Analog Electronic Circuits
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Lecture – 30
Diversion: 2 - pole systems phase margin

Welcome back to Analog Electronics and this is lecture number 30. And in the last class last 2 classes actually last many classes we what we did was we worked over the common source, common gate, common drain amplifiers with capacitance. Then we looked at the cascode amplifier and as well as the cascaded amplifier with the presence of capacitance we worked out all of these over the last several classes. Then at the end of the last class we also discussed Miller's theorem approximation right? And we figured out that when you have a floating capacitor how to deal with it right that might not be the right way all the time.

In fact this Miller's theorem works only under some circumstances in some very specific places. So do not just use it here and there it is generally you avoid it until I say until I tell you that this is one place where Miller's theorem is going to give you a reasonable answer only then use it over there. All right now today what we are going to do is we are going to take a slight diversion. Now the diversion that we are going to take is related to bode plots.

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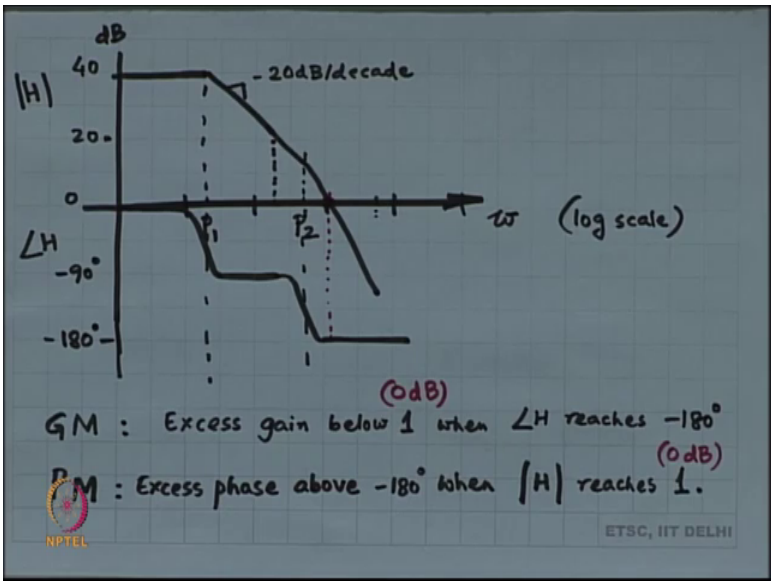

So earlier you had seen bode plots right, when there are 2 poles at each pole there is going to be a 20 dB per decade negative slope extra right this is your bode plot. So, if my pole is over here so suppose this is 40 20 and 0 dB. Then over a decade you are going to get a drop of 20 dB per decade right that is the idea after the pole. And suppose you have a second pole somewhere over here ok then, now over a decade so this is a decade you are going to get a slope of you are going to get minus 40 dB per decade right.

So minus 40 dB decrease something like this fine, I am taking advantage of the boxed paper that I have grid lines are there. Any way this is how the bode plot is going to be and similarly you are going to have a phase plot. So for the phase plot at the pole you are going to get minus 90 degrees at every pole you are going to get minus 90 degrees. Something like this all right this is going to be your bode plot of a 2 pole system.

Now, why we are going to do the 2 pole system is because we saw that in the cascode amplifier as well as the cascaded amplifier right whenever there are 2 stages you are going to see that you are going to get a 2 pole system. And both in case of the cascode as well as the cascade the denominator was did not give any closed form poles pole locations; actually you could have done it, you could have factorized using Shridar Acharya but you know it is going to be such a gigantic expression that it is not worth writing ok.

So that is why we are looking at 2 pole systems today all right. Now there are 2 important things, one is when you are making an op-amp why are we doing all of this? Why are we studying this course? Most of it is towards trying to build an op-amp. So when we are trying to build an op-amp we always must have an understanding that tomorrow after we have finished making this op-amp tomorrow we are going to use this op-amp how in the presence of feedback ok.

So the op-amp is always going to be used in the presence of feedback alright this understanding has to be there. Remember like we discussed you need DC negative feedback for the operating point to set to be set ok. So feedback is absolutely required you are never going to use the op-amp without feedback ok, in fact in special very special situations where you start using an op-amp without feedback in such situations the op-amp is not even called an op-amp it is called a comparator alright.

So we stop calling it an op-amp when we do not have feedback. So as long as it is an op-amp we have to understand that in future after we have finished making the op-amp we are going to use it in the presence of negative feedback. Now, there are some very important tools in control theory that analyze systems that are going to be used in negative feedback. Remember, the op-amp itself might have poles in the left half plane all the poles of the op amp might be in the left half plane. But when you use it in negative feedback tomorrow are the poles still in the left half plane. The poles of the overall system complete system right the system is going to be you know some forward loop h in the presence of feedback h divided by 1 plus h something or h times g some complicated stuff.

Now, is are the poles of that system in the left half plane can you guarantee? Now, in control theory there are some excellent tools that can predict that when I use this system in feedback later on are the poles going to still remain in the left half plane or will something jump to the right half plane. So you probably know the names of Routh Hurwitz criteria Nyquist criteria right and then there is something called related to bode plot right. Then you would have studied so first you would have studied Routh Hurwitz then you would have studied Nyquist criteria, then would have come the time to study bode plots and at the end of studying bode plots they would have talked about gain and phase margin ok.

So in circuits we normally like we like gain and phase margin a lot right, I mean you draw this bode plot. And then you can tell by looking at the plot first of all by looking at this plot offhand you know that P 1 and P 2 are in the left half plane. If P 1 and P 2 are in the right half plane then sorry ok nothing works if a pole happens to go to the right half plane straight away then you know do not bother right that is no good.

Now, the assumption is that P 1 and P 2 are already in the left half plane. Now that they are in the left half plane the op-amp by itself is going to be stable why 2 poles it can have 5 poles right. As long as the poles are in the left half plane the op-amp is stable but, tomorrow when I use the op-amp in a negative feedback system is that system going to be stable or not? That is the question that we want to ask. And the answer to that is that we can examine the gain or phase margin gain and phase margin both tell the same story exactly the same story right. And they are reported from exactly the same plot right you can from the same plot you can estimate gain margin as well as phase margin.

Sometimes it is easier to estimate the gain margin sometimes it is easier to estimate the phase margin but, both of them say exactly the same story. So, the story is that if gain margin is not there then the system is going gain margin or phase margin is not there, then the system is not going to be stable when you place it in unity gain negative feedback tomorrow. Unity gain negative feedback is the worst possible negative feedback, unity gain means the feedback factor is 1.

H is the forward path feedback path is G G is equal to 1 that is the worst case. That is when the most problems happen in control theory and under such scenarios is the system stable or not right normally, you are never going to have feedback gain that feedback path should always be some less than 1 not more than 1. So the largest number you are going to have over there is 1. Why do I say such a thing?

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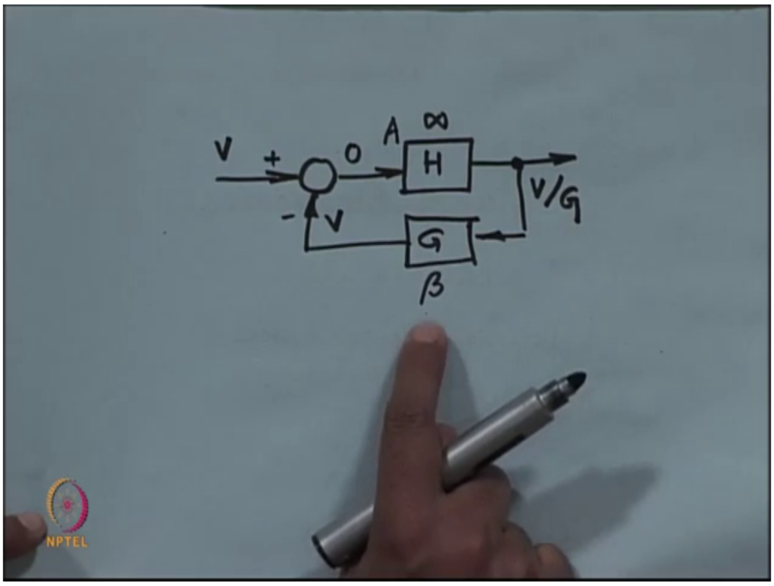

Because, this is what I am talking about so this G is what I am worried about H is my op-amp, H has super high gain, H has gain which is close to infinity all right. Now, if I want this whole thing to work as an amplifier then how does it work? If this is an input V then the error has to be 0 because, H has gain of infinity; that means what has been fed back is also V which means that the output of H is V divided by G ok.

Because that is what is coming into G so H is very large H is infinity. The output has nothing to do with H it has everything to do with G all right so normally we want this feedback factor to be a factor less than 1 right lot of people call it beta right this is called A and this is called beta right.

The feedback factor is beta this feedback factor is to be kept less than 1 because otherwise you are not amplifying. If G is not less than 1 then the output is going to be smaller than the input which seems like a flop. So you want certainly want G to be something smaller than 1 right and therefore the worst case G is 1 all right the largest possible value of G is 1 all right.

So when the feedback factor is 1 is this system stable or not? Is this open loop system this is the open loop system. Tomorrow I am going to place this open loop system in closed loop system with a feedback factor of 1. At that time is it going to be stable or not right I am trying to clarify right, this is a common misconception very common misconception if you read the book you are not going to be sure of what is going on right Is the op-amp not stable? No the op-amp is stable.

But, when I place this op-amp tomorrow in a negative feedback unit again negative feedback system is that system going to be stable or not that is the question that we are trying to answer. And the way we do that is by examining the gain margin or the phase margin. So gain margin is defined as the excess gain below 1 when angle H reaches when the phase reaches minus 180 degrees excess gain below 1 means if it is 1 by 5 then the gain margin is 5 ok. If so if magnitude of H is 1 by 5 when angle of H reaches minus 180 degrees then the gain margin is a factor of 5. If magnitude of H is 0.01 when angle of H reaches minus 180 degrees then the gain margin is a factor of 100 ok.

So that is what I mean by excess gain below 1. How much is the gain less than 1 when angle H reaches minus 180 degrees this is one question this is gain margin. And the alternate one is phase margin and phase margin is defined as what is the phase how much is the phase more than minus 180 degrees when magnitude gain reaches 1, 1 means 0 dB just by the way reminder ok. so what does this mean?

At the point when magnitude of H reaches 0 dB I checked my phase is that phase more than minus 180 degrees if it is more how much more? If it is minus 135 degrees then the phase margin is 45 degrees. If it is minus 120 degrees then the phase margin is 60. If it is minus 200 degrees then the phase margin is negative 20 degrees. So in either case when the gain margin is negative in dB or when phase margin is negative in degrees then the system is going to be unstable when placed in unity gain negative feedback.

However, if both of these are if either of these if one is positive the other will also be positive, if you have got positive gain margin positive phase margin then the system is going to be stable when you place it in unity gain negative feedback. All right so this is the theory this is coming from your control engineering all right we are not going to try to prove it that is not our business over here I am just going to state this. All right now let me look at my particular plot over here I have had 2 poles P 1 and P 2 and if you try to look at what is the gain margin or the phase margin let us try to look at what is the phase margin.

For example so when the at the frequency when the gain hits reaches 1 that is 0 dB at that frequency my phase is minus 180 degrees or it is asymptotically reaching minus 180 degrees. So, either there is no phase margin or slightly slight amount of phase margin something of that order right 0 or close to 0 degrees of phase margin so it might be in fact hard to measure all right. If there was a third pole we would have been sure because it would not asymptotically reach minus 180 degrees. If there was a third pole then you know it would again go and we would know that it is reaching minus 180 degrees all right

So in any case this particular system the phase margin is very little. If this is your op-amp bode plot people are not going to be very happy with this op-amp because they will say that with temperature or with operating conditions if my bode plot changes slightly then my system is going to become unstable. And instability is something that people are not ready to tolerate so the customer is not going to purchase your op-amp fine. If this is the if this is the bode plot of your op-amp no customer is going to be eager to purchase your op-amp alright.

So the question is what where should P 1 and P 2 be such that my customers are eager to purchase my op-amp alright. So a rule of thumb is that a good design is going to have a phase margin of 60 degrees right this is some general guideline.

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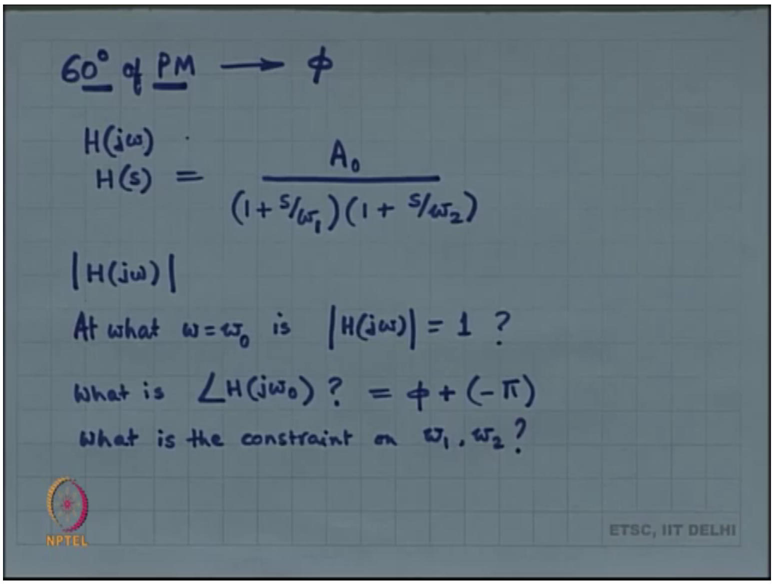

This is general guideline that 60 degrees of phase margin is considered nice by most customers. Some customers are with even lesser phase margin right, some very aggressive customers will say that all right 45 degrees will not be a bad idea right..Somebody whose trying to make a very cheap you know cell phone right he does not care if the his customer is unhappy right you get what you pay for right.

So, he is going to sell his cell phone dirt cheap, so maybe he is going to cut a lot of corners. So this is the corner that he is going to cut he is going to say that all right 60 degrees is too safe right give me an op-amp with phase margin of 45 degrees he is very aggressive right. Then again there are going to be customers let us say you are in the your customer is the military and these military guys will say no 60 degrees is too aggressive let us play it safe right. Our missile should not suddenly point at us and start shooting at us it should not go out of control we need everything guaranteed to be stable give me 90 degrees of phase margin all right.

So, this phase margin number depends on what kind of customer you have. An aggressive customer will ask for 45 degrees very strict customer is going to ask for 90 degrees of phase margin right like the military 60 degrees of phase margin is a nice clean number right. It is safe it is considered safe as a you know you get best of both sides all right so let us evaluate our system.

So, our system is H of j omega, alright and let us say on H of s whatever you want to call it. And let us say it has a DC gain of A 0 and it has 2 poles on the negative real axis 2 poles on the negative real axis, one at minus omega 1 and the other at minus omega 2 fine. So what do we have to do over here? We have to find out magnitude H of j omega right then we have to find out what is omega at which magnitude H of j omega reaches 1 that is 0 dB.

Let us call that frequency omega 0 right, first you have to answer this question then you have to plug in that omega 0 into H of j omega 0 and find the phase right. And you have to make sure that phase is some let us say that the target phase margin is an angle phi so you have to make sure that phase is phi degrees above minus 180 degrees all right so this is the plan we need to do this fine.

So if I want H of j omega to be phi plus minus pi what do I engineer? I have to engineer the locations of omega 1 and omega 2 such that this happens alright. So what is the constraint on all right so this is what we are going to try to do right now. So first we are going to find out we are going to answer this question at what omega equal to omega 0 is modulo H of j omega equal to 1. Then we are going to find the phase then we are going to say what is the constraint on omega 1 omega 2? Such that the phase is equal to phi minus pi alright. So this is how I am trying to set up the problem. So first of all modulo H of j omega is equal to 1.

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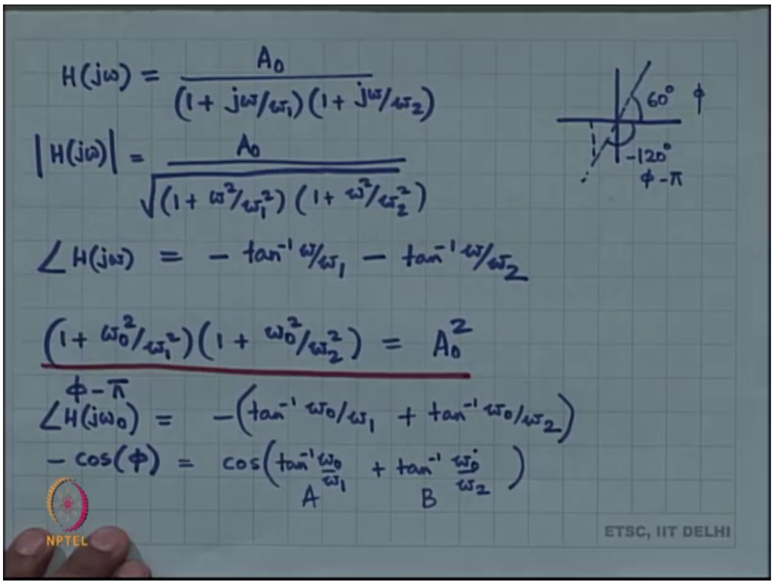

And H of j omega is A 0 I made a mistake no, modulo is what? And what is angle? What is the angle? Minus tan inverse omega by omega 1 minus tan inverse omega by omega 2. Each of these contributes phase right the phase of the first one real part is 1 imaginary part is omega by omega 1 therefore the phase of this is tan inverse of omega by omega 1 divided by 1 alright but this is in the denominator.

So a minus sign comes so the overall phase from this particular component is minus tan inverse omega by omega 1. Component of this is minus tan inverse omega by omega 2 all right this is understood we are doing same trigonometry all right so this is the basic stuff.

Now, we are going to say that at H of j omega 0 modulo is equal to 1 omega 0 is such that modulo H of j omega 0 is equal to 1, which means that if I plug in omega 0 over here then, 1 plus omega 0 squared by omega 1 squared times 1 plus omega 0 square by omega 2 squared is equal to A 0 squared. This is one thing that we know ok.

So omega 0 you can this is a quadratic in omega 0 squared right you will get omega 0 power 4 right you can multiply and then you can solve a quadratic you can waste a lot of time right. You can do that and then you can plug in that omega naught the value that you got from that quadratic into this tan inverse right it is going to be quite horrible right let us not do that all right that is way too much.

Now, instead what we are going to do is let us look at the phase over here let us not try to solve for omega therefore all right. So initially I said that I am going to solve for omega right, but after setting up this equation I am now suggesting that let us give up let us not solve for omega because trying to solve for omega is going to break your back. Alright we are not going to do that we are all engineers right anything is too hard we are not going to do it.

Instead let us look at this from the backside what is the angle H of j omega 0? So angle H of j omega 0 is so much. At omega 0 at omega equal to omega 0 it is equal to just plug in omega 0 so much alright. And now, I am going to do something very special, I am going to take cos of both sides of this expression. So first of all I want angle H of j omega 0 to be equal to what? I wanted to be equal to phi minus pi all right, this is what I want it to be equal, now let me take cos of both sides.

So what is cos of phi minus pi? So let us say this is an angle phi 60 degrees phi minus pi is 60 degrees minus 180 degrees. So that is minus 120 degrees so what is cos of minus 120 degrees? It is going to be all sin tan cos right. If you think of it yes cos is base by hypotenuse, right here the base is negative the hypotenuse still has a solid value it is still positive. Therefore it is going to be cos is going to be negative.

So this is going to be minus cos of phi, cos of phi minus pi is nothing but minus cos of phi and if I take cos of minus something then that is the same as cos of the sum. Now, look it is been a long time since I studied trigonometry I have completely forgot10 all my trigonometry. I remember very little. But I am going to try to do this in a way to help you out I do not remember I cannot do cos of tan inverse A minus plus tan inverse B. That is expecting too much from me, I do not remember this.

But let us try to understand that cos of something plus something is nothing but cos something cos something minus sign of something sign of something right, cos of A plus B is cos A cos B minus sin A sin B that much I remember. So, if I want to evaluate this then all I have to work out is what is cos A what is cos B what is sin A what is sin B where A and B are these tan inverse things all right.

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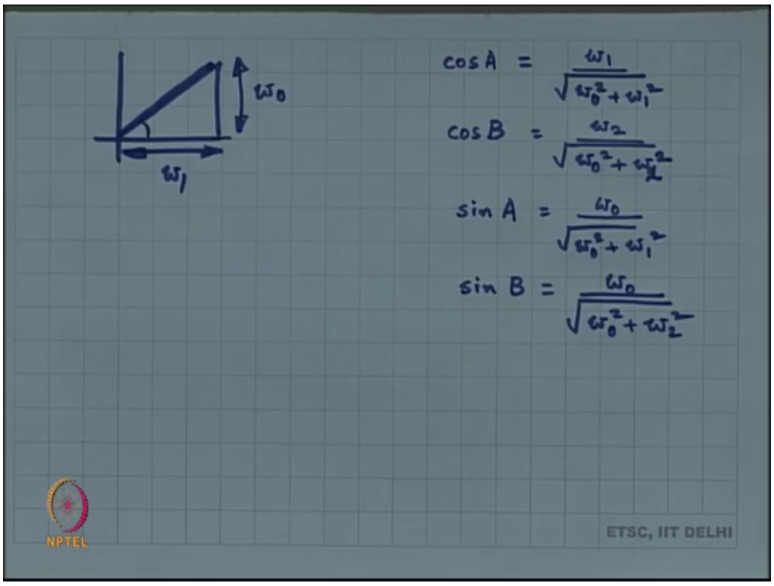

Now think of a situation, so what does tan inverse omega 0 by omega 1 mean? It means that you have got a triangle where the height is omega 0 and this is omega 1 and this is the angle we are interested in. Now, what is going to be cos of the same angle? Cos of the same angle will be omega 1 by the hypotenuse. So, cos of the first one so let us call this capital A let us call this capital B so cos of A will be equal to omega 1 by square root of omega 0 squared plus omega 1 squared am I right I think I am right.

And likewise cos of B will be equal to omega 2 by square root of omega 0 squared plus omega 1 squared omega 2 squared because, B was omega 0 by omega 2 right. And by the same argument sin A is going to be omega 0 by root over omega 0 squared plus omega 1 squared and sin B is omega 0 by root over omega 0 squared plus omega 2 squared all right. So, I have just writ10 down these expressions on the side and then I am going to do cos of A plus B equal to cos A cos B sin A sin B right.

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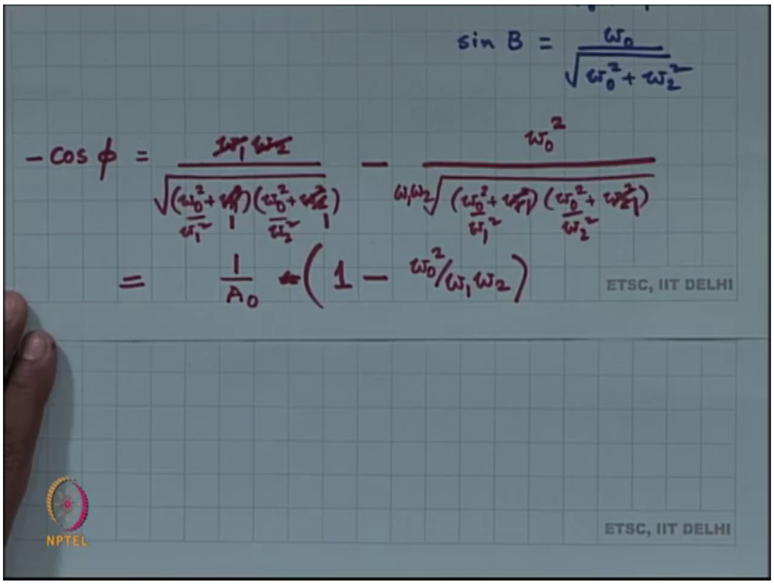

And therefore, I get minus cos phi is equal to omega 1 omega 2 by square root of omega 0 squared plus omega 1 squared omega 0 squared plus omega 2 squared minus sin A sin B. Where, sin A and sin B are is this and now the magic is going to happen. Look at this omega 1 omega 2 let us let us divide the denominator by omega 1 this becomes a 1 I get omega 1 squared.

This becomes a 1 I get omega 2 squared fine and likewise let us take omega 1 omega 2 common over here. This becomes a 1 this divides by omega 1 squared this becomes a 1 this divides by omega 2 squared. And then comes even more magic from my previous relationship I know that this product is actually equal to A 0 squared is this.

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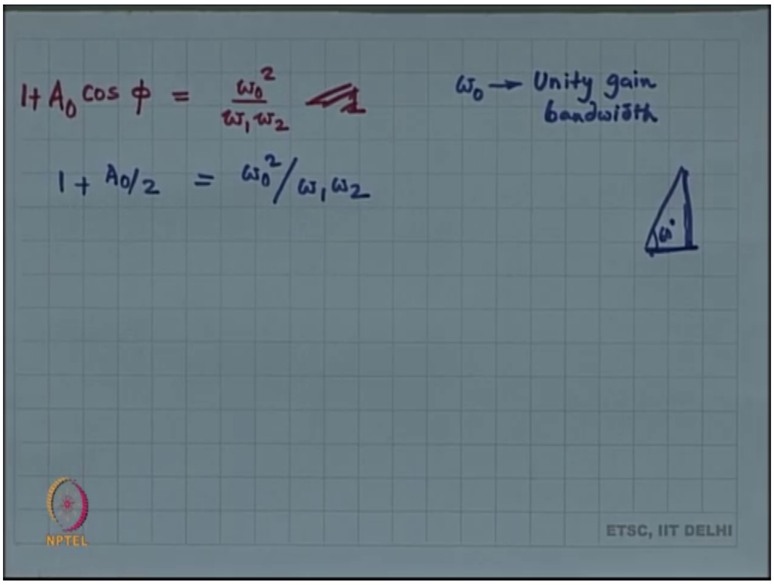

So, this is it so if I want the phase margin to be an angle phi then minus cos phi has to be equal to 1 by A 0 times 1 minus omega 0 squared by omega 1 omega 2 or you could rewrite this as cos phi has to be equal to A cos phi A 0 cos phi has to be equal to so this is a key relationship you can place this 1 on the other side. The product of omega 1 omega 2 have to be such that omega 0 squared by omega 1 omega 2 has to be equal to 1 plus A 0 cos phi where omega 0 is the unity gain bandwidth right. The frequency at which the gain was 1 so it is called unity gain bandwidth all right.

So, at that frequency the gain is equal to 1, I call it unity gain bandwidth magnitude H of j omega was equal to 1 that was my definition of omega 0 fine. My 2 poles have to be arranged in this fashion. If I do not then I would not get a phase margin of phi, if I want a phase margin of phi then 1 plus A 0 cos phi has to be equal to so much or in other words let us try out some numbers let us say phi is 60 degrees What is cos phi? Cos phi is root 3 root 3 by 2. 1 by o this is 60 degrees I am sorry I have really I am really bad at trigonometry base cos 60 degrees is 1 by 2.

So 1 plus A 0 by 2 has to be equal to omega 0 squared by omega 1 omega 2 so this is a relationship that we have to follow. And where has this come from nothing from some transfer function. From a random I mean I have assumed nothing no assumptions have been made right all that I have said is that this transfer function has 2 poles. I have worked out I did not I could not quite get to this point what is the value of omega 0? I kind of gave up all right but then I try to link the two of them together and I came up with a constraint on omega 1 and omega 2 all right. So, this is what we have done so far and what we are going to do is we are going to take this forward in the next class. We are going to see what this means in terms of our transfer functions right, the transfer functions that we have what does it mean in terms of that. So let us stop here and we are going to carry on with this in the next class.

Thank you.