Analog Electronic Circuits

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Lecture – 28
Cascode, cascade – revisit with capacitance

Welcome, back to Analog Circuits. Today is lecture – 28 and we are going to discuss the Cascode and the cascade amplifier. It is actually going to be a revisit of what we had done long back and this time we are going to revisit it with capacitance in mind.

So, in the last class we last two classes actually we had done the common source amplifier, the common drain amplifier, the common gate amplifier with capacitance effects taken into account we saw that the common source amplifier gave us one pole in the right left half plane and one zero in the right half plane, the common gate amplifier gave us only one pole no zeros and the common drain socket gave us a pole as well as a 0 right it gave us a pole as well as a 0 the 0 was thankfully in the left half plane the pole was also in the left half plane.

We further examined the common drain amplifier output impedance, reexamined we said that the common drain amplifier is going to be used in a situation where the source impedance is large, and we applied a source impedance and reexamine the output impedance of the common drain amplifier and found that it is inductive in nature; even though there is no inductor in sight in the circuit there is no inductor at all. However, when you look into the output of the common drain amplifier, it behaves as if it was an inductor ok. So, this is the stuff that we have seen so far.

Next on the agenda is to see multi transistor circuits and our two more important multi transistor circuits are the cascode amplifier as well as the cascade of two common source amplifiers. The cascode is also two amplifiers a common source followed by common gate, ok. Common source followed by common gate is called cascode and then we will do the common source followed by common source as well.

So, this is the overall outline of today's lecture. So, we are going to start with the cascode amplifier.

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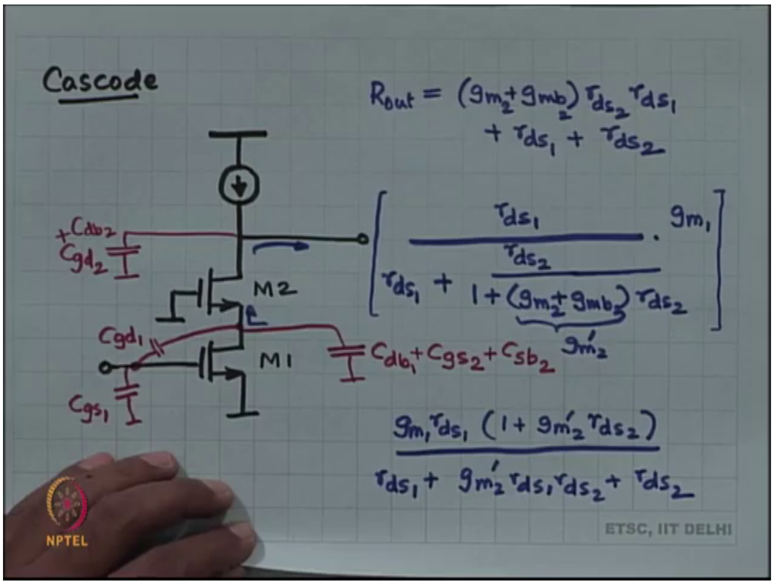

And, the cascode amplifier has to be analyzed with a current source on the top, if you do not put a current source if you only put a resistor, then it is unreasonable, ok. Ideally what you want is a cascode of n MOS devices as well as a cascode of p MOS devices. If you do not want to draw the cascode of the p MOS devices right away then at least place a current source over there, do not place a resistor because that just kills that just destroys the purpose of the cascade. The purpose of the cascode is to get a very large output impedance, ok.

So, this is the cascode circuit it is a common source amplifier followed by a common gate amplifier, ok. The common source amplifier is a voltage controlled current source, the common gate amplifier is a current controlled current source, ok. So, it is input is a current and the overall output is an even better current source than that of the common source amplifier ok, that is the idea. If I look at the output impedance so, the way we analyze this is first we always analyze DC, then we place the capacitors we find the poles and zeros and then we write out the transfer function that has been the strategy so far, ok.

So, let us try to do that of course, this is going to be a situation where that precise strategy is not going to work you need to you know you will need to be a little more flexible about it. So, first we are going to analyze DC. DC analysis involves two things; one is output impedance the other is short circuit current ok.

So, if I look at the output impedance at DC it is a straight forward it is an application of our formula first of all you have to apply 0 volts at the input. So, as soon as you have applied 0 volts at the input this MOSFET now contains g m is gone g m b was gone anyway only r ds. So, this MOSFET M 1 has only r ds on it and now you apply the formula the formula is very straightforward right looking into the drain of M 2, the impedance that you are going to see is equal to intrinsic gain of M 2 which is g m plus g mb times r ds 2 that is the intrinsic gain of m 2 times what is on the drain what is on the source which is r ds 1 plus r ds 1 plus r ds 2 all right. This is the result, this was the formula done output impedance is done.

See by now you would have realized that those two formulae are very important, ok. If you want to make any headway in this course if you still have not memorized those to formulae do memorize because they are vital for this course. Next we are going to use the find out the short circuit transconductance. Now, for the short circuit transconductance what am I going to do? I am going to connect this output to a ground alright and you do not have to draw all of this all the time imagine that you have connected this to ground and we are now going to measure the current in that short circuit.

So, first of all of this on top does not matter I apply an input over here v in, let us say I have applied v in over here M 1 is going to respond with it is g m, right. So, it is going to create a current g m times v in g m 1 times v in inside g mb is not required because body is at 0 volts yeah. So, g m times v in current is created over here through M 1. Now, this current sees r ds 1 which I have not drawn and it also sees an impedance looking into the source of M 2, ok.

The current now has to distribute itself between these two impedances one is r ds of M 1 which I am not drying and the other is whatever you are looking up into the source of M 2 what do you see looking up into the source of M 2? It is 1 by g m 2 in parallel with r ds 2, or you can apply your formula the second formula right and say that drain is terminated with 0 ohms right with a short circuit.

So, r d is equal to 0 and that second formula was r d plus r ds divided by 1 plus g m intrinsic gate g m plus g mb times r ds ok. So, that is the impedance that you will see looking into the source of M 2, you will see an impedance r ds 2 plus r d which is 0 divided by 1 plus g m 2 plus g mb 2 times r ds 2. This is what you see looking into the source of M 2 , one second I apply v in over here, M 1 responds with a current g m times v in g m 1 times v in. This current has to choose between two paths one path is r ds 1 to ground, the other path is this impedance to ground this is ground anyway right this is between this and ground, ok.

So, this current has to share and how does current share you know the current sharing relationship right I goes a current I comes it faces 2 resistors r 1 r 2, the current in r 1 will be proportional to r 2 divided by r 1 plus r 2. So, the current going into the source of M 2 is going to be equal to r ds 1 divided by r ds 1 plus so much, this fraction of g m 1 v in write v in times g m 1 was the current that was created I am not writing v in g m 1 times v in was the current created that current had to share between r ds 1 and this impedance looking into the source of MOSFET number 2, right this is the current sharing relationship right r 1 divided by r 1 plus r 2 is the current going into r 2, ok.

This entire current is going to now go into the short circuit; right whatever current came that entire current will now go into the short circuit. So, this is my short circuit transconductance, and see I am writing this huge massive expression just by inspection, we did not do KVL, we did not do KCL we are not doing anything we just memorize those two formulae, right we did not even expand the circuit right we are just looking at the circuit talking and writing the expression, ok. So, that is how you need to do it the more KVL and KCL you write the less you know you with the less understanding you will get the less intuition you will develop.

Now, what is the intuition here? The intuition is that this quantity ok, this second quantity is small why? Because 1 plus the intrinsic gain is more or less equal to the intrinsic gain. Intrinsic gain of a MOSFET hopefully you have biased the MOSFET well, right, correctly hopefully you know you have got you are in the flat region of the MOSFET, which means that the intrinsic gain of the MOSFET is some large number 20 40 50 right, something and therefore, 1 plus that intrinsic gain should therefore, be more or less equal to the intrinsic gain which means you can ignore the 1.

Now, r ds 2 cancels out and therefore, this entire thing is just 1 by g m 2 plus g mb 2 right transconductance of the second MOSFET is hopefully a larger than 1 by r ds 1 or in other words r ds 1 should be much larger than 1 by the transconductance ok.

In other words this entire term is going to be more or less close to 0 compared to r ds 1 which means that this entire fraction is more or less equal to 1. This fraction is 1 and therefore, the transconductance is just g m 1, ok. So, that is how the cascode operates. You create a current over here and the second cascoding transistor takes all that current and makes a current source of it, right. So, I created a current with M1 and the second common gate circuit took that current and convert made it into a current source that is the idea of the cascade, right.

So, M 2 made it into a current source and therefore, the transconductance is just g m one that is the intuition. So, I have got g m 1 is the transconductance or my r out is some large impedance because it is a current source right once again r out is large because of this first term the first term is large over here, the second and third terms do not matter you have got intrinsic gain times r ds 1.

Obviously, that is going to be much larger than r ds 1 and r ds 2. So, the first term is the dominant term in the output impedance and therefore, once you have the transconductance g m 1 and the output impedance you can write out the voltage gain if you want to write out the voltage gain it is the product of the transconductance and the output impedance.

So, g m 1 times this first term g m one times r ds 1 times g m 2 plus g m b 2 into times r ds 2. So, intrinsic gain of the first device times intrinsic gain of the second device that is approximately your overall voltage gain, all right. This is DC. Now, our job is to throw in the capacitors and what do we do what all capacitors do we have? We now have two devices. So, we have to discuss 2 into 4, 8 capacitors. First we have C gs 1 fine, then we have C source to body of one which is irrelevant because source is at ground, body is also at ground so, no current goes through it anyway would not write.

Next, we have C gate to drain of 1 and next we have C drain to body of 1 ok. So, that is four capacitors for MOSFET number 1 then let us talk about MOSFET number 2. We have C gate to source of MOSFET number 2. Now, C gate to source is between source and ground, and we had a similar capacitor over here. So, it just adds on to this. Next we had C gate to drain of MOSFET number 2, all right then we have C drain to body of MOSFET number 2 between drain and body and that just adds up with C gd 2 and lastly we have C source to body of MOSFET number 2 source to body, body is at ground and we have a similar capacitor over here. So, it adds up, is this all right, great. So, these are all the capacitors.

Now, instead of writing these complicated expressions, I am going to simplify a bit I do not like these complicated expressions. I am going to redraw my circuit in this fashion.

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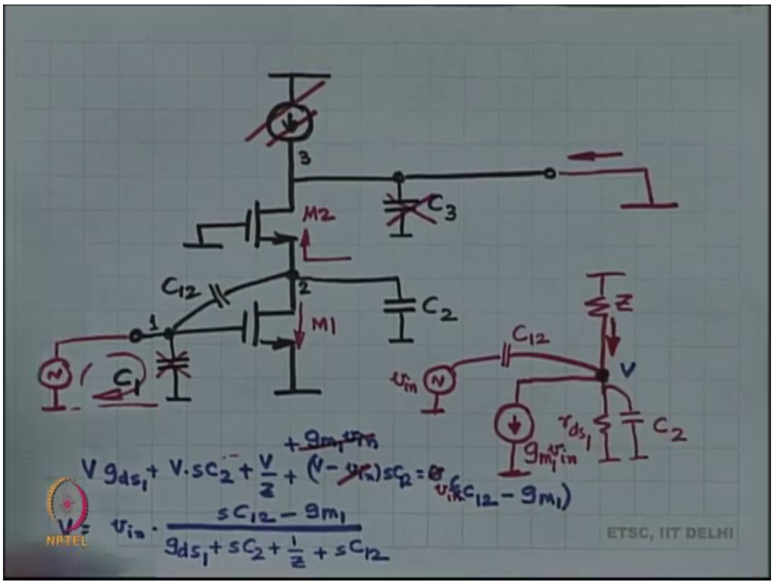

So, let us call this node 1, let us call this node 2 and let us call this node 3, ok. So, I have a capacitor between node number 3 and ground which is actually C db 2 plus C gd 2, I am going to call it C 3. I have a capacitor between node number 2 and ground which is actually C db 1 plus C gs 2 plus C sb 2 I am going to call it C 2. I have a capacitor between node number 1 and ground which is C gs 1 I am going to call it C 1 and I have one last capacitor between nodes 1 and 2 I am going to call it C 1 2, it is actually C gd 1, ok.

So, what you are going to notice by the way is that this gate to drain capacitance is very annoying; this one this C 1 2 is very annoying you are going to see this you have already seen this before it created a right half plane 0 in the common source amplifier, right. This gate to drain capacitance is going to create trouble; we do not like it, alright. If you ever get into device design, right if you get into device physics, you will see that a lot of effort is put into reducing the gate to drain capacitance, we do not like it ok. The good news is that the device design guys are not doing a bad job when the MOSFET pinches off beyond pinch off C gate to drain actually goes all the way down it goes becomes smaller and smaller, right.

So, they are not doing a bad job they are doing a good job. So, C 1 2 over here is actually a small capacitance, it is the smallest of the lot. However, even small capacitances are annoying right just this location this the place where it is you know everything else is with respect to ground this capacitance is floating in between two nodes, it creates you know misery, alright. So, we are going to see some you know approximations later on how to deal with this gate to drain capacitor how to deal with capacitors that are floating around we will do these approximations later on, but for now let us try to analyze this particular circuit, ok.

This is my output I already have the DC expressions. Now, what do I have to do I have to do my same two experiments one is the short circuit current and the second is the output impedance; by the way this current source is not there right in the AC small signal this DC current source can be thrown out, ok. So, we are going to do what do you want to do first, short circuit current or output impedance? You want to do the output impedance first short circuit current, ok. Let us do any one, it does not really matter let us do the short circuit current, ok. So, for the short circuit current we are going to apply a voltage at the input and we are going to short this output and measure the current this is the plan.

Now, let us see C 3 is out, it cannot take any current anyway it is useless both sides of C 3 are at ground. So, C 3 is not going to come into my equations at all, alright. So, we do not have to worry about C 3. What about C 1? C 1 is going to take current, but this is some private loop between the voltage source and C 1, right a loop of it is own right if the current through C 1 is well defined. It is just coming from this voltage source it is a loop of its own, this current has nothing to do with this short circuit current alright. So, whether C 1 is there or not it really does not matter it is not going to come into my equations, fine.

So, let us forget about C 1. So, all I have right now is C 1 2 and C 2, ok. So, C 1 2 seems like it is providing some additional path and C 2 it looks like it is changing something over here some something is going to change let us see what happens. So, first we apply my our voltage v in and then that creates a current g m times v in. g m times v in can now choose to go between many different paths, ok.

So, this is g m times v in g mb times v in is any way out. There is an r ds, there is now a C 2, there is also a C 1 2 and there is the impedance looking into the source of M 2. Now, the impedance looking into the source of M 2 is unchanged from the DC, right. In DC whatever you had computed it is the same old stuff all right same thing just that now there are two more competitors, there is C 1 2 and there is C 2 and we just have to work out what is the portion of current that goes into M 2 into the source of M 2, fine this is our setup.

So, in other words my setup is like this some Z, where Z is really 1 by g mb plus g m of device number 2 write approximately we know what that is. So, this is what I want to do what I want to solve, ok, I have simplified. So, I have my v in over here. This v in input source creates a current g m 1 times v in this g m 1 times v in has to choose between the following paths it has to choose between r ds 1 C 2 C 1 2 and Z and whatever goes through Z is my answer, my final answer is the current that goes through Z that comes through Z rather fine, great.

Now, looks like this C 1 2 is creating a complication over here, it looks as if what if C 1 2 was not there would you have been able to write the answer by inspection? It is just that now the competitors are Z and r ds 1 parallel with C 2, right you could combine r ds 1 and C 2 write them in parallel and then write out this expression. So, this is now going to become r ds 1 in parallel with 1 by sC 2 this is going to be r ds 1 in parallel with 1 by sC 2 the rest of the stuff is going to remain the same and that will be your answer ok.

So, if C 1 2 had not been there this is what would have happened you could also say that C 1 2 is almost with respect to ground. Suppose, you think that C 1 2 is with respect to ground right, imagine you are doing superposition it is not right it is not right, do not do superposition, it is not right in this case imagine you are doing superposition g m 1 times v in current and then you do this experiment once.

So, C 2 and C 1 to come in shunt with each other and then later on you are going to just apply v in and check same answer, can you do such a thing? So, technically you cannot; however, if you specify the value of this current you can. So, instead of drawing a diamond if you draw a circle like I have drawn and you say that this current source is g m 1 times v in.

Now, you can now you can do superposition to find out this current. These are now two independent sources; one source is g m 1 times v in the other sources just v in, ok. So, you can actually do superposition in which case what is going to happen, I apply g m 1 times v in current over here and then find out what is this current and that is going to be the same expression, but here instead of r ds 1, I will write r ds 1 in parallel with 1 by s C 2 plus C 1, fine done. The second experiment is that I apply v in, now I do not apply this. This current source is no longer there I just apply v in and see what is the current going into z, alright.

Now, that is going to be a little harder fine you could do that another possibility is to just do a short KCL in this particular case the short KCL is going to work to your advantage right, it might work out faster. So, you look at this circuit you say that I do not know this voltage V let us say this voltage is V ok. So, the KCL is like this V by r ds 1 or rather V times g ds 1 plus V times sC 2 plus V by Z plus V minus V in times sC 2 is equal to 0, ok. This is your short KCL and from this KCL your V in is going to come on this side everything else is proportional to V C 1 2 thank you, ok.

So, v in time C 1 2 can be moved to the other side and therefore, now you know what is V.

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Thank you. Thank you for pointing out there is a major, ok. So, g m 1 goes to the other side. So, this becomes minus, fine. So, now, I have got V and guess what this current is just V by Z minus V by Z. In fact, ok. So, my short circuit current is v in times 1 by Z and what is Z? We have forgotten what is Z; Z was this complicated thing, all right. You can I mean in this particular case you can write the whole thing it is not too hard.

Let us call this is annoying let us call this g m 2 prime, all right that extra bracket looks funny to me, ok. So, just let us write it as g m 2 prime. So, Z is nothing, but ok. So, I will do it on the next page.

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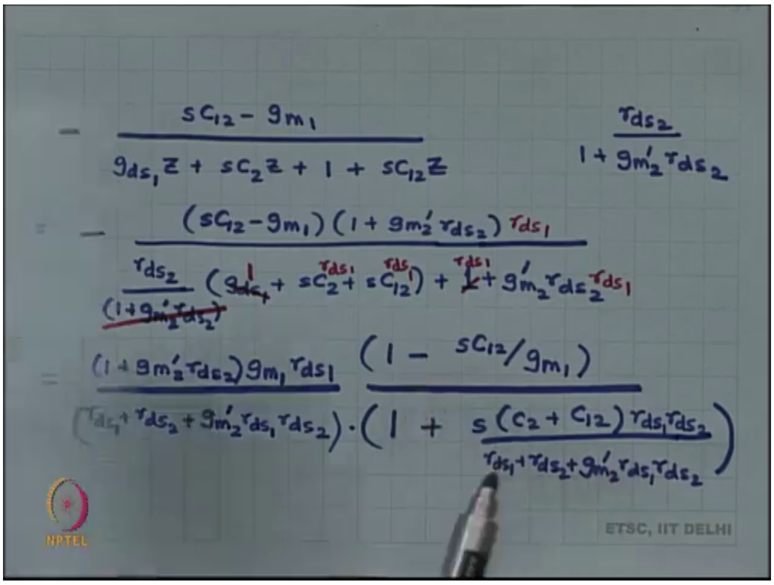

So, transconductance, ok. So, the transconductance is minus because I am looking at the current coming in. So, therefore, a minus times v in goes away in the transconductance expression this is the expression for V I was looking for V by Z all right.

So, s C 1 2 minus g m 1 divided by Z times the denominator so, g ds 1 times Z plus sC 2 times Z plus 1 plus s C 1 2 times Z, ok. So, this is the short circuit transconductance and Z I will plug in as r ds 2 by 1 plus g m 2 prime r ds 2 ok, where g m 2 prime is understood to be g m 2 plus g mb 2. And, now what do I do I have got this ok. So, the easiest thing to do is to multiply numerator and denominator by 1 plus g m 2 prime r ds 2 fine. This is how it is we do not like things in this complicated fashion right we would like to take the DC term out and what was the DC term? The DC term look like this maybe the DC term deserves a little simplification.

So, the DC term was g m 1 r ds 1 in the numerator times 1 plus g m 2 prime r ds 2 divided by r ds 1 plus g m 2 prime r ds 1 r ds 2 plus r ds 2, ok. This was the DC term, fine. Now, this DC term has to be taken out look over here carefully g m one r ds 1. 1 plus g m 2 prime r ds 2 1 plus g m 2 prime r ds 2 appear straight away. g m 1 is right over here and an extra r ds 1 has to be taken out 1 by r ds 1 or rather r ds 1 has to be this is wrong. No, this is fine; this is correct why is this correct the numerator has dimensions of 1. Denominator has dimensions of resistance. So, transconductance should have dimensions of 1 by resistance Siemens.

So, just quickly check the units right we need to take this portion out we also have to take out g m 1 from the numerator and then what remains in the numerator 1 minus sC 1 2 by g m 1, and that is very nice because it starts with a 1, right. There is no coefficient for the term without an s is only 1 and then from the denominator you have to take out the DC term. So, in the denominator I have got r ds 2 gds 1 which is DC, this is s these two are s. So, that is not DC, 1 is a DC. So, I have got 1 plus g m 2 prime r ds 2 g ds 1 g ds 1 is annoying, all right. I think we have to multiply numerator and denominator by r ds 1.

So, that is why there was a slight inconsistency ok. So, I have taken out r ds 1 from the numerator as well and now, from the denominator I need to take out r ds 2 plus r ds 1 plus g m 2 prime r ds 1 r ds 2 and when I do that I am left with a one at DC because I have taken out r ds 2, r ds 1 and this whole thing right. So, if I take that whole thing common then all I am left with is a 1 plus sC 2 plus C 1 2 times r ds 1, r ds 2 divided by whatever you forcefully took common, right. You have taken something common forcefully because not really common. So, that has to be divided ok.

So, this is the template, it looks awful, but this is where we are going to stop and declare victory we have identified a 0 is this 0 left half plane or right half plane? This is the right half plane 0 the right half plane 0 is expected whenever you have stuff like this, this annoying gate to drain capacitor has created that right half plane 0 C C 1 2 you see it is just C 1 2. The location of the 0 is g m 1 divided by C 1 2 that is the location the frequency at which the right half plane 0 is. It is at g m 1 divided by C 1 2 ok.

So, if C 1 2 had been 0 for example, then this would have been at infinity basically it would not have been there, ok. Next we have got a pole there is a left half left half plane pole which is very nice simple pole on the real axis negative real axis, ok. C 2 plus C 1 2 times r ds 1 r ds 2 divided by a gigantic resistance check the dimensions check that this is indeed s times or time constant s C r is it? It is r times r divided by r ok. So, the dimensions are that of a resistance, fine. So, this is it I have got my 0, I have got my pole and I have got the DC term out, alright.

So, this is where we stop dealing with the transconductance. This is the short circuit transconductance.

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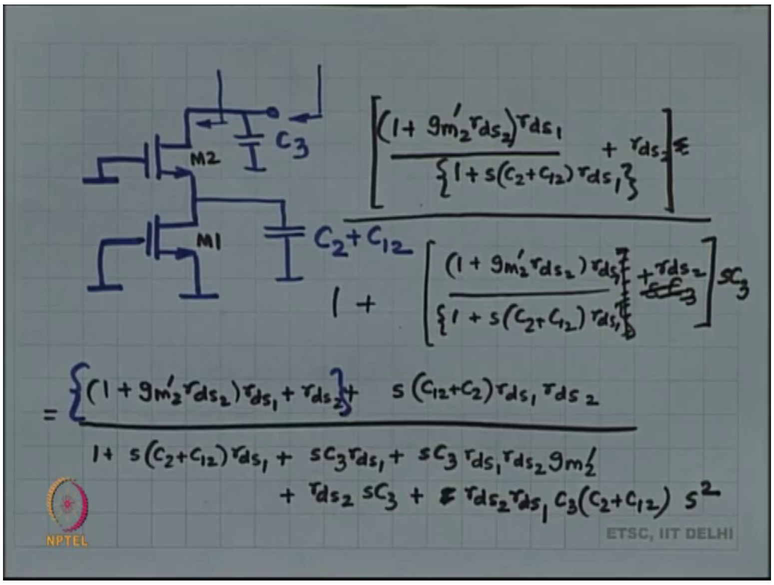

Next we have to find the output impedance and to do the output impedance I am going to connect the input to ground, and when I connect the input to ground remember I had three capacitors. One was C 1 2, one was C 2, I had C 1 2 and I had C 3 ok. So, C 1 2 is going to now come in shunt with C 2. So, instead of writing C 2 you can write C 2 plus C 1 2, fine. This is easy; C 1 does not matter anyway because it is now grounded on both sides, ok. There was a C 1 over here that is now shorted grounded on both sides. So, it cannot take any current, alright.

Now, we have to find the output impedance of this output impedance of this is 1 by sC 3 in parallel with output impedance looking into the drain ok. So, it is just sC 3 1 by sC 3 is just the extra stuff, let us do that later let us place that parallel piece later on. What else is there? What else is there looking into the drain it is the same formula. The source is now terminated with r ds 1 in shunt with C 2 plus C 1, instead of being terminated just with r ds 1 now you are terminated with r ds 1 in shunt with C 2 plus C 1 2 and let us see if we can write the expression by inspection.

So, by inspection the expression is going to be intrinsic gain of MOSFET number 2 which is g m 2 prime r ds 2 times this is the intrinsic gain of MOSFET number 2 times what is on the source. What is on the source is r ds 1 in parallel with C 2 plus C 1 2 fine I have written by inspection this is the first term plus r ds 2 plus whatever is on the source. It is done, this is it. This whole thing comes in parallel with C 3.

So, you can write it as this divided by 1 plus this times sC 3, ok. This is the complete expression. Of course, we are not happy with this kind of an expression we have to simplify. We have to simplify to a point where we can take the DC term common outside and we know what our DC term was DC term was just this alright g m to prime this was g m 2 prime g m 2 prime r ds 2 r ds 1 plus r ds 1 plus r ds 2, that was our DC term. So, we are only going to simplify till that point.

Now, what do we do? First thing we can do is multiply numerator and denominator by this portion, by this portion fine. So, if you multiply numerator and denominator by that you get. I am feeling I am making a mistake, I am right there was an r ds 2 inside this which I seem to have forgotten, yeah, all right. I have done all the multiplications also in the denominator, all right.

Now, the first thing you see is that this is the DC term in the numerator, and you can safely take this common outside and when you take this common outside, this whole thing is left with a 1 and then plus s C 1 2 plus C 2 r ds 1 r ds 2 divided by that DC term. So, it is 1 0 in the left half plane.

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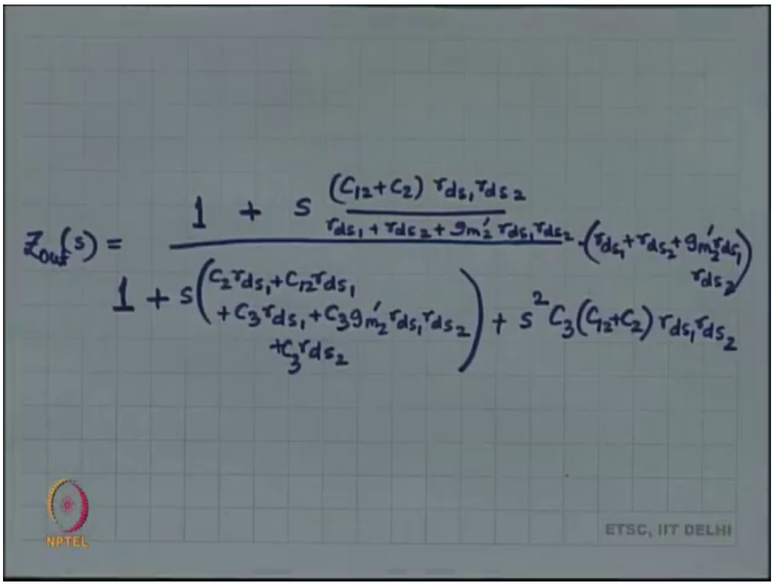

Sorry, I forgot to write the DC term r ds 1 plus r ds 2 plus g m 2 prime r ds 1 r ds 2, is the DC term. This whole thing times 1 plus s C r dimensions are correct s C r this is the numerator and in the denominator I have got 1, which is the only DC term plus s times C

2 r ds 1 C 1 2 r ds 1 C 3 r ds one then C 3 g m 2 prime r ds 1 r ds 2 r ds 2 C 3 r ds 2 plus s squared C 3 C 1 2 plus C 2 r ds 1 r ds 2 ok.

So, it is a second order denominator numerator is first order denominator is second order and I am sorry to tell you that if you try very hard to factorize you would not be able to factorize this, ok, trust me you cannot factorize this. You can do your Sridhar Acharya equation you know you will end up with some nightmare, but no need you cannot do it, alright. What is the point why did I do all of this if you cannot do it.

So, one important observation your transconductance had a right half plane 0 and a pole; these two, this was your transconductance expression DC term, right half plane 0, pole, ok. The output impedance the surprise, surprise has the same pole has the same 0 as this pole, they cleanly cancel out the 0 of the output impedance cancels out with the pole of the trans conductance. So, the transconductance effectively provides only this 0 right the pole of the transconductance will cancel out with the 0 of the output impedance when you multiply them together, ok.

The overall poles are contributed only by the output impedance. So, if you work out only the output impedance, right if you look at the output impedance and work out just the poles of the output impedance you know the poles of the system. If you work out the transconductance 0, that gives you the 0 of the system.

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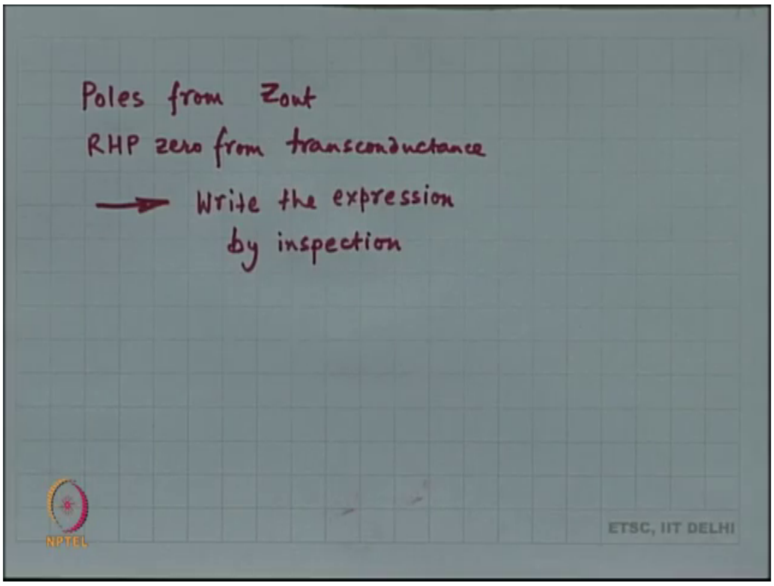

So, let us let us think of it this way instead of doing all of this you could have try to find poles from output impedance RHP 0 from transconductance, and anyway you know the DC quantity, right; by the way look at the DC quantity also there is some something interesting in the DC quantity also. This denominator cleanly cancels out with this right even in the DC that some little bit of magic is happening this quantity this DC term cancels out with this denominator which means you only need the numerator from the transconductance and that is your answer.

Overall voltage gain at DC is just this much coming from the numerator of the transconductance. Do not even have to look at the denominator the denominator cancels out with the output impedance, ok. So, that is also something interesting. In any case let us assume you know the DC stuff the DC stuff is easy to do then after doing the DC stuff you look at the output impedance and work out the poles just the poles, ok.

If we had a shortcut if we had a shortcut to find out just the locations of the poles, alright and if we had a shortcut to find just the location of the RHP 0 then I can just write out the whole expression, ok. So, this is the plan is it possible to do such a thing. Can I find just the poles from 0, just the 0 from the transconductance and write the rest of the stuff by inspection. The answer is it is possible to do such a thing we just have to know how to do it correctly all right.

So, let us stop here and we will continue in the next class.