Analog Electronic Circuits

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Lecture – 32
Compensation

Welcome, back to Analog Electronic Circuits. This is lecture 32 and today, we are going to continue from where we left off in the last class. We are going to talk about Compensation. So, the topic is called compensation; however, it is just what we were doing I did not really tell you that the topic is called compensation in the last class. In the last class we were still working on a diversion, right.

In the last class, we just showed that the ratio of omega 2 and omega 1 in a two-pole system if suppose, you forget about the existence of the 0's and so on and so forth. Suppose, you have a strict two-pole system forget about the third, fourth other poles if they exist forget about. Now, if you have a strict two-pole system and you plan to use this two-pole system within a feedback loop, then you require some phase margin all right and depending on the phase margin that you require you need a certain ratio between the two-poles.

So, if the two-poles are at omega 2 and omega 1 then we showed that omega 2 by omega 1 has to be some large number, right and you can arrive at that large number by solving a certain quadratic, right lambda squared plus lambda times something which was a function of the phase margin as well as the gain, the open loop gain, A naught as well as the phase margin plus 1, right. You can solve this quadratic and you will arrive at the ratio of the two-poles.

Now, this particular ratio of the two-poles has to be large right and this pre knowledge helps us to easily solve the quadratic, instead of solving the quadratic we can just say that that middle term 1 by of that middle term is nothing, but the ratio of the two frequencies, alright. This is what we had done in the last class, right. Now, continuation of the same so, this particular topic is now going to be called compensation; when you manage to engineer the two poles to be a certain ratio that you want that that engineering that exercise is called compensation.

So, let us see suppose you know the boss has said that give me design with a phase margin of phi when the open loop gain is a naught, right. Automatically you plug in your numbers into yesterdays equation in the last classes equation and you find that the ratio of the two poles has to be a certain number. You arrive at this number and keep it in the stack, right, maybe 1500 something we had done some crude computation, we did not really do it accurately we just plugged in some numbers we got a number like 1500. We said let us say the gain is 1000 and let us say the phase margin required is 60 degrees roughly 1500 was what came out.

Now, you look at your system, alright. What is our system? As an example so, we were looking at two different circuits, one was the cascade amplifier the other was the cascode amplifier. So, actually we have discussed the cascade substantially in the last class, but let me I will do it again all over again let us discuss the cascode because we have not really discussed it that much.

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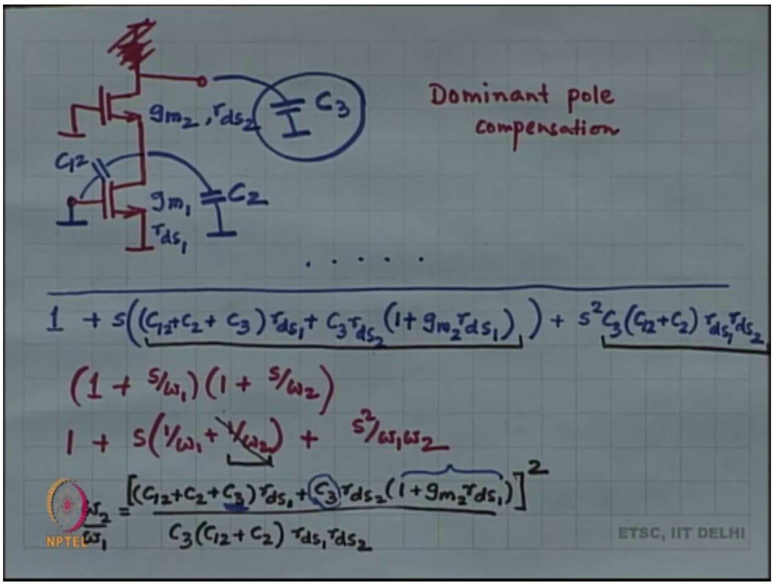

So, as far as you remember the cascode circuit I am just drawing the small signal equivalent. We are going to assume that this resistance is so large that it does not really matter. So, let us say it is not really in the picture invariably what you are going to do is instead of that resistance it is going to be another cascode with p-moses right. So, it is just going to double up, alright. So, this is our cascode and over here when we did our analysis we had placed some capacitors. So, first of all this is g m 1 this is g m 2 and then we had placed some capacitors C 1 2 between 1 and 2 and then there was C 2 and then there was C 3. This is this is all that was there; this is what was there. And each of these devices g m 1 also has r ds 1 and g m 2 also has r ds 2, this was our system that we had analyzed earlier and when we did the analysis we found that the system has some 0's and the 0's are going to be computed straight from the short circuit transconductors.

This system has a couple of poles and the poles are going to be computed straight from the output impedance, alright. Then we had done an analysis of the output impedance of the circuit and doing the analysis of the output impedance is not very hard because this is going to be at ground. So, C 2 and C 1 2 will appear together and then you know you it is just an application of one of our old formula, looking in from the output you see the impedance at the source of MOSFET number 2 times the intrinsic gain of MOSFET number 2 plus they impedance at the source of MOSFET number 2 plus r ds 2, this whole thing appears in parallel with C 3, right.

So, you do all of this computation it is not a very difficult computation, and then we arrived at an expression and I am not going to do the computation again because we have already done it. I am just going to write the denominator polynomial that we arrived at and the denominator polynomial that we arrived at purely based on the output impedance calculation looked like this. So, it was 1 plus s times something plus s squared times something, alright.

So, this is what it was. This is something in the numerator which we are not interested in right now. Right now, we are going to forget about the numerator we are just going to assume that it is a plain vanilla two-pole system and we are going to learn how to deal with these two-poles. In fact, invariably what is going to happen is that the 0's are going to be at much higher frequencies than the poles if the system had to go unstable it would go unstable even before the frequency hits the 0 frequencies.

So, let us not worry about the 0's for now, We will also learn how to tackle the 0's a little later. Sometimes the 0's are going to be the right half plane 0's are going to poles as problems, right. So, we are going to learn how to tackle them separately right that is a separate exercise, for now forget about the 0's.

Now, in this situation we have to make sure that the ratio of the two-poles is the set number that we had arrived at before based on the bosses specifications. Now, if that is the case if the ratio of the two poles is indeed so large then what, how will you be able to factorize this denominator polynomial, right. This denominator polynomial is nothing, but 1 plus s by omega 1 times 1 plus s by omega 2 or in other words it is nothing, but 1 plus s times 1 by omega 1 plus 1 by omega 2 plus s squared by omega 1 omega 2. This is what it was, this is the denominator polynomial.

Now, if this is the denominator polynomial and I am telling you in advance that omega 2 and omega 1 omega 2 is much much larger than omega 1, larger than omega 1 by a factor that we have computed of the order of 1000, alright. In such a scenario what can you tell about this polynomial or what can you tell about omega 1 and omega 2? You can see that omega 1 is going to be 1 by omega 1 is just going to be this coefficient because 1 by omega 2 is much smaller than 1 by omega 1, right.

So, we know in advance the value of the ratio, right. You can used you can see for yourself that this value of the ratio is some large quantity if it is not large then you are probably doing something wrong right or you are probably dealing with a phase margin that is not sufficient, correct. So, immediately you can tell that 1 by omega 2 is not really a concern, in which case 1 by omega 1 is nothing, but what you have got in coefficient number 2, done, right. So, we do not really know how to factorize this analytically, but we know how to factorize this how to arrive at the two poles by inspection looking at the quadratic equation, right.

So, my 1 by omega 1 is nothing, but what I have got and if I know 1 by omega 1 then 1 by omega 2 is a no brainer because the product of omega 1 and omega 2 is 1 by of the third coefficient. So, omega 2 is also a no brainer and then you have to set omega 2 by omega 1 equal to whatever that gigantic number you had. So, let us just work it out over here, let us do the algebra omega 1 over here is.

So, 1 by omega 1 is so much, omega 1 omega 2 is 1 by of this which means omega 2 this is 1 by omega 1 omega 2 sorry, this is omega 2 times omega 1 omega 2 times omega 1 times 1 by omega 1. So, this happens to be your omega 2 and 1 by omega 1 is just the numerator part of this which means that omega 2 by omega 1 is nothing, but the square of the numerator part, fine? This is what it is, alright. So, omega 2 by omega 1 is so much C 1 2 plus C 2 plus C 3 times r ds 1 plus C 3 r ds 2 plus times 1 plus g m 2 r ds 1 the whole squared right this whole thing squared divided by whatever you have in this expression, fine.

Now, this ratio has to be set to a certain predetermined constant a large number, how will you do the engineering over here? What will you tweak that will get you the maximum benefit? Which one of these will you tweak to get the maximum mileage? If you look at it carefully which first of all you are not allowed to tweak the values of g m 1 g m 2 r ds 1 r ds 2 because this is what gives you the DC gain. The DC gain is all important, if you change the DC gain then guess what your set value is no longer set right that also changes alright that is a function of A naught that set value.

So, you cannot now tamper with the DC gain, the DC gain is fixed right; that means, g m 1, g m 2, r ds 1, r ds 2 you are not going to touch the only things that you are allowed to touch are C 1 2 C 2 and C 3 right and by touching what I mean is that you can increase these values you cannot really decrease the values, because the MOSFET is coming with a certain amount of parasitic capacitance C 1 2, C 2, C 3 depend on that you cannot really dial those parasitics down right beyond a certain point. I am sure you would have already dialed it down as far as possible right you cannot dial it down any further, right.

And, now, your only option is to increase these capacitor values. Now, if I allow you to increase the values of C 1 2, C 2 and C 3 which 1 will you work with to give you the maximum value for money? Look at your expression carefully, right if you look at your expression carefully you will see that the maximum value for money is going to come from C 3 and the reason why the value for money is going to come is because it is attached to a factor which is large.

So, C 3 is attached to a factor that is large. Therefore, a slight increase in C 3, remember C 3 is there in the denominator also. So, if you take C 3 common from the numerator you get C 3 squared in the denominator you have only C 3, right. So, it is going to be roughly proportional to C 3, alright. So, C 3 over here is the dominant capacitor, all these terms have C 3 in them C 1 2 and C 2 are can be held to be small compared to C 3 you add extra C 3 on top right from the outside you add extra C 3 and therefore, C 1 2, C 2 are all parasitic capacitances. If you ignore them then the numerator term is proportional to C 3 squared. The denominator is proportional to C 3 which means overall this ratio is proportional to C 3 which means you increase C 3 you can increase the value of the ratio that you desire, fine.

So, this is going to be the compensation technique. The compensation technique is going to be to add capacitance out here right at the load. At the load we just dial in more and more and more capacitance till the requirement of phase margin is met is satisfied, alright. So, this cascode amplifier requires extra load capacitance and this is the way you are going to compensate it. This kind of compensation is called dominant pole compensation.

So, this particular style of compensation where I add extra capacitance at the load it happens to be called dominant pole compensation. Although all compensation is dominant pole compensation, right. There are some techniques where you add a 0 we are not going to discuss those in this course it is out of bounds, right. All the other polar type compensation techniques are dominant pole compensation. However, in circuits this specific way of adding extra load capacitance till your phase margin requirement is met this is called dominant pole compensation.

So, C 3 is the load, right you can add as much load capacitance as you want outside the chip, outside the circuit and that is called dominant pole compensation, that we will meet the requirements of phase margin and now you can use your amplifier in a feedback loop fine, alright. So, this is the cascode circuit. Now, let us look at the cascaded circuit the cascaded circuit we had actually halve discussed right I do not know how far you followed, but we will do it again.

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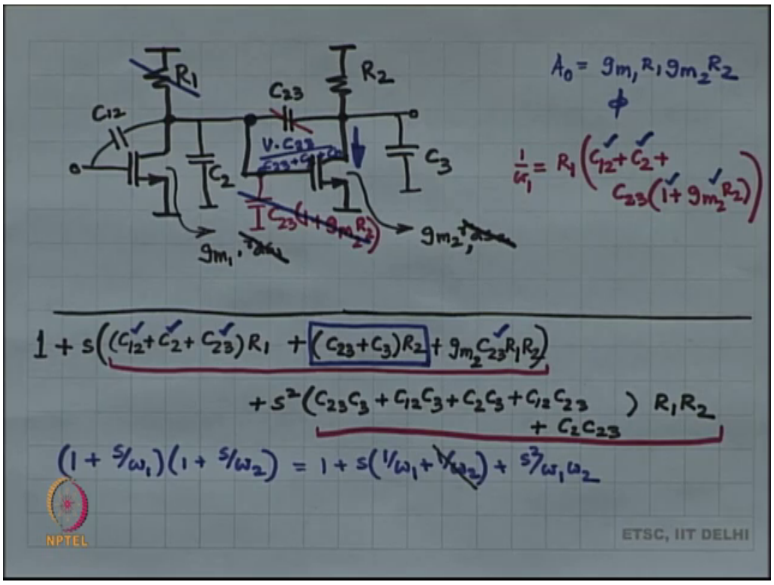

So, as far as my notes are going when we discussed the cascaded amplifier this is what we did and my label seem to be as follows; I called this C 1 2 and this was called as C 2, this was called C 2 3 and this was called C 3 R 1, R 2 and my devices had g m 1, r ds 1 g m 2, r ds 2. This was the circuit that we had analyzed, alright and in this particular circuit we did not really write out the complete analysis right I said that the complete analysis will take a lot of time and it is not really what the while; however, I am just going to write out the expressions, right. As far as you should remember you should remember that the this particular circuit had two right half plane 0's and the 0's came explicitly from the short circuit current analysis short circuit transconductance analysis, do the short circuit transconductance analysis you will get the 0's.

Then, the poles had nothing to do with the short circuit analysis the poles had everything to do with the output impedance. So, if I look at the output impedance I will arrive at a two pole, a second order denominator polynomial and the denominator polynomial looked like this. So, what I am going to do is I am going to make 1 simplification I am going to lump r ds 1 into R 1, just so that my expression is simpler. I am going to lump r ds 2 into R 2. So, R 2 is the old R 2 in shunt with r ds 2 R 1 is the old R 1 in shunt with r ds 1.

Let us just put that in over there and then I am going to write out the denominator polynomial, the denominator polynomial looks like this it is 1 plus s times plus s squared times, and let us start with the s, that is the s term and s squared term looked like this, alright. So, this was my gigantic expression and in this R 1 is the old R 1 in shunt with r ds 1 R 2 is the old R 2 in shunt with r ds 2. So, this is implicit, alright.

Now, once again your boss asked for a certain phase margin and a certain gain open loop gain and a certain phase margin. The open loop gain is nothing, but g m 1 times R 1 times g m 2 times R 2 and it is set. You are not going to tamper with g m 1 g m 2 R 1 and R 2, these are fixed quantities they based on your DC design. Now, based on this value of A naught you arrive at and based on the phase margin that you need you arrive at a certain ratio of omega 1 and omega 2. Omega 2 by omega 1 has to be some large number which is a function of A naught and phi, the phase margin.

Now, after arriving at this large number you are going to say that therefore, my denominator polynomial that I have this is the exact expression by the way, this does not include any of the millers approximation and so on and so forth. This is the exact expression you just take the output impedance, find out, work out the output impedance of the circuit, precise expression there is no nothing in this no hand waving alright. And in this expression therefore, one possibility would be to factorize this somehow using you know your quadratic solutions right minus b plus minus square root of b squared minus 4 a c by 2 a you can do all of that, but doing all of that is not going to be worth your time. Much easier understanding would be that omega 2 by omega 1 is already a large number which basically means that if my denominator polynomial look like this.

If this was my denominator polynomial then this would look like 1 plus s into 1 by omega 1 plus 1 by omega 2 plus s squared by omega 1 omega 2. I am doing this repeatedly, so that you get the hang of it right some of you will find this boring because it is the same thing that I am repeating over and over again, but you know this is the way you learn through repetition. So, over here 1 by omega 1 and 1 by omega 2, the sum of these two is this particular term, the middle term and omega 2 by omega 1 is a very large number which means that 1 by omega 2 is small compared to 1 by omega 1, the reason why is because omega 2 is much larger than omega 1 omega 2 much larger than omega 1 means that the second term 1 by omega 2 term is very small. So, you ignore it.

As soon as you ignore it immediately you know the value of 1 by omega 1. So, the value of 1 by omega 1 is nothing, but the first term, sorry, the second coefficient right and as soon as you know 1 by omega 1 omega 2 is also not very hard to find out because the product of omega 1 and omega 2 is 1 by of the second term alright. So, this is the plan; now we are going to execute the plan and we are going to find out the ratio of omega 2 and omega 1. So, omega 2 by omega 1 sorry, omega 1 omega 2 is 1 by of this, right. So, omega 2 is 1 by of this times omega 1 by omega 1.

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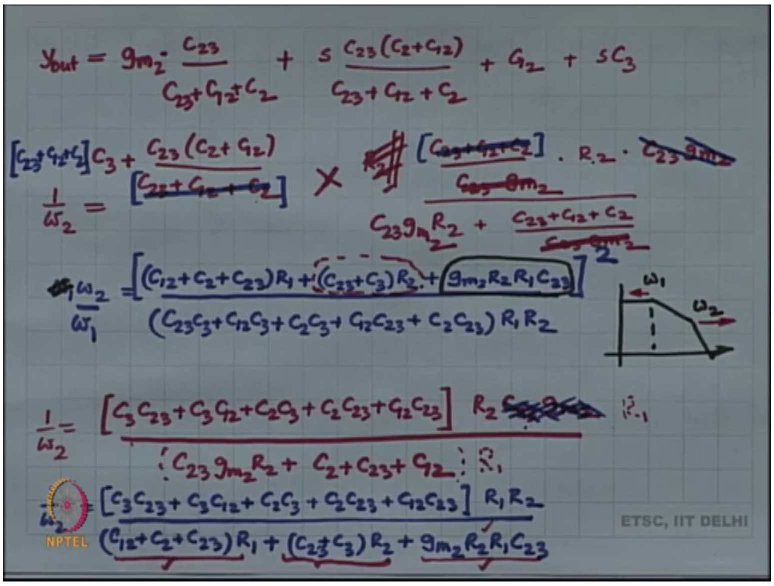

So, this is omega 1 omega 2, but I am not interested in omega 1 omega 2, I am interested in omega 2. Omega 2 is omega 1 omega 2 times 1 by omega 1 and 1 by omega 1 is nothing, but this second coefficient. So, this is omega 2 we are not even interested in omega 2, we want to find out omega 2 by omega 1 which is nothing, but omega 2 times 1 by omega 1, fine. So, an expression similar to what we had found for the cascode.

So, in the cascode we had found an expression that looked like this, and in the cascade we have found an expression which is similar sum square divided by something else. Now, what do you want to do with this? What would you want to do with this? This has to be equated to some large number. Just by the way we had also done the Miller's approximation, right what result does that give? Does that also give something similar? Because this is the time to do it right, this is this is the junction in time when you should also think of the Miller's approximation.

When we did the Miller's approximation what we thought of was that at the first pole at the lower pole frequency, C 2 3 can be broken up as C 2 3 times C 2 3 times 1 plus g m 2

R 2 over here. So, this was Miller, and therefore, as far as my first poll is concerned as far as omega 1 is concerned, omega 1 is less than omega 2, as far as omega 1 is concerned 1 by omega 1 would be equal to R 1 times C 1 2 plus C 2 plus C 2 3 times 1 plus g m 2 R 2.

Now, we also found some omega 1 right 1 by omega 1 for us was this middle coefficient, how far are we from I mean between Miller and this Miller this middle coefficient what is the difference. So, I have got R 1 C 1 2, I have got R 1 C 2. This term is not there over here C 2 3 times R 1 does not really show up over here, but I have got C C 2 3 times R 1 is also C 2 3 times R 1 times g m 2 times R 2 is over here. So, this particular one and then C 2 3 times R 1 is also over here. So, all of these terms are matching.

This stuff is just the little bit that is missing from the actual value of 1 by omega 1 that I estimated with Miller. Now, when I did the Miller estimation right you can immediately see that this g m 2 are 2 times C 2 3 times R 1 is the largest , the same thing is there over here, right. So, if you really think about it these other things are not that important. These this little inefficiency of miller is not that important you can keep it aside. Yes, this is the accurate version, but the miller version is giving you very close to the actual result it is very close, alright. Now, this is 1 by omega 1.

When we did Miller we also had to do, we also did in a different way we found out the second pole, right. The second pole was not so, straightforward; when we talked about the second pole we said that all these capacitors are now going to behave as short circuits and there is going to be some ratio of C 2 3 and C 2 C 1 2 plus C 2 that voltage is going to appear here and that is going to pull some current through the MOSFET. So, you are actually going to see an effect of g m 2 coming into the second pole, alright. So, this is what we had suggested. So, this value of 1 by omega 1 is actually very close, right, this is the only discrepancy between Miller and the exact expression.

Now, let us try to find out the value of omega 2, right. So, when we worked out omega 2 with Miller, when we worked it out with Miller we said C 2 3 is a short circuit, C 2 plus C 1 2 is also a short circuit, the. So, now, we forget about this was only for the forward direction. When these are all short circuits R 1 is not very interesting anymore R 1 r ds 1, all of these go away, g m 1 is anywhere dead, it is not relevant.

So, if I apply a voltage v over here then the voltage over here is going to be v times C 2 3 divided by C 2 3 plus C 2 plus C 1 2 it is a voltage division between capacitors. What I have written is C 2 3 plus C 2 plus C 1, that is the voltage appear. Now, if that is the voltage appear then g m times that voltage is drawn through the MOSFET and when I draw g m times that voltage, when I draw that much current then that adds on to the output conductance.

So, the output conductance is therefore. So, this is only for the second pole calculation not anything else. So, only for the second pole calculation the output conductance is g m 2 times, as far as this MOSFET is concerned plus we have the series path between C 2 3 and C 2 C 1, right this path is also there. So, that is also going to create output conductance or in other words this voltage is v this voltage is so much v times C 2 3 there is going to be some current through this capacitor also.

So, series combination of C 2 3 and C 2 plus C 1 2 then, there is current through R 2 and there is current through C 3 or in other words. I have got a capacitor of value C 3 plus C 2 3 into C 2 plus C 1 2 by C 2 3 plus C 1 2 plus C 2 this is the capacitor and I have got a resistor of value R 2 in shunt with so much and therefore, 1 by omega 2 is this R C time constant we are just trying to compute 1 by omega 2 omega 2. The pole frequency as far as the pole is concerned you are looking at 1 resistance which is the parallel combination of R 2 and this and 1 capacitance which is a shunt combination of C 3 and this.

So, the pole frequency is just 1 by R C, 1 by the pole frequency is just R C, fine. So, in other words 1 by omega 2 that we got using this technique would be the product of these two. Now, to do the product of these two I think it would be better to combine these two parallel quantities R 2 and so much. Then you have to multiply the C with the R, this is the big multiplication 1 the first thing that you are going to do is you are going to multiply by C 2 3 g m 2 numerator, denominator. The next thing you are going to do is you are going to cancel out this factor.

So, you have to cancel these 2 out and multiply it over here, fine. That will leave you with a gigantic result, the result is going to be, fine. So, this could be the result of our so called shortcut analysis, the analysis which did not require the actual calculation of the output impedance, right. I mean this particular expression was something that I wrote from our actual calculation very hard work, right. This particular full-fledged expression was something that we wrote from hard work correct, but this expression is something that you can arrive at much faster right if you know the values of these individual capacitors then this particular expression is going to come out much faster do not worry about it. And in such a scenario what is the difference between our original accurate expression and the expression that we have got now.

So, 1 by omega 2 that we have computed accurately is actually this blue denominator 1 by omega 2 is the blue denominator divided by what is inside the square. The numerator part that is inside the square, so, the blue denominator divided by the numerator part inside the square that is the accurate value of 1 by omega 2. So, 1 by omega 2 accurate let me write it down. So, this is the calculation from our quick analysis and this is the accurate expression. I think we have made a mistake, the dimensions are not right in the red one. Red one the numerator polynomial is C squared actually C cube R and g m is nothing. So, C cube divided by C. So, this is coming out to be C squared so, that is wrong, it should be time 1 by omega 2 should be should look like R times C not C squared. So, let us check.

So, if dimensions are wrong you immediately stop. So, we checked our dimensions over here, they are not correct, we go back. Where did we make the mistake? We had found Y out Y out was g m 2 times ratio of two capacitors fine, plus s times capacitor which is fine, plus G 2 fine, s times C 3 this was fine. From here I took out the capacitor portions C 3 plus C 2 3 C 2 plus C 1 2 divided by this, this is 1 capacitor right, this is the capacitor and the resistor is R 2 in parallel with g m 2 C 2 3 by C 2 3 C 1 2 plus C 2, R 2 in parallel with g m 2 C 2 3 and in the numerator C 2 3 C 1 2 C 2.

So, resistor resistor product divided by resistor resistor sum, then we multiplied numerator denominator by C 2 3 g m 2 this should not have come. So, that is the mistake we made there is an extra C 2 3 g m 2 over there C 2 3 g m 2 just cancelled out, we wrote it all over again. So, C 2 3 g m 2 yeah, this is now correct, fine. So, now, it is R CC by C which is R C, great. Now, let me just write this down. So, from the accurate expression I was writing out 1 by omega 2 and over here we are going to have. So, this is what it is, alright.

Now, what you are going to see is that in this denominator expression in the in this 1 by omega 2 expression once again C 3 C 2 3 matches C 3 C 1 2 matches, C 2 C 3 matches, C 2 C 2 3 matches, C 1 2 C 2 3 matches, R 2 matches, but there is an extra R 1 in the accurate expression, all right. So, you can think of multiplying this by an R 1 and multiplying the denominator also by an R 1, just so that the numerator now completely matches up. Now, if we look at the denominator I have got C 2 3 g m 2 R 2 times R 1 which is matching, then I have got C 2 R 1 C 2 3 R 1 C 1 2 R 1. So, this entire term is matching this entire term is matching; the only term that is left out is what I had over here.

So, there is a slight discrepancy right, it is the same discrepancy even as far as 1 by omega 1 was concerned we had a little discrepancy over here, 1 by omega 2 we have the same discrepancy over here otherwise they look pretty much the same whether you do the exact analysis or you use the Miller's approximation right you get more or less the same result.

So, then what do you do once you have this ready, what do you do? Supposing you work with any one of these two expressions write omega 2 by omega 1 is over here what are you going to do as far as engineering is concerned? You cannot touch g m 2 R 1 or 2 g m 1 these are not things that you can touch you can only touch the capacitors and if you look at the capacitors the 1 that is going to give you maximum value is this term. Because, it is it has a multiplication with a large number and it is coming inside a whole square.

If you look at it very carefully omega 2 if I increase the value of frequency of C 2 3 then omega 2 is actually going to decrease omega 1 is going to increase. I am sorry, 1 by omega 2 is going to decrease which means that omega 2 is actually going to increase, omega 1 is going to 1 by omega 1 is going to decrease I am sorry; 1 by omega 1 is going to increase omega 1 is going to decrease. In other words when I tamper with C 2 3 this was my two-pole system, omega 1 omega 2 omega 1 is going to decrease in frequency, omega 2 is going to increase in frequency and the net effect is going to be that the ratio of omega 2 and omega 1 is going to be managed, alright.

So, this is where we are going to stop for today. We have been talking this exercise, this entire engineering exercise is called compensation, right. So, we have been we try to do two different circuits the cascode as well as the cascaded amplifiers and in the next class, we are going to try to incorporate all of these.

Thank you.