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Lecture – 35 Power amplification

Welcome to the Analog Electronics course. This is lecture number 35 and today the plan is that I am going to start talking about a new module; this is this module is about Power Amplifiers.

So, we are going to start with basic power amplification; now when we started this course at that time also we discussed this; very briefly. We had discussed why we are doing analog circuits at all; right, when you have a transformer for example, the transformer is a linear passive device right. You can amplify the voltage with the transformer right; let us say on the primary you have a voltage V 1 and then the turns ratio is 1 is to 100; then on the secondary you will have a voltage 100 times V 1.

You can amplify the voltage like this, but while doing this step up of the voltage; your current is going to step down right. Net power is going to be conserved V 1 times I 1 is going to be equal to V 2 time I 2 whatever happens right. Because there is going to be conservation of power and this is the linear passive device nothing extra is going to happen.

So, the entire point of learning this course right the only the one and only reason why we learnt this entire set up with MOSFETs and so on and so forth amplifiers right. There is only one reason why we learnt it; so that we can make V 2, I 2 more than V 1; I 1 ok; so, that we can somehow manage amplification in terms of power right. Now when you say amplification in terms of power; I do not ever mean that you are going to violate the law of conservation of energy right.

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You have to satisfy; so somehow from outside you are going to input some DC power to satisfy the entire setup ok.

So, we are going to have a 2 port network right; imagine your transformer right, you have got V 1 I 1 and you generate V 2 and I 2 ok. So, we want V 2 times I 2 to be much more than V 1, I 1 right and how is that going to happen? That is going to happen only if I apply extra power from outside; let us say all these are grounded ok.

So, I apply extra power from outside and this power is going to be a DC; this is going to be a signal right. So, the idea is that you are going to take my voice from the microphone right; you are going to take my voice from the microphone and then. So, this is the microphone this point is the microphone right.

And then you are going to amplify the power in my voice; how are you going to do that? With the aid of an external power supply; so, you use an external power supply you take its help, amplify my voice from the microphone and play it on the loudspeaker ok. So, that is the entire plan right; this is the holy grail of analog circuits, this is why we are learning analog circuits ok.

If you cannot amplify the power in the signal, then there is no need to do analog circuits at all you should do transformers fine great. Now with this understanding we are going to proceed further; we are going to look at our first our very basic common source amplifier.

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So, the plan is of course, that I am going to apply the input signal at the gate of the common source amplifier alright. So, this is if it is common you are planning to apply input at the gate of the amplifier.

Now you need a load; the load is my external load right; I do not want to impose the DC bias conditions on the load. The load should only take signal power and for that normally what we do is; we put a huge coupling capacitor fine or a decoupling capacitor over there ok. Now this capacitor is so big right that it is a short circuit for all AC signals for all signals, it is a short circuit; whereas if you are taking of DC then the capacitor is naturally and open circuit fine.

Now, we have to bias the MOSFET and to bias the MOSFET you can bias it with the current source; for example, right. You can use a current source over here at the drain you can use the current source and bias it. But to keep matters very straightforward very simple let us for now use an inductor to bias this MOSFET. Now this inductor let us choose this inductor value to be so large; the inductor is so big that for all DC signals, it is a short circuit.

For DC, the inductor is always a short circuit ok, but we are going to choose the value to be so large that for all AC signals; it is an open circuit alright. So, inductor impedance is j times omega times l; omega times l, we are going to choose l so large that whatever the value of omega is j omega l is very large and this is as good as an open circuit fine; so, this is the plan. So, the inductor is DC short AC open the capacitor is an AC short DC open ok; these are huge valued capacitors and inductors.

Now automatically the bias point at the drain is VDD; the DC operating point of this MOSFET, V DS is equal to VDD ok. V GS let us come to V GS; so let us supply V GS such that I get maximum swing ok; so let us supply some V GS; let us not worry about it. So if you remember our convention small letters; subscript capital letters is something which is the combination of both the smalls incremental signal plus the DC operating point.

Small subscript small was just the incremental signal, capital subscript capital was the DC operating point ok. So, this was our convention; the only one that is not over here is capital subscript small and capital subscript small is nothing, but the Laplace transform of small subscript small just reminding you; refreshed alright.

Now, for this capital V GS and so for this DC operating point capital V GS is the gate to source voltage and capital VDD is the drain to source voltage. So for this DC operating point suppose I get a certain current capital I subscript capital D. And remember the inductor is a short circuit for DC, but an open circuit for AC which means that the incremental current cannot come through the inductor ok; as far as the incremental current is concerned it is an open circuit.

The current through the MOSFET is small i subscript capital D, which is equal to capital I D plus small i d. And naturally what is going to happen is that the incremental current is going to come slowly through the capacitor small i subscript small d and remember the capacitor is a short circuit for incremental signals. So, the incremental current will happily come through it, but it is an open circuit for DC; which means the capital ID no portion of capital I D can come through the capacitor ok. So, we have cleanly separated the signal from the DC operating point over here; it is very clean alright.

Now, if you think of the voltage the voltage over here is right; there is a DC operating point part which happens to be equal to V DD plus a small signal and incremental part.

Let us not call it small signal I am sorry I am not going to call it; small signal small or large is irrelevant, incremental part right. So, there is a DC part, there is an incremental part and the sum of the two is the actual voltage at that node.

Now, what is going to happen? As far as the DC is concerned, the inductor is the short circuit which is perfect, but as far as AC is concerned that is incremental is concerned; the capacitor is the short circuit and the capacitor is an open circuit for DC which means the voltage over here is going to be small v d right. The current through the load is small i d alright.

So, this is our set up; it is a very straightforward circuit right nothing much is over here. There is only a MOSFET and there is an inductor capacitor and then there is the load and that you are exciting with the help of a signal and a bias voltage and here is a power supply you cannot have components lesser than this right. Which are the components that dissipate power over here, does the inductor dissipate power? No.

Does the capacitor dissipate power? No ok; these are all passive loss less components inductor and capacitor do not dissipate any power ok. So, VDD is generating power; the MOSFET can consume power, the load is going to consume power, but of course, right as far as the signal is concerned; these are not producing any power at all because the current into the gate is always equal to 0; assuming you know forget about all the capacitors and so on and so forth right; let us forget about the capacitors; parasitics.

Let us forget about those then these 2 are not really generating any power this is 0 power ok. So, VDD is producing power, MOSFET is consuming, load is consuming fine. So, this is the setup over here; if you want you can add a resistor in series with the signal source; it is not going to make any difference because the current is 0. Now, let us see what is going to happen in the circuit.

So, suppose small v gs is a sine wave alright and suppose just suppose; that this sine wave is going to produce a response in small i d; which is also a sine wave. Ordinarily, it is not because the MOSFET is a square law device right, but if you assume that the higher harmonics are not there; let us say I filter out the higher harmonics through some mechanism ok. Then I can assume that small i d is also a sine wave; the current in the load is a sine wave ok. So, this is the assumption we are going to make right away.

Now when small v gs goes up what happens to small i d? If I increase small v gs; what is going to happen to small i d? It is going to increase right? g m times small v gs approximately use small i d if it is small signal. If I decrease small v gs; then small i d is going to decrease right below 0. If I make small v gs go below 0, then small i d is going to go below 0 right. Then the net i D is going to go below the bias current right, what is the smallest small i d; what is the smallest drain current that you can have, you can tolerate?

When you talk about swing limits; remember we did swing limits for long time back right. When you talk about the swing limits; the small is drain current is always equal to 0 ok. You cannot have a current smaller than 0; you cannot go negative alright that tells me that the maximum amplitude of small i d is equal to capital I D ok.

So, if you plot; suppose this is capital I D and I am plotting small i sub capital D, then when I increase V GS this increases, but when I decrease V GS this decreases and the maximum amplitude that I can have is equal to capital I D. So, basically you are going to start from capital I D, you are going to go all the way up to 2 I D, come back to I D, come down 0 right that is the kind of waveform that you can have; this the largest possible amplitude you can have right.

So, I can choose the amplitude of small v gs such that I get this largest possible waves alright. So, let us do that let us say that small v gs is such that I get largest possible wave from over here fine. Then just like this we are going to start looking at the voltage v D; so let us look at the voltage v D.

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 $2V_{DD}$ 示 mos Available input power = $\frac{1}{2}$ $\frac{1005}{4R_0}$ Pomer supplied by DC source = Power delivered to load Max possible power in load Max

So, small v sub capital D; now small v sub capital D has a DC operating point value of how much ok? So, the DC operating point value is VDD; now when I increase v gs; if v gs goes up, i d goes up right. When i d goes up what happens to small v d? Small v d actually becomes negative right which means that small v sub capital V DD is going to decrease. So, when the input voltage goes up; the drain voltage is going to go down naturally right.

How much can it go down, what is the best case scenario? The best case scenario is the overdrive voltage right. V DS has to be more than V GS minus V T to make sure that the MOSFETs remains in saturation alright. So, let us say we abide by that rule that MOSFET is going to be instauration ok. In that case this is the kind of waveform we are expecting; best case the largest possible wave form for small v sub capital D, where this much is nothing, but V GS minus V T. Remember these are things that you did when you work with the swing limits of the MOSFET.

Now, let us just simplify V GS minus V T could be 0.2 volts right; let us simplify alright, let us say that it is 0; just super simple. So, I am simplifying I am saying that let us says this is not even V GS minus V T, this is equal to 0 alright. So, in that case the maximum amplitude of small v small d is equal to; so, this is small v small d right VDD and the extra stuff. What is the extra stuff? The extra stuff has an amplitude of V DD; so, it starts from V DD, comes down to 0, goes back to V DD, then goes all the way to 2 V DD, comes back to V DD ok.

If I make this assumption that it is not even V GS minus V T not even 0.2 volts; it is 0 volts, it is the minimum fine; so far so good great. Now, let us look; let us look back again, let us look at the following things. We are going to first look at what is the available power from the input source, what is it? Available power from the input source is source squared divided by 4 R S. So, this is the maximum power that you can draw from the input source; this is small v gs; if you say that it is the amplitude, then it is not even 4 R S; it becomes 8 R S; it is a sine wave ok. And we are not talking about the root 2 adjustment and so on; small v gs is just the amplitude this is fine, this is the available input power.

Now, let us see how much power is supplied by the DC power supply? So, there is a DC power supply over here of value VDD and how much current is it pushing? It is pushing I D amount of current, which means that the power supplied by this DC power supply is VDD times I D; and these are at DC right; VDD and I D are both DC quantities. So, there is no half for anything great; then let us find out what is the power delivered to the load? So, the load sees a voltage small v d across it and a current small i d through it ok. And naturally the power in it, but small v d and small i d both sine waves; so the power in the load is half of small v d times small i d ok.

Now, what is the maximum power that you can possibly deliver to the load ok, what is the maximum power in the load? We have found that the maximum possible amplitude of small v d is nothing, but V DD. And we have also found that the maximum amplitude of small i d is nothing, but capital I D; not 2 I D, 2 I D, where it is reaching the amplitude; the signal part; the amplitude is capital I D. And this is maximum possible V DD is not even correct; it is actually smaller than V DD right. The actual amplitude if you; if you take into account V GS minus V T; then it is smaller than V DD, it is not even V DD fine.

Now, you can define efficiency as the amount of power that you have drawn from the power supply right; you take that into account and you take into account how much power did you deliver to the load ok. So, power delivered to the load divided by power drawn from the power supply is defined as the efficiency. So, efficiency; how much is that? Maximum efficiency is V DD I D by 2 divided by V DD I D; which is equal to half or in other words 50 percent.

So, if I draw energy from the supply; only half of that is going to be delivered to the load, where is the remaining half gone? The remaining half; obviously, has gone into the MOSFET; the MOSFET has must have burnt it as heat. Because there is no other power consuming device over here; this is just the load and the MOSFET. Let us check what is the power consumed by the MOSFET? How will you do it? How will you work out the power consumed by the MOSFET? The power consumed by the MOSFET is the voltage small v sub capital D times the current small i sub capital D.

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Power in Mos = $\frac{1}{T} \int_{0}^{T} i y^{(k)} dt$

= $\frac{1}{2T} \int_{0}^{T} (T_b + T_b \sin \omega t) (v_{DD} - v_{DD} \sin \omega t) dt$

= $\frac{1}{2T} \int_{0}^{T} (T_b v_{DD} + v_{DD} T_b \sin \omega t - v_{DD} T_b \sin^2 \theta) d\theta$

= $\frac{1}{2T} \int_{0}^{T} (T_b v_{DD} + v_{DD} T_b \sin \omega t - v_{DD} T_b \sin^2 \theta) d\theta$

= $\frac{2}{T}$ $= I_0 V_{DD} - \frac{1}{2\pi} \int_{0}^{2\pi} \frac{V_{DD} I_D \sin 2\theta}{2} d\theta$ $\left(\frac{1}{2}\right)^{V_{\text{DD}}I_{\text{D}}}-\frac{V_{\text{DD}}I_{\text{D}}}{2}+\frac{1}{2\pi}\cdot\frac{V_{\text{DD}}I_{\text{D}}}{2}\cos 2\theta d\theta$

Right; this is the instantaneous power, but we have to work out the power consumed over 1 full cycle right. Energy consumed over 1 full cycle because it is changing with time and then look; for the sine wave you do not have to do this integration at all because you know the answer ok. But how did this by 2 answer come? The by 2 came because somebody right or maybe it could have been you did this calculation a long time back and you recall it ok. But in this case we have not yet done this calculation or maybe we have let us just break it up ok.

So, this is going to be integral over one period and then the average fine and if you look at it carefully; small i sub capital D is nothing, but the DC part which is capital I D plus the AC part, the DC part which is capital I D plus the AC part which is capital I D sin omega t ok.

Now, you can always generalize and say omega is 1 hertz or 1 radians per second in which case this omega will go away and this t will become 2 pi right. Suppose omega I mean there is not going to be any difference whether omega is 1 radian per second or 2 2 and a half 1000 radians per second; it makes no difference the answer is going to be the same. Therefore, you can always say that let us do the calculation with 1 radians per second in which case these become set numbers; you do not have to manipulate too much ok.

So, this is i D of t and then v D of t what is that; v D of t likewise is capital V DD minus capital V DD sin omega t ok. So, this is what we have and this is just an exercise; we already know the answer the answer is going to be what? The answer is already there here 50 percent power is delivered to the loads. So, 50 percent power has to go to the MOSFET; we already know the answer this is just an exercise; so let us do this exercise. Now what you want to do? You want to take I D times V DD and then you want to take V DD; I D sin t, then there is minus V DD; I D sin t which cancels this term out and then finally, you have got minus V DD I D sin square t ok.

And lot of people do not like calling it t at all; they just call it theta instead of t, you can call it theta and certainly lot of people will feel more comfortable ok; somehow sin theta is more comfortable than sin t right. So, I D times V DD integral of that these are both constants; integral from 0 to 2 pi is just going 2 pi ok. And then when you divide it by 2 pi; it just comes out and then the remaining part and then sin square theta can always be broken up into cos 2 theta; yes how do you do that? 1 minus cos 2 theta by 2 is sin square theta.

And then once again; what is going to happen is I have got V DD I D by 2 integral d theta, that is going to give me from 0 to 2 pi that is going to give me a 2 pi and 2 pi and 2 pi are going to cancel out and what is going to come out? From the 1; instead of the 1, I will have V DD I D by 2 coming out ok; I still have one more integral to do integral cos 2 theta d theta right.

You can replace d theta with d 2 theta for example, or you can replace 2 theta with some alpha right substitute 2 theta as alpha, this becomes pi in that case and you get a half outside. So, now, you are integrating cos alpha from 0 to pi right; so, integral of cos alpha is sin alpha when you plug in sin alpha to be 0; you get 0 when you plug in sin alpha to be pi alpha to be pi you again get 0. So, the net result of this integral is going to be one big 0 ok. So, therefore, you have V DD I D minus V DD I D by 2 and that leads you to the answer that you expected V DD I D by 2; so there, that is how you actually do it.

So, what did we figure out? We figured out all of the following; we had the simplest of all sockets. Common source amplifier right, biasing was done with the help of an inductor, capacitor was used to decouple the DC operating point from the load fine. Input was applied on one side what did we see? We saw the following; we first saw that the available input power was so much but the power delivered to the load was so much ok.

If R S is large; suppose R S is large then I immediately can deliver a much larger power to the load, than the original input source could have ok. This can be made to be significantly larger V DD I D by 2 can be significantly larger than half v gs squared by 4 R S; if R S is I mean all you have to do is have a weaker microphone R S is larger alright smaller v gs.

Now, weak microphone you can always get power amplification right that weak microphone would not have been able to drive the loudspeaker with the help of this circuit now you probably can; this is number 1 observation. Next power supplied by the DC source is V DD times I D; nothing else, I D is the current, V DD is the voltage; V DD times I D is the power supplied by the DC source.

The power delivered to the load is small v d small i d by 2; I have not made any assumptions in this right. When I say that may be the largest amplitude of small v d is V DD, then I make an assumption that the MOSFET is in saturation and you know the smallest voltage that you can tolerate is 0 ok.

Otherwise this is always correct, no matter what there are no assumptions there is no assumption in this also right these 2 are assumption free; given the circuit this is always going to happen, this is always going to happen. Then from here to here small i d, the maximum amplitude is capital I D and small v d, the maximum amplitude is capital V DD; I have made certain assumptions in this right.

And then after making those assumptions; those are all pretty general assumptions nothing very significant in those assumptions. All that we are assuming is that we have got a sine wave over here number 1 and then you know the this is overly optimistic for example, capital V DD is too optimistic, the amplitude is not really going to be so, large.

So, what does that mean? That means, that the maximum efficiency has an upper bound of 50 percent and it is not even 50 percent; we know it smaller right. For example, this one is overly optimistic; V DD is overly optimistic. So, 50 percent is overly optimistic you are never going to reach 50 percent ok. So, think about it; you are trying to play something on your cell phone, speaker phone right, you are trying to play some music on your speakerphone on the cell phone right.

You are trying to make the entire audience listen to that song as much power as is going out of that speakerphone that much power is going to be burnt as heat inside the power amplifier ok. So, your battery let us say is producing 1 watt right, half watt at best half watt is going out to your audience half watt is going to burn just as heat alright. I mean clearly you try to sell this power amplifier to you know the best company Samsung or whatever Motorola and they are going to laugh at you because this is no good right.

And this power amplifier is there everywhere right; it is not just at the speaker this is also there at the antenna. Suppose you are trying to talk to a base station which is far away from your cell phone right, I am trying to give the example of the cell phone again and again because the cell phone today is the most popular electronic object in your pocket right.

So, your cell phone is trying to communicate with a base station that is far away; you need an antenna and to drive the antenna you are going to use a power amplifier. Because you are going to push a lot of power through this antenna; as much power as you push at least that much power you are going to burn as heat alright. So, this is just not good; this does not cut it, but unfortunately this is how things are all right.

Now, we had a constraint over here and the biggest constraint that is causing, this 50 percent is this. We were telling ourselves that we want to keep the MOSFET in saturation alright; there is nothing else over here. V DD times I D is a golden truth, you cannot violate; small v d, small i d by 2 you are not going to violate ok. So, the only thing that gives me these numbers is I apply this constraint; I get these results. So, this must be something that we have to relook at alright.

Let us take another example; let us take a second power amplifier circuit and instead of the common source amplifier; maybe you can use the common drain amplifier ok; that also has large input impedance. You probably will not bother about the common gate amplifier that much because the input impedance in the common gate amplifier is small not large. Even if you do bother about it then you are going to make sure that the input is a current source, not a voltage source.

So, the strategy is going to change; let us not worry about the common gate, let us do a common drain amplifier.

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And let us not worry about body effect. So, we will assume that V T is constant and everything and once again what we are going to do is; ordinarily you are going to put a current source over here or you are going to put the load over here over there at the source.

Instead we are going to put the load AC coupled to the source and we are going to achieve this bias point ordinarily through a current source. In this case, let us achieve the bias point with the help of a huge inductor right and we make this inductor of such large value that it is a short circuit for DC, but an open circuit for incremental signals. The capacitor is a huge capacitor such that it is a short circuit for incremental signals, but open circuit for DC fine.

So, this is our setup and then again we do the same design as before right; this is going to take a current capital I D; the current over here is small i sub capital D, which is the superposition of capital I D and small i small d. And the current going through the capacitor is only small i small d; fine and then now it is not the drain voltage anymore; now it is the source voltage. The drain voltage is fixed at V DD; the source voltage is 0 because of the inductor right. The DC operating point of the source voltage is 0 and then there is a small signal an incremental signal part of it which makes it equal to small v s.

The capacitor is a short circuit for this particular incremental voltage. Let us not call the input voltage V GS; let us call it the bias voltage let us call it V G and let us call the input voltage as v in just so, that we do not make a mistake. And you can have any series resistance over here ok; so this is our setup and let us see how it amplifies power if at all or what is going on in this circuit.

Now how we going to do that? We are just going to first look at what is the power supplied by the only source of power over here? It is VDD times small i sub of capital D alright; so it is no longer like our earlier case. In the earlier case, we had V DD times I D right; in this case the power is being supplied if it is V DD times small i sub capital D.

So, power supply is V DD times small i capital D alright; what is power delivered to the load? If you assume sine waves, then the power delivered to the load is small i d times small v s divided by 2. And then what is the power consumed by the inductor? Is power consumed by the inductor? Never; inductor is not going to consume any power; is power consumed by the capacitor? No; so these 2 are 0s ok.

The other one that is going to consume power is the MOSFET and we will have to work out the efficiency. How are we going to make the efficiency; work out the efficiency? The efficiency is going to be the power in the load divided by the power supplied by the DC power source fine. So, this is how it goes; now let us make out plots.

Let us try to find out what is the maximum what is the maximum amplitude of small i d that you can possibly have? Small i sub of capital D is the sum of capital I D and small i d. So, the maximum amplitude of small i d has got to be capital I D because small i sub capital D can never be smaller than 0 ok. Amplitude means it is going both positive as well as negative right. So, the maximum amplitude of small i d is capital I D; so I am not even bothering to draw, you understand from our just like our previous discussion right the graph pretty much looks the same fine so far so good.

Let us look at this source voltage or in other words let us look at; let us try to find out what is the maximum amplitude of small v sub small s ok. What is the maximum amplitude at the source, what is it? This can rise all the way at best up to VDD right; the source cannot go beyond VDD right, it is going up and down up and down; this voltage is going up and down incrementally right. When it goes down, it is going down below ground; when it goes up it has to maintain this MOSFET to be in saturation which means that at best it is going to be V GS minus V T less than V DD.

If I say that V GS minus V T is nothing minute at best is going to go all the way till V DD; which means that the maximum amplitude of small v sub small s is nothing, but V DD and that too that is overly optimistic. So, too optimistic; which means that the maximum power possibly; maximum power you can deliver to the load is nothing, but V DD, I D by 2 that is the maximum possible power that you can deliver to the load.

What is the power that is being drawn from the source? The power that is being drawn from the source is VDD times small i sub of capital D; which is equal to V DD times capital I D plus small i d ok. And then when you have to find out the power this small i d is changing with time. So, you have to integrate over one period right, but when you integrate small i d over one period you get 0 right; small i d is nothing, but capital I D cos or sin omega d right and then integral of sine from 0 to 2 pi is going to be a big 0. So, this is not really going to give you any contribution; you can do the integral all you will get is 0 ok.

So, the power drawn from the DC source is V DD times I D; average power. The maximum power that you can deliver to the load is V DD times I D by 2. Therefore, what is the efficiency? The maximum possible efficiency is once again just about 50 percent; in fact, it is lesser than that because we have got one of the 2 the voltage amplitude this is too optimistic right.

We know it is not even V DD it is actually smaller than V DD alright. So, look whatever we are doing our efficiency seems to be 50 percent right half of the power seems to be being burnt in the MOSFET itself. So, this kind of strategy is not quite working out ok; however, this is the general principle of power amplifiers; power amplifiers work on this principle only nothing else right. Power amplifiers are simple amplifiers that are going to be used to blast power into the loudspeaker or into the RF antenna or wherever power is required ok.

So, let us stop here and then in the next class, we are going to look at more efficient waves of doing the same thing right. Typically the constraint that we have even here the constraint is the same why did we get V DD I D by 2? The reason we got it was because we wanted to make sure that the MOSFET remains in saturation. So, we now have to start violating that condition right.

So, far we have held that condition to be golden right till lecture 35; we have said that you know let us make sure that the MOSFET is always in saturation. In lecture 36, we are going to stop saying that we are purposely going to make it not remain in saturation and then we are going to see what happens ok.

Thank you.