Analog Electronic Circuits
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Lecture - 12
Multi - transistor amplifiers

Welcome back to analogue electronic circuits. This is lecture 12 and we are going to continue from where we left off in the last class. We are going to talk about multi transistor amplifiers.

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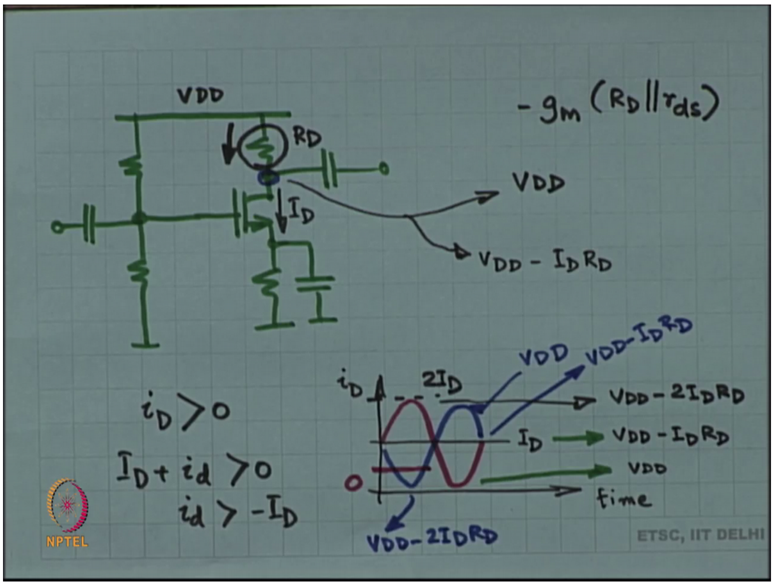

Now in the last class, we saw that when you have got your standard common source amplifier, this your standard common source amplifier. We saw, what we noticed was that a lot depends on this resistor R D right. The gain of the amplifier was minus g m times capital R D in parallel with R d s. So, if R D is small, then the gain of the amplifier drops but then, the signals swing that I have at the output that is also dependent on R D right. So, the, if I have got a current I D over here, then the voltage over here DC voltage over here is V DD minus I D R D right. If the current is capital I D, then the maximum signal current assuming a sine wave is small is also capital I D right.

Because the signal current, the total current cannot go below 0; alright total current cannot go below 0, at all types small i sub capital D has to be greater than 0 at all types; this was the second condition for evaluating swing limits; small i sub capital D is capital

I D plus small i d which means small i d always has to be more than capital minus capital I D alright; which means that if you say that the current is a sine wave writing on the top of. So, the signal current is a sine wave if you say the total current is a sine wave writing on top of capital I D if you say that. Then the least value is 0, therefore the maximum value is 2 times I D, maximum value is the 2 times I D. What is the minimum value? Minimum value is 0. If the minimum value is 0, if it is writing on top of capital I D, then the peak value is 2 times I D, is not it? Yes ok.

So, the peak value is 2 times I D, minimum value is 0. Automatically, that is the current coming through R D. So, when the current coming through R D is 0, the voltage over here is just V D D. When the current over here is I D, then the voltage over here is V D D minus I D R D. And when, so this is the mean. So, at this point of time, you got V D D minus I D R D as the voltage. When I D is so much, you have got a voltage of V D D at the drain and when i d is 2 times I D, you have got voltage at the drain of V D D minus 2 times capital I D R D is this. So, if small i d is a sine wave, then the maximum amplitude of the drain voltage. So, the drain voltage is nominally V D D minus I D R D and then it is riding on top of that ok.

So, if you ask me to draw a waveform for the drain voltage, it is going to look like this. This peak is at V D D. The nominal value is V D D minus I D R D and the lowest value is the trap is at V D D minus 2 I D R D. The amplitude is just I D R D; is it ok? Of course, I have thrown out the load right. There is no load over here. Once you put in a load things will change, agree it. So, the amplitude of the drain voltage is nothing but capital I D R D.

So, effectively, what it means is that you want R D to be large to get good gain right but when R D is large, other strange things happen; I D becomes small; for example, right. In this calculation, we did not look at V D S at all right. V D S also has some limits. So, this is this is assuming that V D S does not limit you. V D D minus 2 I D R D is assuming that V D S is not going to limit you right, for this particular situation, V D S could have limited you ok. So, in any case, R D is something of great importance over here. R D needs to be large to give you good gain and if R D is large unfortunately, V D S starts limiting you. So, you do not get good spin although I D R D seems like a it is going to be the amplitude. However, V D S is going to cut you off over here right. So, that is not really going to be the amplitude, final amplitude, alright.

So, we have this problem right; gain is being limited effectively. What can will do? One simple thing that we can do is have two amplifiers alright, have a cascade of two amplifiers.

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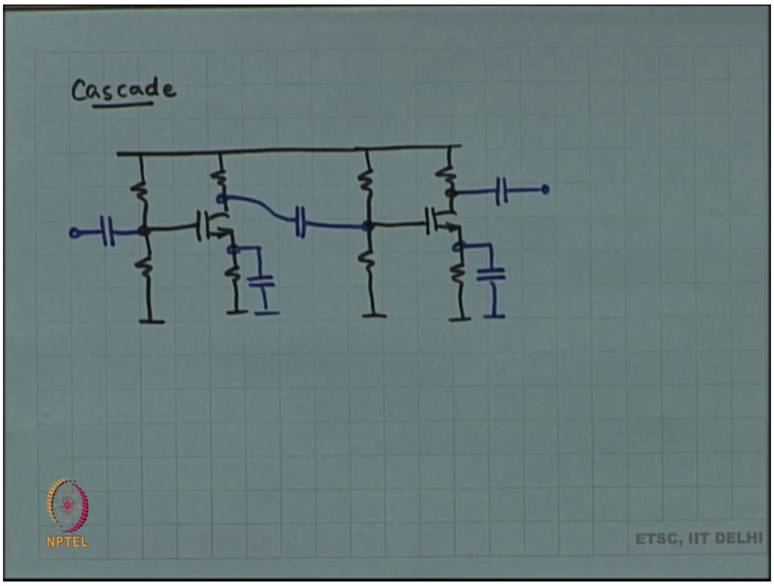

So, this is your bias circuit. Use the same bias circuit for the other one right and then couple the signals in an out ok. So, this is a possibility. So, now, you get the benefit, you get gain of one stage is so much; gain of the second stage is so much. So, the total voltage gain is the product of the two voltage gains. Is this ok? There is some little bit of loading over here which you will have to account for; this, this resistor over here; these two can be made arbitrary large.

So, this should not matter. You can make these mega ohms, Giga ohms, nothing is going to change. This is ok; this is a cascade of two amplifiers. Now this is one possible strategy. What about the swing? The swing problem is just the same right. If you can swing so much, I mean nothing changes as far as the swing is concerned. You are limited by R D, I D times R D right. On one hand you want I D to be small right; R D to be large. On the other hand swing is being limited alright by both I D and R D ok. So, this is one possibility in any case. There is a second possibility which is just like the cascade but we call it the Cascode.

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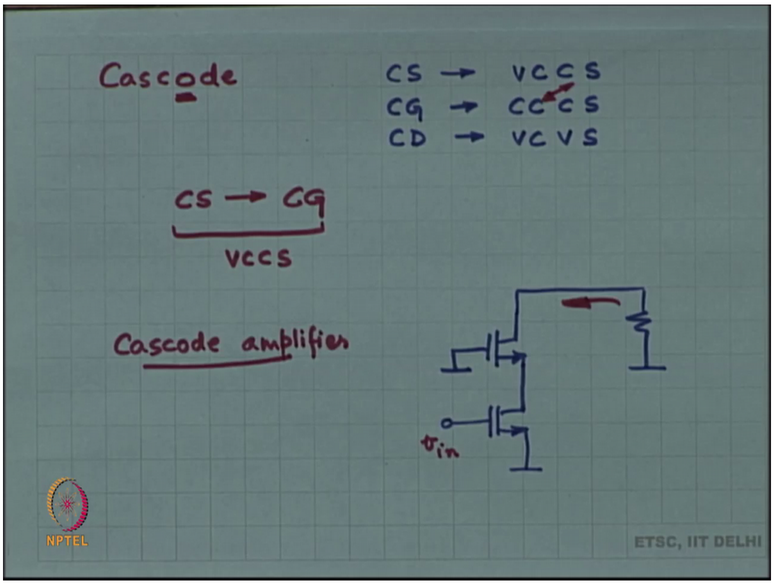

Cascode

CS → VCCS
CG → CCCS
CD → VCVS

$\underbrace{CS \rightarrow CG}_{VCCS}$

Cascode amplifier

There is an o over there and you would not find this word in any dictionary right. This is not an English word. This is a word that is used only in analog electronics. It is call the cascode. And what we do over here is, remember we had one point of time had discussed these voltage sources and current sources.

We said that a voltage source is a circuit which has a low output impedance, a current source is a circuit which has a high output impedance; likewise, we said that something which is voltage controlled will have a high input impedance; something which is current controlled will have a low input impedance right. And then, we referred to the common source as possibly a Voltage Controlled Current Source and then we referred to the common gate as possibly a Current Controlled Current Source and then we referred to the common drain as a Voltage Controlled Voltage Source right. Should be had these three possibilities. We did not really make a current controlled voltage source but that is you can make with the combination.

Now, what do we do? One possibility is to have a common source upfront followed by a common gate. So, common source is a voltage controlled current source, common gate is a current controlled current source. So overall, you will still have a voltage controlled current source. So, this is V C C S, this is C C C S. So overall, you will get a voltage controlled current source hopefully a better one, alright. So, this is also a multi transistor amplifier. So, I have got my, I am drawing the only the small signal picture right now because I first need to understand what the circuit is, then I will draw the biasing and so on.

So, this is the common source amplifier and this is where you place the load but instead of load what we are going to do is we are going to put a common gate structure over there and the common gate structure looks like this. And then finally, you have a load ok. So, the third process is that if I have a common gate structure followed following common source structure then, maybe I would have made a better current source, a better controlled current source alright. So, this is going to be our next topic, the cascode amplifier. Now, this may or may not solve the earlier problem with R D. In fact, it is most probably not going to solve this problem of R D at all but what we are going to see is that we do not really always have to worry about voltage gain; maybe, I mean for example, in this circuit here input is a voltage output is a current right. It is a voltage controlled current source right.

So maybe, we can even work in the current domain instead of the voltage domain right and if you are working in the current domain, may be if the swing is not going to be that much of a problem as before. Because we were working in the voltage domain over there swing was a problem and R D of limiting us right. If I start working in the current domain, may be things are not going to be as difficult right. But anyway this will open up a possibility of that happening. Why we are also studying this is because this is an alternative that needs to be explored it. So, happens that the cascode amplifier is a very popular circuit ok, alright anyway.

So, this is the structure; I am going to apply input over here and the output is this correct. Alright, if you want the output to be a voltage, so be it right. It is going to be this current times this resistor; that is the voltage. But right now, let us worry about only about the current, worry only about the current.

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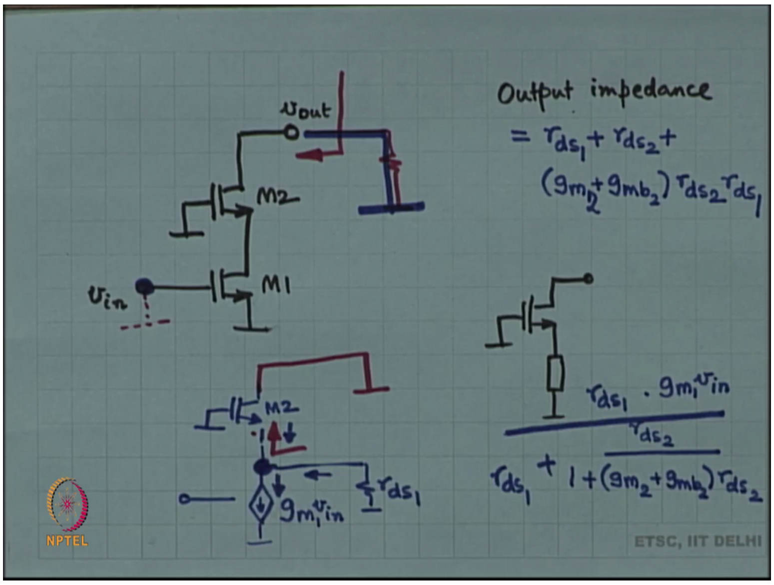

Output impedance

$$= r_{ds_1} + r_{ds_2} + (g_{m_2} + g_{mb_2}) r_{ds_2} r_{ds_1}$$

$$\frac{r_{ds_1} \cdot g_{m_1} v_{in}}{r_{ds_1} + \frac{r_{ds_2}}{1 + (g_{m_2} + g_{mb_2}) r_{ds_2}}}$$

So, the where we work in this course is using the Norton equivalent method. And what is the Norton equivalent method? It comprises of two things; evaluating a short circuit current and evaluating an output impedance. So, let us start with the output impedance. What are you going to do? You are going to null the input voltage source and then you are going to looking from the output and see how much resistance do you see, do you find.

Now, when I null this input voltage source, what happens to this MOSFET. This MOSFET has two elements inside; one is a g m times V G S, the other is r d s, there is also g m b but both body and source are at ground for this particular MOSFET. So, far M 1, g m b, v b s; v b s is equal to 0. So, g m b does not matter for the output impedance calculation gate and sources are both at 0. So therefore, gate to source voltage is 0. Therefore, g m times v g s is also 0. So, it does not matter.

So, all that this MOSFET has as far as output impedance is concerned is an r d s, r ds 1 and then you are looking in from the drain and hopefully you have memorized the formulae that I had asked you to memorize. You did right ok. So, if you look in from the drain, the gate is terminated to ground the resource is terminated to ground through a resistor r ds 1 and therefore the output impedance is nothing but r ds one plus r ds of this MOSFET plus the intrinsic gain of this MOSFET times r ds 1 remember the formula. So, when I looking from outside I see gm of this MOSFET times r ds of this MOSFET times this register plus r ds of this MOSFET plus this resistor ok. So, intrinsic gain of this MOSFET times this resistor plus r ds of this plus this resistor. That is exactly what I did; intrinsic in gain times r ds 1 because this MOSFET is only r ds 1. There is nothing else in it plus r ds 1 plus r ds 2 ok. You need to memorize this formula, you have not clearly done, output impedance is done.

Now, what we have to do we have to find out the short circuit current, short circuit trans conductance or short circuit current whatever you want. So, to find out the short circuit current, we are going to first ground this; then, we are going to apply a input v in and see what happens. Right, now when I am applying this input v in if you think about this MOSFET, there is g m 1 inside, this which is going to respond with g m 1 times v in because v g s is v in. So, g m 1 times v in and then there is r ds 1 to ground alright. And the rest you are just looking at the source of M 2. So, let us redraw. I do not like this redrawing. You should draw this in your mind. This g m 1 times v in and there is r ds 1; g m b of number 1 device does not matter because both body and source are at ground.

So, I am not going to draw that. Thank you; and then above this there is a source with the gate at ground. So, you can use the other formula to think about what is on top right. So, as far as this structure is concerned, looking up from here what do you see? What do you see looking up use your second formula, you will find that looking up you see some impedance right. We will do the second formula. So, then you redraw this is some impedance looking up. So, if I have g m 1 v in over here, this is the source of current. This current is saying two things; on one hand, it is seeing r ds 1. On the other hand, it sees some impedance right.

So, there is going to be a current division; a portion of this current is going to go the other way into this source and whatever goes into the source, eventually makes it is way into the short circuit right. So, that is your answer. So, g m 1 v in and then you have to do a current division between r ds 1 and what you see into the source. What is it that you see into looking into the source? So, if you think about the other formula looking into the source, you normally see r ds of this MOSFET plus whatever was on the drain; nothing is there on the drain. So, it is just r ds 2 divided by 1 plus intrinsic gain of the MOSFET ok. This is the impedance that you see looking into the source of M 2 ok.

So, how is current going to divide? This current is going to divide into two pieces; one portion is going to go through r ds 1 and another portion is going to come through M 2. What is the portion that is going to come through M 2? It is proportional to the other resistor r ds 1 ok. So, this fraction of the whole thing is going to proceed into the short circuit and the whole thing is nothing but g m 1 times v in. You see how I constructed the expression. I started with g m 1 v in and then this going to be a current split right, current division current is going to divide proportional to r ds 1, right. So, g m 1 v in times r ds 1 divided by r ds 1 plus this impedance, in this impedance is so much. So, I just wrote it down alright and then you go step further and simplify.

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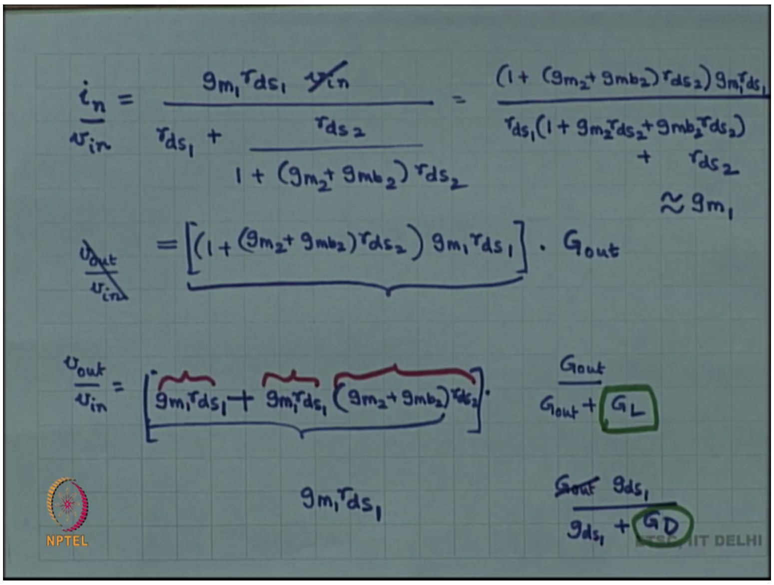

$$\frac{i_n}{v_{in}} = \frac{g_{m_1} r_{ds_1} \cancel{v_{in}}}{r_{ds_1} + \frac{r_{ds_2}}{1 + (g_{m_2} + g_{mb_2}) r_{ds_2}}} = \frac{(1 + (g_{m_2} + g_{mb_2}) r_{ds_2}) g_{m_1} r_{ds_1}}{r_{ds_1}(1 + g_{m_2} r_{ds_2} + g_{mb_2} r_{ds_2}) + r_{ds_2}} \approx g_{m_1}$$

$$\cancel{\frac{v_{out}}{v_{in}}} = \underbrace{\left[(1 + (g_{m_2} + g_{mb_2}) r_{ds_2}) g_{m_1} r_{ds_1}\right]} \cdot G_{out}$$

$$\frac{v_{out}}{v_{in}} = \left[\underbrace{\overbrace{g_{m_1} r_{ds_1}} + \overbrace{g_{m_1} r_{ds_1}} \overbrace{(g_{m_2} + g_{mb_2}) r_{ds_2}}}_{g_{m_1} r_{ds_1}}\right] \cdot \frac{G_{out}}{G_{out} + \boxed{G_L}}$$

$$\frac{\cancel{G_{out}}\ g_{ds_1}}{g_{ds_1} + \boxed{G_D}}$$

So, the Norton equivalent current is nothing but. And you can multiply numerator and denominator by this factor and this is the trans conductance and then the next step is to multiply this whole thing with the output impedance and that will give you v out should you that; no, the output impedance and the load. So, over here we did not consider the load right. We kept the load outside.

So, let us not do this v out by v in. Let us stick to what we have right. We have the short circuit current. So, this is the trans conductance ok. The trans conductance of the circuit is so much. The output impedance is so much. So, this is the complete Norton equivalent model of the circuit all right. Now, if you want to find out the gain then the output impedance has to be modified a little bit by the load impedance right. I mean you can bring the load impedance inside and then find the output impedance, if you want to look at the voltage gain ok.

So, let us just quickly do that. So, it is this output impedance in parallel with the load impedance. Now, notice something, this output impedance over here happens to be r ds 1 plus r ds 2 plus g m 2 r ds 2 r ds 1 plus g m b 2 r ds 2 r b s 1. So, this output impedance is the same as this denominator ok, just pointing it out alright. So, if you want to write this as g out or rather if you want to write this as r out, you have some numerator ok. I, I need to point out one more thing, hang on. So, this numerator and divided by r out fine. The output impedance happens to be this in parallel with the load. So, the total output impedance is 1 by g out plus g l.

So, this quantity times g out output conductance plus load conductance ok. So, this is how it is going to look like, more or less. Now, I want to point out one more thing. Look at this expression, this expression is 1 plus g m 2 plus g m b 2 r ds 2 1 plus g m 2 r ds 2 plus g m b 2 r ds 2 times r ds 1. So, if you look at this numerator, then this denominator is the same numerator by g m 1 plus r ds 2. Also notice that this factor is very large, r ds 1 suppose the intrinsic gain of a MOSFET is 20, suppose ok. So, g m 2 r ds 2 is something like 20 ok. It is more than 20, 100 fine.

So, let us say intrinsic gain is 100. So, this entire factors is of the order of 100; 1 plus 100 right, g m b 2 is smaller right; 100 times r ds 1 plus r ds 2 fine and what have you got here? You have got exactly the same thing 100 times r ds 1 times g m 1. So, you have got 100 times r ds 1 plus r ds 2; 100 times r ds 1 times g m 1; r b s 2 is also you know a large number, but not as large as 100 times r ds 1. So, therefore, the entire denominator is dominated by this first portion; r ds 2 is nothing compared to 100 times r ds 1 which means you can forget about the second one, second term altogether in which case these two are going to cleanly cancel out and all you are going to be left with is just g m 1. What does this mean? This means that the output trans conductance the short circuit trans conductance is just g m 1. What is going on over here? I will be missing something, take a look again. How did we find out the output trans conductance the short circuit trans conductance; those of voltage v in it got converted to a current g m 1 times v in.

Now, g m 1 times v in saw r ds 1 on this side and it is saw another impedance on the other side and there was a current division. On this side, it is r ds 1 on the other side, it is r ds 2 divided by the intrinsic gain which is very small alright; so, 100 times smaller than r ds 2. So, one side it is r ds 1; on the other side, it is saw an impedance 100 times smaller than r ds 2. Which side is it going to pick, it going to go through out the source right. So, all of the current almost all of the current is going to go into the source because current like the lower impedance ok; very small portion is going to go through r ds 1 because r ds 1 is much much larger than whatever you see into the source ok, 100 time larger. So, 1 percent current is going to go this way, 99 percent current is going to go this way, 99 percent current is going to go that way; which means that this g m 1 times v in almost all of it is going into the short circuit. So, I applied v in, I got a current g m 1 times v in going into the short circuit which means that the trans conductance is nothing but g m 1 alright.

So, you see what happened over here? You saw what happened, going into the source of M 2. So, I applied v in, it produced current. Now this current is going to be shared; portion of it will go through r ds 1, portion of it will going to the source. But r ds 1 is always much larger than what you see looking into the source; so, almost all of it is going to go into the source of M 2 ok. So, whatever current it produced all of it went into the short circuit, fine. So, therefore, the trans conductance is approximately g m 1. Is it ok? Do you do you understand what happened over here, alright. So, this is something very important that happened, you almost sent all of the current.

So, this is a current controlled. This is this is what happened over here, M 2 is a current controlled current source; remember, this is a common gate structure. So, whatever current went inside, it came outside right. You apply the current; you got a certain current at the output the same current. The input impedance of M 2 is low right. That is why it was controlled. M 1 is a voltage controlled current source. It produced the current right, it produced the current and that current has the nice low impedance to go through, it came out through the drain ok.

Now effectively, what happened the other thing that happened is that the output impedance got multiplied by a large amount. The output impedance of the entire structure became 100 times. So, it is a nice current source because it is output impedance is very large. So, every, at every step you are seeing over here that it is a very nice current source. It is current controlled over here and then it is a current source because the output impedance is very large. Overall, the circuit becomes a nice voltage controlled current source ok. I have applied voltage, I got g m times the voltage as the current and the output impedance is very high which means it is a very nice voltage controlled current source alright. Now, we just have to work out the gain.

Let us assume that G L is very large ok, in which case this is equal to 1; just let us assume. Then, what is the voltage gain? The voltage gain is just this other factor which is g m 1 r ds 1 plus g m 1 r ds 1 times g m 2 plus g m b 2 r ds 2 ok. So, it is the intrinsic gain of transistor 1 plus the intrinsic gain of transistor 2 times the intrinsic gain of transistor 2. So, intrinsic gain of transistor 1 plus intrinsic gain of transistor 1 times intrinsic gain of transistor 2. So, this is the intrinsic gain of transistor 2, this is. So, let us say each of these intrinsic gain is a factor of 100.

So, 100 plus 100 times another 100 right. So, what have you got? You have got 10000 ok. So, this entire potion is a factor of 10000 times G out by G out plus G L and I already know that G out is very large, oh no G out is very small. I am sorry G out is very small, R out is very large, R out is very large, G out is very small. So, it all depends on G L. So, if I make G L; for example, if I make G L, 10 percent of G out 0.1 of G out, then this is more or less a factor of 0.9 or something and I get I end up with the voltage gain of 10000 any questions so far. You got to have one question. I just did something funny, yes.

So, just like last time, when we did the common source circuit, everything kind of depended on the value of r d, all of you know the intrinsic gain was g m times r ds but then at the end, there was a parallel with r d ok. So, if you think of it as an intrinsic gain kind of situation, then for the common source amplifier your intrinsic gain would have been g m 1 r ds 1 and the output impedance would be you know whatever your G out is, G out would be something, it is actually just r ds g d s, ok.

So, this G D would create spoilsport, would be a spoilsport. This would spoil the entire thing ok. So, the same problem is cropping up over here. How are you going to make this high resistance? So, what we are going to do is, we are going to take a break from the study of these multi transistor amplifiers. We have a problem right; our fundamental problem is how are we going to make this large impedance R D or this large impedance in this case, in this case R L right, how we going to make this and yet satisfy you know some reasonable swing limit and so on right. All the other requirements of the amplifier;

you have to satisfy those as well right. How are we going to satisfy both of these requirements? Number 1, a large value over here and number 2 some reasonable swing limits of the of the amplifier ok. So, we have this quandary and the answer to this quandary is to avoid the use of resistors altogether right. You can not stops, stop using these resistors ok. Do not use resistors, instead let us try to use a MOSFET; for example, think of the following.

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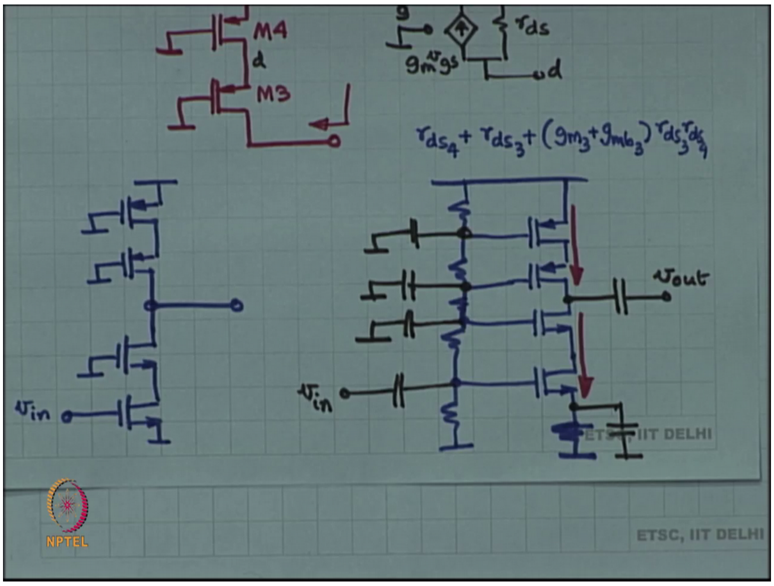

Let us think about this ok. I have replaced the n mos with p mos devices right it is still a cascode kind of structure. I have replace the input voltage with a ground and this is in the small signal picture ok. This is in the small signal picture. What is going to be the input impedance of this? It is going to be, so this MOSFET is going to behave just like a resistor r b s 4, the small signal picture of the n mos and the small signal picture for the p mos terminal for terminal is exactly the same. There is no difference; do not interchange the arrows, do not make them opposite; no nothing right terminal for terminal, it is exactly the same.

So, here I have got source on the top right; the small signal picture of this particular p mos should look like this. Gate is at ground, drain is over here, this is the drain and the sources is at ground right and g m times v g s should point upwards because that is the way does right, g m times v g s, it always points towards the source ok. This is the first time I have drawn a p mos, alright p channel MOSFET, just like the n channel MOSFET.

Just that the current the absolute current direction is reversed ok, the dc operating point current direction is reversed ok. So, do not worry about that, the dc current flows from top to bottom but the small signal picture terminal for terminal is exactly the same ok. And what this means is that the gate is at ground, the source is at ground therefore, this g m v g s is useless. It is not doing anything which means between drain and source all you have is r d s.

So, M 4 is nothing but R D 4 and then you are looking into the drain of M 3, source of M 3 is terminated by r ds 4; use the formula to find out the impedance looking in and the impedance looking in with the same old formula is nothing but r ds 4 plus r ds 3 plus intrinsic gain of M 3 times r ds 4. So, here I have a situation where I can create a large impedance. Why not place this large impedance over here? Why not place that large impedance on the top side? You see what I mean, you do not have to make a resistor using a resistor. You can make a resistor using MOSFETs, what is stopping you. So, for example, over here, I could have had this structure ok.

So, this my old thing, this is the cascode amplifier and this is the load of the cascode amplifier. I needed a very large resistor as the load of the cascode amplifier right, comparable to the output impedance of the cascode amplifier. The output impedance of the cascode amplifier was r ds 2 plus r ds 1 plus intrinsic gain of these MOSFET times r ds 1, the impedance of the load over here is r ds 3 plus r ds 4 plus intrinsic gain of MOSFET number 3 times r ds 4 ok. So, these are going to be comparable to each other. Why, because the same bias current is going through right. If lambda is similar, then r ds is going to be similar ok.

So, now, let us draw the complete circuit right. This is understood so far. So, this is the overall, this is the small signal picture only right because all these are connected to ground. These should not be connected to ground. These should be connected to some reasonable voltage right. I do not yet know what that voltages but maybe, may be just maybe, you could think of some resistive network which generates these voltages. You need one more resistor ok. You want to have source degeneration on the bias ok, very good. And then, in the small signal picture, you want it looking exactly like this. So, this has to be capacitively connected to ground; this one capacitively connected to ground; this one capacitively connected to ground and then this source degeneration has to be shorted out in the small signal picture. And then, you have to capacitively connect this to the input and you take the output correctly through a capacitor. Is this ok? this is 2 daunting.

So, from one transistor, I have suddenly jumped to four transistors ok. You will see even bigger jumps later on but I need you to absolutely memorize formula 1 and formula 2 right; looking into the source what is the impedance that you see? Looking into the drain what is the impedance that you see? Those two formulae are vital if you want to proceed with this course. If you are not willing to memorize those two formulae, then drop out ok. It is just to formulae right and they are quite simple. All you have to remember is, looking into the source the impedance drops by the by a factor which is 1 plus the intrinsic gain of the MOSFET. Looking into the drain, the impedance multiplies by a factor right which is the intrinsic gain of the MOSFET. This is all that you have to remember and then accordingly you can write out the formulae's, but do memorize them and they are important right.

This entire analysis of the circuit is resting upon those two formulae right. The output impedance of the top portion is high. The output impedance of the bottom portion is high. Then the short circuit current the top portion is shorted out for the short circuit current here, the short over here. So, the top portion is irrelevant but as far as the bottom portion is concerned, that is relevant right. You are going to look into the source and there is going to be a current division portion is going to come out through r ds 1; portion is going to go into the source of MOSFET number 2 right. Looking into the source of MOSFET number 2, you will see a low impedance therefore, the current would love to go into that note right, go into the source right.

So, g m 1 times V g s 1, g m 1 times v in is the current generated in M 1 that entire current, almost all of it goes into the source of MOSFET number 2 and comes out through the short circuit. So, the trans conductance is just g m 1. The output impedance is large in parallel with large. The overall voltage gain is g m 1, the trans conductance times the output impedance which is large in parallel with large. So, you have a large voltage gain. So, this is the idea.

So, this is a kind of set up of the cascode amplifier. I am not yet promising that this works right. We have to really think about what this bias network looks like, I am in maybe it is just five resistors like I have drawn, but it is probably likely that these five resistors will have to be worked on a little bit because we have to make sure that all of these carry the same bias current right. The value of the current has to check out otherwise very strange things are going to happen right. It has to have equal bias current through all of them, not I mean you cannot have some bias current through this one and some other bias current for the next one. That is not really going to work out right and then at the same time, you have to make sure that V D S 1, the drain to source voltage of each of these devices is enough to make sure that the devices remain in saturation at all types. So, this we have to think about a little bit.

We will do that in the next class, and then we will move on to the next topic.

Thank you.