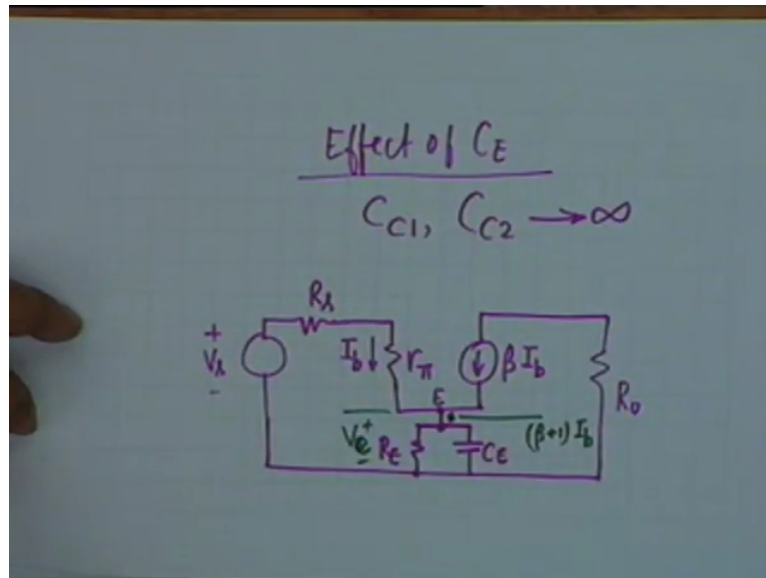


Introduction to Electronic Circuit
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Lecture 36
Small Signal Amplifiers (Contd.)

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36th lecture in which we will continue a discussion on small signal amplifiers and we will start with the effect of CE that is the bypass capacitor and as I said engineers do not like to mix many things up. So for this discussion we assume that CC1 in considering the effect of CE, we consider we assume that CC1 and CC2 both tend to infinity and they both act as short-circuits.

This is only CE that is effective in controlling the low frequency gain, now if I do that then my equivalent circuit becomes Vs Rs CC1 is taken as a short-circuit therefore I have rpi and for a change let us use instead of capital V, let us use the base current the RMS value shall be represented by I sub capital V, there is reason why I am using the current here instead of the voltage.

Then in the collector circuit, collector to emitter, this is the emitter I am not connecting it to the other end of Vs because I want to consider the effect of RE and CE in parallel. This current generator is beta Ib this goes to CC2 is also a short-circuit and therefore RC and RL they come in parallel and therefore they go to ground through a resistance R0. From E we shall have a parallel combination of RE and CE.

It is the effect of CE and we used to consider, we have already assumed that CC1 and CC2's go to infinity. Now the reason why I used current here is now obvious, the current that flows here obviously beta plus 1 I sub b, alright. And therefore the drop that is V_e, no, I should use something else, have I used this symbol correctly? No, I am trying to find the root mean square value.

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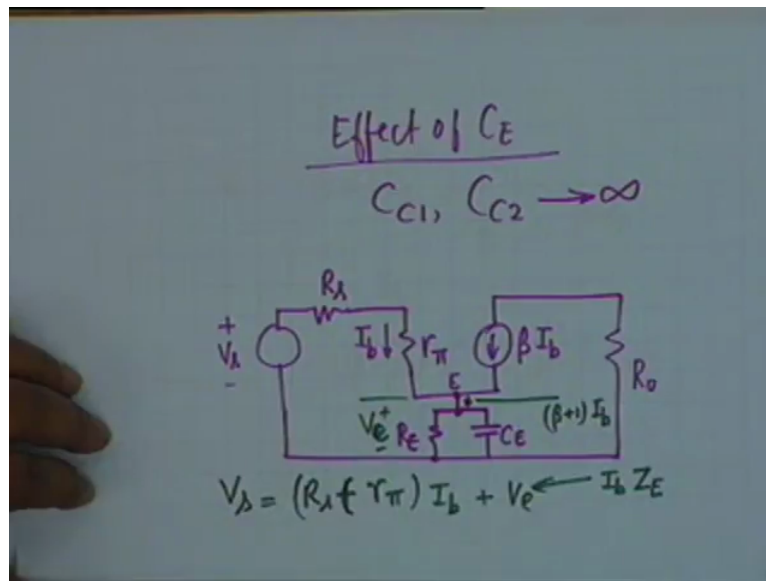
$$V_e = (\beta + 1) I_b \frac{1}{\frac{1}{R_E} + j\omega C_E}$$

$$= \frac{I_b}{\frac{1}{(\beta + 1)R_E} + j\omega \frac{C_E}{\beta + 1}}$$

Small e, okay, V small e therefore let us find this out, V small e equal to the current beta plus 1 I_b multiplied by the impedance, impedance of R_E and C_E in parallel which means that 1 over the sum of admittances that is one over R_E plus j Omega C_E, is it okay? It would have written down the impedance directly R_E divided by j Omega C_E R_E plus 1 but I am doing it in steps.

It is impedance is 1 by admittance 2 elements are in parallel and therefore admittances add for the resistance the admittance is 1 by R_E, for the capacitance it is j omega C_E this I can write as I sub b multiplied by, now I divide both numerator and denominator by beta plus 1 and 1 over beta plus 1 R_E, alright plus j omega C_E divided by beta plus 1, is it okay? Is that clear?

(Refer Slide Time: 5:13)



Now if I write the kvl equation the base circuit, if I write the kvl equation in the base circuit then V_s would be I_b times R_s plus I_b times r_{π} plus V_e , if I can write V_e as the product of I_b and an impedance then obviously I can decouple the input and the output circuit, is that clear? No. Well, my case here KVL is V_s equal to R_s plus r_{π} times I_b plus V_e and V_e have expressed as I_b multiplied by an impedance let say Z_e .

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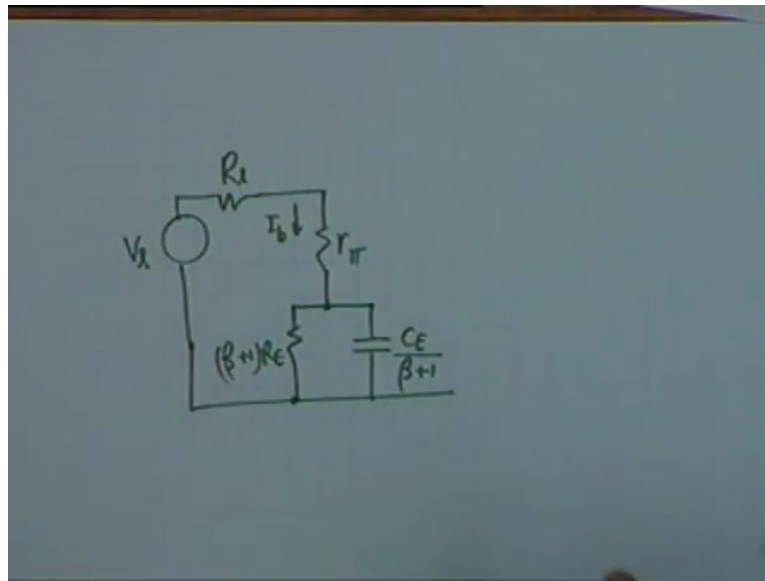
$$V_e = (\beta + 1) I_b \frac{1}{\frac{1}{R_E} + j\omega C_E}$$

$$= \frac{I_b}{\frac{1}{(\beta + 1)R_E} + j\omega \frac{C_E}{\beta + 1}} = I_b Z_E$$

I have done that in this simplification I write this as I_b times Z_e but Z_e is this quantity 1 by this whole thing alright. And therefore I equivalent circuit as far as input is concerned simply a series combination of R_s r_{π} and Z_e , what is Z_e ? Obviously Z_e is a parallel combination of

a resistance which is equal to $\beta + 1 R_E$, not simply R_E and a capacitance was value C_E divided by $\beta + 1$, alright.

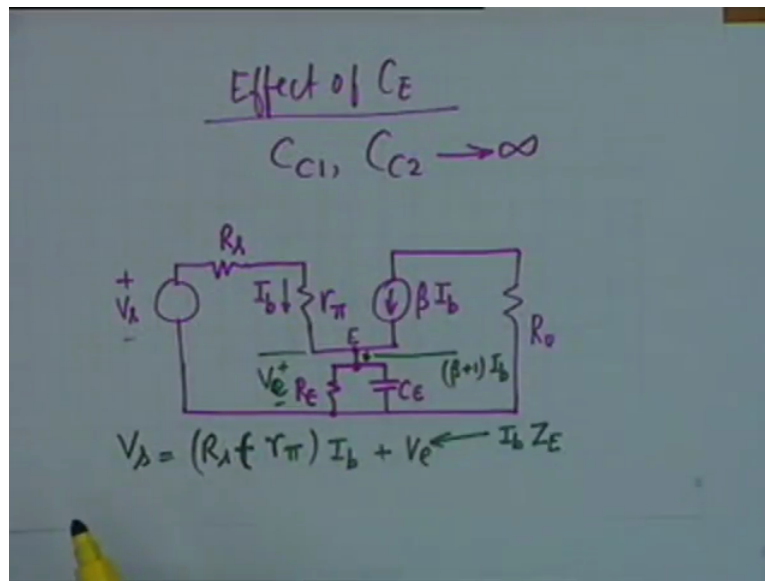
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And therefore by input circuit effectively becomes like this is $V_s R_s r_{pi}$ and then I have a parallel combination of resistance beta plus 1 R_E and a capacitance C_E divided by beta plus 1 alright. And this current is $I_{sub b}$, you see the advantages that I have decoupled the output circuit, now what will be the effective output circuit? Just let us look at that, is this point clear?

That I have allowed the current I_b to flow through this, so what I have done is, I have jacked up the resistance by beta plus 1, I have reduced the capacitance by beta plus 1 is and this is an exactly example we have made no approximation, alright. Now let us look at the output circuit, have a really decoupled?

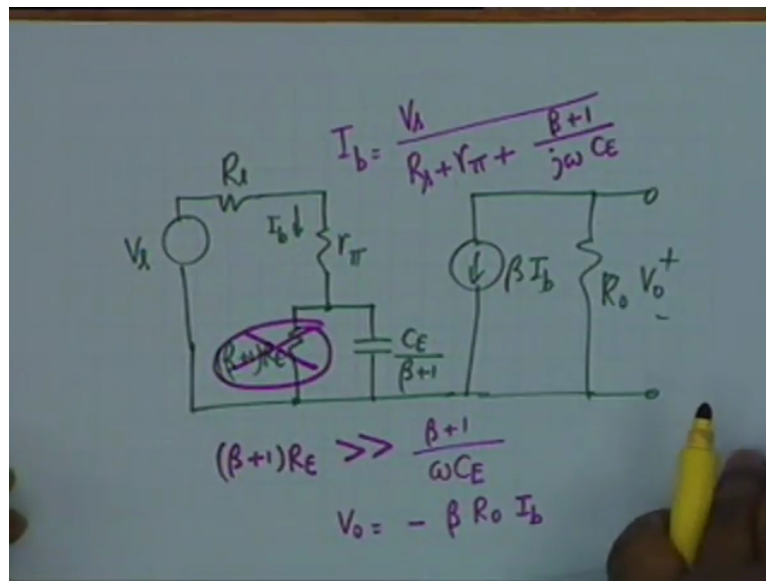
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Interestingly enough you notice that in the output circuit whether current generator βI_b in series with a resistance R_0 and this parallel combination of R_E and C_E . Now what would be the voltage across R_0 ? You know it is simply minus $\beta I_b R_0$ and therefore this is independent of what I connect here, why does this happen? Because the constant current generator delivers current irrespective of what happens to the external world.

What you connect here the current generator simply ignores, it defies, you can connect anything I shall still send the current βI_b and therefore as far as the output circuit is concerned we can safely ignore $R_E C_E$ there is no need for that, is this point clear?

(Refer Slide Time: 8:55)



Okay and therefore my output circuit simply becomes beta I_b and ignoring RAC and simply R_o this will be my V_o . It turns out that in practice the impedance of this that is beta plus 1 R_E is usually much greater than the impedance of the capacitor that is beta plus 1 divided by ΩC_E , is the point clear? C_E is usually chosen large such that the impedance of the capacitor is much less than the impedance of the resistance which means that safely you can ignore this part of the circuit, is this point clear?

If I have 2 impedance is in parallel, one of them is very large compare to the other than effected impedance is that of the smaller one and therefore the equivalent circuit simply becomes this from which you can now calculate the gain that is V_o by V_s . now obviously V_o is equal to minus beta R_o times I_b , alright. From the output circuit beta I_b flows like this and what is I_b ? I_b is equal to V_s divided by R_s plus r_{π} plus 1 over $j \Omega C_E$, well being beta plus 1 here, alright.

(Refer Slide Time: 11:02)

$$\frac{V_o}{V_s} = \frac{-\beta R_o}{R_s + r_{\pi} + \frac{\beta + 1}{j\omega C_E}}$$

$$= \frac{-\beta R_o}{R_s + r_{\pi}} \cdot \frac{1}{1 + \frac{\omega_L}{j\omega}}$$

$$\frac{A_L(\omega)}{A_o} = \frac{1}{1 - j\frac{\omega_L}{\omega}}$$

$$\omega_L = \frac{\beta + 1}{C_E (R_s + r_{\pi})}$$

Now if I combined these 2 then obviously I can get an expression for the gain that is V_o by V_s and you can see that V_o by V_s simple algebra is minus beta R_o divided by R_s plus r_{π} plus beta plus 1 divided by $j\omega C_E$, alright which I can write as, well, just combine those 2 terms I can write this as minus beta R_o divided by R_s plus r_{π} which precisely is what? The mid band gain A_o .

1 divided by 1 plus ω_L is divided by $j\omega$, okay. I take R_s plus r_{π} common then what is ω_L ? Beta plus 1 divided by $C_E (R_s + r_{\pi})$. Can you see why I call this ω_L below frequency 3dB because it was by design; I assumed C_{C1} and C_{C2} to go to infinity. So C_E now determines the low frequency 3dB point or the low frequency cut-off point, alright.

And this is A_o and therefore the low frequency gain now $A_L(\omega)$ divided by A_o , alright. The normalised gain is simply equal to 1 divided by 1 minus $j\omega_L$ divided by ω , alright. C_E , now determines the low frequency 3dB point. Provided C_{C1} and C_{C2} has been chosen sufficiently high, okay. It is sometimes convenient to determine the low frequency 3dB point by C_E rather than either C_{C1} or C_{C2} .

(Refer Slide Time: 13:34)

Handwritten calculations on a whiteboard:

Left side (given values):

$$R_s = 2\text{ K}$$
$$R_E = 1\text{ K}$$
$$C_E = 50\text{ }\mu\text{F}$$
$$R_B \gg r_{\pi} = 1.5\text{ K}$$
$$\beta = 50$$
$$f_L = ?$$
$$R_C = 5\text{ K}, R_L = 10\text{ K}$$
$$C_{C1}, C_{C2} ?$$

Right side (calculations):

$$f_L = \frac{\beta + 1}{2\pi C_E (R_s + r_{\pi})}$$
$$= \frac{51}{2\pi \times 50 \times 10^{-6} (2 + 1) \times 10^3}$$
$$\approx 45\text{ Hz.}$$
$$\frac{(\beta + 1) R_E = 51\text{ K}}{C_{C1} C_E} = \frac{51}{2\pi \times 50 \times 10^{-6} \times 10^6}$$
$$\approx 3.3\text{ K}$$

Let us illustrate this with the help of an example, suppose we have a transistor amplifier in which R_s is 2k, let R_E be 1k, alright. These are very practical values C_E has been chosen arbitrarily as 50 microfarad, alright. R_1 and R_2 are very large that is R_B is much larger than r_{π} , why? This we had assumed right at the beginning, r_{π} is given as 1.5 K and beta is given as 50. r_{π} and beta are given, so you can find out g_m , even find out $I_{sub c}$.

If you recollect, to calculate R_1 and R_2 and the question is estimate f_L , what is the low frequency 3 dB point? And if R_C is equal to 5 K, R_L equal to 10 K then what should be C_{C1} and C_{C2} , do you understand the question? This is what is given, this is the data that is given, what is the low frequency 3dB point? In other words you are being asked to design a circuit, is it is a low frequency 3 dB point is predetermined by C_E and then you are asked what should be your values of C_{C1} and C_{C2} ?

Okay, the design proceeds like this, since C_E determines the low frequency 3 dB points therefore Δf_L as equal to beta plus 1 divided by $2\pi C_E R_s$ plus r_{π} , alright. And you substitute the values of beta 51 2π times 50 times 10 to the minus 6 R_s is 2k and r_{π} is 1k, so 2 plu 1 times 10 to the 3 and this calculates out to approximately 45 hertz.

“Professor -Student conversation starts”

Professor: If you want 5hertz and then you have to use what value of capacitance? What?

Student: 450 microfarad.

Professor: 450 microfarad, yes. Even 1000 microfarad is available this electrolytic capacitors, alright. And therefore if it is a stereo amplifier and this is what is given to you, you are saying your design is bad I will increase the capacitor 9 times are then I will get 5 hertz as the cut-off, alright. And these things should be very obvious to you where to change and what to change, alright.

“Professor-Student conversation ends”

f_L is 45 hertz, now $\beta + 1 R_E$, if you recall we had ignored compared to the impedance of the capacitor, okay. Better plus 1 R_E is how much here? 51 times R_E is 1K, so 51k and $\beta + 1$ divided by $\Omega L C_E$ is 51 divided by 2π times 45 times 50 into 10 to the minus 6, alright. This calculates up to approximately 3.3 K and you see why we ignore $\beta + 1 R_E$, it is about 17 times, 16 times the impedance of the capacitor and therefore that approximately is valid in practice.

(Refer Slide Time: 18:20)

$$C'_{C1} = \frac{1}{2\pi \times 45 (R_s + r_{\pi})} \cong 1 \mu F$$

$\uparrow \quad \uparrow$
2k 1.5k

$$C'_{C2} = \frac{1}{2\pi \times 45 (15k)} \cong 0.22 \mu F$$
$$C_{C1} = 10 \mu F \quad C_{C2} = 2 \mu F$$

The next point is how to choose CC1 and CC2? If we recall what we did, the design philosophies like this with this omega L you calculate the required value of CC1 and CC2 and call them CC1 prime and CC2 prime than in the actual circuit we use at least 5 times this value for example here in this example CC1 prime would be 1 over omega L that is 2pi times 45, yes.

Rs plus rpi, Rs is 2k and rpi is 1.5k and this calculates out approximately to 1 microfarad, 3.5, 7, yes, so it is approximately 1 microfarad. Similarly CC2 prime, we will calculate like this 2pi times 45 then RC plus RL therefore given so it is 15k alright. And this calculate (()) (18:59) approximately 0.22 microfarad. So in this circuit what we use is 5 times this, 5 microfarad, you do not get 5 you get 4.7, alright, or 6.8, I will use 6.8 use 10 microfarad, why not?

Even that is available, incremental cost maybe a little more which is worth it, okay. And CC2 it is at least 5 times this or 1 microfarad is a good figure or slightly higher, 2 microfarad is also available, it is 2 microfarad.

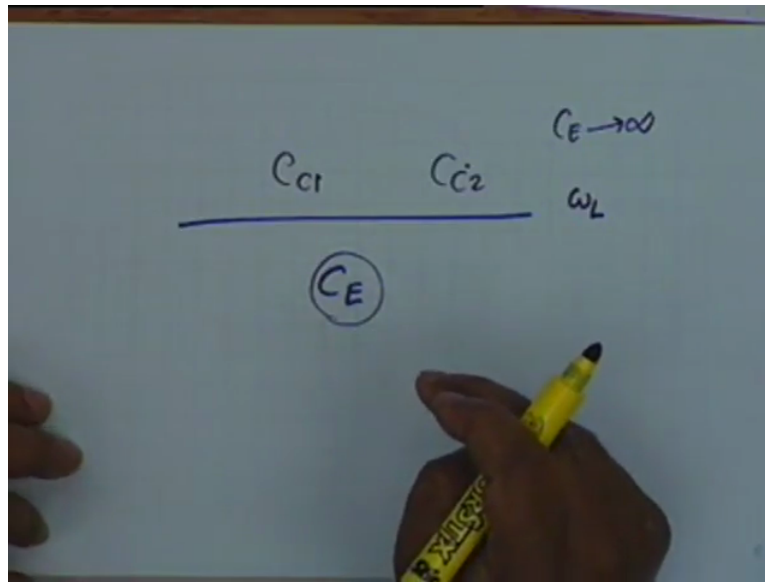
“Professor -Student conversation starts”

Student: Larger than 2 and the second one...

Professor: Larger of the 2, no here the situation is slightly different, here the situation is 3 capacitors, we have already chosen C_E that is the largest value 50 microfarad, larger of the 2 is when C_E is assumed to be infinity.

“Professor-Student conversation ends”

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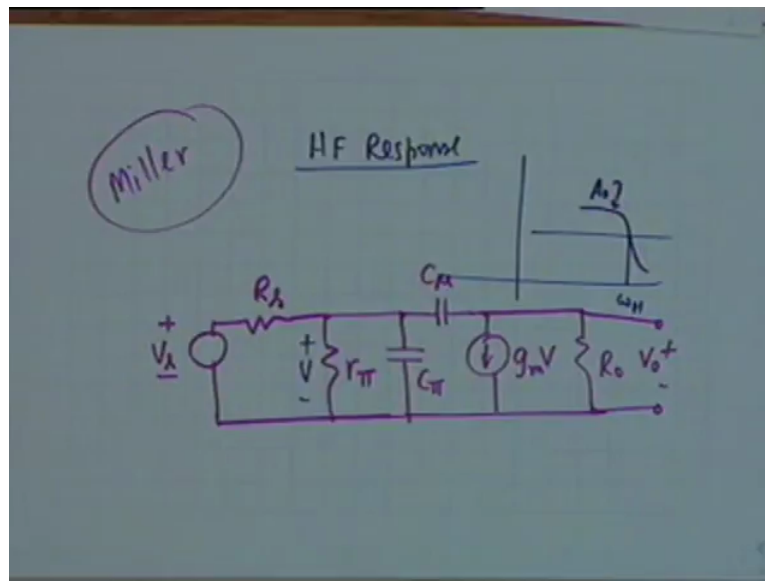
Let me recall it, what I said if you have a choice to determine ω_L , can I have your attention? If your choice is determined ω_L by either C_{C1} or C_{C2} assuming C_E to go to infinity, C_E is 1000 microfarad we have already done that then you calculate from the specified ω_L , you calculate C_{C1} and C_{C2} , have the larger of the 2 is the value that is calculated, the smaller one you multiply 5 times that is a different situation.

In our situation you are determining ω_L by C_E and trying to find out what should we have done with C_{C1} and C_{C2} , whatever you calculate C_{C1} and C_{C2} from the given ω_L we multiply 5 times than it will make sure that C_E , ω_L determined by C_E is the largest of the 3 and so this will determine the low frequency 3dB cut-off point. Now one can say why do not you optimise? Why do not you optimise by using the minimum possible capacitances that is I use all the 3 together and then try to find out by solving sixth order algebraic equation numerically what ω_L shall be?

It is not worth it, alright. Because large electrolytic capacitors are available in plenty, if is a problem you might have to do that but then if space is a premium what you do is, you use very small power supply also for example a circuit to be put in space in a satellite, well, you

are dependent on solar battery the power supply small, so you slow resistances and if you use slow resistances the capacitors have to be sufficiently high but the current level is small and therefore use semiconductor capacitors you do not use electrolyte these are facts of lives, compromises of lives that one has to make, alright.

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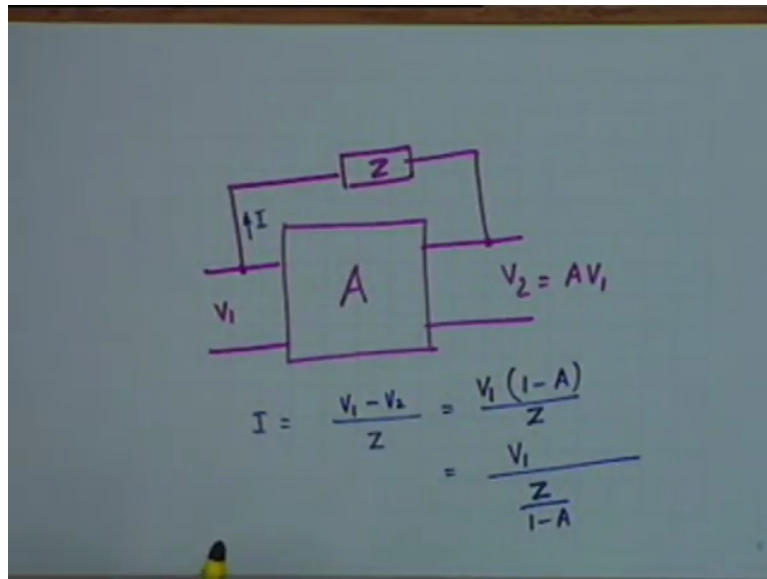


Now so far so good about mid band low frequency, what we do at high frequency? How do you determine at high frequency as you remember ω_H and this is A_0 , how you determine the high frequency 3 dB cut-off? At high frequencies the C_{C1} , C_{C2} and C_E all of them are shorts, what is effective is the internal capacitances of the transistor that is C_{pi} and C_{mui} .

Let us see how C_{pi} and C_{mui} affect this circuit. The equivalent circuit if you draw it will be V_s the it is resistance R_s inevitably present we ignore R_B as we have done earlier, so you shall have r_{pi} , now C_E is short and therefore this is the grounded r_{pi} and in addition we must have the capacitor C_{pi} this voltage is v and then we have between C_{pi} between this and the collector we have the capacitor C_{mui} , the current generator is $g_m V$ and then we have C_{C2} is short therefore R_C and R_L are in parallel and so R_O and this is V_O .

Now since you have taken a course from a (()) (24:15) I take it that you are very good in circuit analysis. Now you can make model analysis, you can make loop analysis and so on but try to do this for the simple circuit terms of symbols life becomes miserable because there is a controlled current source here and so as electrical engineers in common with other engineers we try to simplify matters and this is where a gentleman by the name CJ Miller contributed very substantially. Miller was a practicing engineer and he found this kind of an analysis troublesome, he was not good at loop analysis, load analysis, so he said I will simplify the matter.

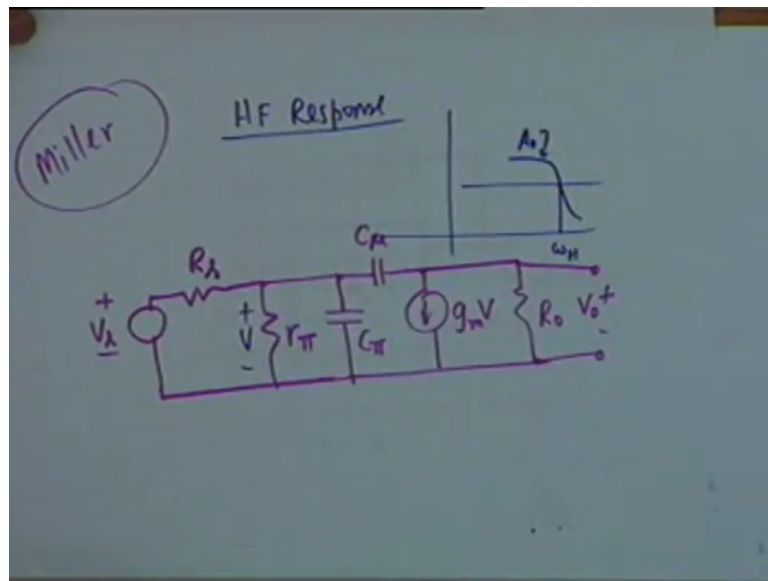
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Let us look at Miller's philosophy little carefully then we shall go back to this circuit, please try to follow this carefully. Suppose we have an amplifier or 2 port, suppose we have 2 port in which the output voltage v_2 is equal to A times V_1 and suppose we connect an impedance Z here that is we have a bridging impedance. The bridge is the input port to the output port, alright.

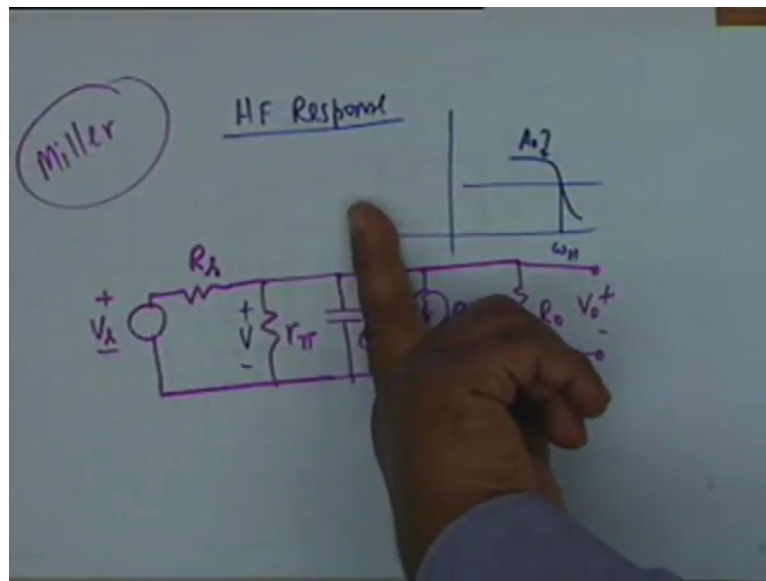
What is the current through this I ? What is the current through this, obviously I shall be equal to V_1 minus V_2 divided by Z and since V_2 is A times V_1 we get V_1 times 1 minus A divided by Z , alright. Look at this in the city of the concept then I write this as V_1 divided by Z by 1 minus A , what does it mean? It means that as far as the input port is concerned the effect of the bridging impedance is simply that you get $(\frac{Z}{1-A})$ (26:55) and impedance which is not Z but Z by 1 minus A , is that clear?

(Refer Slide Time: 27:16)



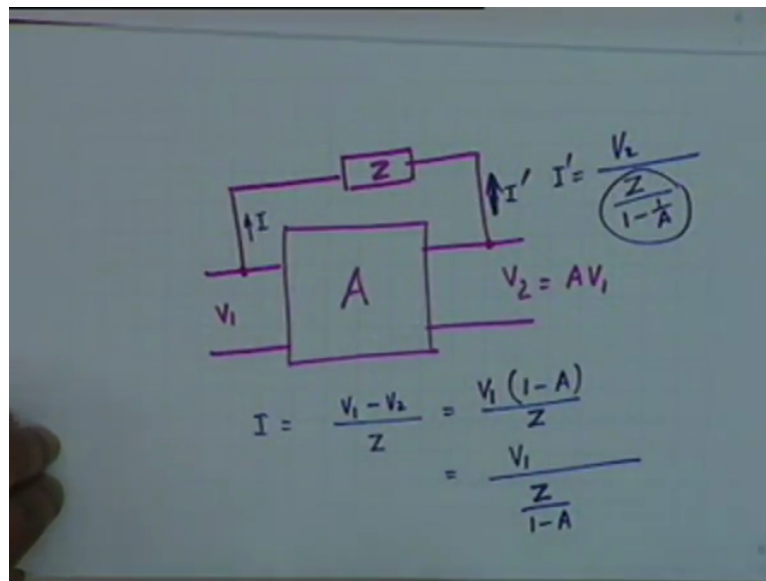
No, you see I want to convert this circuit into 1 in which the bridging impedance is not there and the clue is why did Miller think so? Miller found this very disturbing C_{μ} , it is bridging the input and output.

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You see previously in our mid band as well as low frequency $C_{m\text{ui}}$ was not there and analysis was absolutely by inspection, commonsense nothing else, no load analysis, no loop analysis nothing we just looked at the circuit and wrote down the expression, with this you cannot do this. So he said can I replace is bridging impedance by an equivalent impedance at the input and an equivalent impedance of the output this is what he argued.

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And so what he said is what does this bridging impedance do while it extracts and additional current I . If I can account for this current then obviously the bridging impedance can be ignored as far as the input is concerned, so at the input what we shall have is, since this current is only proportional to V_1 , what I will do is, between this point and this point I will connect Z by 1 minus A , alright.

Similarly let us look at the output, what does the output do? Output there is a current I .

“Professor -Student conversation starts”

Professor: And this current I , well, let us call this output as this current is I prime, can you tell me what is I prime V_2 divided by what?

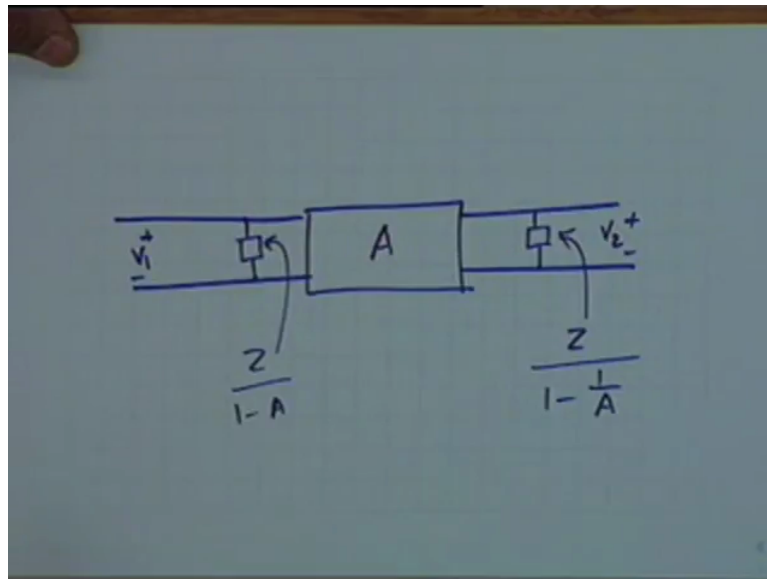
Student: (()) (28:45)

Professor: Pardon me. Z divided by 1 minus 1 by A , is not it right?

“Professor-Student conversation ends”

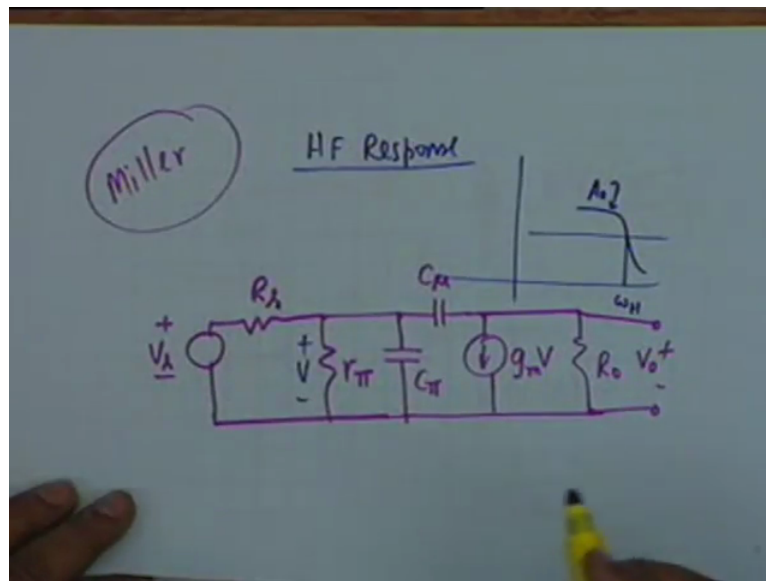
Okay and therefore at the output the bridging impedance acts as if the impedance is divided by a quantity 1 by 1 minus A , yes, there is perfectly all right, okay.

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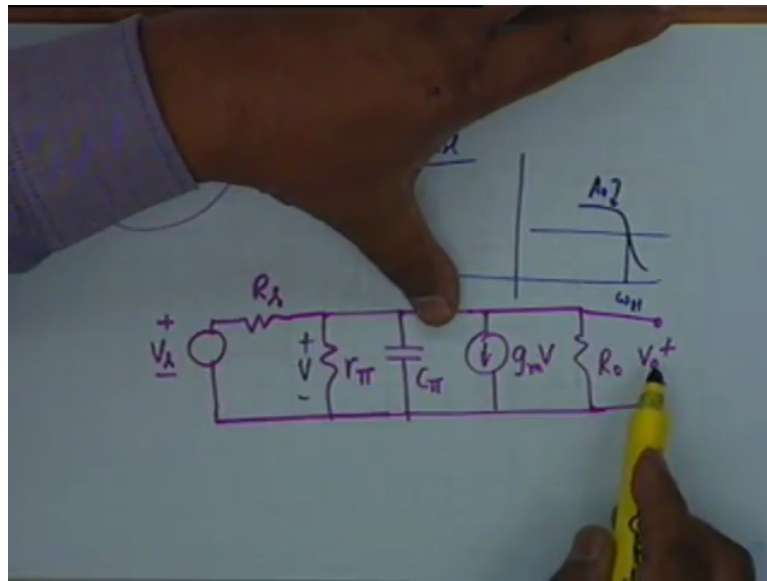
So equivalently then I can replace this circuit which had a bridging impedance by 1 which does not have a bridging impedance and there exact equivalence have made no simplification no approximation. What I have here is an impedance Z by 1 minus A and what I have here is that divided by 1 minus 1 over A , alright. The question is what is A ?

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From this transistor circuit then you for a moment go back to the transistor circuit, let us look at the transistor circuit once more. What is A ? You see A is V_o divided by V and this is where Miller say, let us play a little smart, in calculating A let us ignore $C_{m\mu}$, if I do that then what is A ? This is V and this is $g_m V$, $g_m V$ minus $g_m V R_o$ is the output voltage, output voltage divided by input voltage that is V , so the gain is simply minus $g_m R_o$ this is approximate, alright.

(Refer Slide Time: 30:59)



So, yes, please repeat, Miller said in calculating A we require this value of A that is V_2 equal to A times V_1 , in calculating the value of A for the transistor circuit lets ignore $C_{m\text{ui}}$, alright. If I do that then the output divided by the input V_0 by V is simply minus g_m times R_0 this is an approximate picture, alright.

(Refer Slide Time: 31:20)

$$A \approx -g_m R_o$$

$$Z = \frac{1}{j\omega C_\mu}$$

$$\frac{Z}{1-A} = \frac{1}{j\omega C_\mu (1 + g_m R_o)}$$

$$\frac{Z}{1 - \frac{1}{A}} = \frac{1}{j\omega C_\mu (1 + \frac{1}{g_m R_o})}$$

So A, for our case A approximately equal to minus gm R0 you understand this approximation calculating capital A we have ignored Cmui, alright. What is Z in our case? The bridging impedance is 1 over j Omega Cmui, alright. This is the bridging impedance and therefore at the input Z divided by 1 minus A is equal to 1 over j Omega Cmui times 1 plus gm Rnot, is the point clear?

However effective input impedance reflected input impedance is 1 over j Omega Cmui 1 plus gm Rnot and you see what happens, Cmui is a small capacitor of the order of 300 (pF) (32:16) and Cmui is now multiplied by 1 plus the gain of the circuit, if the gain is 99 and Cmui is 300 (pF) (32:28) equivalent it is reflected and the input across Cpi as 30000 (pF) (32:37), is the point clear?

Even a small capacitor can cause devastation in the input circuit because it can swamp Cpi, Cpi is only of the order of 300 (pF) (32:49) and if the gain is 1000, well, 300000 (pF) (32:54), Cpi can be safely ignored, this is what the effect of Cmui is even though it is a very small capacitor it can be reflected into the large capacitor. Now what happens to the output circuit? At the output circuit it is reflected as Z divided by 1 minus 1 by A which means it is 1 over j omega Cmui times 1 plus 1 over gm Rnot.

And if gmRo is of the order of 100 then safely this can be ignored therefore at the output Cmui remains Cmui, is the point clear? And this is set or this method of calculation this effect is known as the Miller effect, that is a small capacitor, a small impedance is reflected, a small

capacitor is reflected as a large capacitor this is called the Miller effect and the whole theorem is known as Miller's theorem.

If I take account of Miller theorem then you see my equivalent circuit now becomes $V_s R_s r_{pi}$ C_{pi} and then, pardon me.

“Professor -Student conversation starts”

Student: you said (()) (34:28)

Professor: No, let us keep it for the time being it is dependent the value of the gain.

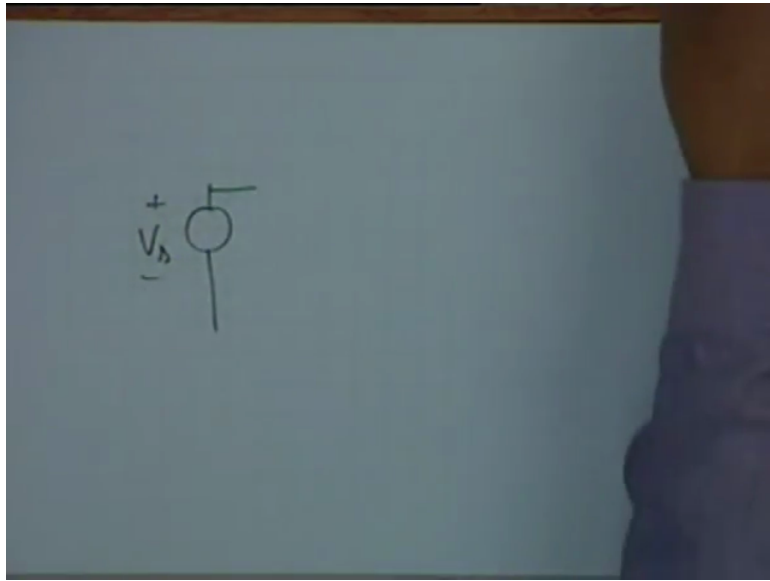
“Professor-Student conversation ends”

C_{pi} and then the Miller reflected capacitor which you shall call as C_M , C_M is C_{mui} multiplied by $1 + g_m R_{out}$. Okay, there are 2 capacitances now and the 2 together we shall represent C_T total input capacitance C_T is C_{pi} plus C_M and voltage across this is V , since you have taken account of C_{mui} the bridging capacitance our input circuit is decoupled from the output circuit.

So we have $g_m V$, now we go happily $g_m V$ but you must reflect Z by $1 - 1/A$ that is C_{mui} here, there is C_{mui} and in parallel with R_0 . Now R_0 as you know is of the order of a few cases, okay. Maybe parallel combination of 4 K and 4 K makes only 2k where is C_{mui} is a very small capacitor 3(()) (35:47). So the impedance of C_{mui} shall in all probability be very large compared to (()) (35:53) and this C_{mui} can be ignored, these are practical simplifications.

A large impedance shunting a small impedance has no effect and so we can ignore this and we can combine these 2 into a single capacitor C_T which gives me the equivalent circuit, we are following equivalent circuit.

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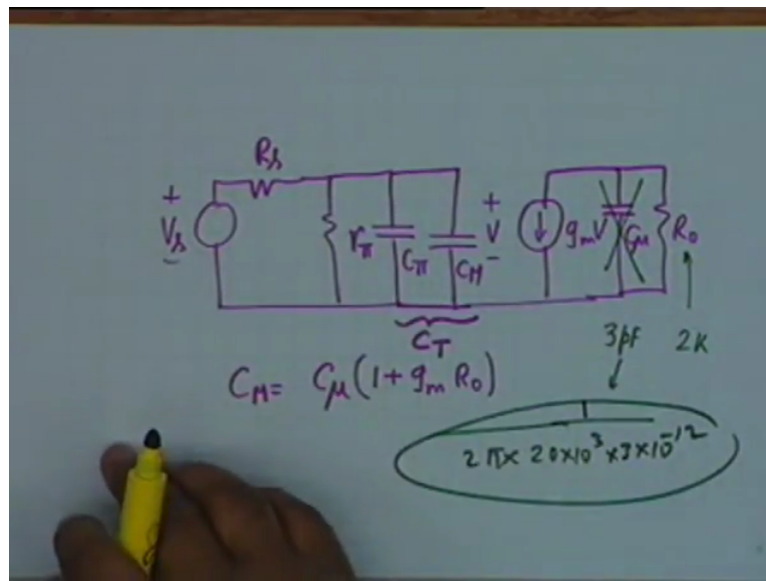


We have V_s ...

“Professor -Student conversation starts”

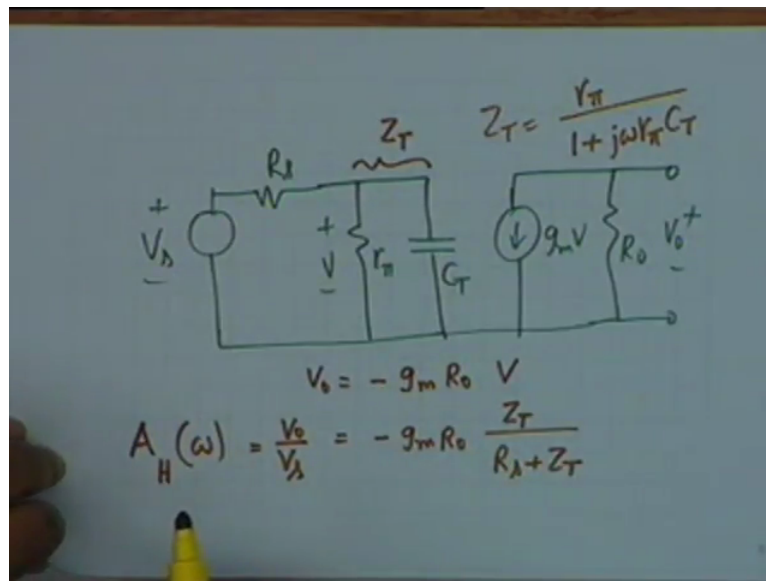
Student: Can you repeat your thought on ignoring C_{mi} .

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Professor: Ignoring C_{μ} , okay. I said that R_o is of the order of a few Ks, let say it is 2k, C_{μ} is a 3puff capacitor, alright. Let see the end of the audio band 20 kilohertz, so frequently at 20 kilohertz the impedance of 3 puffs is 20 times 10 to the 3 2π times this multiplied by 3 times 10 to the minus 12 and if you calculate this, this would be very large compared to 2 K. It will be of the order of (()) (37:03) and therefore affect of C_{μ} can be ignored. And large impedance shunting a small impedance has no effect therefore we ignore that.

(Refer Slide Time: 37:19)



The equivalent circuit now becomes V_s , R_s , r_{π} and C_T , r_{π} and C_T this voltage is V and in the output circuit we have $g_m V$ multiplied by not multiplied (\emptyset) (37:33) with R_o and this is the voltage V_o to calculate the gain V_o by V_s you notice that V_o is minus $g_m R_o$ multiplied by V , alright. Now look at the simplification that comes into effect minus $g_m R_o$, V is let us call this impedance as Z_T the impedance of r_{π} and C_T in parallel let us call it Z_T .

Then it would be Z_T divided by R_s plus Z_T multiplied by V_s , so we take V_o by V_s (\emptyset) (38:27) and you call this A gain as a function of frequency at high frequencies, A_H ω given by minus $g_m R_o$ divided by Z_T plus R_s plus Z_T , okay. Let us write this expression, what is Z_T ? Z_T you can easily show this is r_{π} divided by 1 plus j ω r_{π} C_T .

(Refer Slide Time: 39:10)

$$A_H(\omega) = \frac{-g_m R_o Y_{\pi}}{1 + j\omega Y_{\pi} C_T} \cdot \frac{R_s + \frac{Y_{\pi}}{1 + j\omega Y_{\pi} C_T}}{R_s + Y_{\pi} + j\omega Y_{\pi} C_T R_s}$$

$$= \frac{-\beta R_o}{R_s + Y_{\pi} + j\omega Y_{\pi} C_T R_s} \cdot \frac{1}{1 + j \frac{\omega}{\omega_H}}$$

$$\omega_H = \frac{1}{C_T (R_s \parallel R_s)}$$

If I substitute this, let us see what happens, $A_H(\omega)$ becomes equal to minus $g_m R_o$ divided by $1 + j\omega Y_{\pi} C_T$ divided by $R_s + Y_{\pi} + j\omega Y_{\pi} C_T R_s$, I have simply substituted for Z_T , alright. Now I multiply both numerator and denominator by $1 + j\omega Y_{\pi} C_T$ and you also notice that in the numerator have g_m and r_{π} , so I can write minus βR_o divided by R_s , R_s multiplies this R_s and r_{π} is left alone $R_s + Y_{\pi} + j\omega Y_{\pi} C_T R_s$, is that okay.

So now I take this out, you see my purpose is to bring the mid band gain in some manner or other, its band gain is minus βR_o divided by $R_s + Y_{\pi}$ and this I can write as one plus $j\omega$ by ω_H , where ω_H is defined as what? $1 / (r_{\pi} C_T R_s)$, no, not quite, into $R_s + Y_{\pi}$ and do not you see that this is simply $1 / (C_T (R_s \parallel R_s))$, this therefore is the high frequency 3 dB point at ω equal to ω_H the gain becomes $1/\sqrt{2}$ times the value at mid band that is my normalised gain.

(Refer Slide Time: 41:41)

$$\frac{A_H(\omega)}{A_0} = \frac{1}{1 + j \frac{\omega}{\omega_H}}$$

$$\omega_H = \frac{1}{C_T (R_T \parallel R_S)} \quad \leftarrow \quad \omega_\mu = \frac{1}{C_\mu R_0}$$

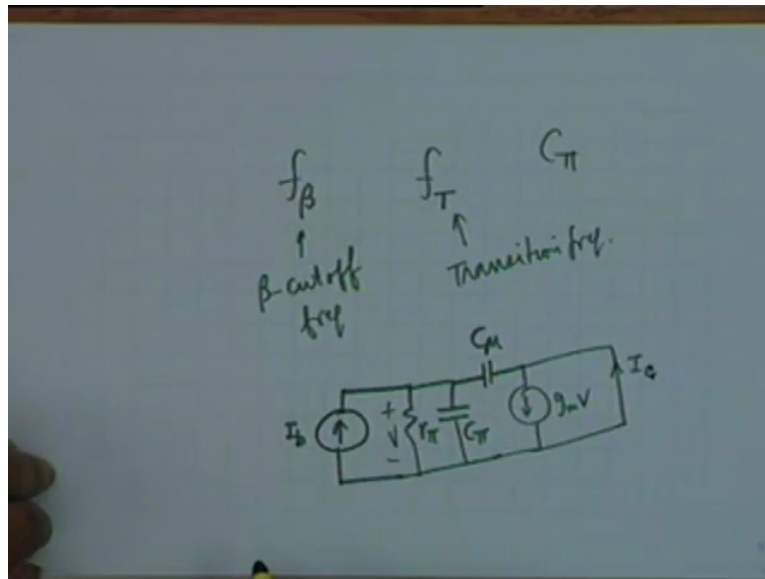
ω_{11} ω_{12}

My normalised gain now $A_H \omega$ divided by A_0 becomes simply equal to $1 + j \omega$ by ω_H . Is it okay? There are no other capacitors which are giving the ω_H , why not? We have started with 2 capacitors how come we are left with only one because we ignored, C , no, CC_1 and CC_2 and CE have no business to show their faces (()) (42:18). $C_{\mu i}$ and the output (()) (42:22) we ignored it.

Suppose we include a defect, what would have happened? ω_H , now from it is strictly π parallel R_s if we had included $C_{\mu i}$ we would have got some $\omega_{\mu i}$ which is equal to $1 / C_{\mu i} \times R_0$ and constantly there will be $C_{\mu i} \times R_0$ and this $\omega_{\mu i}$ would have been much greater than ω_H because $C_{\mu i}$ is a very small quantity, is the point clear? Now between 2 values of ω , ω_H and $\omega_{\mu i}$ the lower one shall determine the high frequency 3 dB point, is this point clear.

In the low frequency case, you had ω_{11} and ω_{12} and the higher one was determining ω_L because if this is ω_{12} and this is ω_{11} , it is this which is close to 1 by root 2 (()) (43:43). On the other hand at high frequencies it is the smaller one that shall determine and therefore this is the high frequency 3 dB cut-off point and we could determine this because Miller was there but it is not that you cannot determine this we can do this numerically if the values are given we will solve the equation, an algebraic equation we can solve it but you can see the analysis has become almost by inspection now.

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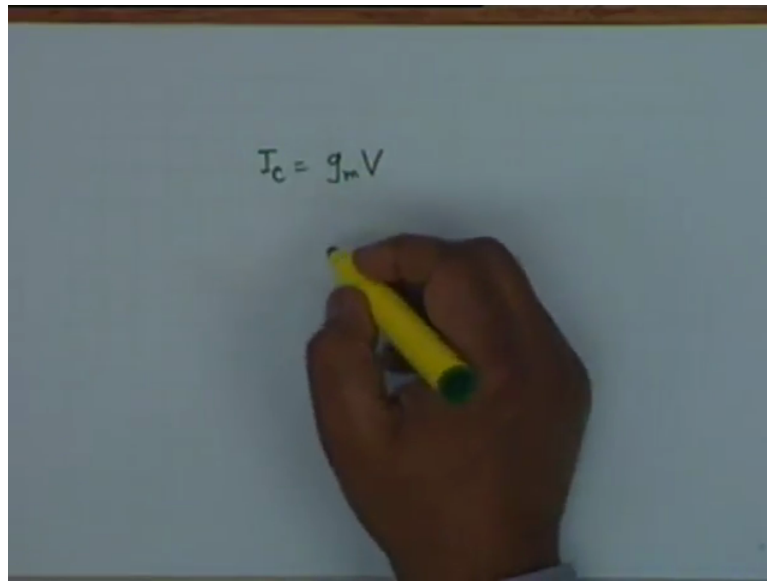


In the remaining few minutes I shall introduce 2 terms and then let you go and have these concepts simmered 2 terms a transistor manufacturer usually does not specify C_{π} . It specifies a quantity known as f_{β} or a quantity known as f_T while the names of these are beta cut-off frequency and this is called the transition frequency and the definition comes like this, suppose you drive a transistor with a current source I_b , alright.

And you find out the short-circuit output current, you take a common emitter transistor drive it at the signal source I_b current generator short the output and find out the current. Well, in terms of the equivalent circuit you see in a base you have r_{π} C_{π} then you have C_{μ} then you have $g_m V_{be}$, this is V and as I said you short-circuit the collector than this current would be I_c the output current, the collector current, alright.

And the ratio of I_c to I_b would be the current gain or current amplification factor and to indicate that this is done under short-circuit conditions, we say this is the short-circuit current amplification factor, alright. If I calculate this you see obviously I_c is equal to $g_m V_{be}$, is not that right?

(Refer Slide Time: 46:47)



I sub c is equal to gmV, alright. No, pardon me.

“Professor -Student conversation starts”

Student: Value.

Professor: I sub C is not equal to gmV.

Student: It is a (()) (47:00)

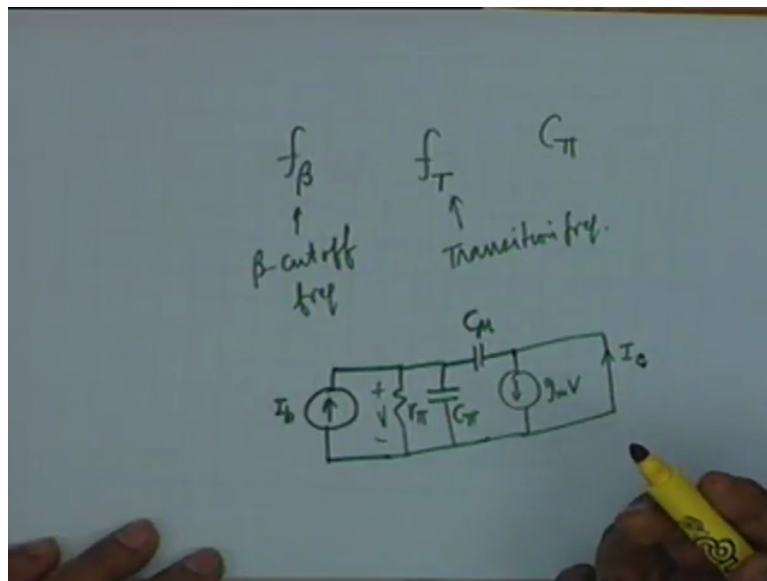
Professor: It is a short-circuit and therefore the current generator, why should it send through impedances, it finds short-circuits, so it sense all its current here, no.

Student: That is (()) (47:13)

Professor: There is no approximation here this is exact. The current generator also looks for the shortest possible part like every system in the world goes to its less potential energy, why should the current generator be oblige to send the current to Cmui, who will tell me? It sense all its current to the short-circuit.

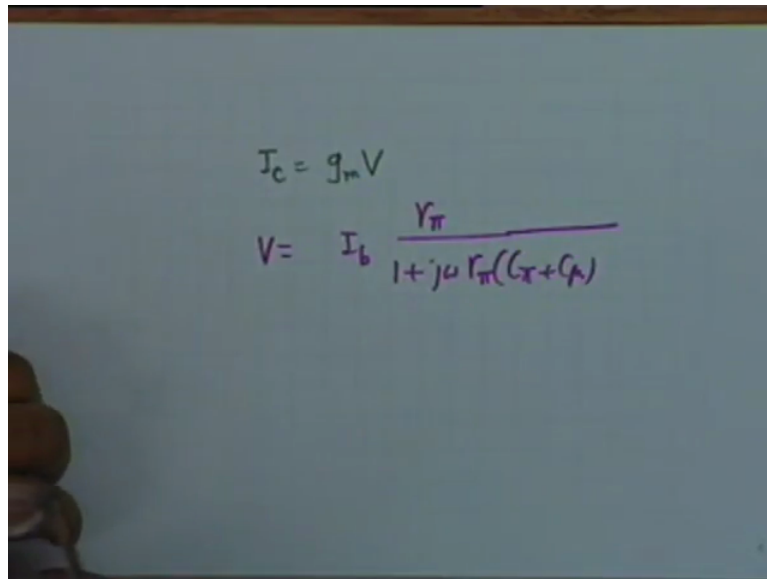
“Professor-Student conversation ends”

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So $I_{sub c}$ is gmV but what is V? You see this point is virtually grounded that means if I look at the input circuit it is as if we have capacitor C_{pi} plus C_{mui} .

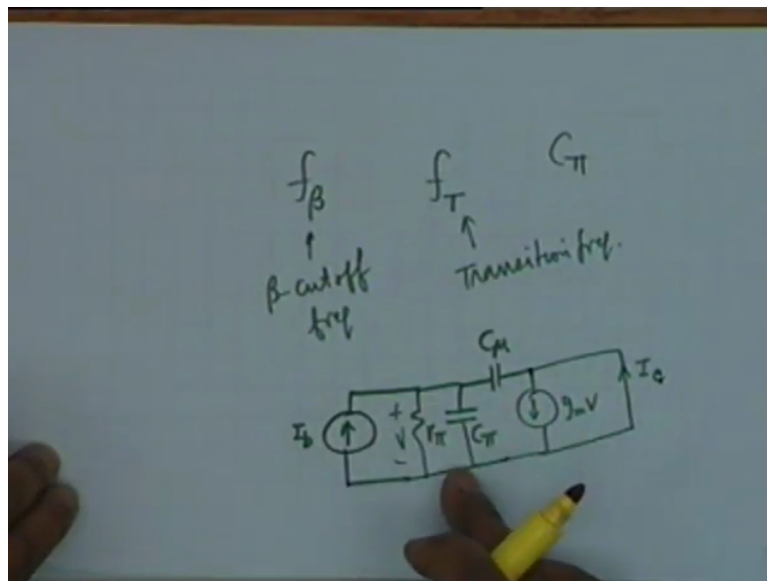
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The image shows a whiteboard with two equations written in purple marker. The first equation is $I_c = g_m V$. The second equation is $V = I_b \frac{r_{\pi}}{1 + j\omega r_{\pi}(C_{\pi} + C_{\mu})}$.

And therefore V is I sub b multiplied by r_{π} divided by 1 plus j ω r_{π} C_{π} plus C_{μ} , alright. Is the point clear?

(Refer Slide Time: 48:17)



I sub B this current generator flows to this impedance and this impedance consist of r_{π} parallel C_{π} parallel $C_{m\mu}$ the current generator does not come into effect alright.

(Refer Slide Time: 48:28)

The image shows handwritten mathematical derivations on a whiteboard. At the top, it states $I_c = g_m V$. Below that, the voltage V is given as $V = I_b \frac{r_{\pi}}{1 + j\omega r_{\pi}(C_{\pi} + C_{\mu})}$, with an arrow pointing to the denominator term $(C_{\pi} + C_{\mu})$ and the text $\rightarrow 0$. The main equation is $\beta(\omega) = \frac{I_c}{I_b} = \frac{\beta_0}{1 + j \frac{\omega}{\omega_{\beta}}}$. To the right of this equation, $f_{\beta} = \frac{\omega_{\beta}}{2\pi}$ is circled. Below the main equation, $\omega_{\beta} = \frac{1}{r_{\pi} C_{\pi}}$ is also circled.

This is what I have written and you also know that C_{μ} is of the order of a few pF and C_{π} is of the order of 100 pF, so you ignore C_{μ} alright. So you can write now I_c by I_b , okay. Short-circuit output current divided by the input current base current and this quantity is represented as beta but beta as a function of frequency $\beta(\omega)$ and you can see that this is simply equal to $g_m r_{\pi}$ which by definition is β_0 but since beta is never being represented as a function of frequency we should call that beta as the mid band beta that means we are going to represent this by β_0 , alright.

$g_m r_{\pi}$ shall be represented as β_0 and the other quantity I shall represent as $j\omega$ divided by ω_{β} where obviously ω_{β} is equal to $1 / r_{\pi} C_{\pi}$, alright. And f_{β} is $\omega_{\beta} / 2\pi$, can you now give a definition of f_{β} ? f_{β} is a frequency, well if you measure beta from low frequencies to extremely high frequencies then the high frequency at which beta falls by 3 dB is the frequency f_{β} and therefore f_{β} is known as the beta cut-off frequency, alright.

(Refer Slide Time: 50:35)

Handwritten notes on a slide showing the frequency response of a transistor. The main equation is $\beta(\omega) = \frac{\beta_0}{1 + j\frac{\omega}{\omega_\beta}}$. Other equations include $\omega_\beta = \frac{1}{r_\pi C_\pi}$, $Y_\pi = \frac{\beta_0}{g_m}$, and the definition of transition frequency $\omega_T \triangleq$ freq. at which $|\beta(\omega)| = 1$. The final result is $GBW = \omega_T = \beta_0 f_\beta$.

Let me write this again beta omega is equal to beta 0 divided by 1 plus a omega by omega beta, omega beta is equal to 1 by rpi Cpi. The manufacturers not measure Cpi, what the measure in an automatic measurement setup, you see there do not make one transistor a day they make millions a day and one cannot go on measuring million transistor's beta and Cpi and all this.

So what they do is, automatic transistor comes immediately and machine a robot hooks it up to a test instrument and it measures it displays the beta versus frequency path and it measures f beta and this is the quantity that is specified. If you want for your design the given f beta even find out Cpi from there because you know rpi, how do you know rpi? Pardon me, rpi is beta 0 by gm, how do you know gm? $40 I_c$ and beta 0 is specified by the manufacturer they specify beta they do not specify rpi, they do specify omega beta.

Most of the times manufacturers specify, do not even specify omega beta, this specify a frequency called omegaT, omega T by definition is a frequency at which the magnitude of beta omega is equal to 1, alright. Magnitude of beta omega is equal to 1, what does it mean? It means that the current application just ceases beyond omega T, is not that right? If omega is greater than omega T and beta omega will be less than 1 which means there is no current amplification factor this is why omega T is called a transition frequency.

Transition for amplification to no amplification and it is a matter of simple of algebra to show from here, you prove magnitude beta omega equal to 1, to show it from here that omega T is nothing but beta 0 multiplied by f beta, omega T is beta 0 f beta it can be very easily shown

from here you will take this as the tutorial problem or maybe you will set it in one of the examinations, okay. It is very easy to show.

Now what does this mean? It means that even if f_{β} is not given f_T is given, you can still calculate C_{pi} all this is directed towards C_{pi} and f_T has a physical significance that beyond this frequency the transistor is useless, is not that right? If $I_{sub c}$ is equal to $I_{sub d}$ what use is of such a transistor? So this is the absolute upper limit of frequency that is a transistor should be considered useful.

You also notice that β_0 has the dimension of gain and f_{β} is the so called bandwidth, that is a band of frequencies within which the transistor β remains to within 70.7 percent of its mid band value and therefore this is the product of gain and bandwidth and another name for ωT is GBW gain bandwidth product (()) (54:30) are very fond of many terms for the same quantity, they also call it the figure of merit of a transistor that can be understood.

If you have 2 transistors in which you have T in 1 megahertz and 10 megahertz and you want to make a system at 5 megahertz, naturally you will choose the 10 megahertz band that transistor is more meritorious than the 1 megahertz one, alright. With this we conclude today