## **Analog Electronic Circuits Prof. Shouribrata Chatterjee Department of Electrical Engineering Indian Institute of Technology, Delhi**

## **Lecture – 18 Differential amplifiers-II**

Hello, and welcome to Analog Electronic Circuits, this is lecture 18. And today we are going to talk about Differential Amplifiers, we will continue from where we had left in the last class.

(Refer Slide Time: 00:45)



So, in the last class we were talking about how to analyze the differential amplifier from a brute force point of view that is by treating the two MOSFETs as a pair of common source and common gate circuits. And, then doing superposition one by one and finding the result, this is what we were working on in the last class ok.

So, ordinarily what happens is there is a current source over here, tail current source this is drawn like this, right I have unfolded it and drawn it in one vertical column. So, it is the same circuit it is drawn in a slightly unintuitive way and, when I draw it in that unintuitive way, by the way this MOSFET.

You can think of this as a small signal PMOS also it makes no difference whatsoever gate and source right, it is just the organization of where is the source where is the gate that is all. Whether this is NMOS or PMOS it does not matter, in the ordinary differential amplifier this is an NMOS device that is why I have drawn it as an NMOS, but then when I draw it like this just for drawing sake it might make more sense to have it as a PMOS it does not matter ok.

In either case what we are going to do is, first we are going to apply  $v_1$  and ground  $v_2$ measure the 2 outputs, then we are going to apply v 2 ground v 1 and measure the 2 outputs this is the plan ok. So, let us ground v 2 and apply v 1 and, I need to measure the 2 outputs, this is the objective ok.

So, now what you have is no let us do the actually let us do the opposite let us not do this. Let us apply v 2 and ground v 1 ok, the reason why is because in this case it is a the second stage is nothing, but a common gate the gate is at ground all right fine. So, we are going to apply signal add v 2 measure the response over here, and then apply signal at v 2 measure the response over here.

Now, it so, happens that just one of these two experiments is enough, why because as you see the same current flows right through ok. So, let us recalibrate our thoughts. So, I am going to apply v 2 over here think of this as a PMOS, if you are not comfortable with this drawing ok. I am going to apply v 2 signal at the gate, I need to measure the voltage over here and, then what is on this side there is some gigantic resistance on this side, which is a lumped version of whatever you see looking in from the source all right.

So, in other words we need to do these experiments we need to so, this is what where we had left off in the last class, we need to figure out what is the impedance looking in at this point, what is the impedance looking over here. We need to also figure out what is this impedance right and what is this impedance all of these we need to figure out.

## (Refer Slide Time: 05:09)



So, let us do it once again, we need to figure out what is the impedance looking in over here ok. So, in either case we have to do the we have to follow the Norton equivalent model, in which case we have to first to the short circuit current, when we have to find out the output impedance of then we know the different voltages, this is the general plan of the Norton equivalent method.

Now, if we want to do that, then I need to find out the impedance looking in over here, when v 1 and v 2 are both had ground, I also have to find out the impedance looking in over here, when v 1 and v 2 are both at ground and these two are exactly the same. So, just one of the two experiments is sufficient so, what is the impedance looking in over here, looking up I see R and, looking down I am looking down into the drain right, when the source is terminated with this compound resistor with this compound impedance.

And what is that compound impedance? That compound impedance is also one of the two formally so, that is looking in from the source drain is terminated by R. So, I will see R plus r d s divided by 1 plus g m plus g m b let us call it g m prime g m plus g m b, it is just annoying to write that bracket and so on ok. So, this is the impedance looking down and by the way this is the same impedance looking up over here all right.

Then in the next step you are actually looking down from here right. So, what is the impedance? The impedance is whatever is on the source times the intrinsic gain plus that impedance plus r d s. So, if this is  $Z$  so, let us call this  $Z$  1, then the impedance  $Z$  2  $Z$  1

into the intrinsic gain plus  $Z$  1 plus r d s ok. So, it is one plus g m prime r d s times  $Z$  1 plus r d s, which is nothing, but plug in I mean this denominator will nicely cancel out it becomes R plus two times r d s ok.

Then finally, this is this is the impedance looking down, there is also an impedance looking up which is equal to R. So, this is what we have computed fine R times R plus 2 r d s divided by R plus R plus 2 r d s ok. So, this is the output impedance and, the output impedance is the same looking in from both of the outputs, the output impedance is computed when v 1 and v 2 are both ground. Now, we are going to do the short circuit current.

(Refer Slide Time: 10:02)



So, we can do either of the two short circuit currents and, we have to do the short circuit current ideally on both the outputs, but soon you are going to see that it is not required. So, let us work out the short circuit current, when I apply the input over here and let us measure the current over here. So, when I apply the input over here, what do I do with the other one, the other one has to be grounded I n is the short circuit current which I need to measure ok.

What are you going to do first? First what we are going to do is we are going to treat this whole thing as a compound resistor and, we already know it is value it is value is how much it  $Z$  1 R plus r d s by 1 plus intrinsic gain ok. So, this is some compound resistor, in which case my problem is simplified, my problem actually looks like this.

If you are not comfortable with the PMOS, you can always switch to an NMOS, but remember it is terminal for terminals. So, now this compound resistor will still be at the source and, you are trying to measure this current I n, this is something you can do is this a problem that you can solve should be able to solve this, this is a what is this a common drain circuit right and the voltage over here.

So, the common drain circuit had again, the gain was g m divided by it was smaller than 1 remember g m plus g m b also was came into the picture plus g d s plus g 1 all the non idealities ok. So, that was the voltage over here and, if that is the voltage that times G 1 is I n fine. So, this is my short circuit current I n is equal to g m times G 1, where G what is G 1, G 1 is R  $Z$  1 is this ok.

So, let us you know work with it a little bit let us modify for example, let us modify numerator and denominator by  $Z_1$  times r d s, g m plus g m b is nothing, but g m prime ok. And what is this it is the same thing as before right, it is g m prime times Z 1 g m prime times r d s plus 1 times  $Z$  1 ok, 1 plus g m prime r d s times  $Z$  1, where  $Z$  1 is this all right. So, this net denominator is nothing, but R plus 2 r d s, you after plugging in the value of Z 1 great.



(Refer Slide Time: 15:26)

So, what do you have finally, you have I n this is the short circuit current ok, when I have applied  $v$  2 the short circuit current is this times  $v$  2 is equal to  $v$  2 times g m r ds by so, much when I applied v 2 is this ok. So, what all have I done, I have looked at the circuit from this point of view, I applied v 2 over here and, then I found the output impedance looking in from this point and, I also found out the short circuit current going into the circuit ok, these are the two things I found out.

And therefore, what is the model the model is that you have got you apply v 2 and, as a result you are going to produce a voltage, which is minus of this current times the output impedance ok. So, this is the model, this is the Norton equivalent to Thevenin equivalent transformation ok.

And therefore, what is the voltage over here the voltage over there is minus of I n times the output impedance, what was the output impedance once again ok. So, this was the output impedance this factor nicely cancels out and, I end up with the output voltage to be equal to minus v 2 times g m r d s by 2 into R plus r d s ok. So, what have I done so, far.

(Refer Slide Time: 19:15)



Let us again recapitulate because, a lot of you are probably feeling a little lost over here. So, that is why I am recapitulating again and again, I have two inputs to this circuit v 1 and v 2 ordinarily, instead of two inputs I am going to do superposition so, I am going to do inputs one by one.

So, first we connected v 1 to ground and, we are now experimenting only by applying v 2. Now, when I applied v 2 I measured the voltage here and, I found that this voltage is minus v 2 times g m r d s by 2 R plus r d s this is what I have found so, far ok. This is R so, therefore, what is this current this current is just this voltage by R and, where is this current coming from it is coming from the top. So, therefore, what is the voltage here this is also R.

So, once again I have got a voltage v, let us say this voltage is v forget what it is value is I have a voltage v, this current is v by R the same current is flowing from the top that is also v by R. It is dropping across R and therefore, this voltage is v and therefore, what is the voltage here you started from 0 therefore, this is minus v ok, which means that this is plus v 2 times g m r d s by 2 into R plus r d s is this great.

Now, what we are going to do is we are going to apply v 1 and ground v 2 and when I apply v 1 and ground v 2, the same thing is going to happen just that, now it is because of v 1 and, now this node is going to respond with a voltage minus v 1 times g m r d s by 2 R plus r d s; And they because this node is now minus v 1 g m r d s right. So, now, I define this current this current is going to flow right through over here and therefore, the voltage over here is going to be plus v 1 times g m r d s by 2 into r plus r d s ok.

Now, I apply superposition and I apply v 1 and v 2 at the same time. So, now I will have effect of  $v \vee v$  1 as well as the effect of  $v \vee 2$  which is the sum of the two expressions all right. So, now, my entire problem is solved so, my entire problem now has I applied v 1 and, I applied v 2 on one side I got v 1 times g m r d s by 2 R plus r d s minus v 2 times the same factor, on the other side I got  $v$  2 times the factor minus  $v$  1 times the same factor.

In other words this was my circuit, when I applied v 1 over here and ground over here, I got a response minus of that response, I applied v 2 over here ground over here, I got a response over here and minus of the response over there ok, net superposition is the combination of both fine. If I apply v 1 minus v 2 as my input signal the signal part of the input is v 1 minus v 2. The interference part of the input is the average of v 1 and v 2 ok, this was the understanding that we had.

Now, if you look at each of these individual voltage output voltages, they are proportional to the signal both of them right because, these two factors are the same. So, one node is minus v 1 minus v 2 so, minus of v 1 minus v 2 times g m r d s by 2 R plus r d s, the other one is plus v 1 minus v 2 times g m r d s by 2 into R plus r d s.

If you will now look at the difference of the 2 voltages right, when the two factor will go away, this two factor will go away. So, it is now this top 1 the bottom 1 right, v 1 minus v 2 is common everything else is common so, the two factor will go away, it becomes double right the difference becomes double and, it becomes the difference of the 2 outputs is divided by the difference of the 2 inputs is nothing, but g m r d s by R plus r d s is this you understand.

So, we get exactly the same result, as we did in the earlier classes in the earlier classes, we took a shortcut we used symmetry in the circuit to come up with the answer. In this we did not use that shortcut right, we did a more brute force kind of analysis, but when I do these brute force analysis, I am I did not really draw the small signal model of the MOSFET did I, did I draw the small signal model of the MOSFET never right. The small signal model of the MOSFET was not even required right, I avoided it altogether right, I did not expand the small signal model of the MOSFET at all.

So, do not you know when you try to analyze something by brute force, even brute force can be smart it is not, it is not like you immediately split up the MOSFET into g m, g m b, r d s and then start solving the circuit no, that is not what we are going to do. We are going to look at impedances right, we are going to look at impedances what is the impedance looking in from the drain, what is the impedance looking in from the source and, accordingly we are going to analyze the circuit.

For example suppose this current source was not ideal what would you have done. Now, the general idea would be once again you are going to do both short circuit current as well as output impedance right, that is the general idea.

Now, when you are doing output impedance you look up you see R, you look down into the drain right, you see that the source is terminated by some impedance what is the source terminated by, on one side it is terminated by the impedance of the current source. On the other side it is terminated by whatever is looking up into the source.

So, you make put these two in parallel right combine them, now that is what is terminating the source now, when you look in from the drain, it is 1 plus the intrinsic gain times what is on the source plus r d s ok. And that whole thing is now the output impedance looking in from a node right is that ok, that is how you are going to analyze, it you are still not going to break it up into the small signal models of the individual MOSFETs ok.

Then in the next step you are going to say ok, now what is the short circuit current. So, you are going to apply v 2 over here and, then measure the current in this, how do you measure the current first you are going to apply v 2 and, then you are going to lump all of this as 1 resistor. And we already know how to lump all of this as one resistor, it is this resistance equivalent resistance, in shunt with the resistance looking into the source we know how to do that so, it is now lumped.

And therefore, now I know the voltage over here because, this is a simple common drain circuit. If I know the voltage over here, then I know the current that is the short circuit current ok. You see the line of thought we are not going to break it up into the small signal models at all right, these now there could be mismatch, these two MOSFETs could have been different that is ok.

These two resistors could have been different that is also right, nothing is going to bother you, because you are going to do it step by step, first you are going to find out the short circuit current over the output impedance over here, then the short circuit current over here, that will tell you what is the voltage we are here, when I applied v 2 and grounded  $v<sub>1</sub>$  ok.

Now, that you know the voltage over here, now you know the current the same current is going to flow, now you know the voltage here as well. So, you have two output voltages one over here one over here when you applied v 2 ok. And we did not worry about symmetry in fact, the circuit could be totally asymmetric this, these two MOSFETs need not be ideal the they equal, these two resistors need not be equal, this current source need not be an ideal current source.

The technique remains exactly the same, do you understand this ok, because sometimes when there is imbalance, this might be the easiest way to proceed forward ok.

Suppose these two MOSFETs are so, different that one is a MOSFET, the other is not even a MOSFET it is a b j t, will you be able to solve this, you should be able to solve it, because nothing much has changed ok. You are you are doing exactly the same technique over and over again, you are always trying to find out impedance levels according to

impedance levels, you are going to work out the currents and proceed forward all right. Now, next what we are going to do is we are going to talk about this resistor.



(Refer Slide Time: 31:36)

So, we have seen this before that this resistor is an irritate, it is a very annoying piece over, there why is it annoying because the value of that resistor should be as large as possible ok, largest possible value is infinity which means that resistor becomes an open circuit. But if you have an open circuit there, then the rest of the circuit does not work because you do not get a g m on the MOSFET ok.

So, that resistor is rather irritating, because for DC conditions you want a certain current to flow through that resistor, but for AC conditions you would rather have you know an infinite value ok. For the signal you would have you would like to have a very large value for that resistor right. So, it is a little annoying how do we proceed.

So, this resistor creates swing problems right, I mean you try to trade off on one side you have to have reasonable swing, on the other side you have to have bias current on the other hand you have to have gain in the amplifier right. So, all of these will conflict with each other, you would not be able to find a reasonable value for the resistor given all your specifications of the gain as well as the swing as well as the current and so, on and so forth. So, this resistor is rather annoying any way to replace this resistor.

So, what property do we want as far as DC goes operating point goes we want current going through the resistor ok, but as far as signal goes the higher the value of resistance the better for us because, then we will get more gain ok. So, these are this is what we need for that resistor over there, what can we do about the resistor.

What is it that you want to do? You want to replace it with an inductor is a is not a bad idea, if you replace this resistor with an inductor, then as far as DC operating point goes, it can carry any current that you want right the inductor is a short circuit ok, but for signal which is AC the inductor effective value is j omega L which is a very large impedance you pick your L value as large as possible and you will be fine ok.

So, this is certainly a solution it is a valid solution a lot of RF circuits, you use this technique, because in RF the value of omega is large and, when the value of omega is large you do not need very large values of inductance to achieve the desired response right j omega L is the impedance for the signal right, where omega is the frequency of the signal.

Now, at very large omegas you do not need a gigantic value of L; however, if you are not if your omega is not very large, if your omega is 1 kilo hertz 1 kilo radians per second, then you need a very large value of L going even into all the way into henrys right. And that will make the inductor very big and bulky and, in such a case it is not a good idea.

So, this is a solution it is a valid solution replace the resistor with an inductor, it is a valid solution when we go for RF circuits that is at very high frequencies, at very high frequencies you do not need large values of inductors. So, for example, giga radians per second 10's of giga radiance per second 100's of giga radiance per second.

In such a scenario you know let us say you want 10 kilos ohms over there right, 10 kilo ohms and you have your omega is 10 giga radians per second, the Z value you require is 10 kilo ohms in that case the L value 1 nanohenry is sufficient is that ok, 1 nanohenry no 1 picohenry. So, 10 giga radians per second j omega L forget the j part 10 kilo ohms.

So, L is 10 k by 10 giga the 10's cancel out kilo by giga is how much kilo by giga is 1 by mega so, 1 microhenry ok. So, 1 microhenry over there will be sufficient to give you 10 kilo ohms of resistance ok, 1 microhenry is a little too much, but you know 10 nanohenry's is reasonably small that we normally go ahead and implement, in which case you do not get 10 k you only get 100 ohms ok, but that is a solution that that is workable, this is something that is workable at very high frequencies.

At low frequencies this is not workable because, the inductor value becomes so, large that it is not you know it becomes big and bulky. So, it is not a workable solution at low frequency.

So, what do we do we still have the problem, we need to replace this resistor by something a little smarter, for operating point I need to conduct current and, but for the signal I need a large value of impedance over there, what is the answer, yeah inductor is done inductor works for RF what else any other solution current source great.

(Refer Slide Time: 39:19)



So, current source will work. So, something like this ok, this current source at DC it pushes a certain amount of current which is the bias current required for the device, but for a signal as far as signal is concerned, this current source is going to behave like a large resistor and that is perfect ok. The larger the resistor the better, the more ideal the current source the better, because then the output impedance on the top side.

So, if you look at the output impedance looking in from the node, you have the on the top side, you have very high impedance of the current source, on the bottom side you have r d s. So, your net output impedance is r d s and therefore, the gain is going to be

minus g m times r d s ok. So, this is the general strategy, this is a strategy that we are going to use.

So, instead of a resistor we are going to use a current source, how will you make a current source, how will you make a current source with a current mirror what is what is inside a current mirror a MOSFET ok. So, let us think of this as our common source amplifier, instead of having an R on top, let us have this structure on top. Once again this is the small signal incremental picture, I have drawn I have not drawn the entire circuit ok, we will draw the entire circuit. Let us first see if this small signal incremental picture is good is good enough for us ok.

So, what is the short circuit current, if I connect this to the output straight away, what is inside this MOSFET inside this MOSFET there is r d s and, there is a g m, but the g m does not matter because, gate and source are both had ground. So, v g s is ground. So, this MOSFET only has r d s oh came to ground.

Now, I put a short circuit to ground; that means, that r d s to ground is now irrelevant, this MOSFET also has r d s to ground and, that is also irrelevant. This MOSFET has g m because v g s is the input voltage. So, the it conducts a current g m times v g s and, this entire g m times v g s is going to come right through the short circuit all right that is the short circuit current experiment.

The second experiment is the output impedance. Now, when you look at the output impedance, you look in from outside you can look up you see r d s, you look down you see r d s. So, net effect is going to be r d s 1 in parallel with r d s 2. And therefore, the gain of the circuit is going to be oh, you are not happy with the gain of the circuit is it ok.

So, we do this Norton equivalent model by computing the short circuit current and the output impedance and, then in the next step if you want a voltage gain, then you convert the Norton equivalent into a Thevenin equivalent. And the way we do that one easy way to do that is to imagine the Norton equivalent model and, just see what is the voltage that you get.

This is the Norton equivalent model this is what you have modeled here, this current is g m 1 times v in, this impedance is r d s 1 in parallel with r d s 2, what is the voltage over here yeah forget the load, there is no load over here, we are trying to find the voltage gain. If you want to place a load go ahead place a load that will change the voltage gain right, now it is going to be r d s 1 parallel with r d s 2 in parallel with the load that is ok.

So, it is going to be the product of the transconductance g m 1 and the output impedance. So, the product of the transconductance and the output impedance, with the appropriate sign is going to be voltage gain, appropriate sign this sign could be positive or negative depending on the direction of this short circuit current, transconductance times output impedance. So, that is the voltage gain short circuit transconductance all right.

So, here the voltage gain is minus g m 1 r d s 1 in parallel with r d s 2 ok. So, this is going to be our plan instead of having r d s 1 in parallel with R, I am going to have r d s 1 in parallel with r d s 2 this is nicer because, R has limited values possible right, when I when I increase R. For example, the current decreases if I decrease the current then g m changes, all kinds of funny things happen they are all interrelated. So, let us not worry about R let us throw out R, let us replace it with a current source right, or in other words let us replace it with a MOSFET.

In which case my gain becomes minus g m 1 r d s 1 parallel r d s 2, both r d s 1and r d s 2 are comparable to each other, these are similar in terms of value hopefully all right. So, therefore, this is a small signal diagram that we want, this is the small signal incremental circuit that we want. If this is the small signal incremental circuit what is the full circuit. So, let us go back to few steps let us think of the common source amplifier and, then we will change it to the differential amplifier.

(Refer Slide Time: 46:54)



So, for the common source amplifier, we applied the input over here, I did not know the bias voltage of the input ok, I have to work out the bias voltage input is coupled on to the over there and, then I have to work out the bias voltage at the gate of this MOSFET that is also something I have to work out. So, let us say I have a current reference. So, I pull this current through this MOSFET this is the desired this is the desired drain current of the MOSFET to give me the right amount of g m of this particular MOSFET ok.

It is the current mirror now I also have to create the right biased voltage here. So, maybe what we can do is we can have it set up like this, is this yeah, I have thrown out the source degeneration all of that has been thrown out. Let us say I work out what is the right current that I need to achieve a certain g m, I make that current source. Now, I use that current source to work out what is the right value of the gate for the PMOS, I use the current source to work out what is the right value at the gate of the NMOS and, bias them is this ok, this is the overall plan ok, this would be a good plan for the single ended structure.

Now yeah so, this floating current source is a problem we have not really learned how to make a floating current source, we have current sources so, far we have are all terminated on one side to ground, but that is you can make a copy of the current and then copy it again.

So, for example you could do this so this is your final device. And then in the interim you can create another copy of it is this better, do you like this more there is one more job that needs to be done. This is one more job that needs to be done and that is to isolate the signal from the bias and to do that we have to place a large resistor in this path ok.

So, that the signal does not go inside the bias, otherwise the signal goes inside the bias and starts waving around this current, or you know the this voltage over here that should not happen. So, you place a large resistor over there, so, that the signal does not leak into the bias side here as well.

Is this rest of it is good all right. So, this would be a nice amplifier right. So, we have gotten rid completely gotten rid of all those bias resistors right, earlier we had you know four bias resistors RD RS RD was also the load right RS, then R 1 R 2 all of those are gone, we have just one resistor over here of arbitrarily large value ok, otherwise it is all **MOSFETs** 

You create the current such that the g m is good and, then you mirror that current create copies of the current right, why is this good you are using so, many devices so, many MOSFETs, earlier you had 1 MOSFET now you have. Now, 1 2 3 4 5 MOSFETs plus what is inside the current source and other four MOSFETs inside the current source 5 MOSFETs inside the current source. So, 5 plus 5 10 MOSFETs is this good. The thing is that this is great it is not just good this is great, why because you could throw out resistors and, on an integrated circuit resistors are no good just not good ok, MOSFETs are fantastic on an integrated circuit.

They are easy to make they require less steps, they are reliable they have all the good properties, a resistor means an extra mask it means extra cost it is gigantic, it is resistivity is not very large. So, it is going to be big all kinds of problems are associated with resistors, if it is possible to avoid a resistor, then please avoid the resistor so, this is actually great.

The next thing is in this the value of g m is controlled with the help of this current source ok, in the earlier design the value of g m was uncontrolled right. So, the entire structure is much nicer in this particular case all right. Now, what do we do in the in case of the differential amplifier.

## (Refer Slide Time: 53:48)



So, the differential amplifier we are just going to replace the resistors on top with MOSFETs that is it right, rest of the stuff remains exactly the same bias voltages of these two nodes are not a worry, why not we discuss this they are not a worry as long as they are such that this current source remains in saturation ok. It is dictated by the bias voltages over here are dictated by the current ok.

So, whatever the bias voltage here is the bias voltage here is going to be v g s t plus V T less than the bias voltage at the gate, bias voltage at the source operating point voltage at the source is v g s t plus V T less than the operating point voltage at the gate and, that is because this is the current source that is perfectly fine. So, the exact dc operating point voltage is that the gate are not really very important it as long as it is above a certain value ok.

Now, what about these P MOS's these P MOS need to conduct a certain amount of current, in which case we have to mirror just the right amount of current and, what is if this current source is I naught, then each of these have to be I naught by 2 ok. So, it is a current mirroring exercise, that is a I have a reference current I naught and, then in the ratio 2 is to 1, I need to create I naught by 2 in each of these ok.

And then I have to create this also. So, from the same reference current I will create one more copy and, then copy it again ok. So, it is now a copy of a copy so a structure like this, this is the general plan ok. This is the overall differential amplifier now right no

resistors only MOSFETs right, all the bias voltages are taken care of with the help of current biasing we are now biasing with the help of a current source and current mirrors.

Now, what is the value of this current source, this value of this current source is going to come from some constant g m biasing circuit, again 4 MOSFETs one resistor remember we discussed that ok. So, the entire structure is now current biased not biased with voltages anymore, they are biased with values of currents, the operating point is set by currents ok.

So, let us stop here this is basically the overall discussion of differential amplifiers right, we have to do one more very important differential amplifier structure, this kind of a differential amplifier is called an active load the load of the differential amplifier is active almost all differential amplifiers today are active load. So, the next what we are going to do is we are going to do a self biased active load, differential amplifier and that will close this particular topic ok.

Thank you.