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Lecture - 36 Power Amplifiers 2

Welcome back to Analog Electronics. Today is lecture 36, and we are going to continue with our discussion on Power Amplifiers. Yesterday we were talking about power amplifiers and we had a few revealing insights. We started with the common source amplifier, made it as simple as possible with a big inductor and a big capacitor right. We also worked with a common drain amplifier where the same thing happened, we had a big inductor and big capacitor. Then we applied an input looked at the output and then we figured out the ratio of the power that we manage to deliver to the load to the power that we absorbed from the DC power source.

And we found that there is an upper bound on this ratio, the upper bound is 50 percent and that too this is an optimistic overly optimistic upper bound right. We I already we already discussed the lacuna why this even this 50 percent is little overly optimistic. Anyway, so, let us say best case scenario is 50 percent. So, 50 percent is the efficiency of the circuit, the ratio between the power that we can deliver to the load to the power that we absorbed from the DC power source; which effectively means that if I want to deliver 1 watt to the load, 1 watt is going to be burnt extra as heat ok.

And this is generally a very bad thing. I mean, you do not want to waste power as heat. Heat is the worst form of energy right. So, once energy is burnt as heat, you cannot get it back, you cannot mostly not usable anymore, alright.

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So, our circuit we had I will just redraw once again. We had one MOSFET, we had an input, and here a big capacitor at the output. The biasing was through the inductor ok. So, this is the structure of the circuit alright. And more or less this configuration is the configuration of a power amplifiers ok. We also discussed an alternate version of this, where the MOSFET is common drain as asupposed to common source. The common drain form is sometimes used, but it has other issues.

Namely, you remember we had discussed a long time back when we were studying the common gate, common source, common drain amplifiers; after we had done the capacitive stuff after figuring out the capacitances. We are redoing the common source common gate and common drain circuits right. Now during that revision of the common drain circuit, we figured out that the output impedance of the common drain circuit is inductive, it looks like an inductor which basically means that as frequency increases the output impedance increases. Now this is true for all voltage buffers anyway.

So, we were seeing that if there is capacitance in the load, then the inductive output impedance and the capacitance in the load resonate with each other and it can cause instability ok. A lot of times therefore, the common drain circuit is not going to be preferred right especially if you know beforehand that the load is capacitive. If the load is not capacitive, if you are sure there is no capacitance in the load then no problem, but if the load is capacitive, then beware right you can still use common drain circuit as long as you know what you are doing ok. So, you should convince yourself before jumping into the common drain circuit. Convince yourself that this is going to work without causing instability.

Instability means when there is resonance in the circuit, then somehow you know the poles might end up in the right half plane, and then you know you are going to get large oscillations. So, you might see these things in the lab a lot of time for example. A lot of times in the lab when you are trying to draw a large current from a power supply ok.

There are these power supply boxes right in the lab, all of you have gone to the lab have used some lab or the other. And typical equipment in the lab are there are oscilloscopes, there are power supply, there are function generators right. And a lot of times you know the power supply is either going to look like a box like this or a box like this something where you connect right.

Now, when a lot of times when you try to draw large impulsive currents or large currents ok, transient currents, large transient currents ok. Suppose I try to draw a current waveform which is you know is outside the boundary of this course right. So, I am just discussing sketchily.

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Suppose you are you have got this power supply box, and from the power supply box your trying to draw current that looks like this, ok. And this amplitude is very large, and this time also is very small alright. And invariably what happens is that loads are capacitive. Remember, recollect your class 11 or class 12 electrostatics when you studied electrostatics in class 12, you would have studied that metallic sphere has some capacitance ok. Just a metallic sphere has capacitance ok, anything made up of metal has capacitance ok. Capacitance is everywhere between any 2 objects there is some capacitance, you can work it out as if you know conductor insulator conductor.

There is capacitance between my 2 hands ok. So, these capacitances are there everywhere, it does not really have to be a physical capacitor ok. You have implemented something on a circuit board. Now there is a wire going to the circuit board, there is another wire coming out of the circuit board, between the 2 wires there is going to be capacitance ok. So, invariably even if you did not deliberately have a capacitive load, invariably your load will have some form of capacitance. A little bit of capacitance ok, if not a lot. So, for example, in the lab we have measured that if you take one of those integrated circuits, the capacitance between any 2 pins is of the order of several picofarads ok. So, capacitance is there everywhere.

Now, suppose you take this large current, transient current you draw this transient current from the power supply. The power supply is a voltage buffer of sorts; we are going to discuss power supply soon. But it is a voltage buffer of sorts its output impedance is going to be heavily inductive. And you are driving some sort of a capacitive load, you have got a heavy inductive output impedance, right. If you look at the overall Thevenin equivalent impedance, you will have one inductor one capacitor right. And you have this large transient waveform it is going to cause a huge amount of. So, you are going to have poles which are a pair of poles, which are very close to the imaginary axis, if not on the right half plane.

Sometimes these poles go to the right half plane then it is disaster. Even if they do not go to the right half plane, if they are close enough to the imaginary axis, you are going to get a response that looks like this ok. If you have got a pair of poles, very close to the imaginary axis ok. And whenever you have got this l and c coming in shunt with each other right this is something to be afraid of alright. So, generally speaking therefore, we avoid this kind of a setup.

Especially, when it comes to power amplification; however, the concept is the same ok. And we discussed it in the last lecture the result is exactly the same. Just that in the last class we did not incorporate capacitance between a gate and source right, if you through in capacitance then poles are going to pop up, and then suddenly you might start observing that things might become unstable or near unstable, both are bad enough ok.

So, generally speaking this is avoidable unless you know what you are doing right. A lot of times if you really know what you are doing then this is alright. So, our power amplifier circuit for now is going to be the first one. We are going to stick to this power amplifier circuit. And even with this power amplifier circuit what you saw was that if I want to maintain the MOSFET in saturation at all times, then the maximum theoretical upper bound of efficiency is 50 percent, fine. So, this kind of a power amplifier is called a class A power amplifier ok.

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And in the class A power amplifier, the MOSFET is in saturation all the time throughout the cycle of the waveform. So, the cycle of the waveform if you think of one sin wave right, throughout this one sin wave this current waveform is going to; so, if the voltage over here is this, and the current is the opposite. So, throughout this waveform the current is positive non-zero right, at best it is hitting 0 right, but it is not going below 0, throughout the waveform. So, we say that the MOSFET is conducting current through all 360 degrees of the current phase of the phase of the signal right. The entire period of the signal the MOSFET is conducting current right. We say that the MOSFET is conducting current for all 360 degrees of the phase of the signal ok. And, in this case we found that the maximum efficiency is 50 percent, alright.

Now, when I freeze it like this it kind of gives you a hint as to what to do next ok. So, the hint is that we are going to make keep the, I am, we are not going to make keep the MOSFET conducting for all 360 degrees ok. That is going to be the hint right. So, if not 360 how many degrees? We are going to try different options ok. So, let us try the following.

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So, our circuit remains the same ok. This is the circuit; for all the power amplifiers the circuit is the same. Only later on we are going to make some slight modification and arrive at newer circuits, but right now the circuit is fixed alright. We are going to change make some slight tweakings over here, what we are going to tweak is the value of V GS, capital V GS ok. We are going to change the value of capital V GS.

So, yesterday we call this v n or v g I think we called it v gs or v g, v gs it does not matter ok. What we are going to tweak right now is this one. So, earlier the combined gate voltage of the 2. So, if you look at the combined gate voltage of the 2, it has some value and around the value you are going up and down ok. And earlier what was happening was, the minimum value as well as the maximum value, this entire range of values would be above the threshold voltage of the MOSFET, such that there is current through the MOSFET all the time, fine. This was the strategy earlier. Earlier my value of capital V GS which is capital V GS, the centerline is capital V GS ok. Small v gs is the amplitude around that ok.

The net waveform is small v sub capital gs ok. The combined waveform is small v sub capital gs. Now earlier we were making sure that the MOSFET always conducts current, no matter what ok. What does that mean? That means, that is small v sub capital gs is more than a certain threshold all the time v t of the device, all the time. Through all 360 degrees of the signal phase right, this is what we were looking at earlier. Now let us stop doing that ok, let us cut the MOSFET of for part of the duration, let us cut it off. So, what are we going to do? We are going to reduce the value of V GS to a point.

So, earlier my current waveform ok, first let me draw my earlier current waveform, earlier the current waveform was like this. I had some value and around the value, I was going up and down, like a sin wave. And I was making sure that I do not go below 0, fine. Now what we are going to say is, let us cut the MOSFET of for some part of the cycle. Let us not keep it on through out ok. So, we are getting the MOSFET off. Let us say we cut it off for half of the time. So, we are going to conduct only during this time, and keep the MOSFET off the remaining time. Let us say that is the plan, then I am going to make a current waveform that look maybe looks like this, I am sorry.

So, this was the earlier current waveform. Now the new current waveform would look like this; is this ok? Fine, this is going to be the shape of the current waveform. Now this is small i's of capital D, alright, let us formalize it. Let us say small i's of capital D this time t equal to 0, let us say this is pi, this is 2 pi 3 pi and so on ok. So, small i's of capital D is equal to some i naught times sin.

So, let us without any loss in generality, let us call this theta instead of time ok. Just I am just changing the alphabet over there ok. Something like this. So, let us say i naught sin theta is the current. So, this current has a peak of i not, peak value of capital I naught right. And between 0 and pi, it follows i naught sin theta, and between pi and 2 pi it follows 0. Again between 2 pi and 3 pi, it follows i naught sin theta, again 3 pi and 4 pi it will be 0 and so on and so forth, alright.

And how did this happen? This happened just by reducing the value of the gate bias voltage V GS ok. This is what we have done. Now the inductor is a short circuit, it is so big that it is a short circuit for DC and a completely open circuit for AC ok. What does that mean? This current is still capital I D, alright. And this current is still small i's of capital small i small d ok. This is true no matter what. It does not matter what is the waveform of small i capital D, this relationship is true, the DC part is going to come through the inductor because it is an AC open circuit. And the AC part is going to come through the capacitor, because it is a short circuit for AC alright. So, this is true no matter what.

Now, how will you split up small i D into a DC part and an incremental part. The DC part is easy you just find out the average ok. What is the average current when the function looks like this, it is periodic right? How do you find the average? Some of you will rattle this out already, because this is like a half wave rectified sin wave right. If you do not remember how to do it, this is how you do it.

You integrate and then you divide by the total time, right the total time is 2 pi and you integrate one divide by 2 pi. This is the average ok, and 0 to 2 pi if I want to integrate from pi to 2 pi it is 0 anyway. So, we do not have to bother, we just have to integrate from 0 to pi, and that too we have to integrate I naught sin theta d theta from 0 to pi ok. And pi to 2 pi is 0 anyway. So, it does not matter, integral of sin theta is minus cos theta.

And if I plug in cos theta equal to pi, I get minus 1, and if I plug in cos theta equal theta equal to 0, I get another minus 1. So, net is minus 2 over here there was a minus sign upfront. And therefore, the final answer is I naught by pi ok. And if you remember your half wave rectifier, then you have a voltage right voltage of 220 volts, you half wave rectified what do you will get you get 220 divided by pi, anyway. So, this is the result alright. So now, what we have figured out is the DC part. So, small i D must be the rest right, if I subtract the DC part out of the signal then I get the incremental part alright. So, small i D must be small i's of capital D minus capital I D.

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 $i_{p}(t) = (\frac{T_{0}}{\pi}) + \sum_{n=1}^{\infty} a_{n} \sin(\frac{2\pi n \theta}{\pi}) + \sum_{n=1}^{\infty} b_{n} \cos(\frac{2\pi n \theta}{\pi})$ $i_D(\theta) = I_{\theta/\pi} + \sum_{n=1}^{\infty} a_n \sin(n\theta) + \sum_{n=1}^{\infty} b_n \cos(n\theta)$ $a_n = \frac{1}{2} \frac{\lambda}{L} \int i_b(\theta) \sin(\frac{\pi n \theta}{\lambda}) d\theta$

So, what is my small i D? Something like this, fine. If you draw the waveform, then small i D, I am sorry, this is how small i D looks like. This shift is right and this distance is i naught, so far so good, great.

Now this small i D does not resemble a sin wave at all. I mean, it looks like a sin wave for only half the cycle. If you look at the this is the periodic signal, you can look at the Fourier series of this periodic signal you will get a sin wave component at the fundamental, but it will have harmonics lots of harmonics. So, I am going to place the load over here in the circuit ok. But before I place the load let us think a little bit right. Is it right to place just a resistive load over there? Because small i D is going to come through that resistive load, and it is not going to look like a sin wave at all the voltage is not going to look like a sin wave ok, and that is not something very nice.

Instead what a lot of people do is place a filter ok. You can easily place a filter, let us say nose filter to make sure that none of the harmonics go through. So, you can have nose filter at the second harmonic, third harmonic at all the harmonics. Or you could place a low pass filter over there to make sure that none of the higher frequencies go through at alright. So, there are variety of techniques.

Let us imagine that there is some sort of a filter like that, maybe that filter looks like this ok. So, imagine this current has components at the fundamental and at the harmonics, alright. It has components at the fundamental and at the harmonics. As far as the harmonics are concerned or let us say second harmonic is the dominant harmonic, and you do not want to worry about the other ones ok.

So, at the second harmonic, I make sure l and c together form a short circuit. J this is j omega l and this is 1 by j omega c which is which can be adjusted in value right, if this is some 5 times j, then this could be adjusted c could be adjusted to be minus 5 times j exactly at that frequency. And as a result the net impedance of these 2 can be made to be 0; which means that the second harmonic is going to go through this ok because that is a short circuit. So, for the second harmonic it is going to go through this, right. Likewise, you can build a short circuit, at third harmonic, a forth harmonic whatever whichever harmonics you are worried about ok.

So, we are not going to discuss this in detail at all. We are in fact; we are not going to discuss it at all. I am just trying to say that you can always conceive a filter that looks like that and place it over there, alright. And as you see it is not terribly difficult to do that, fine. Then what happens to the current over here that comes to the load? All the harmonic currents are going to come through this through the short circuits, right. And the load can carry the current which is the fundamental of this periodic wave. So, pause, let us pause a bit. So, this is my i D, this is my small i D ok, small i D equal to something between 0 to pi small i D is something else between pi and 2 pi. This is the periodic signal right. What happens between 0 and 2 pi this is going to repeat forever ok, between 2 pi and 4 pi 4 pi and 6 pi and so on and so forth, it is repeating itself.

So, this is the periodic current. This periodic current can be broken up as a Fourier series. You know how to do Fourier series, right? Any periodics any periodic wave form can be broken up for example, i d of t, I know i d of t is periodic ok. Here I am writing t where it is actually just theta, t is the same as theta for me. i d of t is periodic I know, right i d of t can be written as some constant plus in our case the constant is already 0, we have already taken out the DC part ok. In fact, you could have work directly with is of capital D of t, in which case this would be I not by pi, plus some a n sin plus b n cosine, and these have to be summed from n equal to 0 to infinity, you have all these components. I am sorry not 0 n equal to 1 to infinity ok. The 0th term has already come out, fine? This is the Fourier series.

Now, in our case capital T is 2 pi, small t is theta. So, we have a little simplification over here. This is just theta; capital T is 2 pi. So, this cancel out, ok. Just for clarity I am re writing. And do you know how to figure out the coefficients? Small a n and small b n? That is what you learnt in your maths class, in your Fourier series class right. And as far as I remember a n was given in the maths class as 2 by capital T integral from 0 to capital T x of t, I am sorry, is for sin a n this for sin and b n this for cos.

So, for the cos term you have to do a cos, for the sin term you have to do a sin ok, and integral with the sin. This is what I remember from the Fourier series maths class, alright. Now in our case in our specific case once again capital T is 2 pi. So, these to cancel out, this comes 1 by pi, t is nothing but theta ok. And for now I am really interested in the fundamental component that actually goes out through the load; which means that I am really interested in n equal to 1 ok.

Just the fundamental of this Fourier series is what is most interesting to me. All the other harmonics are garbage right. I do not like this harmonics, I do not interested in them. I am interested in the fundamental, why? Because my input voltage was at the fundamental, right. I made the sound at the fundamental, I expect the loudspeaker to respond at the fundamental. So, I am interested only in the fundamental, the garbage is not very interesting to me. So, I am interested in n equal to 1 ok. And we have already taken out the DC component. So, whether you use small i's of capital D or whether you use small i's of small d it is up to you, you will get the same answer ok. So, let us just do it.

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 $a_1 = \frac{1}{\pi} \int_0^{2\pi} i_p(\theta) \sin \theta \, d\theta = \frac{\frac{1}{\pi} \int_0^{\pi} \sin^2 \theta \, d\theta}{\frac{\pi}{\pi} \theta}$
= $I_{\theta/\pi} \int_0^{\frac{\pi}{2}} (1 - \cos 2\theta) \, d\theta = \frac{I_0}{\pi} \cdot \frac{\pi}{2} = I_0/2$
 $b_1 = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} i_p(\theta) \cos \theta \, d\theta = \frac{I_0}{2\pi} \int_0^{\frac{\pi}{2}} \frac{\sin \theta \cos \theta}{2}$ **ETSC. IIT DELHI**

I am interested only in a 1 1 by pi integral 0 to 2 pi i d of theta small i's of capital D was a niter function ok. It looks more attractive then smaller i small d. So, let me just use small i's of capital D ok. I think my answer is going to come faster if I use small i's of capital D the result is going to be the same you can use small i's of small d as well. And if you remember small i's of capital D is i naught sin theta from 0 to pi. And the rest of the time it is just 0; which means this integral you do not have to do go all the way from 0 to 2 pi, from pi to 2 pi small i D is already 0. So, the product of 0 with something else is 0 and sum of all of that is going to be a big 0.

So, therefore, you just have to deal with 0 to pi and in that 0 to pi range small i's of capital D is nothing but I not sin theta, times another sin theta ok. And sin square theta can be broken up into 1 minus cos of 2 theta, half of that ok. This is cos square plus sin square and this is cos square minus sin square.

So, the cos square portion cancels it out, sin squared and minus of minus sin squared 2 sin squared half of that sin square, just to check that, this is correct ok. Then what happens? You can do integral of cos 2 theta d theta right, from 0 to pi, you what are you going to get. You need to replace substitute 2 theta with some alpha right. And then suddenly you are going to integrate from 0 to 2 pi right, to double the frequency; which basically means that this the second half of the integral is going to be a 0, and therefore you need not do it.

You only have to work out the first half of the integral, which is an integral of 1. And therefore, the answer is just pi by 2 coming out of this ok. So, this is our result, and then there is a 2 and then a 3 and so on. What about b 1? B 1 is also at the fundamental. So, you need to check b 1 as well ok, and i d of theta from pi to 2 pi is all 0. So, once again you note that out ok; sin theta cos theta is actually half of sin 2 theta fine.

And then again you substitute you say let us say let us make 2 theta equal to some alpha, then theta is alpha by 2 ok. And the integrals are going to be from when alpha is equal to 0 to when alpha is going to be equal to alpha is 2 times theta. So, 2 pi and if I integrate sin alpha from 0 to 2 pi right, do not worry about this factor of half. If I integrate sin theta from 0 to 2 theta, then I get a big 0. So, this entire result is 0. So, b 1 is a clean 0, alright.

And then b 2 b 3 etcetera can be worked out, but we are not interested in b 2 b 3. We are only interested in a 1 and b 1; which means, that the current that I am going to bring through the load. Remember, I have shorted out all the higher order harmonics ok. So, the higher are order harmonics are not going to bother me. The actual current that is going to come through the load is just i naught by 2. So, i naught by 2 is a 1 ok, a 1 sin one times theta. So, it is a 1 a 1 is i not by 2 i not by 2 sin theta is the correct. Is this fine? This is clear? I hope this is absolutely clear so far so good. We have not yet done in electronics, we have only you know kind of separated the current, we have split up the current small i's of capital D into a DC part, higher order harmonics and a fundamental ok.

And I have figured out that if the current waveform looks like this if it only goes from 0 to pi; that is, if conduction happens only for 180 degrees, and not all 360 degrees, if it happens only for 180 degrees, then the DC part is i naught by pi, and the fundamental is i naught by 2, ok. The amplitude of the fundamental is capital I naught by 2 the DC part is i naught by pi. This is all that I have figured out so far; so far so good? Now let us look at the voltages. So, when I do the voltages this current is going to come through the load right, if you think of the load as a resistive load, then the voltage is going to dip by the this resistance times the current right.

So, the voltage over here, the signal voltage over here actually looks like a sin wave ok. So, the voltage at the drain is also split up into a DC, this is the DC plus the signal, small v s of capital D. And this is only the incremental part small v d, now the small v d actually looks like a sin wave right, why? Because a sin current is coming through a load, fine? You can always have think of the load as a pure resistor. Even if it is not a pure resistor, even then it is a sin wave right, in the steady state. So, this voltage over here is a sin wave. In that case, what is the maximum amplitude of this voltage. So, what is small v s of capital D? Small v s of capital D looks exactly like before. It is a voltage riding on top of V DD from sin wave which is riding on top of V DD.

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So, the voltage at the drain ok, maybe my phase is incorrect, right. If the current goes up the voltage should go down, ok. This is probably the correct, ignore the black line alright. This is how small v sub capital D is going to look, alright. And now tell me: what is the best case amplitude of this voltage. The amplitude has got to be V DD right. So, this amplitude has got to be equal to V DD. So, the best case amplitude of the voltage waveform is V DD, fine. Ok now let us do some numbers, but what is the current drawn from the power supply? I D, DC it is a pure DC current I D. So, the power drawn from the power supply is V DD times I D, and I D is capital I naught by pi.

Then what is the power delivered to the load? The voltage across the load is small v d sin wave, the current through the load is I naught by 2 sin theta, that is also a sin wave. Therefore, the power delivered to the load this is also this is you ignore the sin theta of here part, right the amplitude is I naught capital I naught by 2, this has an amplitude of small v d. Therefore, the power that is delivered to the load is small v d times the amplitude of the current capital I naught by 2 divided by 2 ok. And the best case the largest amplitude of v d is what we figured out is capital V DD, alright. So, the best case efficiency, the best possible efficiency is when this is largest, fine.

And all these factors cancel out very nicely. And you get pi by 4, alright, any idea about pi by 4 is like? Pi something like 3 ok. So, 75 little bit more than 75 you get about 78 percent, 3.14 divided by 4 right, 0.78 approximately. Now what is this? What is 78 percent? 78 percent is my new efficiency, right it is the efficiency if I conduct the MOSFET only for 180 degrees out of the full 360 degrees, right. If I say that let me restrict myself to just half of the cycle right. And during that half of the cycle current goes through the MOSFET. Remaining half of the cycle MOSFET is off, right cut off, completely cut off, because of gate bias. Then the best case efficiency in a scenario where the amplitude of the voltage is maximum right, this is also an over estimate right overly optimistic.

Because this cannot go all the way down to 0 right, but even then even with an overly optimistic view, I get at least I get 78 percent, I am not stuck to 50 percent, fine alright great. So, this kind of an amplifier, I have not done anything as far as the circuit is concerned. The circuit is the same, all that I have done is tweaked V GS to a point such that half of the time the MOSFET is cutoff, alright. So, such a scenario is called a class B power amplifier, and here the MOS is in saturation, saturation is wrong is conducting, right.

And let us not call it 360 degree let us call it 2 pi ok, radians. So, in the class B power amplifier the MOSFET is conducting for pi radians. And here the maximum efficiency is of the order of 78 percent, actually pi by 4. And that is the theoretical maximum efficiency ok. So, nothing much has been done. We have studied 2 kinds of amplifiers, power amplifiers; the class A power amplifier, the class B power amplifier. And in the next lecture we will study a few more, ok.

Thank you.