Introduction to Electronic Circuits. Professor S.C. Dutta Roy. Department of Electrical Engineering. Indian Institute of Technology, Delhi. Lecture-6. Signal Waveforms.

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Professor: $(0)(1:01)$ you know according to sanskrit it should be, if hindi is derived from sanskrit, then that is wrong. This is $6th$ lecture in 110 and today is 11-01-94 and our topic would be signal waveforms. What we are going to discuss today are the various kinds of signals that arise in electronic circuits. And the general topic is signal processing circuits. As part of this general topic signal processing circuits, we shall $1st$ discuss some of the signal waveforms. It is important to understand what is signal is and what processing means so that we can devise electronic circuits to perform the processing.

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 $v(t)$ * $i(t)$ Signal

A signal by definition in electrical engineering, a signal is either a voltage or recurrent. A signal is a voltage or a current which varies with time, that is the signal, signal is either time varying voltage or a time varying current, as simple as that, which, just any time varying voltage or current is not enough, which carries some information, that is called a signal. A signal is the voltage or a current which varies with time in such a manner that it has to convey an information, all right. For example the current that you receive in a telephone receiver, it varies with time and that is what is converted or transduced into sound and that sound here is the telephone conversation.

So that is a signal or the ones and zeros or the dashes and dots in a typical telegraph system, the morse code, these are time varying voltages or currents which carry information and therefore that is also a signal. Something interesting at the door? Okay, why do not you open? So a signal is a time varying, signal is the time varying voltage or current which carries some information, that is $2nd$ part is important, it has to carry information. Any time varying voltage or current is not enough, also no voltage or current which does not vary with time can carry an information. A steady current or steady voltage does not carry any information, so that is not a single.

It is important to distinguish between signals and non-signals. All voltages and currents are not signals, all right, in order to qualify as a signal it must be time varying, $1st$ qualification, $2nd$ qualification is it must carry an information or a message. As i said the current in a telephone receiver or the morse code that is transmitted along telegraph lines, these are all signals. And the question of processing a signal arises when the signal that is available to you, when a signal that is available to you is not the desirable, is not the desired signal, you want to convert it to some other form, some other form.

For example if you are given simply the current in the telephone receiver, for that is not your desired signal, you want to process the signal so that it produces a sound, in other words you wish to transduce the electrical signal into a sound signal and that is the form of, that is the desired signal. So this, the telephone receiver or the micro or the microphone that acts as a signal processing element. The signal may be very weak, all right, for example an old record for example which has gone very weak and you wish to amplify this, all right.

The amplification is a processing or you might have a slowly varying signal like this and we wish to find out where it is varying at the most rapid rate. Then what do you do, you differentiate the signal, all right. So differentiation is processing or you might wish to have the integrated effect of a single, in other words you might want to integrate a signal, that is integration is also signal processing. Anything that you do to the signal, another example could be that the signal is mixed with noise which is almost always the case. A signal is the desired information, it is always make with some undesired information or noise.

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If you wish to separate the signal from the noise, then you have to subject the signal to a signal processing circuit which gets rid of the noise and retains the signal only. So these are the various examples of signal processing. We shall come to signal processing circuits later but let us study some of the, some of the common signal waveforms. A signal waveform is said to be continuous or so-called dc, they ordinate here could be either a voltage or current,

all right because signal is a voltage or current, time varying voltage or current. This could be v or i, we shall not write this again and again, excuse me, and the independent variable is the time.

So a signal which remains constant, voltage or current which remains constant for all values of time, this is called a dc waveform or also called a continuous waveform, continuous signal waveform. But as i said in non-time-varying voltage or current does not carry any information. So in that sense this is not a signal, this is simply a waveform, dc or continuous waveform. Consider now a battery, let us say voltage v in series with a switch and a resistance r, all right. And suppose the switch is put on at time t equal to 0, then if you measure the voltage across this v of t and you plot the voltage v of t versus t, this is equal to 0, then for t less than 0 into 0 and for t greater than 0 it is equal to capital v, is not that right.

So t greater than 0, 10 equal to capital v, in other words the signal now the voltage rises from 0 to a value capital v abruptly at t equal to 0, such a waveform is called a step waveform, step, it just, it does look like a step in a staircase, all right. As i said it could be the voltage for the current. If you measure this current i, obviously the current i shall also be of the form of a step, all right. I would be equal to simply v by r, v divided by r. So the current as well as the voltage in this circuit shall be a step. If the amplitude or the height of the step is unity, if this is 1 volts or 1 amperes, then you call it a unit step, all right.

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A unit step therefore is a function which is 0 for t less than 0 and which rises to 1 at t equal to 0. And this function is given a special name of unit step functionality denoted by u of t, this symbolic u is reserved for a unit step. I therefore the voltage in the previous case, in the previous circuit which had a step of hide capital v, if you have voltage waveform of height capital v, voltage step v of t versus t, then obviously be of t can written as capital v times u of t. U of t is a unit step and this symbol is more or less universal, we shall reserve the symbol small u for unit step. It could be, it could be a voltage, it could be a current, it could be a force, it could be acceleration or it could be anything.

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But u simply stands for unity height and when you write a u of t, it means the abrupt rise or the step others at t equal to 0. T equal to $0 - it$ is 0, at t equal to $0+ it$ is unity, this is the symbol that is reserved for this. Now suppose we have battery, we consider another circuit, battery in series with a switch and then an r, resistance r, you measure the voltage vt across the resistance r for the current that is flowing through and they are simply related by ohms law. And what to do is you operate this circuit on and off. Let us say it is on for 0 to say t1, all right, that is it is put on at t equal to 0, then it is kept on up to time t1, then it is put off, that is t1 to let say t2, then again maybe it is put on, let us say t2 to t2 plus t1 and off from t2 plus t1 to some value you calculate, all right.

Suppose this goes on, that is this switch is put on for a time t1, then put off for a certain region of time t2 - t1, then again put on for t1, again put off for t2 - t1 and so on. Suppose we go on doing this, then what would be the waveform, what would be the shape of the signal. If i plot v of t versus t, then we shall have for the on period the voltage shall be v, for the off period the voltage shall be 0, again for the on period the voltage will be v, for the off period the voltage is 0 and so on. Such waveform, they look like rectangles, they are called pulses,

pulses. So this is a pulse waveform and you have seen how it can be generated, pulse waveform.

This is also a special kind of signal which is, this is a signal because it is time varying and if you call the on pulse, the on voltage as 1 and the off voltage as 0, then this forms the basic waveform of a digital computer. A digital computer after all works on the basis of just 2 levels of voltage or current, one level is called 1 and the other level is called 0, ones and zeros, all right. So a pulse is in the heart of operation of a digital computer.

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There are other kinds of this forms which are also of interest, for example you have waveform, once again let say this is a voltage waveform. You have waveform which rises linearly up to a certain instant of time and then it falls abruptly, okay, then again it rises linearly, falls abruptly and so on. So this waveform looks like the teeth of a saw and therefore it is called a sawtooth waveform. It is just that similarity, there is no other, no other interpretation associated with this. Now i sawtooth waveform is extremely useful in a cathode-ray oscilloscope, in cro. The electron beam that you see on the face of a cro, have you seen a cathode-ray oscilloscope?

The electronbeam that you see, well it starts from the left-hand, it starts from the left-hand, it goes linearly across the screen and then in almost 0 times it comes back. So it is coming back you cannot see, all right. Then again it goes back like this, then again comes back like this, this is a waveform to switch it is subjected, that is the electronbeam is subjected to a force which varies like this. So therefore it goes linearly up to a certain point and come back

immediately, again goes, again comes back. The tv scanning is also there exactly by a similar waveform, sawtooth waveform.

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The other kinds of waveforms which are useful in electrical engineering and electronics are the exponential and the other one is sinusoidal. And exponential waveform simply varies exponentially with time. For example you could have of t which is v e to the power at, all right. It is exponential, that is it varies with the power of e. Well if a is 0, a is 0, then you see that it is a dc, all right, continuous waveform. If a is greater than 0, then it is a growing waveform, if a is positive, then at time t or the amplitude or the value of the voltage continues to increase, all right.

If this is, let us say t equal to 0, this is t, then when a is greater than 0, it is a waveform like this. If t equal to - infinity, it goes to 0, as t goes, what is the value here, it is obviously capital v, when t equal to 0 in this capital v and then it continues to rise. So this is called a building exponential, and exponential which builds or which grows, building or growing exponential, all right, that is a better term i suppose, growing exponential. On the other hand if a is negative, if a is negative, then obviously the exponential will be a decaying exponential. If a is negative, then see what happens is, it decays like this.

Again v of be equal to v2 e to the at, that this voltage, this level shall be equal to v to, all right, this level is v2 and as t increases, it became. So this is called a decaying exponential. All these are not hypothetical quantities, growing exponential and decaying exponential occur very frequently in circuits, in electronic circuits and also in passive circuits.

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For example, for example let us consider a capacitor, which is charged to a voltage let say v, all right. The capacitor c which is charged to voltage v, so the charge on the capacitor shall be equal to c times v. Let us have a switch here and a resistance here r. This switch is closed at t equal to 0, the charge is q, let us call this a q0. Q0 is the charge on the capacitor at t equal to 0 -, all right. So q0, all right, let us use a different colour, at t equal to 0 - the charge is q0 and when the switch is put on, the charge on the capacitor now finds a path for flowing, capital r, this resistance allows a path to flow.

If the voltage is v, then obviously the current through the capacitor shall be v by r. And you know when, when current flows through a resistor, heat is generated, so the resistor consumes energy and therefore the charge in the capacitor decreases, so does its voltage. And this decrease as you shall see later is exponential, the decay is exponential, all right.

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Or let us say, let say we have battery v, we have a switch and then we have a resistance and then we have a capacitor c. Suppose we measure the voltage across this v of t across the capacitor c, this is a resistance r and this which is put on at t equal to 0. All right. At t equal to 0 -, that is when the switch is off, the capacitor voltage is 0, capacitor has no charge in it, so the capacitor voltage is 0. So if i plot the capacitor voltage v of t, t equal to 0 - it is 0, then at t equal to 0, at equal to 0 the switch is on, the capacitor cannot change its energy instantaneously and therefore t equal to 0+, the voltage across this will still be 0 and therefore the current v by r passes in the circuit and tends to charge the capacitor.

As soon as accumulates a little charge, voltage appears across it and as it continues to accumulate charge, the voltage across the capacitor rises as you shall see exponentially. And if sufficient time is allowed for the capacitor to charge, which capacitor will ultimately charge to a voltage equal to that of the, that of the battery v. That is in equilibrium condition when the capacitor has been fully charged, the current in the circuit shall be equal to 0 or the capacitor voltage shall be rising to a value of v, all right. So this is a case of the growing exponential, and this is the case, the previous one was the case of a decaying exponential.

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In exponential as i said, if i consider a decaying exponential, that is v e to the - at, let us make it specific where a is greater than 0, all right. Suppose we wish to find out the value of time at which v of t, you see v of t diminishes with time, it starts at capital v and its elimination with time. Suppose we want to find out the current at which the voltage reduces to v by e, that is the eth fraction of the original voltage. Then let us call this time as t1. So t1 would be given by v by e equal to capital capital v e to the power - at1. And therefore we shall have e to the -1 leaves equal to e to the - at1. V and v cancels, 1 by e is e to the -1 and therefore one should be equal to at1 or t1 should be equal to 1 over a, all right.

This time t1 is given a special name, it is called a time constant of the exponential waveform. And it is denoted usually by capital t, capital t the time constant and therefore in terms of the time constant of the exponential waveform be good rewrite our exponential waveform as v e to the power - t by capital t, which makes it obvious that when small t is equal to capital t, then the value of the waveform shall be equal to v by e, all right.

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It is a matter of simple calculation to show that a waveform like an exponential waveform like this e to the - t by capital t. Let us consider the normalised waveform, that is normalised with respect to the value at t equal to 0, all right. In other words vt by v is normalised and it starts from 1 and then it decays like this. All right, it is very simple to show that if t is equal to 5 capital t, then the value vt by v decays from 1 e to the power -5, that is 1 by e to the power 5 which is approximately 0.0067, less than 0.7 percent, all right. Less than 0.7 percent, which is approximately, which is smaller than the accuracy of the ordinary instruments available in the laboratory.

If this is a voltage, you cannot measure the voltage to an accuracy less than 0.7 percent and therefore 0.0067 is for all practical purposes can be taken to be 0. And this is the reason why electrical engineers say that an exponential waveform has a lifetime of 5 times the capital, the time constant. Is the phrase understood? It is very loosely said, it is not exactly 0 but since the value is less than the accuracy of an instrument available in the laboratory, we say that for all practical purposes, an exponential they decaying waveform has decayed to 0, it means it is dead after 5 times the time constant.

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And that is why the lifetime of an exponentially decaying waveform is approximately 5 times the time constant. Exponential waveforms are extremely interesting, not only they are important, they are also interesting because as you know d dt of an exponential waveform e to the at is also exponential, is not that correct? Integral of e to the at dt is also exponential , it is e to the at divided by a, all right, indefinite integral. So this makes it, this makes life very simple to work with exponential waveforms as you shall see later. And to illustrate the importance of exponential waveforms, let us consider an example.

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We have a current generator which generates a 2 times e to the - 250 t milliamperes, okay, current generator which generates not a constant value but it varies with time, that means it is

a current signal 2 e to the -250 t milliamperes. What is the time constant? 1 by 250, that is 0.004. Okay. And this is passed through a series combination of a resistance r which is 3k and then inductor l which is 4 henry, what you are required to find out if vr, it is voltage across r, vl, the voltage across l and the total voltage v across the combination, all right. As you can see vr is simply ir, it is simply the current multiplied by r and this is milliamperes and this is kilos, so it will be 6 times e to the power -250 t, unit, volts.

Do you understand why it is 6? 3k is 3000 and 2 milliamperes is 2 times 10 to the -3 and therefore 3 times 2. Now vl similarly would be l, that is 4 di dt, that is 4 times 2 times 10 to the -3, milliamperes mind you, we want to convert it to volts, times d dt of - e to the -250 t, that will be - 250 e to the power -250t, is that okay? Can you tell me how much is this? -2 e to the power -250 t volts. So the polarity that is shown here is not the correct polarity, vl is negative, means that this and this is positive and this and this is negative, all right.

This is because the current is a decaying exponential, so d dt is negative, the slope is negative and therefore v could be simply the sum of the 2 and therefore this will be 4 times e to the -250 t volts. Similarly we could have taken a capacitor and we could have found out the voltage across the capacitor by taking 1 by c and then integral of idt. That would also have been and exponential. This is the beauty of the next potential waveform that an exponential signal generates in linear circuits, if it is non-linear, then of course you have complications.

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But in the linear circuits, and exponential lingual input produces exponential signal at output, because integral and differential of an exponential is again an exponential. Finally we

consider the most important waveform of all, that is a sinusoidal waveform. A sinusoidal waveform is very much a practical proposition. Because if you take a coil, if you take a coil and rotate it in a constant magnetic field, then you know that the voltage and emf is generated will be called, which is sinusoidal. So electricity generation at generating stations, the generated waveform is sinusoidal. And a sinusoidal waveform in general, can you give me another example of sinusoidal, sinusoidal quantities in any, any other field, it does not matter it is electrical on non-electrical?

Student: Simple harmonic motion.

Professor: Who executes a simple harmonic motion?

Student: A pendulum.

Professor: A damped pendulum or an undamped pendulum?

Student: Undamped.

Professor: Undamped obviously. A damped pendulum, all that you have to do is to add an exponential term. A damped pendulum will execute oscillations of this form, sin omega t plus theta where exponential term, if it is included, then this will take care of the decaying amplitude. That is the ideal gradually executes smaller and smaller r, all right. Therefore a pure sinusoid, let us, to bring variety into experience, let us say, we now consider a current i of the, will be of the form capital i, sometimes the college im, sin of omega t plus alpha let us say. It could be seen or it could be cosine all right for it could be let say v of t is equal to vm cosine of omega t plus alpha, sin or cosine, does not matter, both are sinusoidal.

There is a favouritism in favour of sin, we do not say cosinusoidal because it uses more letters, so that is all, economy. So we say sinusoidal, both are sinusoidal. Now various parameters of the sinusoidal waveform are firstly this quantity, which is called the peak value. Obviously this is the maximum of the right-hand side because sin can either be the maximum value is 1 and therefore im is the maximum value, so im is called the maximum value or peak value or simply called amplitude, amplitude, these are the 3 names by which it can be called.

Omega is called the frequency in radians per second, radians per second. In a particular example, simple harmonic motion can describe another particle revolving around the circle. So the number of complete circles it executes in 1 second is the frequency in radians, okay. Omega is the frequency in radians per second, t is the time in second and alpha is called the phase or sometimes, well the actual, actual nomenclature should be initial phase. That is when t equal to 0 , t equal to 0 , mark these words carefully, when t equal to 0 , the angle is simply alpha. That is why alpha is called the initial phase or sometimes loosely simply the phase.

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Actually phase stands for whatever the argument of sin is. That is the total, total quantity of omega t plus alpha is the phase at time t, alpha is the initial phase but loosely it is often called the phase. The sketch of a sinusoidal waveform, let us say vm cosine of omega t plus alpha is very familiar. It is something like this where this amplitude is vm and this is - vm, the maximum in the positive direction is plus vm and maximum in the negative direction is also the same, it is - vm. And this value would be vm cosine alpha, this is t equal to 0, what is the value of the time corresponding to this?

Obviously, okay this is omega t, then this would be - alpha, agreed. What is the difference between this time and this time in terms of omega t? What is the difference of omega t between this point and this point, pie by 2. All right. So what is this value then? Pardon me?

Student: - alpha plus pie by 2.

Professor: - a plus pie by 2, wonderful. And what is the difference between these 2 points, that is the point at which… 2 pie, omega t equal to 2 pie. So if i denote the corresponding time by capital t, then omega capital t is equal to 2 pie and what is capital t, the time period. And what is the relation between time period in frequency? Inverse. And therefore if i say that this sinusoid has a frequency of f in cycles per seconds, t is the time for one cycle, then the number of cycle perceptions per second is 1 by t. If we say f is the frequency 1 by t, then unit of f is cycles per second or hertz.

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So omega is 2 pie by capital t because omega capital t is 2 by and therefore this is equal to 2 pie f, omega is in radians per second and f is in hertz, after the name of henrique hertz. To illustrate this sinusoidal, sinusoidal will be your bread-and-butter in this course on introduction to electrical circuit, electronic circuits. It will also be extremely useful in the course 120 which is electromechanical energy conversion which, in which we will see that all generators generate sinusoidal waveform. All right. Then what we do with it is somebody else's business but originally you always generate a sinusoidal waveform.

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Let us take an example, a nontrivial example. The example is all sinusoidal waveforms express the following sinusoidal signal as a function of time. The description of a sinusoidal signal is given, you have to express it as an equation or as an expression. And the information that is given in the frequency is 100 hertz, okay, amplitude is, let us say it is a current, amplitude is 10 milliamperes and this is i of t, and it passes through, this is the information that is supplied, it passes through 0, 0 value with positive slope at t equal to 1 millisecond, this is information that is supplied about the waveform. You have to write an analytical expression for this.

What other things given, the frequency is given, the amplitude is given an about phase, obviously what remains is the phase, is not it, about phase out the initial phase alpha, the information that is given is that it passes through 0, the value of the waveform passes through 0 with positive slope at t equal to 1 millisecond, all right. So i can write my i of t as 10 times 10 to the -3, always normalise, always in terms of si, 10 milliamperes, then would you use it sin or cosine? We have not been asked, we have not been told, so we can use anything will is, let say we use a cosine.

So cosine of omega t, omega is now 2 pie times 100, which is 200 pie t, all that remains to be found out is alpha, all right. And it has been told to us that the value is 0, the value is 0, i t is 0 at t equal to 1 millisecond with positive slope. Now the importance of positive slope and negative slope you can see that a cosine waveform is like this, here at this point it is positive slope, when it passes through 0 to positive slope, at this point it passes through 0, it is

negative slope. So it is this point that is given, all right. Now what is the condition for positive slope?

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Let us take a cosine of omega t, a cosine of omega t plus alpha, d dt of cosine omega t plus alpha is - omega sin of omega t plus alpha. So if the flow is to be positive, it is not a trivial question, you follow this carefully. If the slope is to be, if d dt is to be positive, then obviously sin must be negative because there is a negative sign here. Now when is sin negative, when omega t plus alpha itself is negative. And therefore the condition that i t is equal to 0 at t equal to 1 millisecond with positive slope implies that this quantity should be equal to -, this quantity should be negative, - pie by 2, that is correct.

Which will give away alpha. That is our condition would be 200 pie, 1 millisecond, so 10 to the -3+ alpha should be equal to - pie by 2. Is that clear how we found this information out? We argued that the argument omega t plus alpha must be negative and since cosine is 0 the value must be - pie by 2. And therefore alpha is equal to, this is -0.5 pie and this is 0.2, so this is -0.7 pie, is that okay? And therefore my final answer is 10 to the -2, 10 into 10 to the - 4 cosine of 200 pie t - 0.7 pie, so many amperes. Alright, this is the final answer, be careful about handling sinusoidal quantities.

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Depending on the slope and other information it is very easy, it is very easy to make an expression. Now sinusoidal waveforms are special kind of waveforms which go by the general name of periodic waveforms. Periodic, periodic waveform. A function f of t, i will not use f of t because small f we have used for frequency. A function x of t is said to be periodic, is said to be periodic if x of t equal to x of t plus n t, where capital t is a constant and n is an integer, positive or negative including 0. X of t is said to be periodic, the quality $(1)(45:25)$ is that x of t values repeat after every regular interval, this interval of repetition is said to be the time period.

So t here is the time period and n is an integer, n is an integer, we n could also be 0, x of t is obviously identically equal to x of t and n could be any other integer, positive or negative. And for example, a cosine omega t plus alpha is periodic because it is equal to a cosine omega t plus capital t plus alpha, where capital t equal to 2 pie divided by omega. It is therefore a periodic waveform. Now a periodic waveform because it repeats after regular intervals, how do you measure a periodic waveform, how do you devise a meter to measure a periodic waveform?

There are various kinds of measurements, for example you wish to see the periodic waveform, then you have to display it on a cathode-ray oscilloscope. That displays the total waveform, it is a cosine omega t plus alpha, all right. Now if you want a parameter for the periodic waveform, there are various parameters possible, for example one could think of the average value. The average value is always spoken of with regards to one period, all right.

Average value is one period, what is the average value of a sinusoidal waveform then? Average value of a sinusoidal waveform?

Student: 0.

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Professor: 0, if you take sin omega t for example, the average value, equal positive, equal negative, therefore the average value is 0. However if i take let say a triangular waveform like this, a symmetrical triangular waveform like this, let us say this is 0, t and let say i of t, all right, what is the average value? Suppose this is im, yah, im by 2, this is the time period and therefore the average value is im by 2, half of the peak value. In general the average value of a periodic waveform let us say i average is equal to integral, integrate over one 0 to t i t dt and divided by capital t, this is the average value definition.

Now does it matter measure the lower limit as 0 or some other value? Can we have t1? Yes, then the upper limit should be t1 plus t, you could start anywhere but you have to and after interval of capital t. So this average value is independent of small t1, all right. Now since in a sinusoidal waveform the average value is 0 count it over one, in a sinusoidal waveform, the average value is usually calculated over half a period. So for a sinusoidal waveform i average is calculated over half a waveform, that is you take, you take half of the waveform, the positive cycle, find out the area and then divided by, divide by t by 2.

And you can easily show that this is given by twice im divided by pie, you can easily show. We can take sin omega t for example, integrate from 0 to t by 2 and then divide by 2, which is equal to 0.637 im. You must remember the definition of the average value for the sinusoidal waveform. If we take over the one period, it is identically equal to 0 with the peak of half a period. Next time next time would be monday again, is not it, this friday is a holiday, oh, i see a smile on reflex. So monday we shall talk of other kinds of parameters for periodic waveforms. That is all for today.