Analog Electronic Circuits
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Lecture - 11
Swing limits contd., Multi transistor amplifiers

Welcome back to the Analog Electronic Circuits course and today's lecture number 11. And the plan for today is that we were going to continue where we had left off on the problem of swing limits and, we are also going to look at a few multi transistor amplifier circuits ok. So, in the last class we were looking at this particular amplifier structure.

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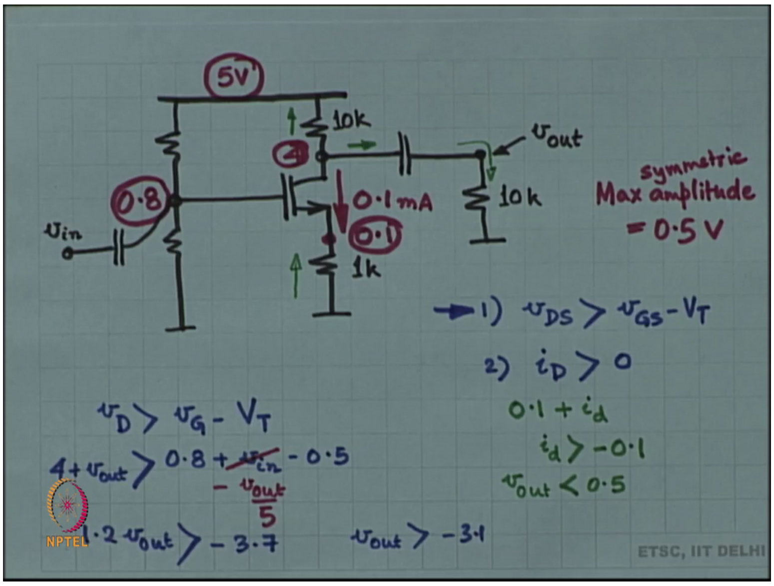

So, we had chosen 10 k over here, 10 k over here, 1 k over here, right and as far as I remember this was 5 volts, this current we had chosen was 0.1 milliamperes and, the voltage over here was 4 the voltage over here was 0.1 and the voltage at the gate was 0.8.

So, this was my setup in the last class and, we are always going to so, this particular amplifier had a gain of approximately 5 it is actually less than 5, but let us assume that the gain is 5 minus 5. If I increase the input by 1 millivolt then the output is going to decrease over here, it is going to decrease by 5 millivolts right. So, you will see minus 5 millivolts at v out ok. So, this was what we had done, we are chosen some values of k and etcetera, but those do not matter to us right now all right.

Now, when you are evaluating the swing limits there are two things that you have to be reminded of item number 1 is that v D S has to be greater than v G S minus V T at all times, this is item number 1. And item number 2 is that i D has to be greater than 0 at all times these are the absolute currents and voltages right, absolute means the sum of the DC operating point and the small signal and the incremental signal.

Now, this in just by the way the incremental signal need not be small all the time right, for small signal analysis to be valid it has to be small, but otherwise it does not really have to be small, it is just an incremental signal. And the incremental portion always goes through the is a short circuit through the capacitor right. So, whatever it is small large it really does not matter ok.

So, from item number 1 what did we get, we said that v D S has to be greater than v G S minus V T of this both the source as well as the drain are moving right. This source is not really static, if there is a signal current on i D then the source voltage will also change; however, if you look at equation 1 carefully you will see that the source voltage will cancel out from both sides, which effectively means that v D has to be more than v G minus V T ok. And the gate voltage is the dc gate voltage 0.8 plus v in V T is 0.5 that is what we had picked in the last class and, what is the drain voltage drain voltage is 4 plus the extra drain voltage incremental drain voltage is v out, but v out is minus 5 times v in ok.

So, instead of writing v in over here let us write minus v out by 5 and, then put the v outs together put the constants together and, what do you end up with. So, you end up with 1.2 v out has to be greater than 3.7 minus 3.7 ok, or in other words v out has a lower limit of minus 3.1 or so, whatever it is right 3.7 by 1.2 fine is this ok. So, this was this is what we had done in the earlier class this was evaluating the first condition ok.

Now, right now I am doing it very mathematically, but these are limits that you can actually observe on the circuit itself, you do not really have to do it mathematically all the time right, this mathematics is not really that important all right. The second one is i D has to be more than 0. So, how do we evaluate that i D is nothing, but capital i D which is 0.1 milliampere plus small i d so; that means, this basically means that small i d has to be more than minus 0.1 milliampere all right, which basically means that the signal current only the signal current going up maximum signal current going up is 0.1 milliampere ok.

Now, where is this 0.1 milliampere go, where is this going to go it is going to go up through the source and, then split in this case two equal portions all right 0.05 milliampere and 0.05 milliampere right, it is going to split up into these two equal portions and 0.05 milliampere is going to drop across 10 k over here for example, and how much voltage will it create at v out 0.05 times 10 k is 0.05 milliamperes times 10 k it is 0.5 volt. So, the maximum amplitude on v out is 0.5 volt because, the maximum current that it can draw this load can draw out of the drain is 0.05 milliampere and, when that 0.05 milliampere comes through 0.05 times 10 k you get half a volt of amplitude is this ok.

So, if v in was to be a sine wave suppose, suppose v in was to be a sine wave, then what would be the maximum amplitude that you would have at v out, v in is a sine wave so, v out is also a sine wave inverted ok. So, gain times all right the amplitude will the amplitude of v out is the gain times the amplitude of v in all right, it is a sine wave purely symmetric.

Now, v out is allowed to go down to minus 3.1 volt on the other hand v out is allowed to rise by a maximum of 0.5 volts. So, the maximum it can dip is minus 3.1 maximum it can rise is 0.5, what does this mean, what is this symmetric maximum amplitude of v out it is only 0.5. So, symmetric amplitude at v out so, we out can go up and down by 0.5 volts so, the largest sine wave that you can see at v out is 0.5 volts, you want see a larger sine wave than that is this all right.

So, this kind of summarizes so, we did two examples one actually more than two three examples, we did 1 without the source 2 of them without the source degeneration 1 with the source degeneration, And we saw in different ways what happens what are the maximum swing limits at the output note, all right there are all kinds of other topologies, you could for example, I have a common drain circuit common gate circuit all of those, they will also have similar computations with swing limits you will have to go through those computations right.

There could also be questions, where you say you declare in advance that I want a maximum symmetric amplitude of 0.5 volts, what are the resistor values ok, or you could say that we need to equalize another problem would be that let us try to equalize these 2 limits. There is no reason for v out to have a larger dip, then a crest right the the size of the crest and, the size of the trough, these should be equal.

So, let us try to equalize the maximum amplitude on the positive as well as negative side ok. And therefore, what are the resistors going to be right. So, there are variety of problems that you can have with these swing limit calculations ok, you will get some practice with these problems in the assignments yeah.

Next what we are going to do is we are going to move on to the next topic, we are going to try to make amplifiers or structures circuit, structures which have more than one transistor.

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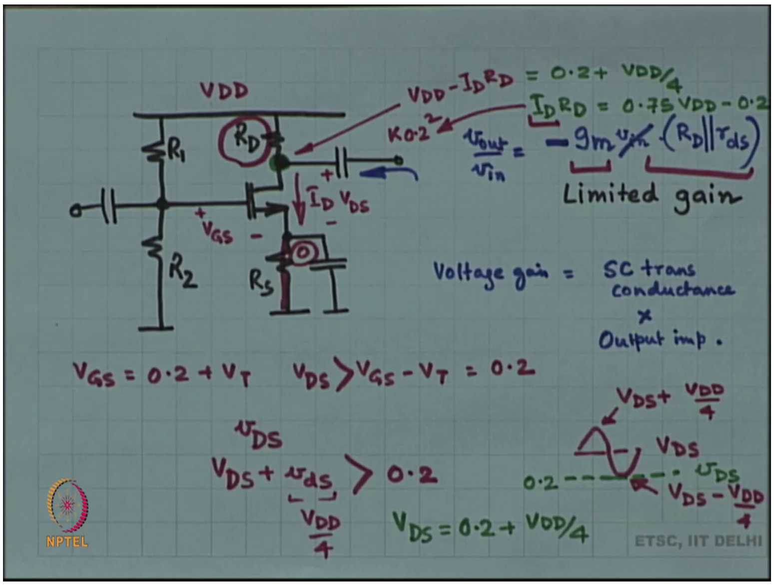

So, first let us try to understand if there is any problem with the common source amplifier is there any problem, this common source amplifier is great is there any problem ok. So, this is the structure of the common source amplifier, can we list out the different problems of this circuit ok.

So, problem number one is that the gain is it a problem the gain is negative no that is not a problem the gain of the structure is minus g m times. So, remember in the small signal picture ok, how do you how do you find the quickly look think about the gain of the structure, very quickly you do the Norton equivalent right do it try to do it in your mind, this is a short circuit all right.

So, experiment one for Norton equivalent is to do the short circuit test I do a short circuit over here. So, R D does not matter R S never mattered anyway, R D does not matter R 1 and R 2 never really matter right. So, I apply a signal at the input what is the current in the short circuit, inside the MOSFET there is r d s, and there is g m that is inside the MOSFET. So, I apply a signal at the input g m produces a current g m times v in that goes into the output short circuit, what about R D S, r d s the output short circuit has grounded on one side and the source has grounded on the other side ok.

So, there is no current in the r d s and that automatically means that the short circuit current is g m times v in all right. So, the transconductance is nothing, but g m that is the transconductance. So, you see very quickly you can do these things, you just have to be help yourself by not revealing not drawing the small signal full small signal picture every time see. This is after all engineering and on engineering we like to skip a few steps.

So, try to skip some steps all right. So, short circuit current is g m times v in and there 2, I am not really writing v in, I am writing the short circuit transconductance that is that is just g m ok, because later on I am going to so, what will you do short circuit current is g m times v in the output impedance is going to be something right. So, v out is going to be g m times v in times the output impedance, then after that you are going to do v out by v in is equal to g m times the output impedance ok.

So, that is the strategy with some sign the sign is negative in this case because, the current is coming this way in the short circuit current to never mind, just focus on the idea. The idea is that when we talk about the Norton equivalent, there are many steps you do the short circuit current, but then do not write the v in at all you just write it as a transconductance g m right.

And then multiplied with the output impedance and that gives you the voltage gain all right. So, this is the strategy that we are going to follow. So, transconductance in my case is just minus g m and, then the output impedance what is the output impedance of this circuit so, if I look in from outside this is a short circuit, I look up I see R D, I look down I see yes you have got a g m and you have got a R D S, but the input is grounded, when you are doing the output impedance the input has been nulled right, nulled means that voltage source has been made equal to 0 ok. So, input is grounded input is grounded means that gate voltage is grounded, if the gate voltage is grounded source is grounded. So, g m does not matter at all.

So, all you have is r d s right. So, looking down I see r d s looking up I see R D, which means that the output impedance is nothing, but R D in parallel with r d s and that is it ok. So, this is how we do the analysis. You would find out the short circuit current or rather the short circuit transconductance find out the output impedance multiply them you are going to get the voltage gain ok. So, this particular amplifier, this common source amplifier has an output impedance R D parallel r d s has a transconductance of g m and therefore, the voltage gain is g m times R D parallel r d s all right.

Now, r d s is limited by the value of g m did you know that no ok. So, when we do the MOSFET, when you look at the MOSFET characteristics.

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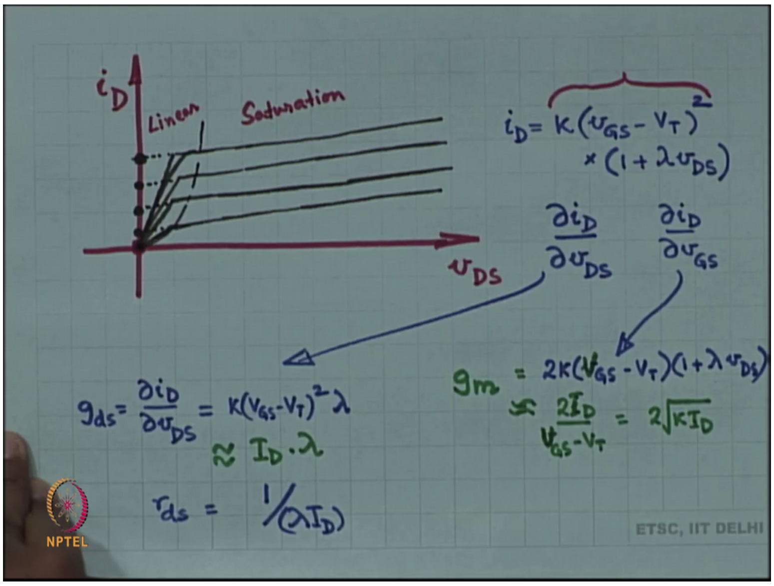

$$i_D = \overbrace{K(v_{GS} - V_T)^2} \times (1 + \lambda v_{DS})$$

$$\frac{\partial i_D}{\partial v_{DS}} \qquad \frac{\partial i_D}{\partial v_{GS}}$$

$$g_{ds} = \frac{\partial i_D}{\partial v_{DS}} = K(V_{GS}-V_T)^2 \lambda \approx I_D \cdot \lambda$$

$$r_{ds} = 1/(\lambda I_D)$$

$$g_m = 2K(V_{GS}-V_T)(1+\lambda v_{DS}) \approx \frac{2I_D}{V_{GS}-V_T} = 2\sqrt{KI_D}$$

You think of it as two straight lines this is one straight line and, then you think of it as another straight line ok. For the ideal MOSFET this second straight line is going to be parallel to the x axis, but no MOSFET is going to be ideal all right. So, this is what we have all right. So, this is the mos characteristics that I have redrawn.

Now, ordinarily in this region in the flat region of the MOSFET, i D you would say that i D is some constant K times V G S minus V T the whole squared ok, this is what you would say ordinarily; however, that is not really the case, there is some slope to it all right. So, what we try to do is we try to model this, as a straight line first of all we try to model this has a straight line. And then we say that i D is K times v G S minus V T the whole squared times 1 plus sum lambda v D S ok. So, this is a constant times a function of v D S constant with respect to v G S right.

Once I fix v G S this entire portion becomes a constant, when I fix v G S this entire portion becomes a constant, this still remains because v D S changes and it starts looking like a straight line that cuts the x axis v D S equal to 0 at the original current ok. So, the original current times 1 plus lambda v D S, the original current times 1 plus lambda v D S. So, this is the model that we have and, hopefully all of these have a similar slope ok. Rather a slope that is proportional to the current fine.

So, when you do partial derivative of i D with respect to v D S what do you get, when you do partial derivative of i D with respect to v G S. So, there are two partial derivatives right i D is now a function of v G S as well as v D S, you can do dou i D by dou v G S, you can do a dou i D by dou v D S. If you do this what do you obtain you obtain 2 times K times v G S minus V T times the rest of the staff ok.

And over here if you suggest that lambda happens to be a small number lambda should be a small number right, lambda is related to the slope of this graph of this flat portion right. So, that should be generally a small number. So, if lambda is a small number, then this is nothing, but 2 K times v G S minus V T right and that is you know 2 i D by v G S minus V T ok.

You could also say that that is equal to v G S v G S minus V T is square root of i D by K ok. So, multiple possibilities you can look at ok. So, in this case you have eliminated K altogether. In this case you have eliminated v G S altogether, in this case you have eliminated i D altogether right. So, you can have so, this is g m ok. So, g m could be thought of as a function of K and v G S is capital V G S g m could be thought of as a function of i D and v G S and not K and, g m could be thought of as a function of K and i D and not of v G S. So, you can manipulate as you please fine.

All that you are making a few assumptions here, you are making an assumption that lambda is small and, that is a very convenient assumption because, otherwise this factor is going to linger on and, it is going to create a lot of difficulties all over the place oh. This is accurate is it, you do not really have to fine.

This expression is correct no matter what all right. So, the these are this is GM, this is what happened when you did a derivative partial derivative with respect to v G S. Now, you could also do a partial derivative with respect to v D S and, what happens in that case. In that case what you get is K times v G S minus V T remains and, then you get just lambda all right.

And K times v G S minus V T whole squared is approximately equal to i D and, you have got lambda, which basically means that the slope of this curve is lambda times i D approximately is this ok. This also means that r d s this was g d s r d s is 1 by lambda i D, what are the units of lambda, what is the unit of lambda look at this expression one plus lambda v D S, what is the unit of lambda is a 1 over here and there is a lambda v D S, your adding two things on 1 hand your adding a 1 on the other hand your adding lambda v D S. So, what is the dimension of lambda 1 by volt it is 1 by volt.

Because only then lambda v D S becomes the number so, the dimension of lambda is 1 by volt 1 by volt means volt is in the numerator, divided by current that is resistance. So, it all checks out all satisfied all right. So, r d s is going to be typically 1 by lambda i D, where lambda is going to be given to you right. So, depending on the value of i D you get r d s, typical values of lambda would be you know 0.1 0.01 0.05 something like that ok, but what is important is that r d s is inversely proportional to i D typically.

All right and I need you to see that r d s is inversely proportional to I D. So, if I jack up i D r d s is going to drop. On the other hand given a device right, if I have given you the device K is fixed, you cannot do much with k all right and given a bias current, if I give you the bias current, then I D is fixed. So, g m is two times square root of K I D you have eliminated v G S altogether you take, whatever v G S you require for that particular I D and that particular MOSFET all right.

So, g m is proportional to square root of I D, r d s is proportional to 1 by I D all right the best possible gain coming out of this amplifier best possible is when r d is very very large is g m times r d s, which means g m times r d s, which means that if I decrease I D look..

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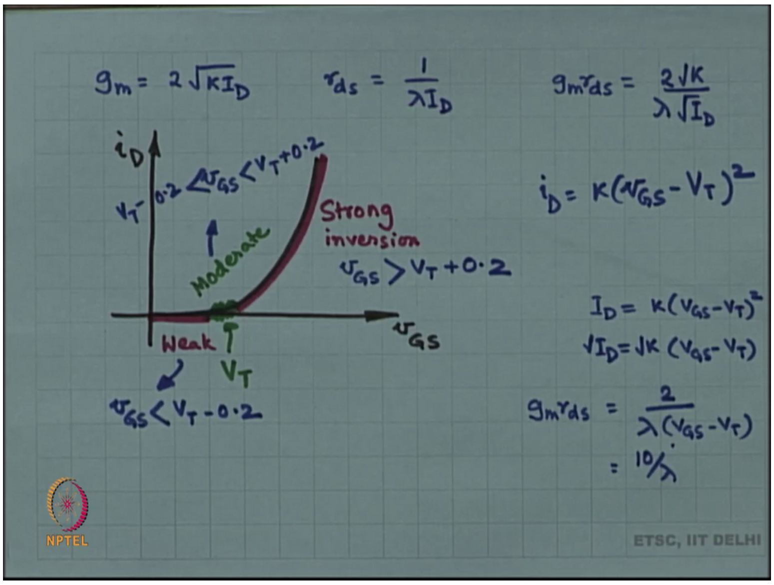

$$g_m = 2\sqrt{KI_D} \qquad r_{ds} = \frac{1}{\lambda I_D} \qquad g_m r_{ds} = \frac{2\sqrt{K}}{\lambda\sqrt{I_D}}$$

$$i_D = K(v_{GS} - V_T)^2$$

$$I_D = K(V_{GS} - V_T)^2$$

$$\sqrt{I_D} = \sqrt{K}\,(V_{GS} - V_T)$$

$$g_m r_{ds} = \frac{2}{\lambda(V_{GS} - V_T)} = 10/\lambda$$

Let us do this g m is equal to two times square root of K I D r d s is 1 by lambda I D ok. The gain of the circuit best possible gain intrinsic gain of the MOSFET is g m times r d s right by the way both of these are slight approximations, but that is ok. So, this is equal to two times root K times divided by lambda divided by root I D ok. So, what this means is that, if I decrease I D, then hopefully the intrinsic gain of the MOSFET is going to increase.

So, if I decrease the value of I D, then hopefully intrinsic gain of the MOSFET is going to increase this is general suggestion ok; however, you cannot do that much, you have to maintain the MOSFET in strong inversion for all these equations to be valid ok. These equations are valid only when the MOSFET is in strong inversion not otherwise all of these modelling all these models right.

So, if I keep decreasing I D, then suddenly the MOSFET is going to drop out of strong inversion what is strong inversion what is strong inversion oh ok, you do not know. So, the MOSFET has 3 regions of operation here, the MOSFET has 2 regions of operation region 1 is the linear region, where it goes like a straight line through the origin ok, that is called the linear region.

And the other region is the flat region. So, this called the saturation all right and these are 2 regions of operation of the MOSFET based on the value of v D S so, here the function is it is a function of v D S, if I change v D S you can go from linear to saturation and vice versa fine.

The MOSFET has three more regions of operation, which are independent of those 2 and, in this in the earlier case you were changing v D S and going from one region to the other, here you are going to change v G S and go from one region to the other all right. And mostly this entire region is called strong inversion; this entire region is called weak inversion. And this region is called moderate inversion all right and, what is this number over here on the x axis.

So, this place is V T remember the current is a function of v G S minus V T you say K times K times v G S minus V T, the whole squared for example, right. So, k times v G S minus V T the whole squared is valid in strong inversion right, when V T equal to when v G S is equal to V T, the current is not really equal to 0 it is some small something small. So, this K times v G S minus V T the whole squared.

So, this equation look at the three regions v G S is much more than V T this is valid, v G S is equal to V T right you get your answer as 0, but when v G S is equal V T, if you take a MOSFET to the lab and apply the right v G S such that it is equal to V T, you will find that there is still some current ok. So, this equation is no longer valid this region is called moderate inversion and, when v G S is much less than V T right that is this region right over here it looks like the current is equal to 0 right, but if you actually measure it in the lab it is not really equal to 0 all right. It is still going to be something. So, that region is called weak inversion.

So, we have three regions of operation, strong inversion weak inversion moderate inversion, strong inversion, moderating inversion, weak inversion. Now, the standard definitions for these regions are you are in strong, if v G S is greater than V T plus 0.2 ok. And you are in weak if v G S is less than V T minus 0.2 ok. These are the standard definitions and, you are in moderate if v G S is in between the 2 ok.

So, the limits are you have V T minus 0.2 you have V T plus 0.2, if v G S is more than V T plus 0.2, then it is called strong inversion if v G S is less than V T minus 0.2 it is called weak inversion in between it is called moderate inversion and, the standard understanding is that this particular equation that I have i D equal to K times v G S minus V T the whole squared, this equation is valid in strong inversion which means that v G S has to be more than 0.2 ok, only in such a scenario this particular equation is valid.

Now, what is the equation in moderate, what is the equation in weak I do not I am not going to reveal these to you right now, because these are all matters of I mean number of people are doing PhD thesis, on these topics how to model the MOSFET what are these equations so on and so forth and, it is it is not a matter of you know teaching in a class right, this is not a matter of classroom teaching. This is not any way that the objective of the course right. The objective of the course is to take the MOSFET and use it, not to you know delve into the physics of the MOSFET and so, on right.

So, we need to understand that what is the take away, the take away is that there is a lower limit as to how low v G S minus V T can be v G S minus V T should be more than 0.2 to be in strong inversion all right, if v G S minus V T is not even 0.2 volts. then maybe you are not in strong inversion maybe this equation is no longer valid, all right is this is this understanding so far all right. So, let us get back.

So, what I am observing over here is that my intrinsic gain goes up, when I D goes down all right, but there is a limit to how low I D can be right, what is what is I D now, I D is K times v G S minus V T the whole squared in strong inversion. So, root I D is root K times v G S minus V T in strong inversion right, you plug this in over there root I D what do you get root K root K cancel out.

So, you get this entire expression g m r d s is equal to two times lambda v G S minus V T. So, there is a limit to v G S minus V T the lowest v G S minus V T that you can have is 0.2 ok, you cannot have v G S minus V T that is lower than 0.2. So, if this is if you want to remain in strong inversion, if all of this analysis is correct right. So, that is 1 limit that I have.

Now, if v G S minus V T is 0.2, then this entire thing is 10 by lambda is this so, far ok. So, g m times r d s is limited to 10 by lambda. Now, what is lambda? Lambda is the slope of lambda I D is the slope of this curve right, whatever the current bias current is I do not know lambda times I D is the slope of this curves, this set of family of curves, all right typical values of lambda is you know 0.1 0.01, 0.01 is 2 too low for lambda 0.1 0.25 these are typical values of lambda, which automatically means that 10 by lambda is going to be is going to have a limit right, how large it is going to be you will have something like a 100 as the intrinsic gain of this particular MOSFET so far so good ok.

So, let us get back, my question to you was what are the drawbacks of this circuit. So, the number 1 drawback that I have is that this circuit has limited gain all right gain of the circuit is limited, first of all the intrinsic gain is g m times r d s the output impedance is not r d s it is much smaller than r d s it is R D parallel, r d s that is number 1 ok. Then you are saying that g m times r d s has an upper limit has a sealing ok. Now, how much lower is R D compared to r d s is there an upper limit to R D can R D be infinite, can I have R D to be infinite over here no why not why cannot R D is infinite. If R D is infinite, then where is the bias current you are not going to have any bias current ok.

So, suppose you have got bias current I D all right. Suppose you need a certain v D S, how much v D S do you need more than v G S minus V T by the swing that you want ok. So, let us say you have kept your v G S minus V T to be at it is lower limit that is 0.2 volts ok. So, v G S is 0.2 plus V T all right because, you want the best possible gain. So, you keep v G S at the lower limit 0.2 plus V T ok; that means, that the lowest value of v D S v d s has to be more than v G S minus V T, v D S certainly has to be more than 0.2 certainly ok.

Now, this is the operating point v D S, the actual v D S has to be even more than even actual V D S small v sub D S, which is equal to operating point v D S plus small v d s, even this has to be now more than 0.2 right, because at all times you need v D S to be more than v G S minus V T.

Let us assume that the gain of the amplifier is so, large that the gate is not really moving around right, the drain is moving, but the gain of the amplifier is large enough that even when the drain moves the gate because, the gain is large the gate is not really moving very significantly. So, forget any signal on v G S as in v G S is not really changing this is more or less constant ok. So, I have therefore, at all times I need to satisfy capital V D S plus small v d s has to be more than 0.2 ok. Now, small v D S is changing with time it goes up it goes down right, it goes positive as well as negative ok.

Suppose I want a good amplitude out of this MOSFET right, some reasonable amplitude at the output, let say you want if your if your power supply is V D D over here, I do not know what the power supply is maybe it is 5 volts maybe not. Suppose your power supply is V D D, I I would like to have you know at least half of the power supply, as no a quarter of the power supply as the amplitude of V D S symmetric. So, V D S is going up and down by a quarter of the supply voltage ok, or in other words small v d s is equal to V D D by 4.

So, the lowest it is going to go is minus V D D by 4, which now means that capital V D S has to be more than 0.2 plus so, this level. So, this is my even this has to satisfy greater than 0.2. So, let us say this is equal to 0.2 right, in that case capital V D S is 0.2 plus a quarter of the power supply fine. So, automatically this is going to reveal what is the value of R D, I am telling you that capital V D S is 0.2 plus a quarter of the power supply all right. Let say this source voltage is 0 all right, let us say this R S is 0 ohms let us make life easier right. Let us not complicate ourselves let say R S is 0 ohms right so, this node is at 0 volts.

Suppose this node is at 0 volts, where is this node at this node is at V D D minus R D times I D, I D times R D and what is V D S, S is at 0, V D S is also equal to V D D minus R D R T ok. And, now you say that you want V D S to be 0.2 plus V D D by 4 all right. So, automatically you are saying that I D R D take I D R D to the other side that is equal to 0.75 V D D minus 0.2 ok.

So, I D times R D has a value automatically all right. Now, you can say let us look let us you know how does this change with the swing. So, if I if I decrease the allowable swing, then this increases. So, I D times R D increases ok. So, if I have declared the value of I D for example, R D is automatically declared if I make R D very very large, then I D is going to be very very small ok, but I D has a lower limit does not it I D has a lower limit, we just saw in the earlier 1 right, to remain in strong inversion I D I mean v G v G S minus V T as a lower limit right.

So, I D has a lower limit K times 0.2 squared is the lower limit for I D. So, this I D over here is nothing, but K times 0.2 squared right, what I am trying to drive at is that there is a limit to how large R D can be R D cannot be infinite ok. If you want to make R D too big, then first what is going to happen is the swing is going to drop for example, if R D is huge what is going to happen, if R D is huge automatically you need this number to increase, there is a limit to how large this number can be can it be point, can it be more than one no it cannot be more than one, because you got this as V D D minus V D D by 4, you said that I want V D D by 4 to be the amplitude over there ok.

At least so, there is a limit to how large this can be if you want 0 amplitude over there, then this is just V D D ok. So, you cannot have R D to be arbitrarily large, at best even when the swing is nothing right, there is no swing that you are going to allow this is V D D minus 0.2 right, you have a lower limit to how low I D can be, accordingly you will have the value of R D is this ok.

So, what do I have V D D minus 0.2 by K times 0.2 squared that is going to be the largest possible value of R D, what I am trying to hint at over here is that this R D is no good right, there is a limit to how large this R D is, if I make R D to be very large, then the swing is going to go down, if I make R D to be very small the gain is going to go down.

So, this R D is playing spoilsport over here, this particular resistor. If I increase it is value swing is going down and, even then I can only increase it so, much there is a limit right, even when there is 0 swing. If I decrease it is value the gain is dropping like a stone ok, the gain is already limited it is its going down even further ok. So, what we are going to do is we are going to take this understanding and, then we are going to try to solve the problem, this problem over here is R D. The gain is being limited by R D; the swing is being limited by R D. Now, let us try to solve the problem can we make better designs ok.

Thank you so, much.