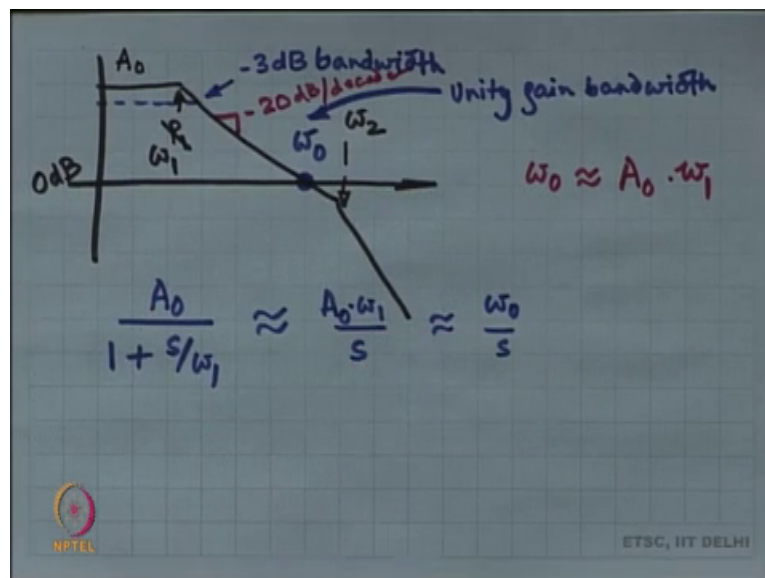


Analog Electronic Circuits
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Lecture - 34
Unity Gain Bandwidth

Welcome back to analogue electronics. Today's lecture number 34 is going to be about unity gain bandwidth. So, this is what we were discussing. In the last class, but we had started talking about it very briefly.

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The idea is that if you look back at the bode plot; you have a system that is going to look like this. This is the open loop Op-Amp.

It is a 2 pole system, probably with 0 that you have right half plain 0 that you have cancelled out. So, you be do not bother about the 0 over here, right. You got a pole P 1 and you have got another pole, not P 1 we called it omega 1 is it, omega 1, and you have got another pole omega 2. And in between you have this point omega naught, and at this point the gain the Op-Amp at this frequency the gain of the Op-Amp is 1 or 0 dB. So, this is called the unity gain bandwidth. The gain at which the amplifier has unity gain the frequency, the bandwidth for which the amplifier has gain more than unity, alright.

Now, commonly used bandwidth expressions are minus 3 dB bandwidth ok. That is going to be over here right. And the other one is unity gain bandwidth. Now there is something special about the unity gain bandwidth, something very special, and we are soon going to see what it is. For now, you have to realise that if you do not worry about ω_2 , then the relationship between ω_1 and ω_0 is; what is the relationship between ω_1 and ω_0 ? Yeah, what is the relationship? At ω_1 the gain was A_{naught} right, and then the gain started dropping at 20 dB per decade.

20 dB per decade means, If the frequency increases 10 times then the gain decreases 10 times. If the frequency increases x times, then the gain is going to decrease x times. At ω_0 the gain has decreased A_{naught} times, the gain is now 1. So, earlier gain was A_{naught} , at ω_1 , at ω_0 the gain is 1. So, the gain has decreased x times; which means that the frequency has increased the same A_{naught} times. So, therefore, ω_1 / ω_0 is approximately A_{naught} times ω_1 .

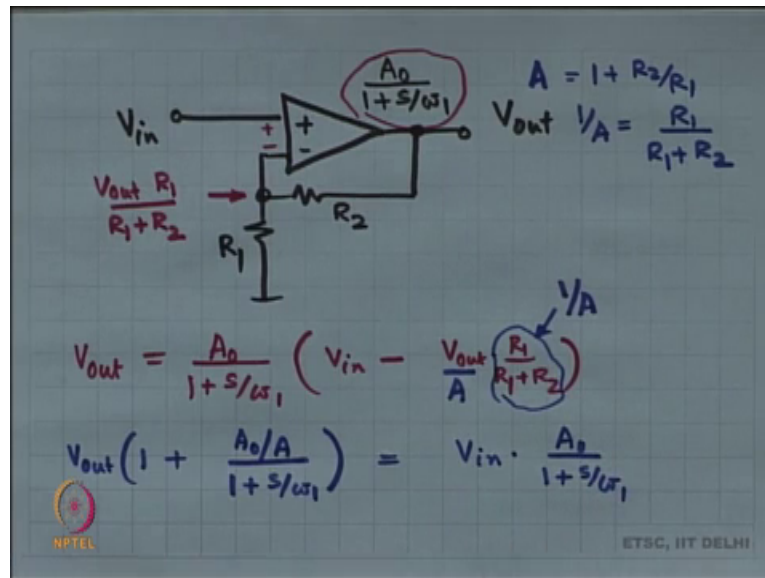
Now, the system could have been expressed as $A_{\text{naught}} / (1 + s / \omega_1)$, if you do not worry about ω_2 , ω_2 is much higher than everything else. Remember, this is you know logarithmic scale; so, little bit here and there is a large leap. If this was your overall approximate transfer function, then you could say that ω_1 is a very small frequency, right ω_1 is usually a small frequency. And for all reasonably large frequencies like ω_{naught} and so on right all these reasonably large frequencies.

s / ω_1 , s is where you are going to plug in the frequency. J times the frequency is s . So, s / ω_1 is going to be significantly larger than 1. At those frequencies and in such a scenario you can start ignoring the 1. And you can approximate this as $A_{\text{naught}} \times \omega_1 / s$, and $A_{\text{naught}} \times \omega_1 / s$ is nothing but,. So, this is the overall setup alright. So, the entire system can be simplified by virtue of the fact that the first pole is a very low frequency. So, whenever you are talking about any frequency, that frequency is going to be typically larger than the first pole because the first pole is a very low frequency.

Few a 100 hertz tens of hertz; if you buy a commercial Op-Amp of the shelf, it is a few hertz, ω_1 is just a few hertz, 5 hertz, 10 hertz something like that, ok. So, any reasonable frequency is going to be above ω_1 . And therefore, the amplifier can be

expressed as nothing but omega naught by s where omega naught not is the unity gain bandwidth, alright. Now let us look at a few systems. One popular system your favourite amplifier Op-Amp circuit is this one.

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Ok and you know that the gain of this system. V_{in} , V_{out} you know the nominally the gain of this system V_{out} by V_{in} is $1 + R_2/R_1$ we know this for a fact. Now the Op-Amp is no longer our lovely Op-Amp, we are going to replace the Op-Amp with its transfer function. And the transfer function let us say the transfer function is A naught by $1 + s$ by ω_1 right, ω_2 is much larger. So, I am not going to worry about it. So, this is my amplifier alright. Now what is the voltage over here? Let us analyse, we need to find out the relationship between V_{in} and V_{out} , given that the Op-Amp does not have infinite gain anymore, it has a gain which is this one.

Right, it is a function of frequency and it has limited gain. So, let us try to analyse the system. We have V_{out} over here; the voltage over here is V_{out} into R_1 by $R_1 + R_2$, fine. Resistive division, potentiometric division; so, the voltage difference between these 2 is V_{in} minus this quantity. And that is going to be amplified by this factor and you are going to get V_{out} . So, therefore, you can write an expression V_{out} is equal to A naught by $1 + s$ by ω_1 times V_{in} minus V_{out} times R_1 by $R_1 + R_2$, fine.

Now, just pause over here a little bit. I am going to call the nominal gain of the amplifier of this circuit. The nominal gain was $1 + R_2/R_1$, remember. So, I am just going to

call that as A, ok. That is the nominal gain of this circuit. That is the gain that you desired right. Let us call that A what is 1 by A right? So far so, good this makes sense because you are comparing V in with V out divided by A. If V out divided by A is equal to V in right, then only it works, these 2 have to be equal. So, therefore, these 2 are this one has to be equal to V out divided by A. So, that the error is 0 ok.

Now, let us solve, we have to solve for V out by V in. So, will take all the V out related terms on one side, and we are going to keep all the V in related terms on the other side. And this is what we get gathered collected all the V out terms on one side V in terms on the other side.

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$$V_{out} = \frac{A_0}{1 + s/\omega_1} \left(V_{in} - \frac{V_{out}}{A} \frac{R_1}{R_1 + R_2} \right)$$

$$V_{out} \left(1 + \frac{A_0/A}{1 + s/\omega_1} \right) = V_{in} \cdot \frac{A_0}{1 + s/\omega_1}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0/(1 + s/\omega_1)}{1 + \frac{A_0/A}{1 + s/\omega_1}}$$

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And now we are only interested in V out by V in, ok. And your natural instinct is going to be to multiply numerator and denominator by 1 plus s by omega 1.

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$$\begin{aligned} V_{out}/V_{in} &= \frac{A_0}{1 + s/\omega_1 + A_0/A} = \frac{A_0}{(1 + A_0/A) + s/\omega_1} \\ &= \frac{A_0 / (\cancel{A_0/A})}{1 + s/\omega_1 \cdot (\cancel{A_0/A})} \\ &= \frac{A \leftarrow DC}{1 + s/(\omega_1 A)} \end{aligned}$$

$A_0 \gg A$

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You want to simplify further? I think so, I think a little bit more can be done we can multiply numerator and denominator by A not even not even that. We can divide numerator and denominator by A naught.

No, not even that you see the way we work is that the denominator should look like 1 plus s times something. That was a template. So, I am sorry we should not deviate from the template. So, what we have got is A naught by 1 plus A naught by A plus s by omega 1 right, this is what we have got. The denominator should look like 1 plus s time something. So, therefore, we should divide numerator and denominator by this factor. Is this ok? Now let us think about it for a little bit. A naught is a large quantity 10,000, ok.

A is a relatively small quantity; you are going to make a limited gain amplifier circuit over here, right. You are going to make a gain of 10, gain of 15, something some limited number. It is not going to be certainly not going to be more than the Op-Amp gain, it cannot be more than the Op-Amp gain, it is going to be significantly lesser than the Op-Amp gain such that the circuit works, right. So, A is substantially smaller than A naught, A naught by A is going to be a substantially large number; in which case 1 plus A naught by A is more or less equal to A naught by A.

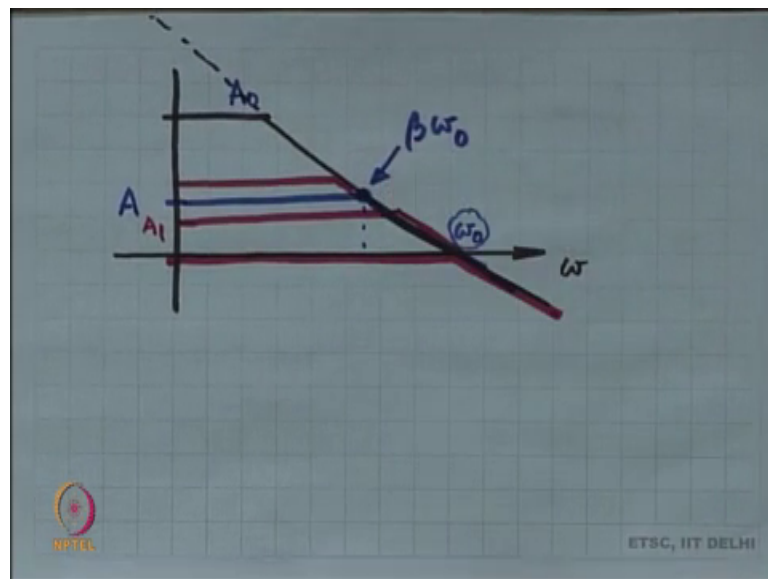
So, you can stop worrying about this one portion right. You can stop worrying about this one portion. You see in engineering, it is important to stop worrying about the small things, you look at the big picture over here. It is not maths, this is engineering. So, then

we can simplify a little bit further, and what happens? A_{naught} and A_{naught} cancel out, very politely, and all you get is $\frac{1}{1 + s/\omega_1}$ A_{naught} what is ω_1 1 times A_{naught} , ω_1 1 times A_{naught} A is the unity gain bandwidth, ω_1 $naught$ by A , alright.

So, this is what we have. What does this mean? This means that A_{DC} at 0 frequency, this s term is irrelevant at 0 frequency, and all that you are left with is A at DC. You were trying to make an amplifier of gain A , your left with a gain A at DC perfect, right. Of course, the gain was not exactly A it was A_{naught} by $1 + A_{naught}$ by A which I have simplified to A by assuming that A_{naught} is much larger than A . And unless you make this assumption you will not get that perfect gain of A , that you wanted. So, a certainly has to be much much much smaller than A_{naught} for this 2 what ok. If you if you have not chosen a to be much smaller than A_{naught} , then I am sorry your circuit is not going to give you the desired performance right.

Now, this is the performance at DC ok. What is the bandwidth? What is the 3 dB bandwidth of the system? The 3 dB bandwidth of the system is not related to ω_1 it is related to ω_1 $naught$, the unity gain bandwidth, it is the unity gain bandwidth divided by A .

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So, let us go back a little bit. I had my system ok, this was my unity gain bandwidth. I start from a DC gain A_{naught} . Actually the value of A_{naught} is irrelevant, right you could have started anywhere on this curve.

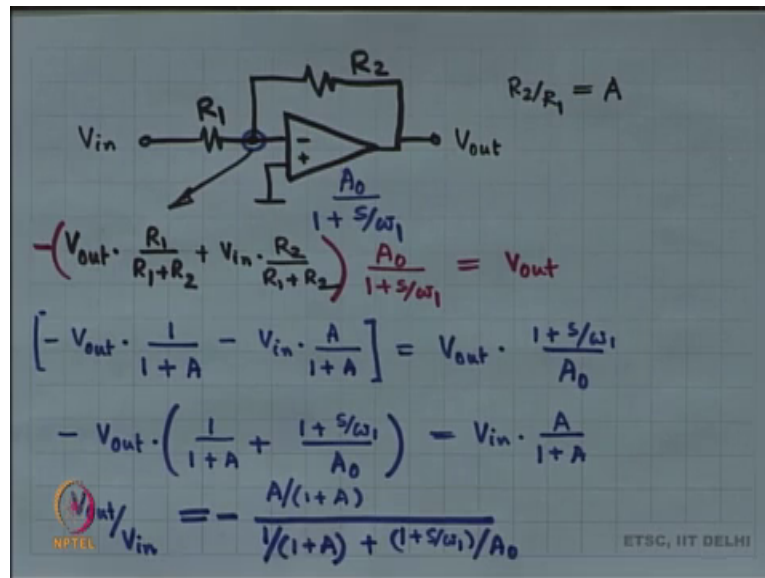
Now, you are saying that I make this beautiful circuit, alright and this circuit has a nominal gain of a at DC. Let us put a over here, a has got to be smaller, substantially smaller than A_{naught} . And guess what, the 3 dB bandwidth of the system is $\omega_{naught} \times A$, ok. It is right at this corner; how do you know that right side the corner? Because if I decrease the frequency by ω_{naught} the gain increases by A ok. So, at the frequency $\omega_{naught} \times A$ the gain is A . So, it is right at this corner alright, any other system suppose instead of a , you design your circuit to have a gain of A .

Suppose $A = 1$ is this one, what is the bandwidth going to be? What is the characteristics going to look like? It is going to look like this. Right will have the same overlapping characteristics, whatever system, you design it is going to have the same overlapping characteristics ok. And the overlap depends on the value of ω_{naught} , the unity gain bandwidth. Suppose your a was equal to 1, maybe right A could be chosen to be one, you could make your circuit to have a gain of one we have we know some utility of having a gain of one, right. It is useful in that case the system will just look like this.

And it will have a bandwidth a 3 dB bandwidth of ω_{naught} , alright. So, that is why this unity gain bandwidth is so important, right. Whatever system you are going to design, when you look at the 3 dB bandwidth of that system, eventually it is going to be related to the unity gain bandwidth of the base amp, the core amplifier the core Op-Amp, that is being used inside that system, alright. Now in our analysis, we ignore this $1 + A$ as soon as you ignore the $1 + A$ you can start calling it calling the Op-Amp as $\omega_{naught} \times s$.

So, when you have a lot of Op-Amps in the system, you can approximate all the Op-Amps, as $\omega_{naught} \times s$ and that will give you a quicker answer to the problem, let us do one more.

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Let us do one more of your second favourite circuit was inverting amplifier, ok. And here we are going to choose R_2 by R_1 is equal to the gain, ok. So, here the gain is minus A in this case, fine. Now unfortunately the same result is not quite work.

. So, let us do it. Let us do it; we need to do it to appreciate that the same result does not quite work. It is going to be similar, but not exactly the same. And then will be generalized from there. What did we do last time? We said that the amplifier has a response that looks like this, ok. So, will start from this, and what is the voltage over here? The voltage over there is related to V_{out} and V_{in} , ok. So, how did I do this?

So, quickly I did it by inspection with the help of superposition. So, forget the Op-Amp over here, first I assume that V_{out} is there, V_{in} is ground, in which case it is a potential divider right, a value R_1 by $R_1 + R_2$. In the next case, I imagined that V_{in} is there, V_{out} is ground, in which case it is a potential divider again, with R_2 by $R_1 + R_2$, ok. So, it is V_{out} times R_1 by $R_1 + R_2$, plus V_{in} times R_2 by $R_1 + R_2$. That is the voltage over here. This voltage minus of this voltage in fact, is the difference between plus and minus, times the gain of the Op-Amp is equal to V_{out} . Fine, so far so good?.

This is fine, but I want to write down R_2 by R_1 to be equal to A , alright. So, let us simplify a little bit ok. I just divided numerator and denominator by R_1 , and here I am going to divide numerator and denominator by R_1 . And then, again you need to collect

all the V out terms together, all the V in terms together, ok. And then finally, you have to find out V out by V in ok. You see the maths is no longer the same as before, but anyway let us proceed. Instinct would say that let us multiply numerator and denominator by A naught times 1 plus A.

So, I am multiplying numerator and denominator by A naught times 1 plus A. Fine, so far so good?

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$$V_{out}/V_{in} = - \frac{A A_0}{A_0 + (1+A) \left(1 + \frac{s}{\omega_1}\right) \left(\underbrace{A_0+1+A}_{A_0+1+A} + (1+A) \frac{s}{\omega_1}\right)}$$

$$= - \frac{A A_0 / (A_0+1+A)}{1 + \frac{s}{\omega_1 \cdot \frac{A_0+1+A}{1+A}}} = - \frac{A}{1 + \frac{s}{\omega_1 (1+A)}}$$

$A_0 \gg A, 1$

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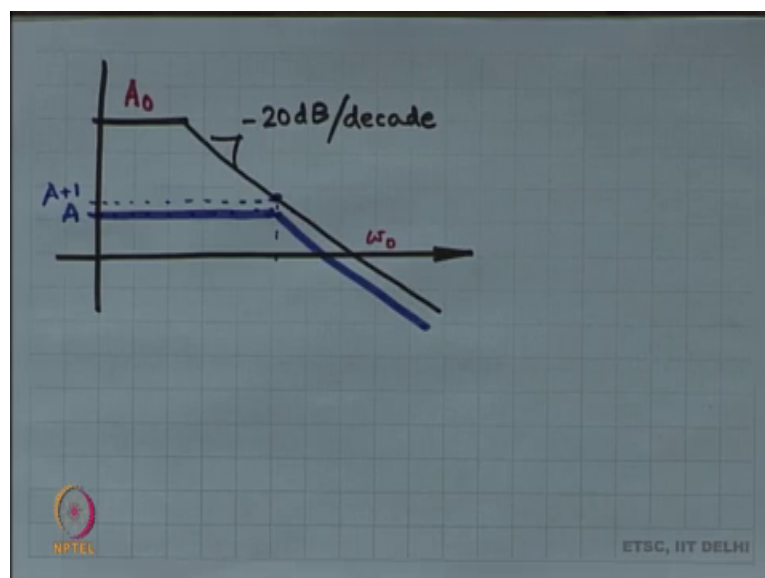
Ok, what you do a sanity check ok, you can do a sanity check, you just plug in s is equal to 0, if I plug in s is equal to 0, this disappears. So, the denominator becomes A naught plus 1 plus A, and A naught is substantially larger than both 1 as well as a which means this whole thing can be thrown out. In which case you get A times A naught divided by A naught which is just minus A.

So, it works the sanity check says that we are on the right track we haven't made a mistake, while doing our calculation. So, this is always very important, sometimes you should pause, and check your answer, right. There are easy ways to check your answer, alright. Next what are we going to do? We need to make sure that the denominator is of the form 1 plus something. Ok right now, the denominator is A naught plus 1 plus A plus 1 plus A times s by omega 1, right this is what the denominator is right now. We do not like it, right we want to make it 1 plus s times something ok.

And what does that mean? That means, numerator and denominator have to be divided by this factor. So, let us do that, and this is what we have, alright. Now once again you can approximate $A \gg 1$ plus A to be approximately A , assuming that A is much much much larger than both a as well as 1 . You do that, then these portions disappear in which case this boils down to A times $A \gg 1$ by $A \gg 1$ is approximately A divided by $1 + s$ by ω_c times $A \gg 1$ is, ω_c times $A \gg 1$ is the unity gain bandwidth ω_c ok.

And what does that mean? That means, that my gain is approximately minus A which is perfect. But the bandwidth the 3 dB bandwidth is ω_c by $1 + A$, what happened here?

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So, if you do a bode plot is started from $A \gg 1$ at ω_c you had your unity gain frequency, and then you are saying that let us build a system with a gain of A , ok. So, you start building a system with a gain of A , but the bandwidth that we have is equivalent to ω_c which is the unity gain bandwidth divided by $1 + A$.

So, not exactly ω_c by A , because ω_c by A would have been this corner frequency, not quite right our corner frequency is that for which you start with $A + 1$ ok. So, this corner frequency is going to be used. So, it does not exactly overlap, it is almost there ok. Instead of dividing by A the bandwidth you are dividing the bandwidth by $A + 1$. So, it is a slightly smaller bandwidth where did $A + 1$ come

here. So, in our first circuit the [vocalised-noise] the non-inverting amplifier, we got a nice gain of A , a nice bandwidth of A times smaller.

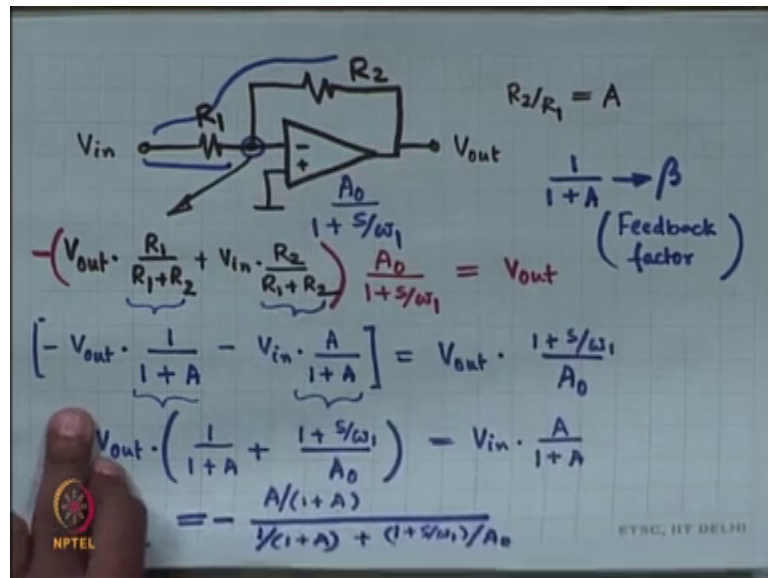
Right in this circuit the bandwidth of the whole circuit the bandwidth of the Op-Amp is the unity gain bandwidth of the Op-Amp is ω_{naught} . The 3 dB bandwidth of the Op-Amp is ω_1 , which is a tiny frequency. But the bandwidth the 3 dB bandwidth of the whole circuit is something which is related to not related to ω_1 at all, it is related to the unity gain bandwidth ω_{naught} , it is the gain of this circuit times smaller than ω_{naught} , ok. So, if you had chosen a gain of 5, then the bandwidth 3 dB bandwidth of the whole circuit would be 5 times smaller than the unity gain bandwidth, ok.

In fact, gain times the bandwidth is a constant 5 times ω_{naught} by 5 is ω_{naught} . Gain times bandwidth happens to be a constant in this circuit. This inverting amplifier is slightly different right, what did we get in this inverting amplifier? We got it to be slightly different, we got it as if the amplifier has a gain of A , and a bandwidth of ω_1 , then it has a unity gain bandwidth of ω_{naught} which is A times ω_1 , A times ω_1 . Now I plug this into a circuit to make a gain of A .

The bandwidth of the circuit is going to be ω_{naught} divided by $1 + A$. Now where is this $1 + A$ coming in from? Ok, that is where it is coming from this one A you see there is $1 + A$ everywhere. It is not just A , it is $1 + A$. By the way A is not a gigantic number, ok. A can be a small number, A can be 1, A can be 2. So, $1 + A$ cannot be approximated as A . But $1 + A$ can be approximated as A ok. Because A is always very large it is 10,000, 100,000,.

So, this $1 + A$ is coming right from here, right it is coming from this ratios, these 2 ratios that we made right. In fact, $1 + A$ is nothing but $R_1 + R_2$. How did we get $R_1 + R_2$, right? The sum of these 2 resistors, $R_1 + R_2$ and this R_1 . So, $1 + A$ is called the feedback factor ok, in popular word name in the literature this is called as the feedback factor.

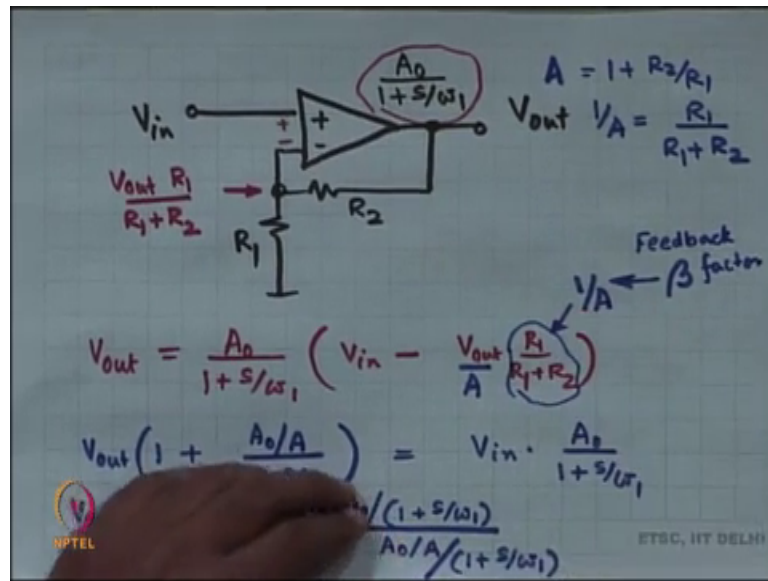
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So, this is called as beta lot of people call it as beta defined as the feedback factor. What does it mean? It means I take V out; what portion of V out is fed back? It is beta times V out that is fed back, alright.

And in such a case, you could write this as minus A by 1 plus s by omega naught times 1 by 1 plus A; so, omega not times beta. So, the bandwidth changes by the factor beta whatever that factor is. Now this happens to be general enough. If you look at the feedback factor in the first circuit, what happens? V out what portion of V out is fed back; it is V out times R 1 by R 1 plus R 2. So, R 1 by R 1 plus R 2 is going to be called beta.

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So, in this case beta is just 1 by A; whereas, in the other circuit, beta is 1 by 1 A, ok.

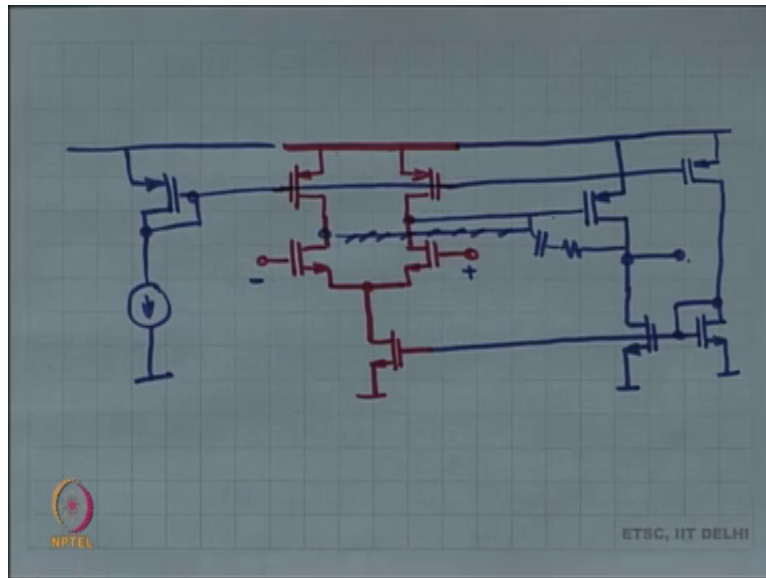
So, in the non-inverting amplifier the feedback factor is just 1 by A. In the inverting amplifier the feedback factor is 1 by 1 plus A, alright. So, slight change, and if you say that the feedback factor is just 1 by A in the non-inverting amplifier. Then, it follows right it straight away follows that, you know, this bandwidth is just beta times omega naught, whatever to beta is right. Earlier we were calling it omega naught by A now I am calling it omega naught times beta where beta is the feedback factor, alright. So, this is the logic, whatever your amplifier structure is, you have to figure out what is the feedback factor. And feedback factor times the unity gain bandwidth will give you the overall bandwidth of that circuit, alright.

So, with this we are going to. So now, you understand why omega not is so important, ok. And not this corner frequency. This corner frequency is of no value. This one has all the value, omega naught has all the value, ok. So, given that we have completed our discussion on amplifiers, we have also completed our discussion on compensation. Frequency response of the amplifiers, how to design the Op-Amp; lot of times after this whole course if you are asked draw an Op-Amp, will you be able to draw an Op-Amp? Lot of people end up not drawing Op-Amps at all.

So, I hope this course is not going to leave you in confusion about what does an Op-Amp finally look like. So, an Op-Amp is finally going to incorporate all these small elements

that we have learnt. We have learnt it in bits and pieces. We started from the common source amplifier right. So, then from the common source amplifier, we built up into a differential amplifier. So, the Op-Amp should have a differential input stage, because there are 2 inputs.

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So, you are going to have 2 inputs ok, and differential input stage means that, at the source you need a tail current. You have also learnt about current sources. So, how does a tail current source look like? It is going to look like a MOSFET.

Fine, then lot of people say, that let us place resistors over here. No, as soon as you place resistors, the A times gain of the Op-Amp is going to become very limited very, very limited. So, you do not want to place resistors at all, you want to place an active load. So, here comes here starts the variation. This part is certain, this is absolutely there. You can not avoid this part right. If you say draw an Op-Amp and you do not draw this part, then you are absolutely wrong, ok. This has to be there then; on top of this you are going to put a load.

What is the load going to look like? If you have an inverse input then the load is going to be p mos, ok. Where is gain? What this, what these gates are doing is left for the future right now. Now for example, these gates to could be hanging connected to some bias voltage right. It could be self-biased for example; this could have been arranged as a

current mirror right. All kinds of things could have been done with these gates let us not draw it right now. That there are some p mos to do the load is what is important.

Now, if you are going to do a single ended Op-Amp as a post to a differential ended Op-Amp, then you are going to take only one output as a post to 2 ok. A lot of people take 2 outputs, and then in the second stage have another differential amplifier, that is also perfect. That is also perfectly fine, that will just give you larger common mode rejection ratio. So, it all this is all the engineering right. Lot of these options are there. One option is that you take just one output from the first stage and create a second stage.

Another option is that you take both the outputs from the first stage and make another differential amplifier as the second stage, right. Maybe this time you can have a pmos differential amplifier. It all depends right you can have a p mos differential amplifier; you can have an n mos differential amplifier. You can take just one output, and have just one common source structure, right all these possibilities are there. Now you build a second stage, and then you take the output from the second stage.

So, this is what the Op-Amp should look like, right. So, I will just give you an example, let us forget about the differential, let us just make a second stage. And for variety I make it a pmos second stage, with an nmos active load ok. This is your output. How will you put the plus minus signs? Well, if this is the output, then this is this will be minus right. And that means, this node has to be plus, because there is one more inversion. Then after this you have to compensate, because now you have made a 2 stage Op-Amp.

So, you better compensate right. And then you need to have biasing for this current source. You also need to bias these 2; these 2 are also like current sources right. So, maybe you can mirror all of these together. So, from one reference current, I mirror create the p mos biased voltages, create the n mos biased voltages. And of course, you have to ratio all of these, right. So, so that this takes double the current as these. These are all the items then, what else do? You have to do you have to make this reference current source. How will you make the reference current source?

We had discussed constant g_m biasing right. So, all of these little little things that we have studied, all of these have to be put together into the Op-Amp, right? And the Op-Amp has to be designed to some specification, alright? Power, bandwidth all of these are going to be part of your specification, alright. Now, any other important things that you

are forgetting over here? Sometimes you want better current mirrors in which case you are going to make cascade current mirrors, right. All of these little things that we have studied; they are all going to be put together in this overall design of the Op-Amp.

So, with this we are going to conclude the study of the Op-Amps, right. And we are going to move to the next topic in the next lecture, we are going to start talking about power amplifiers. And we are also going to start talking about voltage regulators,. So, thank you very much, we will meet in the next class.