

**Analog Electronic Circuits**  
**Prof. Shouribrata Chatterjee**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Delhi**

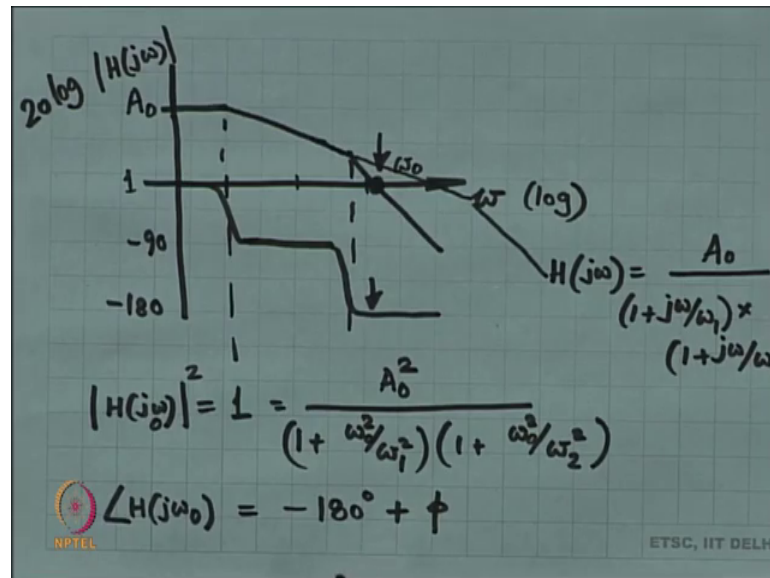
**Lecture – 31**  
**Diversion continued: Two pole systems**

Welcome, to Analog Electronics and today's lecture – 31. We were actually on a diversion on a sidetrack, we were talking about two pole systems and how the poles of a two pole system should behave such that I get a certain phase margin. And, why do I need phase margin, because in the eventuality the system is going to be used in a feedback loop, in a unity gain feedback loop when it is used in the unity gain feedback loop the system should be stable.

So, based on these stability criteria we said that let us make sure that it has a certain phase margin. Based on the phase margin requirement we were analyzing a two pole system because what we have seen is that all our systems have two or more poles, ok. Two poles is a kind of going to be the minimum possible number of poles that we are going to have in a two stage amplifier. Two stage amplifier could be a cascode amplifier could be a cascade of two amplifiers whatever you do you are going to end up with two poles.

So therefore, we analyzing a two pole system and in this two pole system we were trying to work out what is the arrangement of these two poles such that I get a certain amount of phase margin. So, what we had covered in the last class we had two conditions.

(Refer Slide Time: 01:57)



One condition was ok. So, let me just quickly redraw. So, this was my Bode plot I am going to plot mod of  $H$  of  $j$   $\omega$  on the y axis actually it is going to be log scale, because it is in dB. So,  $20 \log$  of mod of  $H$  of  $j$   $\omega$  is what we are going to plot and  $\omega$  is also in the log scale, right which means that they are going to go in steps of 10, 1, 10, 100 and so on. And then we found that if I have a two pole system then at pole 1 I start going down at 20 dB per decade and then at pole 2, I get another 20 dB per decade, right.

So, net minus 40 dB per decade something like this is what the plot is going to be and similarly at pole 1, I get minus 90 degrees and at pole 2 I get another minus 90 degrees right and then came the definition of phase margin the definition of phase margin was that at the unity gain frequency, at this point what is the phase, how much is this phase above minus 180 degrees, ok. So, my first question is what is  $\omega$  naught the unity gain frequency and for that. So,  $H$  of  $j$   $\omega$  was equal to some constant divided by  $1 + \omega/\omega_1$  and  $\omega/\omega_2$ , where the two poles of the system this was my setup and then the first thing that we wanted was mod of  $H$  of  $j$   $\omega$ .

So, at  $\omega$  naught mod of  $H$  of  $j$   $\omega$  is equal to 1. So, this was number one mod of  $H$  of  $j$   $\omega$  naught is equal to square root of  $A$  naught squared square root of 1 is 1. So, we do not have to write that square root of ok. So, this was my setup which automatically means that I am sorry, which automatically means that  $1 + \omega$  naught squared by

$\omega_1$  squared times 1 plus  $\omega_0$  squared by  $\omega_2$  squared is equal to  $A_0$  squared this was my setup number one this is the definition of  $\omega_0$ , and then we said that at the frequency  $\omega_0$  angle of  $H$  of  $j\omega$  is equal to minus 180 degrees plus a certain phase margin ok. This was my phase margin requirement and then what we did was we took cosine of both sides.

Student:  $H$ .

Which one is square root?

Student: (Refer Time: 05:49).

But one square is one fine thank you. So, you put the square over there because 1 square is 1. So, it does not really matter. The next criterion was that angle of  $H$  of  $j\omega_0$  has to be  $\phi$  degrees more than minus 180 degrees to get a certain phase margin and then what we did was we took the cosine of both sides. And, so, angle of  $H$  of  $j\omega_0$  is minus  $\tan^{-1}$  of  $\omega_0$  by  $\omega_1$  minus  $\tan^{-1}$  of  $\omega_0$  by  $\omega_2$  right that is what it was and then we took cosine of all of that. Cosine of minus 180 degrees plus  $\phi$  is equal to minus  $\cos \phi$  and all of this we had done in the last class, right. We have done a lot of work in the last class.

(Refer Slide Time: 06:51)

$$\cos A = \frac{\omega_1}{\sqrt{\omega_0^2 + \omega_1^2}}$$

$$\cos B = \frac{\omega_2}{\sqrt{\omega_0^2 + \omega_2^2}}$$

$$\sin A = \frac{\omega_0}{\sqrt{\omega_0^2 + \omega_1^2}}$$

$$\sin B = \frac{\omega_0}{\sqrt{\omega_0^2 + \omega_2^2}}$$

$$-\cos \phi = \frac{\tan^{-1} \frac{\omega_0}{\omega_1} \tan^{-1} \frac{\omega_0}{\omega_2}}{\sqrt{\frac{\omega_0^2 + \omega_1^2}{\omega_1^2} \frac{\omega_0^2 + \omega_2^2}{\omega_2^2}}} - \frac{\omega_0^2}{\omega_1 \omega_2 \sqrt{\frac{\omega_0^2 + \omega_1^2}{\omega_1^2} \frac{\omega_0^2 + \omega_2^2}{\omega_2^2}}}$$

$$= \frac{1}{A_0} \left( 1 - \frac{\omega_0^2}{\omega_1 \omega_2} \right)$$

So, we had done minus  $\cos \phi$  is equal to  $\cos$  of minus  $\tan^{-1}$  something minus  $\tan^{-1}$  something else then we did cosine of two different cosine of  $a + b$  is equal to

cos A cos B minus sin A sin B then I worked out each cos A and cos B cos sin A and sin B then I plugged in everything and this is the simplified expression that I got, right.

(Refer Slide Time: 07:27)

$$1 + A_0 \cos \phi = \frac{\omega_0^2}{\omega_1 \omega_2}$$

$\omega_0 \rightarrow$  Unity gain bandwidth

$$1 + A_0/2 = \frac{\omega_0^2}{\omega_1 \omega_2} \quad \text{Example, } \phi = 60^\circ$$

$$\omega_0^2 = \omega_1 \omega_2 (1 + A_0 \cos \phi)$$

$$\left(1 + \frac{\omega_0^2}{\omega_1^2}\right) \left(1 + \frac{\omega_0^2}{\omega_2^2}\right) = A_0^2$$

$$\left(1 + \frac{\omega_0^2}{\omega_1^2} (1 + A_0 \cos \phi)\right) \left(1 + \frac{\omega_0^2}{\omega_2^2} (1 + A_0 \cos \phi)\right) = A_0^2$$

$$(1 + \lambda \alpha) (1 + \frac{1}{\lambda} \alpha) = A_0^2$$

And, at the very end I had found that 1 plus A naught cos phi, right this can be simplified and we got this, right. Let us leave this out this was an example ok. So, just plug in phi equal to 60 degrees. Now, this is the relationship that we arrived at in the last class, ok. So, I am going to continue from here this is not the end of the story this is a little more twist. Omega naught squared is still kind of an unknown right what is the value of omega naught squared and the value of omega naught squared is determined from this result, ok. 1 plus omega naught squared by omega 1 squared times 1 plus omega naught squared by omega 2 squared is equal to A naught squared, ok.

So, that actually gives you the value of omega naught squared, right. And that we actually have to plug in over here this value of omega naught squared. So, we right now what we are going to do is one possibility is that we are going to solve for omega naught squared and plug it in over there, alright. The other possibility is basically what we have to do is we have to eliminate omega naught squared from this set up I do not like omega naught squared coming into the way, ok. I want a relationship between omega 1 and omega 2 and that is it there should not be any omega naught squared in between to interfere, ok.

So, I have two relationships; one is this, the other is this we have to eliminate omega naught squared. Now, one way is that you find omega naught squared from this equation plug it in over here, the other ways you solve this and plug it in over there right whichever way you do it you will end up with the same result, come on you have to end up with the same result. So, let us let us do this. This looks easier, right omega 1 omega 2 times 1 plus A naught by 2 is equal to omega naught squared. I am sorry not this, this is just an example: this was the actual result, right. I am just going to plug it in over here and what do I get and this is actually a relationship between omega 1 and omega 2.

Let us simplify matters a little bit, let us call this one plus A naught cos phi something let us simplify it a little bit. What do you want to call it? Let us call this as let us call it as beta, or if you do not like beta let us call it as alpha, fine in which case I can rewrite this I will I am going to make another definition I am going to call omega 2 by omega 1 let us call this as lambda, ok.

So, now I am going to rewrite my expression 1 plus lambda times alpha times 1 plus 1 by lambda times alpha is equal to A naught squared, ok. This looks much simpler than before and what we are going to do is we are going to solve for lambda because lambda is the relationship between omega 2 and omega 1, it is omega 2 by omega 1 it is the ratio, right. It looks like you can solve for lambda from here. So, let us first multiply very quickly.

(Refer Slide Time: 12:43)

$$(1 + \alpha^2) + \alpha(\lambda + 1/\lambda) = A_0^2$$

$$\alpha \cdot \lambda^2 + \lambda(1 + \alpha^2 - A_0^2) + 1 = 0$$

$$\frac{\omega_2}{\omega_1} + \frac{\omega_1}{\omega_2} = \frac{A_0^2 - (1 + \alpha^2)}{\alpha} = \frac{A_0^2 - (A_0 \cos \phi + 1)^2}{1 + A_0 \cos \phi}$$

$$\lambda = \frac{A_0^2 - 1 - (A_0 \cos \phi + 1)^2}{1 + A_0 \cos \phi}$$

$\phi = 60^\circ$   
 $\lambda \approx 15k$

NPTEL ETSC, IIT DELHI

It is  $1 + \alpha^2 + \alpha \lambda + 1/\lambda$ , fine. And if you want to solve for  $\lambda$  I think what you have to do is you have to first take a not squared on this one side and then multiply the entire result by  $\lambda$ , and then solve a quadratic equation, ok.

So, what that boils down to is  $\alpha \lambda^2 + \lambda + 1 + \alpha^2 - A \cos^2 \phi + \alpha = 0$ . And you can make yet another simplification you can divide the entire equation up by  $\alpha$ . So, this goes away this becomes a 1 and this gets divided by  $\alpha$ ;  $\alpha$  is some constant. What is the  $\alpha$ ?  $1 + A \cos \phi$ , I know cause  $\phi$  I know  $A \cos \phi$  is a constant, predetermined constant.

So, this is my equation  $\lambda^2 + \lambda + 1 = 0$ . And this is a very straightforward equation to solve. It gives you two roots, right and the two roots are inverse of each other you remember this something about product of two roots sum of two roots of a quadratic equation. The product of these two roots is equal to 1, right both these coefficients are 1. So, the product of the two roots is going to be equal to 1 which means I am going to get I am going to solve for  $\lambda$  and  $1/\lambda$  at the same time, right.

After all what is  $\lambda$ ?  $\lambda$  is the ratio of  $\omega_2$  and  $\omega_1$  which frequency you choose as  $\omega_2$  which as  $\omega_1$  is up to you. So, both solution are equally good, right. If they are 1 by each other then they are equally good solutions, right. Once you choose 100 megahertz as  $\omega_2$ , 10 kilo hertz as  $\omega_1$  or you could choose 100 mega hertz as  $\omega_1$ , 10 kilo hertz is  $\omega_2$  it does not matter, you get equal and opposite roots for this quadratic equation, alright.

So, that is very good the other thing is what about the sum of the two roots you remember something about some of the two roots it is the middle coefficient  $B/2A$ , ok;  $B$  is this divided by  $A$  is one minus I think there was a minus sign right it was minus  $B/A$ .

Student: A (Refer Time: 16:08).

$A$  is 1, right. So, you are going to get the sum of two the two roots no it is minus  $B/A$ , not  $B/2A$ . It is minus  $B/A$ . So, it is just this negative of this is the sum of the two

roots. So, if I call the two roots as  $\lambda$  and  $1/\lambda$  which they are, ok. One of the two roots is  $\omega_2$  by  $\omega_1$  the other root is  $\omega_1$  by  $\omega_2$ . So, the sum of those two roots is actually negative of this middle coefficient or in other words and of course, you can expand  $\alpha$  over here, ok. Sorry, something like this, ok. You can expand you can put plug in the value of  $\alpha$  over there. So, this is the sum of the two roots, fine.

Now, comes a big engineering question a lot of times it is not worth the while solving even a quadratic, ok. A quadratic means you have to enter things in a calculator and then number crunch and then solve lot of times even that is hard work and we engineers do not like hard work, ok. We will always find an easy way to solve something much harder.

So, even this quadratic has a simple solution and the solution lies in this picture, ok. Remember  $\omega$  is in the log scale that is the clue  $\omega$  is in the log scale. What that why I am giving this as the clue is that if I want this phase to be more than minus 180 degrees then. That means the pole the second pole has to come after  $\omega$  naught hits 0 dB or 1, right. It has to come after the magnitude hits one alright, not before or even if it comes before it should be just before which means that the ratio of these two has to be at least as big as  $A$  naught. Remember we started from  $A$  naught if not more, ok.

So, the order of my answer  $\omega_2$  by  $\omega_1$ , the order of magnitude of that answer is as big as  $A$  naught and we are making an op-amp an amplifier which has very large gain of the order of 1000, 10000, 100000  $A$  naught is that large gain, right 1000. The value of  $A$  naught is large which means that  $\omega_2$  by  $\omega_1$  has to be of that order of magnitude 1000 or more, right. The more the better phase margin you are going to get if you have less than  $A$  naught as your ratio of two poles then even from the Bode plot it is obvious that you are not going to get any phase margin, fine.

So, that means, that the answer I am looking for is large of the order of  $A$  naught which means  $1/\text{answer}$  is nothing. And therefore, the sum of the two roots is just equal to one of the roots, alright. So, this is a huge engineering approximation that you can make and it is very straightforward to make this engineering approximation you should in fact make it, alright. You can say that if  $\omega_2$  is larger than  $\omega_1$  then  $\omega_1$  is going to be much much smaller than  $\omega_2$  and it is not worth your while talking about it at all which means that the sum of the roots is just equal to this, ok.

So, we do not have to solve this quadratic at all. You can solve it if you have a lot of spare time on your hands you can go ahead and solve it, it is not going to get you any intuition, right. The intuition is that the sum of the two roots is almost equal to negative of the middle coefficient because the root one of the two roots is very large.

Now, this is an approximation we are going to use over and over again. In a quadratic equation the sum of the two roots is almost equal to the negative of the middle coefficient if one of the two roots is very large compared to the other, all right and you see look at our picture; our picture tells you that that is going to happen one of the two roots is going to be much bigger than the other and if that happens then automatically I have my answer,  $\omega_2$  by  $\omega_1$   $\lambda$  is equal to minus of this or basically it is equal to this, fine; and now all you have to do is someone.

So, your let us say your company boss your company manager tells you that I need an op-amp of a certain phase margin  $\phi$  and a certain gain  $A_{naught}$   $b$   $c$  gain  $A_{naught}$ , right. Immediately you can set up your works and you can work out what is the ratio therefore, of the two poles that you are going to have in this op-amp, fine. So, this is something fundamental for any two pole system right this is the diversion that we wanted to take. So, we are going to use this result, ok.

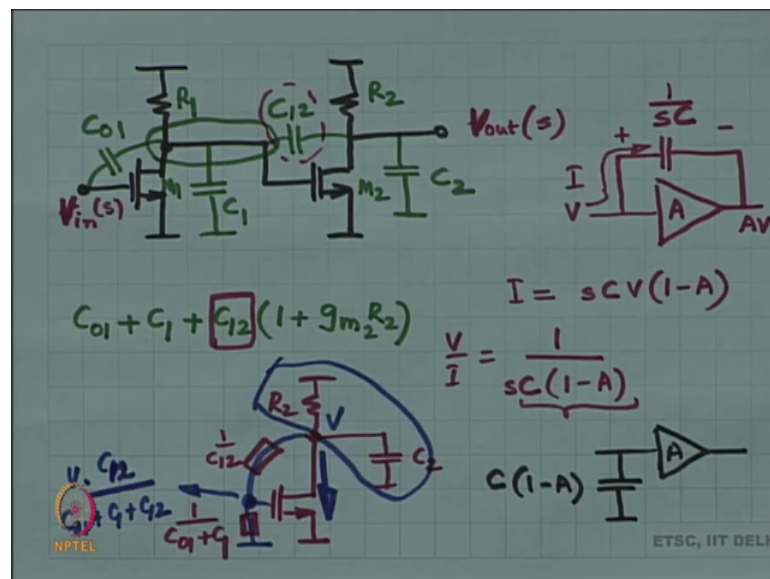
Let us check a few numbers, let us say cause  $\phi$  is half  $\phi$  60 degrees. 60 degrees happens to be a very popular acceptable value of phase margin among many industry managers right they are going to say that give us phase margin of 60 degrees. So, immediately you plug in  $\cos \phi$  of half, what you get? Let us even say  $A_{naught}$  is 1000, ok.  $A_{naught}$  is 1000. So, this is 1 million minus 1 almost equal to 1 million minus 500 plus 1 squared, 500 1 squared 500 plus 1 squared and one 500 1 squared you can you know this is 1 million after alright, 20050000.

So, 1 million minus 250000 is 70050000 divided by 1 plus 500. So, half  $k$  750  $k$  by half  $k$  which gives you 15000  $\lambda$  is equal to 1500, ok. We started with a gain  $b$   $c$  gain of 1000,  $A_{naught}$  equal to 1000, right. We got 1.5 times  $A_{naught}$  as the required ratio of two poles, right. Do you see that we are getting a large number for  $\lambda$  right; you have to get a large number for  $\lambda$  because there is no other way, if you have a small  $\lambda$  then immediately you will get no phase margin at all, alright.



So, now you can actually get back to what we were doing. So, we were working on two fronts we had designed a nice cascaded amplifier, we had also designed a cascaded amplifier right, we had analyzed right, we came up with a denominator polynomial which we said we do not want to factorize, because we do not know how to solve the quadratic or we do not want to solve the quadratic. Once again you see let us not solve these quadratics ok, we are going to try to use this kind of a technique as far as possible, alright.

(Refer Slide Time: 27:19)



Now, what had happened? I had designed this nice amplifier we had some capacitances in place, ok. We had some names for these capacitors this was  $C_0$  one this was  $C_1$ , this was  $C_{12}$  and this was  $C_2$  as far as I remember, is this correct or I am making some mistake? Whatever it is ok, this is probably what it was I had applied an input I had got some output sorry I had applied capital V in of s I had obtained some output capital V sub out of s right and then we did the analysis.

Our analysis showed that the system has two right half plane poles, sorry two right half plane zeros which arise, because of the short circuit current those zeros are because of  $C_{01}$  and  $C_{12}$  and it has two poles. Because of the output impedance only because of the output impedance it has two poles, ok. This is what we had found.

This is where we had stopped right, we had written some massively complicated expression and then we had I had stalled over there I said that; let us not bother with

trying to solve this any further, remember this was the case, right. Thus, we adopted the second strategy we also said that we also looked at Miller's theorem that was our second strategy, ok. Under Miller's theorem what we had said was that this  $C_{12}$  was playing spoilsport as far as the two poles are concerned, ok. If  $C_{12}$  was not there calculating a two poles was very easy if  $C_{12}$  was not there calculating the two poles was just these two the  $R_{21}$  by  $R_2 C_2$  is 1 time constant the other time constant was  $1$  by  $R_1 C_1$  plus  $C_{01}$ , ok.

So, those two poles came out in a very straightforward way because  $C_{12}$  is not there. Now, as soon as  $C_{12}$  is thrown into the picture things go haywire my analysis is very difficult and Miller's theorem was proposed and I said that Miller's theorem is not really a theorem as such, right it has a lot of fallacies it should not be called a theorem at all. So, Miller's theorem said that if I have an amplifier  $A$  across the amplifier there is a capacitor  $C$  if I apply a voltage  $V$  over here, then the voltage at the output is  $A$  times  $V$ , ok. If the voltage at the output is  $A$  times  $V$  then the voltage across the capacitor is  $V$  minus  $A$  times  $V$  that is the net voltage across the capacitor. And therefore, the current in the capacitor is  $C dv$  by  $dt$  ok, where  $V$  is one minus  $A$  times capital  $V$ , fine.

So, the current in the capacitor is really. So, the current going into the system so, if I only current going into the system is that into the capacitor. So, the current going into the system is therefore, the capacitor is a conductance of value  $s$  times  $C$ . On this side it is  $V$  on the other side it is  $A$  times  $V$ . So,  $V$  minus  $AV$  is the net voltage drop across the capacitor conductance of  $s C$  times  $V$  minus  $AV$ , fine. This was the current and therefore, the input impedance I applied  $V$ , I got a current  $I$ . Therefore, the impedance is  $V$  by  $I$  which means that the impedance is  $1$  by  $s C$   $1$  minus  $A$  which means it is as if I have a capacitance of value  $C$  times  $1$  minus  $A$ .

So, Miller's theorem said that instead of looking at this like an amplifier and a capacitor in shunt we can maybe look at it as just an amplifier and a capacitor to ground of value  $C$  times  $1$  minus  $A$ , ok, this was Miller's theorem. However, this is not really a theorem, right or you can still call it a theorem, but beware that whenever you are going to apply this theorem you are going to make mistakes, there is nothing wrong about this so far, right. Whenever you want to apply it you are going to make a big mistake you are going to apply thinking that  $A$  is a constant, right. Unfortunately it so happens that whenever

you apply  $A$  is never really going to be a constant.  $A$  is also going to be a function of frequency, and therefore, this entire thing is going to become very complicated.

However, as an approximation as an engineering approximation for intuition building this is a very convenient tool, ok. You can say that this second stage of the amplifier this is my first stage of the amplifier the second stage of an amplifier has a gain of minus  $g_m$  times  $R_2$ , ok. This has a trans conductance of  $g_m$  this has a resistance of  $R_2$  you can think of it as providing a gain at least at DC it provides a gain of minus  $g_m$  times  $R_2$ , and therefore, at the low frequency. Remember, at low frequencies the gain is almost equal to the DC gain at high frequencies the gain we do not know what is going to happen at high frequencies, right.

Now, for one of the two poles, it is a two pole system right, it better be that one pole is much larger than the other pole, if it is not then what amplifier you designing. So far we have not talked about this, but after this experience we understand that one pole has to be much larger than the other pole. We have a certain idea of how large it has to be in fact, right? You are given  $\phi$ , you have been given  $A_{naught}$  so, you know exactly how large one pole has to be with respect to the other pole that automatically means that one pole is very close to DC the other pole is a very high frequency automatically, ok.

So, at DC at least  $A$  is equal to minus  $g_m R_2$  at the other frequency we do not know what  $A$  is. So, let us not bother right, but at least at DC  $A$  is equal to minus  $g_m$  times  $R_2$  and I have got  $C_{12}$  in shunt with this amplifier and therefore, the capacitance it is going to project is going to be  $C_{12}$  times  $1$  plus  $g_m$  times  $R_2$   $1$  minus  $A$ . So, it is as if the capacitance at this node is effectively  $C_{01}$  plus  $C_{12}$  times  $1$  plus  $g_m^2 R_2^2$ , and now suddenly the problem is tractable, right.

Now, suddenly you see that you have got a pole at  $R_1$  times this; alright it is a much easier problem. Now, this handle can also be used to engineer the two poles to be much greater than one pole much greater than the other, ok. So, some engineering also can be done over here, all right. Let us again go back to the perspective when you try to design your plain vanilla amplifier you do not know the locations of the poles you have designed something now you are going to use this amplifier in a feedback system. Before you use this amplifier in a feedback system you have to make sure that the two poles in

the amplifier are far apart from each other. How far apart from each other? By this much ok.

Now, when you have designed this plain vanilla amplifier you already know the value of  $A_{naught}$  that is fixed we do not want to tamper with it, we do not want to touch it. The boss comes into your room and says that, before you sign off on the design make sure that your phase margin is 60 degrees or something, ok. You have designed some  $A_{naught}$  already you do not want to change it you only now want to tweak the phase margin adjusted to your requirement, this is the situation. So, my amplifier is ready I just want to adjust the location of the two poles such that the ratio of the two poles is what I desire, ok.

So, one handle is this  $g_{m2} R_2$  over here, ok. What you can make sure is that this  $g_{m2} R_2$  this is a large number, right  $g_{m2} R_2$  this is an part of an amplifier this better be large this most some of the gain of the amplifier, right. So, amplifier gain is  $g_{m1} R_1 g_{m2} R_2$ . So, you have got  $g_{m1} R_1$  times  $g_{m2} R_2$  as the gain of the amplifier has large gain that you have designed. So, obviously,  $g_{m2} R_2$  is going to be some large quantity.

So, this large quantity can be used to make sure that one capacitance is much larger than the other capacitance the capacitance at the other node. Or in other words one pole is far removed from the other pole, alright. So, this is your tool at hand which means that  $C_{12}$  is where you can do your engineering,  $C_{12}$  is going to be affected by  $1 + g_{m2} R_2$  which means that from outside; right from outside you can engineer the value of  $C_{12}$  you can add some more  $C_{12}$ , alright that is not going to change the DC gain, but it is going to drastically change the location of the first pole and maybe the second pole, let us see what happens to the second pole. I have not checked the second pole as yet. This is the capacitance at node number 1, right.

So, the location of the first pole is  $1 / R C R_1$  times this capacitance, ok. The second pole I have not really checked so far, fine. Are you able to follow my thought process? My thought process is that from outside I am going to add a certain value of  $C_{12}$  on top of what is already there remember  $C_{12}$  was actually the culprit,  $C_{12}$  is causing a lot of confusion, ok. It is causing me to end up with a denominator polynomial which is so complicated that I refuse to factorize it even though it is second order, ok. But, even though it is complicated it is coming with an amplification right  $C_{12}$  is being amplified

by the second stage which means that at the end of the day when I have to make one pole much larger than the other then my choice should be  $C_{12}$ , fine ok.

Now, where is the second pole? Now, for the second pole you have to understand that we are working at a much much higher frequency, correct. Clearly, is the first pole, this has to be the first pole because this capacitance is large this is not the second pole, ok. So, clearly ratio of first pole and second pole is very large which means that the second pole itself is at a very high frequency. Now, what happens at high frequencies? At high frequencies if you have big capacitors they start behaving like short circuits, because the conductance becomes so high  $s \text{ time } C$  is the conductance right of the capacitor right, if  $s$  becomes very high then  $1 \text{ by } s \text{ } C$  becomes very small which means that the capacitor starts behaving like a short circuit, right. The capacitor is like a parking lot you cannot take out cars move cars inside the parking lot very fast you have to slow down, ok.

So, it does not allow fast changes. So, at if the change is very fast if you push in something from one side immediately from the other side same amount will go out, all right. So, therefore, the capacitor is going to behave like a short circuit at high frequencies.  $C_{12}$  is going to behave like a short circuit at this high frequency at  $\omega$   $2 \text{ } C_1$  is also going to behave like a short circuit  $C_{01}$  it is also going to behave like a short circuit everything it is a gigantic short circuit. Let us not worry about  $C_2$  over here  $C_2$  is my outside this part of my load,  $R_2$  and  $C_2$  are part of my load. So, let us not talk about them.

Otherwise as far as the remaining structure goes so, I have got  $R_2$  and  $C_2$ , but otherwise I have got  $m_2$ , all right and I have got a short over here and  $C_1$  and  $C_{01}$  are also shorts, ok. So, this is rampant confusion, it is a big short circuit. So, let us not again worry about  $R_1 \text{ } C_1$ , ok. Let us throw them out of this picture sorry let us not worry about  $R_2 \text{ } C_2 \text{ } C_1$  to  $C_{01}$  and  $C_1$  are going to behave like short circuits  $R_1$  is no longer relevant at these high frequencies right because  $C_1$  is a short.  $C_{12}$  is a short at these high frequencies.  $R_1$  is no longer relevant it is out of the picture.

Now, when you have so many shorts when it is all the whole thing is shorted up right what do you do you say how heavy is one short with respect to the; how good is one short circuit with respect to the other shot circuit, right how heavy is one with respect to the other right. That is going to tell you how tightly these two nodes are coupled as

opposed to these two nodes being coupled. So, it is as if you have some impedance here and some impedance here these two are both tending to 0, right what is the ratio between these two impedances one is  $C_{12}$  and the other is  $C_{01} + C_1$ . And therefore, if I apply a voltage  $V$  over here then I will get a voltage with voltage division this is just like a potentiometer right I will get a voltage proportional to  $1$  by  $C_{01} + C_1$ , correct, it is like a potential divider.

So, if I apply a voltage  $V$  over here then the voltage over here is going to be  $V$  times  $C_{01} + C_1$  divided by  $C_{12} + C_1 + C_{01}$ . Yes, I mean if you think capacitors then it is inversely proportional, ok. So, if I apply  $V$  over here then the voltage over here is going to be if I apply  $V$  over here then the voltage here is  $V$  times, sorry.

So,  $V$  times  $C_{12}$  divided by  $C_{01} + C_1 + C_{12}$ , ok. If you do the  $1$  by and then simplify this is what you will get. Is this clear? Great and that means, the  $g_m$  times this voltage is the current or in other words this MOSFET is not open it is not a dead MOSFET. It is actually an impedance, it is a diode connected MOSFET it is actually an impedance. So, at this high frequency the MOSFET is going to behave like a resistor of a very low value ok.

So, you see what I mean this MOSFET is just  $g_m$ . So, this is drawing current  $g_m$  times the gate voltage at this high frequency gate voltage is. So, I apply  $V$  the MOSFET starts drawing current, ok. This is what we did earlier also just that now I am throwing out the  $R_1$  and simplifying matters, because at this high frequency  $R_1$  is no longer relevant it is all short circuits right this is exactly what we did earlier as well just that now  $R_1$  is gone, right. Now, that  $R_1$  is gone you will see that things are much simpler to analyze.

(Refer Slide Time: 49:51)

$$C_{01} + C_1$$

$$\frac{g_{m2} \cdot C_{12}}{C_{01} + C_1 + C_{12}}$$

$$C_2 + C_1 + C_{01}$$

$$\frac{1}{g_{m2}} \cdot \left( \frac{C_{01} + C_1 + C_{12}}{C_{12}} \right)$$

$$P_1 \rightarrow \frac{1}{R_1 (C_{01} + C_1 + C_{12} (1 + g_{m2} R_2))}$$

$$P_2 \rightarrow \frac{g_{m2} C_{12}}{(C_1 + C_2 + C_{01}) (C_{01} + C_1 + C_{12})}$$

So, if the MOSFET takes a current it is going to take a current  $g_m$  times  $C_{12}$  by  $C_{01} + C_1 + C_{12}$  this is the amount of current that the MOSFET takes which means that the resistance posed by the MOSFET is one by of this, ok. So,  $1$  by  $g_m$  times  $C_{01} + C_1 + C_{12}$  by  $C_{12}$  this is the resistance that the MOSFET is offering. And remember  $C_{12}$  is some external value right you have added  $C_{12}$  it is not just a overlap capacitance anymore, ok. Ordinarily it is just overlap and plays spoilsport, but in this particular case to split up the two poles to get a large ratio of the two poles you have deliberately added  $C_1$  to over there from my earlier discussion, ok. So, this is the resistance offered by the MOSFET.

Now, let us see I have a node over here, right this is my node at this node I have a certain resistance of this value and capacitance of value  $C_2$  and then series combination of  $C_1 + C_{12}$  and the rest, whenever there is series combination it is smaller than the smallest one right  $C_{12}$  is large over here, because it has been externally added. So, therefore, it is just about equal to  $C_1 + C_{01}$ , because  $C_{12}$  is a large quantity from outside.

So, which means that the effective capacitance at this node is  $C_2 + C_1 + C_{01}$ . This is the effective capacitance and this is the effective resistance the resistance is not  $R_2$ , all right. This is the commonest mistake that you are going to make. The mistake that you are going to make is that when you do the miller you are going to split up miller on one side like this on the other side as  $1 - A$ , right and then you are going to add

that capacitance over here and then you are going to open up  $C_{12}$  all together and then you are going to say that  $R_2$  is the resistance and the capacitance is  $C_2$  and the Miller capacitance, that is the mistake. Why is that a mistake because at this high frequency that we are talking about no,  $a$  is no longer the original  $g_m R_2$  minus  $g_m R_2$  at this high frequency, ok.

At this high frequency all these capacitors are going to behave like short circuits they are going to behave as such good short circuits that  $R_1$  will be gone the entire MOSFET will look as if it is diode connected. Once the MOSFET is diode connected the MOSFET itself is going to behave like a resistor because now, it looks like a diode fine. And therefore, the resistance is not  $R_2$  it is actually much smaller than  $R_2$  which means that the pole frequency which is  $1/R_1 C_1$ . The pole frequency is going to be much higher than you originally thought it would be, all right. So, I have got two poles now this is my setup my setup is that I have got node number one where I have found the capacitance the resistance is just  $R_1$ .

So, my pole number 1 is  $1/R_1 C_1$  plus  $C_1$  plus  $C_{12}$  times  $1$  plus  $g_m R_2$  and then pole number 2 is the combination of these two, ok. Pole number 2 I have got one resistance one capacitance. So, again it is  $1/R_1 C_1$ , fine. These are my two poles and  $C_{12}$  has to be such that the ratio of these two poles is the required ratio of the two poles, alright. So, I am trying to connect up everything all together.

So, this is where we are going to stop today what we did was we worked out the ratio of the two poles required for a certain phase margin. Then we looked at we went back to our 2 stage cascaded amplifier, right. As far as the cascoded amplifier is concerned the same theory will work right nothing different then we found that in the two stage cascaded amplifier  $C_{12}$  we used miller theorem. Because I have really forgotten the analysis that we would have found out the output impedance, we would not have bothered with the short circuit current just the output impedance would have given me the denominator polynomial, right.

In that denominator polynomial I would have looked at the middle term which would be the sum of the two poles the sum of the two poles would be one of the poles in that denominator polynomial the third term would be the product of the two poles I already know one. So, I know the other ok, that would be my technique.



However, I did not do that right now, what I did was I used Miller's theorem and I used it very carefully, ok. This is how careful you have to be when you use Miller's theorem it is not a theorem apply it judiciously. So, our application was that at the first pole  $C_{12}$  will get amplified. At the output these are all going to behave like short circuits Miller should not be bothered with. And now you get the ratio of these two poles, this ratio of the two poles should be what you originally wanted it to be, ok.

So, with this let us close this lecture. In the next lecture we are going to actually solve some of these things, right; we are going to get deeper maybe do a couple of examples.

Thank you.