Analog Electronic Circuits
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Lecture – 04
DC operating point

Welcome back to analog electronic circuits this is the fourth lecture and today we are going to discuss the start making circuits with MOSFETs and we hope to make our first amplifier circuit using a MOSFET. So, in the last class we were brief recap of what we had done and we looked at the characteristics of the MOSFET in the last class.

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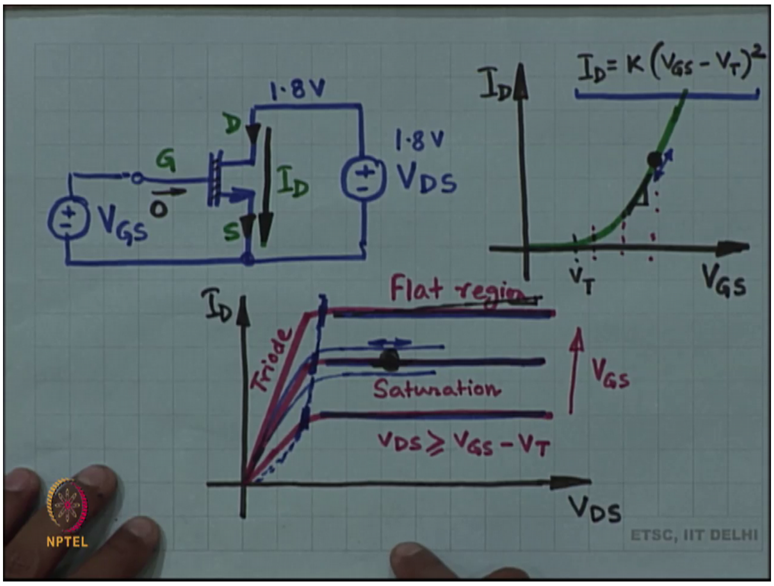

And in the characteristics of the MOSFET we found that there is a triode region which we do not like and there is a flat region of the MOSFET which we like and we are going to operate the MOSFET in it is flat region as far as most analog circuits are concerned and in this flat region the behavior of the MOSFET is like a voltage controlled current source.

The flat region works out when V DS is more than a certain voltage it should be more than approximately V GS minus V T ok. If you are in the flat region of the MOSFET then I D changes only with V GS and this relationship goes by I D is equal to K times V GS minus V T the whole squared where K is some arbitrary constant as of now K actually we know what K is, but let us not worry a bout it.

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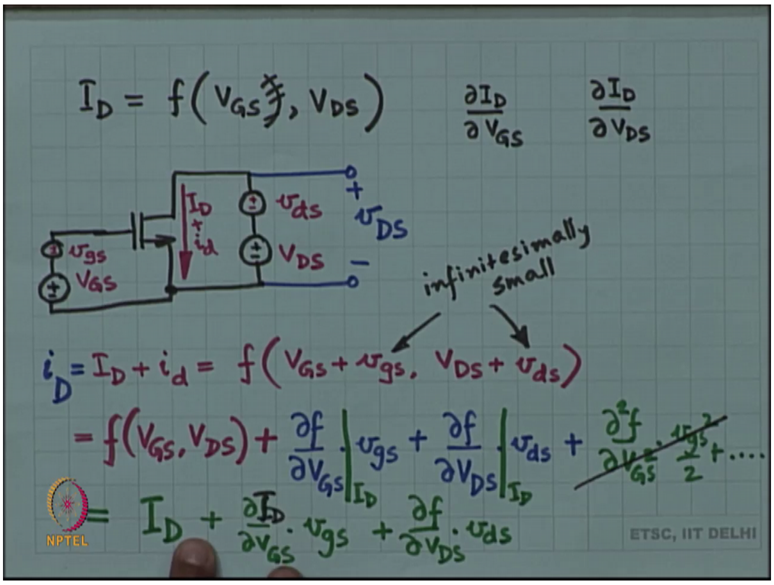

$$I_D = f(V_{GS}, V_{DS}) \qquad \frac{\partial I_D}{\partial V_{GS}} \qquad \frac{\partial I_D}{\partial V_{DS}}$$

$$i_D = I_D + i_d = f(V_{GS} + v_{gs},\ V_{DS} + v_{ds})$$

$$= f(V_{GS}, V_{DS}) + \left.\frac{\partial f}{\partial V_{GS}}\right|_{I_D} \cdot v_{gs} + \left.\frac{\partial f}{\partial V_{DS}}\right|_{I_D} \cdot v_{ds} + \cancel{\frac{\partial^2 f}{\partial V_{GS}^2} \cdot \frac{v_{gs}^2}{2}} + \ldots$$

$$= I_D + \frac{\partial I_D}{\partial V_{GS}} \cdot v_{gs} + \frac{\partial f}{\partial V_{DS}} \cdot v_{ds}$$

So, this is what we did in the last class and then what we did was we applied V GS and V DS on both sides of the MOSFET and then we applied and incremental small voltage v gs and the small voltage v ds and then we did a Taylor series right. I D was some function of V GS V DS then I D plus small i d is going to be the same function of V GS plus small v gs, V DS plus small v ds and then we did the Taylor series.

The full Taylor series breaks up into partial derivatives and we ignore the higher order terms because small v gs and small v ds are really small and then what we got was that the complete current is equal to the constant current at the operating point plus slope times V GS plus another slope times V DS. We modify that to say that the small current the incremental change in current is equal to a slope times V GS the incremental v gs plus slope times the incremental v ds.

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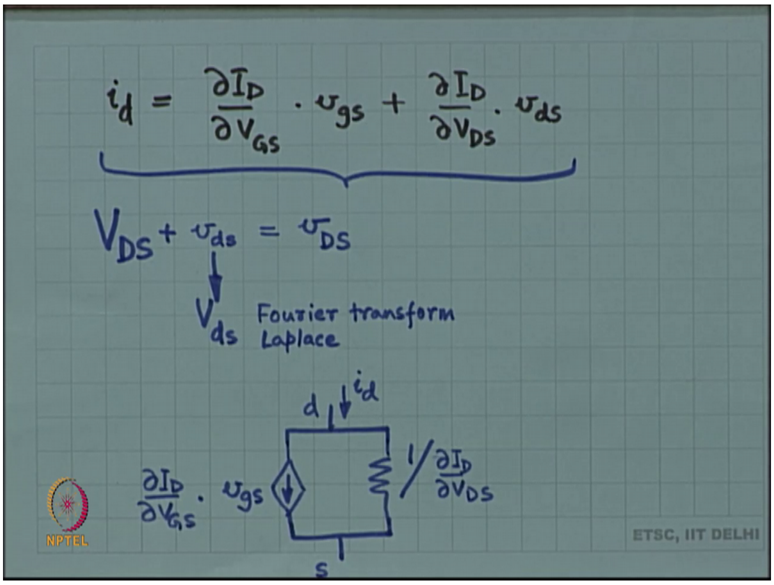

$$\underbrace{i_d = \frac{\partial I_D}{\partial V_{GS}} \cdot v_{gs} + \frac{\partial I_D}{\partial V_{DS}} \cdot v_{ds}}_{}$$

$$V_{DS} + \underset{\downarrow}{v_{ds}} = v_{DS}$$

V_{ds} Fourier transform
Laplace

So far so, good yes and then what we try to do was to interpret this equation above in terms of a circuit and this is really crucial understanding this is absolutely crucial we did this in the last minute of our last lecture. So, I really want to make sure that you understand this, this is an equation I am trying to represent this equation as a circuit all right. So, what I have done is i d is the sum of 2 currents so, it is the sum of 2 currents this one and this one KCL ok.

The first current is dou I D by dou V GS times incremental voltage v gs, dou I D by dou V GS times incremental v gs. The second current is dou I D by dou V DS times v ds all right if the voltage between these 2 is V DS then that this current is going to be V DS divided by the resistor. The resistor is 1 by dou I D by dou V DS so, it is really dou I D by dou V DS times v ds that is the current now you had a question your question was why is this a register and this is the current source why not both current sources or why is this not a register, why not both registers ok.

So, the answer is that this particular path this side is d this side is s the current through this path depends on the voltage between d and s so, that is the resistor. Whereas here the current through this path does not depend on the voltage V DS it depends only on V GS. So, this is not a resistor this is a resistor is this understood all right. So, the model is clear now we have some short hand notation for all of these we are going to present some definitions over here.

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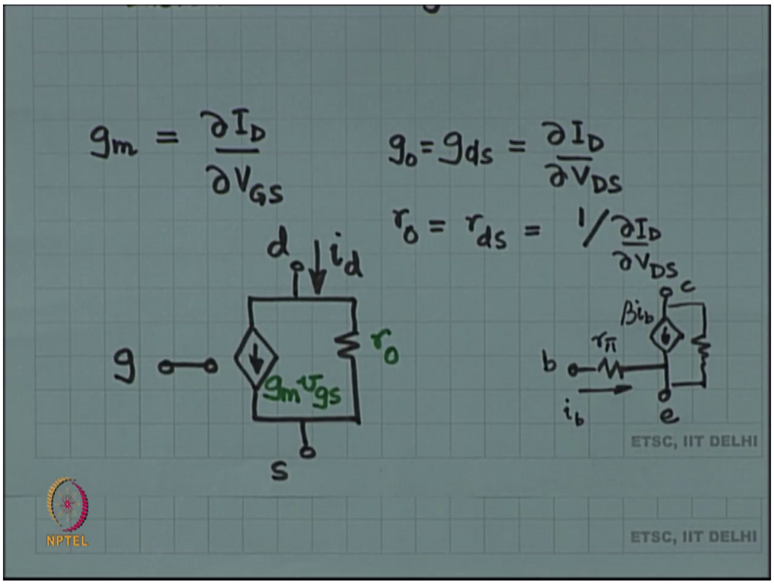

So, our first definition is going to be gm this is going to be called the trans conductance this is defined as dou I D by dou V GS and this is the very important parameter remember dou I D by dou V GS is significant dou I D by dou V DS is very small that slope is very small right this is taking into account a slight slope in the characteristics if there is no slope then nothing happens over there ok.

This particular slope over here this is very very very small. So, in a very good MOSFET the slope will be 0 right that is what the analog circuit designer wants. So, to the analog circuit designer dou I D by dou V DS is going to be a small number the resistance between d and s is hopefully very large because it is 1 by. So, some people call this gds some people call this go right and this is called dou this is dou I D by dou V DS right and if you like writing it has a resistance you can call it ro or rds. So, these are just definitions that we are going to stick to remember gm is very significant ro is hopefully very large it is a large resistance go is hopefully very small gds is very small gm is something significant ok.

So, analog circuits we worry about gm a lot all right. So, our model now does not mean this complicated equations. So, between gate drain and source our model is I have got a voltage controlled current source or value gm times v gs and resistor value ro or rds whatever you want to like to write and I have label the terminals in small letters right because this circuit is not the actual circuit, this circuit is only the incremental circuit. So, with this circuit you will be able to work out only small i small d right this current is not the complete current this current is only small i d fine.

So, this is called the incremental small signal model of the MOSFET and if you have studied the bjt if you are coming from the bjt world and watching this video then of the model is slightly different, in the bjt world you arrive at a hybrid pi model. So, over there it looks like this. So, this is the extra one r pi and this current source is beta times i b base emitter collector and this is some large resistance right usually you do not worry too much about this resistance in the bjt it does not matter, but it still is there just like r o so, this resistance is there all right.

So, far so good so, this this is not really the bjt is not really part of this course, but I am just trying to point it out if somebody is already familiar with the bjt then what where m I in that perspective ok. So, what we have done. So, far is we have applied voltages we have incrementally change these voltages and we have worked out a circuit representation of the incremental change in the current and let us see how we can work with this forward.

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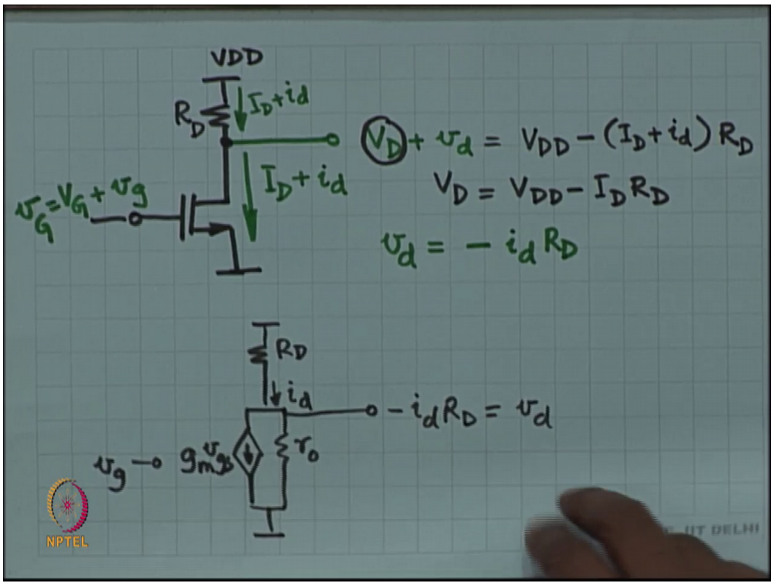

So, let us say today that I have a MOSFET and suppose this is voltage is at V GS plus some incremental voltage so, this is ground right this terminal is ground. So, I am not going to worry about it sources at ground. So, instead of writing V GS I am writing V G plus or small v g, I have applied at the gate and what is this in terms of our convention it small sub capital small sub capital it is capital plus small small now source is constant at 0 volts. So, this is going to cause a current I D plus an incremental change in the current which is small i d, as a result what is going to be the voltage over here, just by the way the voltage over here was capital V D plus and incremental voltage small v d, but if you look at the circuit this current is I D plus small i d suppose this resistor is R then what is the voltage here.

So, the voltage over there is equal to V DD minus so, let us call this R D is this all right. Now if small v d was equal to 0 suppose this small v d. So, or rather small v g was equal to 0 all of this small incremental staff was 0 then you take away small v g take away small i d right what is capital V D, then capital V D would just have been capital V DD minus capital I D times R D. So, this is the absolute voltage over there so, this is this V D which means that small v d is nothing, but minus i d times R D ok.

Look what happened I first applied just a constant voltage V G I obtained a constant current I D, I obtained this current I D and this I D flows through R D and that automatically means that the voltage at the drain V D is nothing, but V DD minor I D R D you have a question the question is that the voltage at the drain depends on I D, but the voltage at the drain is also controlling I D right in a weak sense yes. So, let us let us assume that you know how to do that I am going to show how to how to work it out just now, but let us assume that you know how to do that in that case V D capital V D is just V DD minus I D R D. The incremental change in v d when I applied an incremental voltage v g over here it created an incremental current i d and that created an incremental change in voltage v d.

So, that incremental change is nothing, but minus I D r d and this is coming from KCL and KVL nothing else. So, I am not really looking at any functions or any slopes or what the device characteristics it is I have not seen all of that this is coming just from KCL and KVL and from our definitions ok. So, what this could lead to is as follows, I had the incremental model of the MOSFET which created and incremental current this was the incremental model of the MOSFET I applied and incremental voltage v g over here right it created some incremental currents I D. This incremental current is going to flow through the resistor and come from voltage VDD, but incrementally the voltage V DD is not changing at all.

So, the incremental change in V DD is nothing. So, this is going to be treated as a ground and low and behold you have got 0 volts, you have got minus i d R D over here and this was the drain voltage you see then it is working out properly, you think it is blue no it is not think about it this is what we have done is precisely correct there is no ambiguity in this all right now let us go back to a question.

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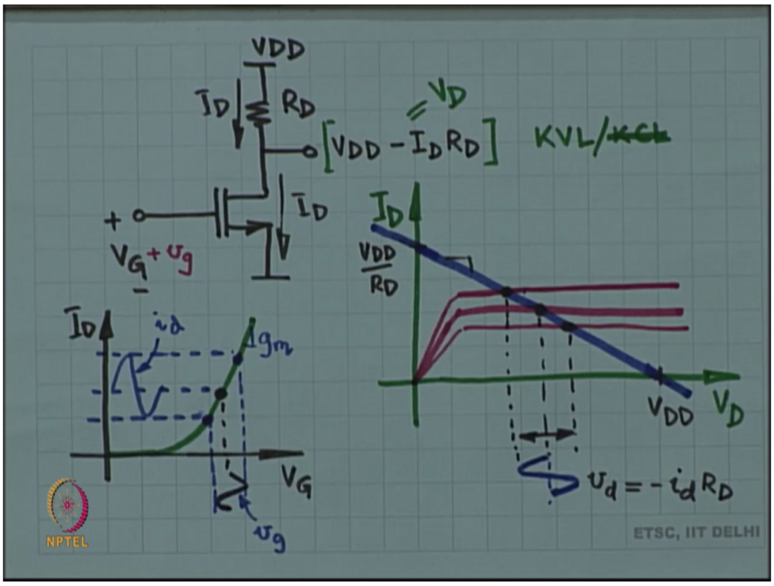

I have applied voltage V G over here, now V G if the device is in it is flat region I do not know whether it is really in it is flat region or not. So, whatever it is there is going to be some current I D. The voltage over here is going to be V DD minus I D times R D now so, far this is from KCL KVL actually no KCL only KVL the voltage over here is V DD minus I D R D this is coming purely from KVL.

So, suppose I draw a graph and on the graph I make the x axis as V D by the way V D is the same as V DS because source is at 0 volts on the y axis I place I D, then how is this going to look V D is equal to V DD minus I D R D how is this going to look this is going to look like a like a straight line right. It is this is the equation of a straight line V D is the x axis I D is the y axis this is the equation of a straight line. So, straight line which cuts the x axis at what point and y axis at what point.

So, if I D is equal to 0 capital V D will be equal to V DD and if V D is equal to 0 then I D will be equal to V DD by R D and then beyond that this is nothing, but a straight line you have no choice about it this is coming only from KVL nothing else and KVL is always correct. So, this is a straight line we have not yet worked out the device over here, now I D is also happens to be a function of V D because of the device, the device has some responsibility to play I D is going to be a function of V D and what is that function depending on the value of V G that you have chosen you are going to pick a curve and the curve that you have picked depending on the value of V G could be this and if it is this then V D and I D have to satisfy KVL which means it has to lie on the blue line it also has to satisfy the device characteristics which mean it has to lie on the red line.

Which means that V D and I D have to be this point this intersection point and that is it that is the only V D and I D choice that you are allowed to have over here which satisfies both the blue line from KVL as well as the red line from the device characteristics if the device had if this redline had some slope. So, be it there is still going to be only one point of interaction now what we are going to do is, we are going to change v g let us apply a small incremental change to the gate voltage if I incrementally change the gate voltage I will have to incrementally select a new curve for this red line for this device line.

So, the new curve for the device line let us marked in read again what about the other the blue line will the blue line change no the blue line never changes right whether you have changed I D or not right I D can change with time, but if this current changes no problem you will still have to adhere to the blue line the blue line is coming from KVL and nothing else. So, whatever is the current, you still have to adhere to the blue line. So, when I incrementally change the gate voltage I would have to shift to this point over here if I pull the gate voltage down I will have to shift to a third point right depending on which way I incrementally change the gate voltage I will be changing my drain voltage.

So, suppose I incrementally change my gate voltage then my drain voltage will also change incrementally. So, for example, suppose I incrementally had a sin wave riding on top of the dc gate voltage then correspondingly I would have a sin wave of small i d amplitude which is riding on top of capital I D. So, this if this has an amplitude of small v g and this has an amplitude of small i d then I D is related to small v g by I D is equal to this slope times v g g m times v g this slope is gm very good.

Now that I need to select different lines over here right I know which lines I D is changing by. So, accordingly I need to select 2 different 3 different lines and accordingly also depending on the slope of this curve of this blue line I will get notice when I D increased the drain voltage decreased. So, I drew the sin wave over here like this so, here I pointed the sin wave in exactly the opposite way. So, this sin wave is going to have an amplitude V D and that is why there is the slope on this line what is that slope yeah it is slope is R D minus R D yeah ok. So, is this does this answer your question.

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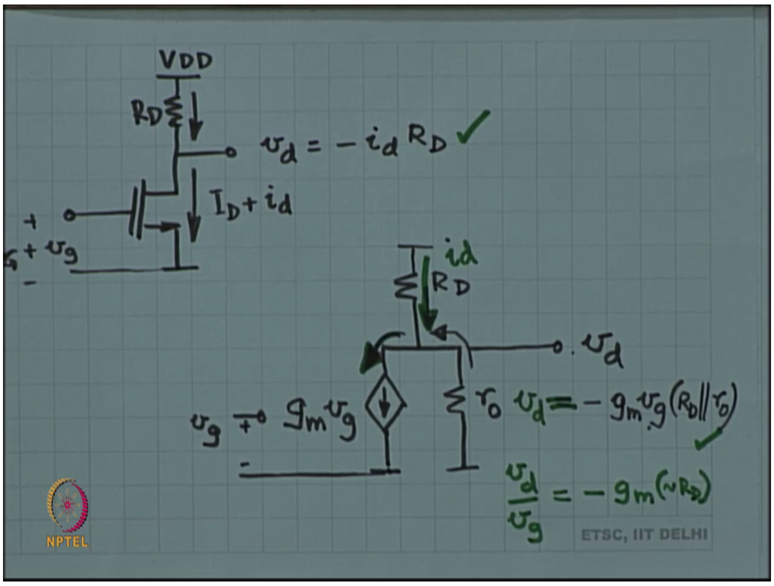

So, this so, far what we have done is a assuming that the slope of the curve of in the flat region the slope is very large that is what we have assumed. So, far but if I do not assume that the slope in the flat region is very large then this calculation is going to be a little hard it is not going to be so, straight forward. So, far it is good V D is equal to minus i d times R D ok, but I need to figure out what is this I D small i d, small i d happens to be a function both of v g as well as V D.

So, it is not very obvious what is the relationship between I D and vg because had it been completely flat think completely flat region then I D small i d would depend only on small v g small i d would be g m times small v g, but not anymore now we have small i d is equal to gm times small vg plus go times small v d or in other words I can redraw my circuit in this fashion. V DD does not change incrementally so, I have marked this as nothing 0 volts when you do not write the value of the voltage it is 0 volts.

The ground terminal is fixed it does not change incrementally. So, this is also marked as 0 volts then the resistor remains a resistor all the linear elements remain exactly where they are the MOSFET is going to be replaced by it small signal incremental model which happens to have gm times v g and ro fine and then I just work out what happens I applied a v g over here it creates a current gm times v g this gm times v g current happens to split up between R D and ro R D and ro appear in parallel to each other right.

So, the voltage v d is equal to minus I D times R D parallel ro sorry this current time R D yeah it is not I D times R D parallel ro anymore I D is the sum of these 2 currents all right, but this current is gm times v g. So, gm times v g times R D in parallel with ro whatever that is that yeah. So, this is also correct, but this is also correct, this is not i d this current is not i d this current is i d ok. So, be careful over here. So, this is correct this is also correct. So, it is gm times v g that is the current and this current is going to come through a combination of R D and ro out of the ground terminal 0 potential terminal.

So, combination of R D and ro is R D parallel ro in you know what that is that is r d times ro divided by R D plus ro I do not want to write it down ok. So, that combination resistor is carrying a current gm times v g. So, the voltage that appears at the drain terminal is minus gm times vg times R D parallel ro. So, v d is equal to minus gm times v g times R D parallel ro, now this is the incremental change in the voltage all right. Now let us look at the scenario where ro is very very large because the device is in it is flat region and let us suppose it is a good device. So, ro is infinite so, R D parallel ro is approximately R D gm is a large number right.

Suppose it is a large trans conductance just to give you an idea of what large is 1 milli siemens it could be a good value for gm 100 kilo ohms could be a good value for R D in which case 1 milli siemens and 100 kilo ohms combined multiply with each other and you get 100 so, 1 milli siemens 100 kilo ohms vg. So, v d by v g is equal to minus g m times r b parallel ro which is approximately R D minus gm times R D, R D is 100 kilo ohms gm is 1 milli siemens we will give you minus 100 what does this mean this means that if small v g is 1 milli volt then small v d is minus 100 milli volt ok.

You have successfully made an amplifier you applied a small signal worth 1 milli volt and the response at the drain was worth minus 100 milli volts right 100 times larger. So, this is this is the function of an amplifier all right this is your first amplifier just by the way it is not complete this amplifier is not even close to complete, but at least you are saying that with the help of a MOSFET you are achieving some sort of amplification that is what we wanted right that is this is the crux of the course making an amplifier is the objective of this course because once you can make amplifiers you can make all kinds of other circuits ok.

So, v d by v g has come to a large number right I have just thrown in some numbers gm is 1 milli siemens R D is 100 kilo ohms and that automatically gives you v d by v g to be equal to minus 100 and this amplification is very nice this is exactly what we want if I apply 1 milli volt signal at the gate I get a response of minus 100 milli volts riding on top of capital V D ok. So, let us understand go back over here I started with a voltage V G right this was my voltage V G and then on top of the voltage V G I applied a small signal small v g. So, this is that small signal that needs to be amplified the capital signal does not need to be amplified that is known as the operating point of the MOSFET of the device.

So, the operating point is capital sub capital always the signal that needs to be amplified is small sub small that is the signal. So, I have the operating point plus a small signal right that creates the operating point creates an operating current it is called the bias current plus signal current small i d, select the line depending on the current right this is the operating point the operating point capital V D plus signal voltage small v d and what we are saying is that this small v d is 100 times larger than small v g. So, if I apply 1 milli volt over here I will get minus 100 milli volts over there.

So, I apply 1 milli volt a sin wave I get minus 100 milli volts sin wave, but this sin wave is riding on top of the operating point is this so, far understood. So, this is our first amplifier and amplifiers unfortunately will all have this operating point they are not going to operate around 0 volts. So, we have 2 businesses over here, one is the first thing that we need to do is I need to work out how to set an operating point.

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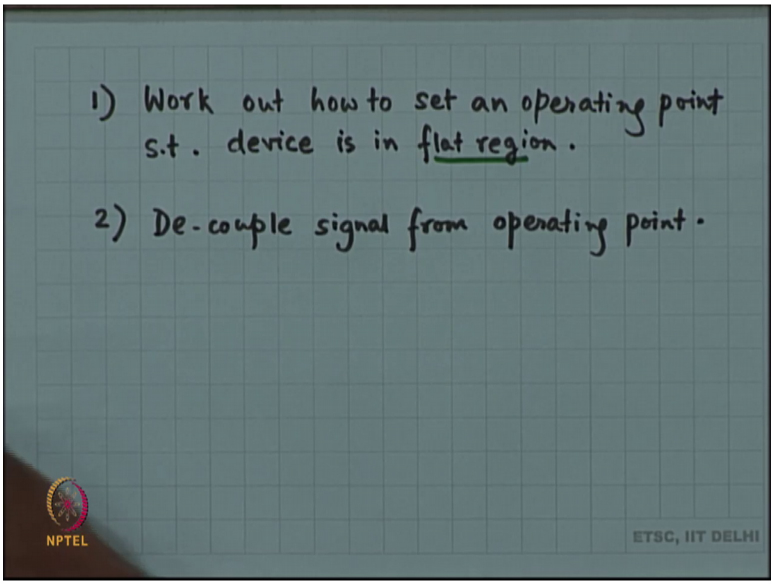

1) Work out how to set an operating point s.t. device is in flat region.

2) De-couple signal from operating point.

Such that devices in it is flat region, why is the flat region important because ro would not have been large had it not been in the flat region. If it was not in the flat region this ro would not have been large it would have been rather small you know you placed 100 kilo ohms plus a small resistor would have been the small resistor right.

So, this large gain this large amplification that you got was primarily because you had a large resistance R D over here in parallel with a very large resistance ro like that very large resistance is no longer large. Then that is going to dictate the value of R D parallel ro in which case you are going to find out that this amplification factor is not going to be significant at all it is going to become rather small. In fact, gm is also going to change, but most important is this region ro needs to be very very high.

So, that is why I want the flat region ok. So, this is something that I need to work out I need to figure out how to set an operating point or rather how to create an operating point how to establish an operating point and then number 2 that I need to do I may or may not need to do this is to decouple the signal from the operating point this is the second thing that I need to do what I mean by this is the fact that over here my output was riding on top of a of the operating point I do not want that I want the output to be just the signal I want the input over here the input was a small input riding on top of the operating point I do not want that my small input is just going to be some small signal that is what I really want ok.

So, these 2 things I need to work out first I need to work out how to set an operating point and the second I want to work out is to decouple the signal from it is operating point and once I understand how to do these 2 things I am all set. So, the first part how do I set an operating point the operating point is the current capital I D and the voltage V DS the current capital I D and the voltage V DS this is 2 comprise the operating point or sometimes you want to have a clear idea of what is the value of I D and what is the value of V DS you also want to establish a voltage at the gate.

So, one simple way to establish a voltage at the gate is with the help of a potential divider ah.

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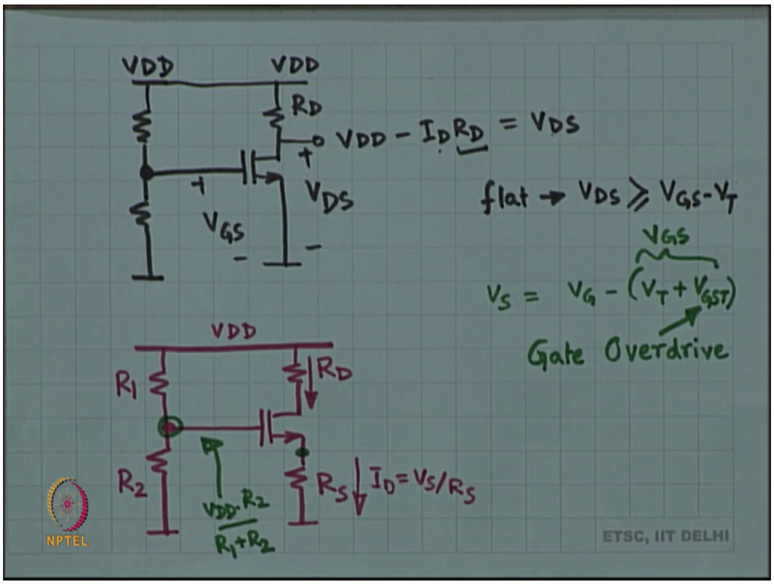

So, far we have been using a voltage source unfortunately these voltage sources are not that easily available right battery is always of a fixed voltage. So, you have to get your you have to create these voltages out of the same old battery ok. So, you have got one battery let us say the battery is 1.5 volts. So, from that 1.5 volts you have to create the desired value of vg which I might be able to do like this and then once I have created the desired value of vg I can apply I can work it out like this. So, now, I have a desired value of vg source is that 0.

So, V GS is established and from the curve from the MOSFET curve V GS I D as a function of V GS I work out what is I D and the voltage at the drain is going to be V DD minus capital I D times R D and that is going to be equal to V DS ok. So, this is the straight forward way of doing things unfortunately this is not very good why is this not good, any idea why would I mean this is very easy I set the value of gate voltage that I want.

And automatically the current is set and then from that I work out what is V DS why this is not good is because the relationship between the gate voltage and the current is very sensitive the current is not sensitive to drain voltage, but it is very sensitive to the gate voltage as long as you are above V T right. In fact, it goes up as a square right. So, if I make a small mistake in choosing the right value of the gate voltage then I am going to be off by a large amount if I am off by a large amount suppose I have you know underestimated the value of V G.

I have applied a slightly larger V G than required if I apply the slightly larger V G than required then I D is a lot larger it is very sensitive right if I D is a lot larger then V DS happens to be a lot smaller and remember R D is something that you want to be large ok. So, the same phenomenon that gave you the gain in the amplifier that same phenomenon is at work over here small error in choosing the value of I D is going to create a large error in V DS. In other words a small error in choosing the value of V GS will create a large error in choosing the value of V DS just like earlier right earlier I apply the small signal of 1 milli volt on V GS I found that V DS is 100 millivolts smaller then it is dc operating point.

Here if I make a an error of 1 milli volt at V GS then the error in V DS is minus 100 milli volts it is 100 milli volts smaller than it should have been and that is not good because that might break your circuit your V DS has to be more than a certain amount, remember right V DS has to be more than a certain amount it might break things this is for flat region right to remain in the flat region V DS has to be more than V GS minus V T.

Now, if you say that small error in V GS 1 milli volt error in V GS has created 100 milli volt error in V DS when are you sure if V DS is correct or not you are not sure anymore ok. So, that is the reason why this establishing the operating point in this fashion is not really very nice it is not good at all ok. The same phenomenon that gave you gain in the amplifier is working against you over here in creating the operating point all right.

Then what do we do so, one strategy is to do something like this now what is going to happen, what is the voltage here is the same as earlier right it is a potential divider I have a voltage over there and then the source voltage is no longer at 0 all right.

So, this is the critical thing the source voltage is no longer 0. So, this voltage is V DD times R 2 by R 1 plus R 2 the source voltage is V GS less than the gate voltage. So, V S is this gate voltage minus V GS, V GS can be written has something more than V T. So, this is what we write it as this is called the gate overdrive.

So, this entire thing is V GS fine is the gate overdrive, this V GST is shorthand instead of writing V GS minus V T we write it as V GST ok. So, this is my source voltage the current I D is V S by R S, but it is also the same over here fine. So, now, what you have to do, is you have to work out an equation your equation is going to look like this.

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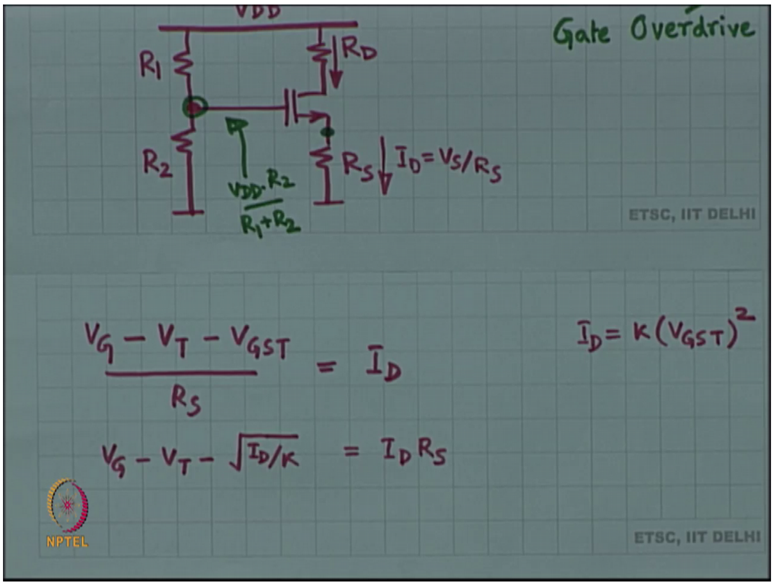

$$\frac{V_G - V_T - V_{GST}}{R_S} = I_D$$

$$V_G - V_T - \sqrt{I_D/K} = I_D R_S$$

$$I_D = K(V_{GST})^2$$

So, I know the gate voltage and the source voltage is the gate voltage minus V T minus V GST V GS minus V T this is the source voltage this source voltage divided by R S is equal to I D, but V GS minus V T that is V GST ok. This V GST is also related to I D through the device equation and the device equation says that I D is K times V GST V GS minus V T the whole squared ok.

So, in other words you write this and now you need to solve a quadratic equation to arrive at the value of I D ok. So, you can crank your quadratic skills and arrive at the value of I D what we are going to do is we are going to stop over here and discuss from this point onwards later on what we have covered in today's class first we started with the model of the MOSFET and then we showed how we can get amplification if I operate in the flat region of the MOSFET right that was our first. So, called amplifier, but it was a very crude amplifier because I was unable to figure out how to set the dc operating point and I was unable to decouple the input and the output signals.

So, then we started working on creating the dc operating point and yeah so, that is where we are. So, our first circuit did not really work because the same principal with which we did amplification the same phenomenon which gave us amplification created an error in the dc operating point and therefore, that is not a very good way to set the dc operating point. So, this is a second thing that we are working on and then we have arrived at a quadratic while working on this circuit we arrived at a quadratic and these quadratic needs to be solved sooner or later ok.

Thank you.