremental drain current that is the idea, fine. Now, how do you compute these partial derivatives at the operating point?

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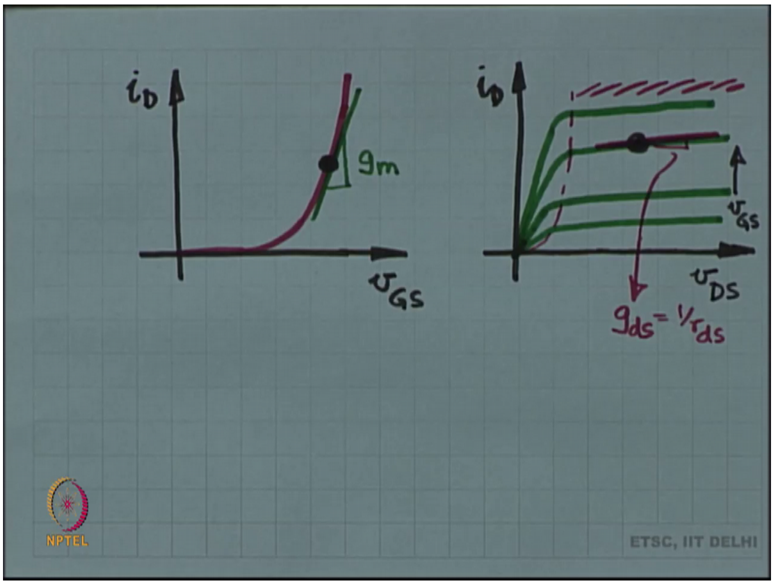

So, we are back to the MOSFET characteristics. So, from the gate depending on the great source voltage we have somewhat of a quadratic relationship, right. Some people think that this is this quadrat this curve looks like id equal to some k times v GS minus V T the whole squared, and then depending on the value of v GS you think there is going to be some current right and then accordingly it also depends on v DS, ok. So, depending on the value of v GS for different values of v GS you pick the right curve, all right.

And then finally, you work out what is the value of v GS depending on the value of v GS you pick the curve and then once you have picked the curve depending on the value of v DS you work out what is the DC operating point. So, this is how you would have computed the operating point, fine. How do I workout r ds and gm from the partial derivatives, right and the partial derivatives come from these characteristics.

So, for example, from the i D v GS characteristics I look at the slope at this point and this slope tells me the value of the transconductance gm, done. And then from the i d v DS characteristics I look at the slope over here and from the slope of this line this slope of this line is going to be how much g ds, equal to 1 by r ds is that ok? Is this understood so far or you do not understand why, where this curve came from?

So, if this curve depends on where you are over here, all right. So, be careful about it this curve over here depends on where exactly are you located in this graph. So, if you are located outside of this region that is in the flat region of the MOSFET, in this region of the MOSFET then only this curve is valid otherwise you will get some other curve, there is some other shape but, whatever that shape is it does not matter you all you have to do is find the slope you plot the characteristics of the MOSFET and find the slope at that operating point at that operating point if I change v GS slightly how much does i D change, that is all that you are working out at that operating point. If I change v DS slightly how much does i D change? this is all that you are interested in, ok.

Once you have done that then you have been successfully able to work out, what is g m v gs and what is r ds and you know everything about this MOSFET, is this so far? Oh, what about the fourth terminal? Great, I did not expect this question, but yes; so the fourth terminal, the body terminal. So, all the MOSFETs come with another additional terminal called body.

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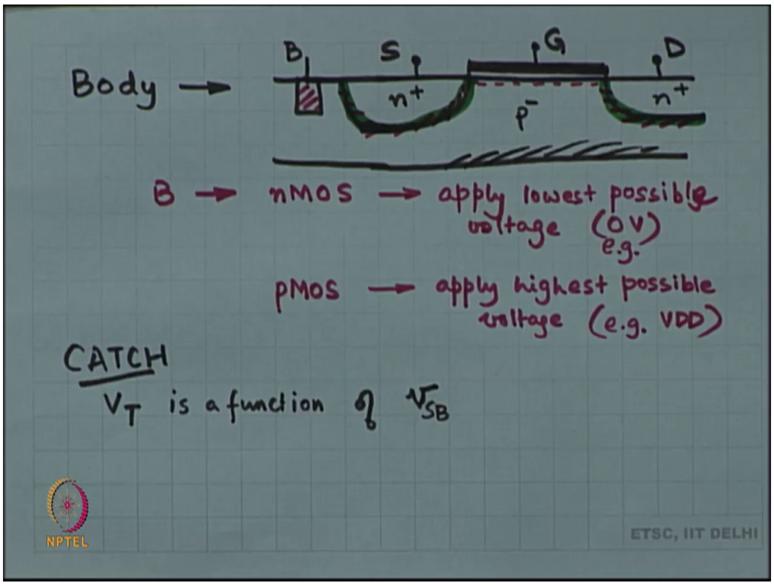

B → nMOS → apply lowest possible voltage (0 V) e.g.

pMOS → apply highest possible voltage (e.g. VDD)

CATCH

V_T is a function of v_{SB}

It creates so, if I apply a potential at the body which is not. So, the body let us start from so, the body is the rest of the stuff, right all of this. So, normally what is done is you take a place the contact of this substrate then that becomes a body, ok, but what we are referring to is the potential of this side.

Now, the body has an effect. The effect is if I apply a voltage on the body which is not equal to the lowest potential so, suppose this is an nMOS device, ok. In an nMOS device you have got two p-n junctions. These two p-n junctions will automatically be have some depletion region, ok. Now, if you are not careful about these p-n junctions then they might get forward biased and that is not something that you want over here because when you forward bias the p-n junctions the MOSFET is not going to behave like a MOSFET anymore the MOSFET action happens over here, right.

You get an inversion layer formed right underneath the gate when you apply a gate voltage the gate tries to full charge to the top of the of the surface right beneath it, right and that creates an inversion layer that is the connection between source and drain right that is called the channel. So, that is how the MOSFET operates, it does not operate like a BJT, all right.

So, what is very important is to make sure that unlike in a BJT, in a MOSFET these p-n junctions should not operate like diodes, they should be reverse biased, it is very important. So, we have to make sure that this p-n junctions are totally dead, not forward biased at all and how do we do that? We have the body terminal what we do is we apply the lowest possible voltage to the body terminal because it is p if I apply the lowest possible voltage to the body terminal then it may guarantee that the source and drain cannot go below the lowest and therefore, this those two p-n junctions are not going to be forward biased at all, ok. So, you will always have a MOSFET over there and not some other pair of diodes all right.

So, this is usually the strategy. So, the strategy is that for the body terminal I in an nMOS apply lowest possible voltage, usually ground for example, ok, but if you are working with the pMOS device then the body is going to be n minus right the pMOS device is the opposite of the nMOS device. Here the body is going to be made up of n and the source and drain are going to be made of p and therefore, if I want to make sure that these two junctions are always reverse biased then I have to apply the highest possible voltage to the body.

And, usually a good example of the highest possible voltage is the power supply voltage, ok. When you are working with MOSFETs the common acronym for the power supply is VDD, fine. So, this is going to be the ordinarily, this is the strategy that nMOS devices apply the lowest possible voltage to the body and be done with it pMOS devices apply the highest possible voltage to the body and be done with it and then forget about it.

So, if you have a chip which has many nMOS devices, many pMOS devices what you are going to do is you are going to apply zero volts to the bodies of all the nMOS devices and VDD volts to all the pMOS device bodies and then leave it at that and then once you are done that all of this small signal model and all of these things the body does not pop up at all. You are not going to worry about the body at all you can treat it as the 3 terminal device. You can treat the device as just three terminals.

However, does the catch, the catch is that V T of the device the threshold voltage of the device is some function of the source to body voltage, ok. So, the source to body voltage if it is not 0 or if it is if suppose the source is changing right then the source to body voltage is going to be changing then, that means, that V T of the device is going to be changed. So, this is the catch, all right. And, this is something that you have to be aware of ok.

So, while we are discussing the small signal model of the MOSFET you have to be aware that V T is a function of V SB. So, let me make this small absolute V SB and likewise the source voltage, if it has some change or let us let us say the body voltage has some change, ok.

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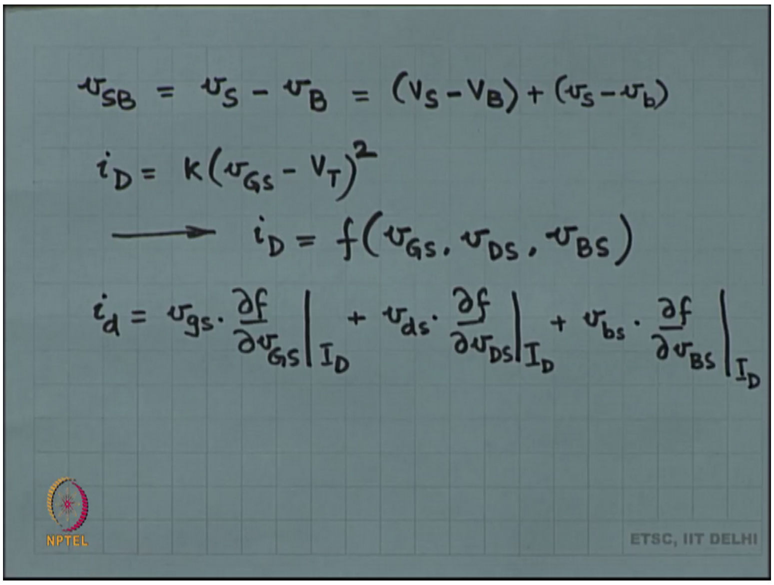

$$v_{SB} = v_S - v_B = (V_S - V_B) + (v_s - v_b)$$

$$i_D = K(v_{GS} - V_T)^2$$

$$\longrightarrow \quad i_D = f(v_{GS}, v_{DS}, v_{BS})$$

$$i_d = v_{gs} \cdot \frac{\partial f}{\partial v_{GS}}\bigg|_{I_D} + v_{ds} \cdot \frac{\partial f}{\partial v_{DS}}\bigg|_{I_D} + v_{bs} \cdot \frac{\partial f}{\partial v_{BS}}\bigg|_{I_D}$$

So, suppose the body voltage has some incremental change. If either the source or the body if these voltages are changing then that is going to change the V T of the device with time and then I know that i D is some function of v GS minus V T let us say K times v GS minus V T, if V T itself has changed with time then i D is no longer just a function of v GS it is also a function of v SB. So, therefore, i D is not a function of just v

GS and v DS, it is a function of v GS V DS and v SB or v BS, right it is all the same thing ok. So, this is the correction.

And, when you make a correction like this it is going to affect the structure of the small signal model and let us take a look at how it affects the structure. So, now, i D is capital I D plus small i d right and if you think about small i d then small i d is going to be the sum of all the first order derivatives right, it is going to be small v gs times dou f by dou v GS computed at the operating point plus small v ds times dou f by dou v DS computed at the operating point plus small v bs times dou f by dou v BS computed at the operating point. So, this is now that extra term that has come up because of the body effect fine ok.

So, how does the model change? The earlier model had two components because there were two terms over here. So, this was g m times v gs were gm is equal to dou f by dou v GS computed at the operating point plus v ds times 1 by r ds, v ds by r ds, where 1 by r ds is dou f by dou v ds. And, now an additional component is going to come and what is that going to be, is it going to be a resistor? No, because this is going to be exactly like the first term. So, the third term will look similar to the first term this is a controlled current source where the controlling parameter is v bs.

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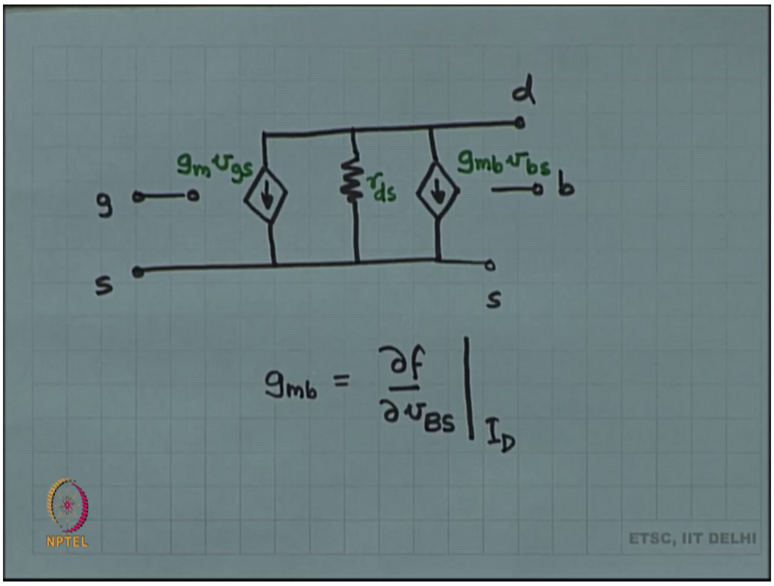

$$g_{mb} = \left.\frac{\partial f}{\partial v_{BS}}\right|_{I_D}$$

So, finally, this is going to be my model. So, I will have three elements, one is g m times v gs, one is V DS by r ds and the other is g mb times v bs; where g m so, g m was g m and r ds are still defined like what I have defined them as earlier. The same definitions continued and then I have got a new parameter over here g mb and that is equal to dou f by dou v BS computed at the operating point fine, ok. So, this is what happens when you take into account the body as well. So, normally we are going to forget about the body when we see that the source voltage is changing with time, the body voltages.

So, our strategy is going to be this, right our strategy is going to be that for every nMOS we are going to apply the lowest possible voltage. For example, 0 volts and that will make sure that the p-n junction is dead, ok, both these p-n junctions have to be dead. For every pMOS we will apply highest possible voltage, for example, VDD and that will make sure that the p-n junctions are dead, ok.

Now, once we have done this the body voltage is constant. So, incrementally just the body voltage is not changing. But body to source voltage could be changing if the source voltage is not fixed if the source voltage is constant then v bs is not changing at all ok, because body is not changing source is not changing, but if the source voltage is changing that automatically means that v bs will have some small signal and if that is the case then we have to fall back to this particular model they this model is not going to work, right. We have to introduce the third component over here is this ok, all right.

So, let us use our model in our first circuit. So, our first circuit is right over here, right this is going to be our amplifier we discussed this is the common source amplifier, all right and you are going to apply a signal over here with respect to ground and then we are going to measure the output over here basically the output from here will go to an oscilloscope or to a microphone or to a speaker sorry not microphone to a loud speaker or wherever it is.

So, I will call that v in, the input node I will call it v in and the output node is going to be called as defined as v out and then of course, the big question is what is v out by v in this is what is called the voltage gain of the amplifier v out by v in, ok. This is what is the most interesting. I apply a 1 milli volts signal, will I see a 100 milli volt signal at the output or not. The voltage gain is what is important in the amplifier and how do we compute voltage gain.

So, how do we analyze this circuit? So, to analyze the circuit what we have to do is first we find out the DC operating point, then we workout g m, r ds and g mb using our definitions, ok, by looking at the slopes of the curves. So, this is part – 2, after we have done this then part – 3 is to replace the MOSFET. So, the MOSFET has to be replaced with its model, with its small signal incremental model and when we are talking about small signal incremental the small signal incremental picture then power sources like VDD have to be nulled because these are not changing with time, right.

So, the small signal incremental VDD is 0, ok. So, every node that we are talking about should have their small signal incremental quantities prescribed. So, if it is a current source it will become open circuit, if it is a voltage source it becomes a ground, all right. So, let us do this; the resistors, capacitors, what is going happen to the capacitors? The resistors remain as it is. The capacitor was chosen to be so big and fat that when a signal is changing with time this capacitor is going to behave like a short circuit. So is this and so is this, right. They are all going to start behaving like short circuits.

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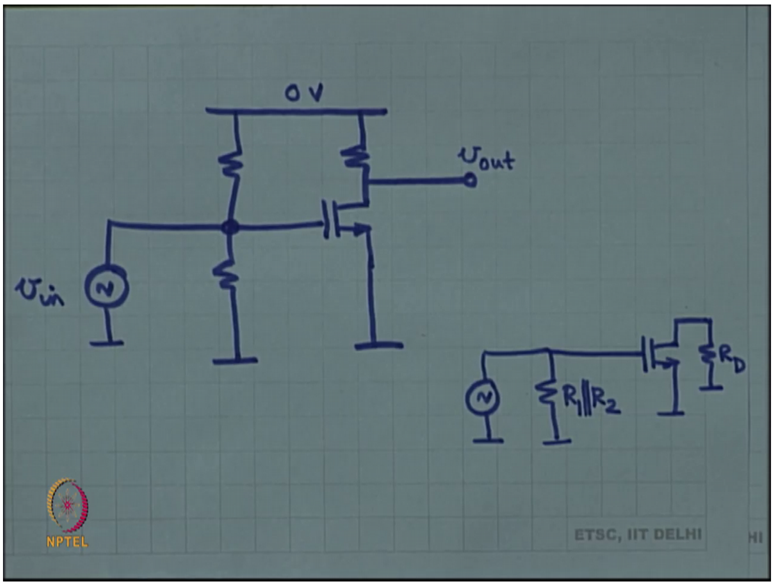

So, now we are in the small signal picture, right. This is not the complete schematic this is only the small signal picture of the schematic, the small signal part of the schematic and here the capacitor this particular one is a short circuit which means R S has been shorted out. Likewise, this capacitor is a short circuit which means this particular node is connected straight away to v in, and the output capacitor is also a short circuit which means this node is connected straight away to v out and then VDD does not change incrementally, it does not change with time which means this particular node is ground, ok. It is no longer at VDD, it is ground, ok.

If you want you could redraw this with all the grounds at the bottom, but this is ok, right some people prefer drawing it in this fashion. Some people prefer drawing it in this fashion I am not necessarily one of those people, but it is ok, right. You can draw it like this if you want to. Is this ok?

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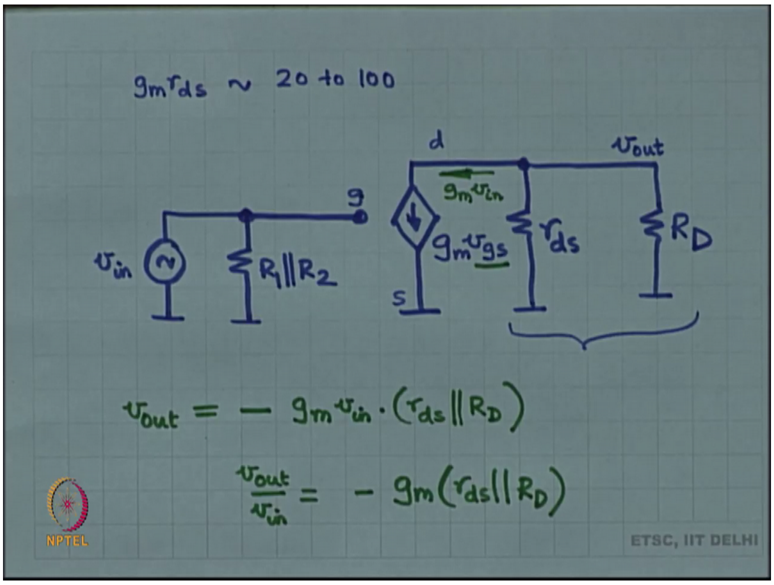

Then what you are going to do is replace the MOSFET with its small signal incremental model and what we are going to land up with is something that looks like this. Now, notice that the source terminal is at ground and the body terminal is always at signal ground which means that the source to body voltage is not changing at all, which means that g mb times v bs; v bs is 0, because body is not changing source is not changing. So, g mb, v bs is 0 which means that this particular current source you need not bother about it because its value is going to be 0, ok. So, 0 ampere current sources; so I am not going to bother drawing it you can draw it, but body and source are both at 0. So, this is what you end up with.

So, you are applied v in you want to measure v out and your R 1 parallel R 2 over here you have applied this voltage to a gate, source is at ground right and then g m v gs is part of the MOSFET r ds is part of the MOSFET right the drain is connected straight away to v out and then there is R D to ground. You have to measure v out and this is straight forward first of all r ds and R D appear in parallel with each other, ok. Secondly, the voltage v in appears straight away across gate and source which means that v gs is nothing, but v in, this is the same as v in which means that this current is g m times v in.

gm times v in current comes through the parallel combination of r ds and R D which means that v out is equal to minus because it is flowing out of ground g m times v in is the current times r ds in parallel with R D or in other words v out by v in is equal to minus g m times r ds in parallel with RD. Is this ok? So far so good, alright, where is the gain in this? The gain in this is in the combination of g m and r ds parallel R D.

If R D is very large r ds is usually large right g m times r ds is known as the intrinsic gain of the MOSFET. So, gm times r ds of a MOSFET is typically large where large when I say large it could be a value between 20 and hundred right depending on how nice the MOSFET is gm times r ds could be some value between 20 and 100. Now, depending completely on the value of RD right with respect to r ds you are going to get your game from this equation, right.

This is this equation is gm times r ds in parallel with RD if RD is infinite then you get minus gm times r ds otherwise you get something lesser than that, ok, which means that your gain is at max equal to minus g m r ds. So, whatever that value is how good a MOSFET you have used I do not know according to that value you will get a gain between minus 20 and minus 100 less than that right because you finally, have RD in parallel, ok.

So, this is the common source amplifier and we have done we have done the whole thing what we have done today is we have explained in detail the small signal model of the MOSFET including the body, right and then we have explained in detail how we have done the small signal analysis of the circuit, right and we have described the entire common source amplifier in this fashion.

Now, in the next classes what we are going to do is we are first going to talk about few conventions right input impedance output impedance and so on, voltage source, current source we are going to talk about a few of those conventions. And then what we are going to do is we are going to talk about the common drain circuit and the common gate circuit, alright those amplifiers are going to be our next objectives.

Thank you. And hope to see you soon.