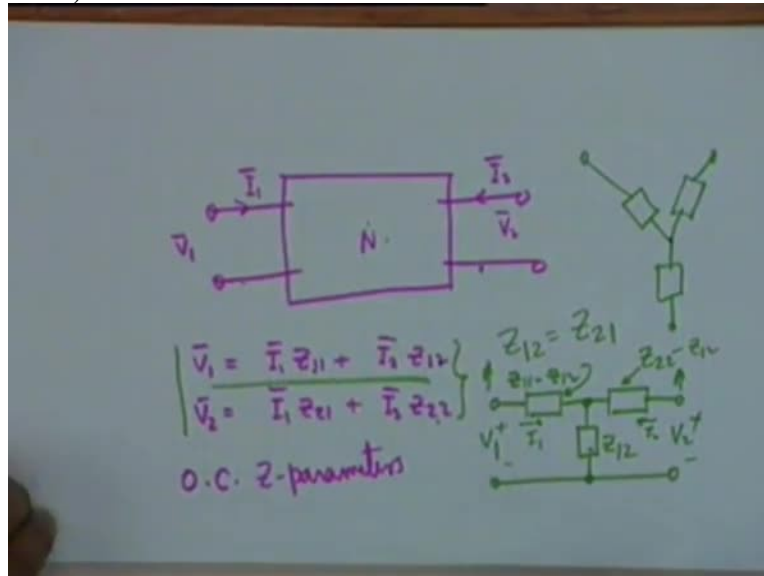


**Introduction To Electronic Circuits**  
**Professor S.C. Dutta Roy**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Delhi**  
**Module No 01**  
**Lecture 24: Two-Port Networks**

Electrical. 24<sup>th</sup> lecture on two port networks.

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As I had discussed yesterday, a two port network is one which has 2 ports, as simple as that. And for sinusoidal excitation, we take the phaser quantities  $V_1$ ,  $I_1$ ,  $V_2$ ,  $I_2$  and as already mentioned, 2 of these variables can be dependent, the other 2 are dependent. And there are 6 ways of expressing these relationships. 2 of them which are most popularly used are the currents as the independent variables and the voltages of the dependent variables and we have  $I_1 Z_{11}$  plus  $I_2 Z_{12}$  and  $I_1 Z_{21}$  plus  $I_2 Z_{22}$ . In order not to write these bars again and again, we will omit the bars. It will be understood in the context that these are phasers.

I have already explained why there are 2 subscripts. 2 subscripts are needed to express the transfer impedances and therefore to be consistent we use two subscripts on the driving point impedances also.  $Z_{11}$  is a driving point impedances with the port 2 open circuited. This  $Z_{11}$  is  $V_1$  divided by  $I_1$ . So it is an impedance at the driving point at port number 1 with port 2 open

circuited. In fact, all these 4 parameters,  $Z_{11}$ ,  $Z_{12}$ ,  $Z_{21}$ ,  $Z_{22}$ , if you measure or define, these will be conditions under open circuit.

For example,  $Z_{12}$  is  $V_1$  divided by  $I_2$ . That means you apply a current generator here and measure the voltage under what condition? Open circuit condition. That is, you have to make  $I_1$  equal to 0 and therefore these are called Z parameters and sometimes you use the adjective, OC open circuit Z parameters. And you also know that because of reciprocity, that is if capital N is a reciprocal network, then you know that the 2 transfer impedances are equal,  $Z_{12}$  is equal to  $Z_{21}$ . There are many other interesting things that can be brought out from these relationships and one of them is that the blackbox, if you represent it by, if you characterise it by  $Z_{11}$ ,  $Z_{12}$  and  $Z_{22}$ , only three out of four are needed if the network is reciprocal, then you can construct an equivalent model for the network like this.

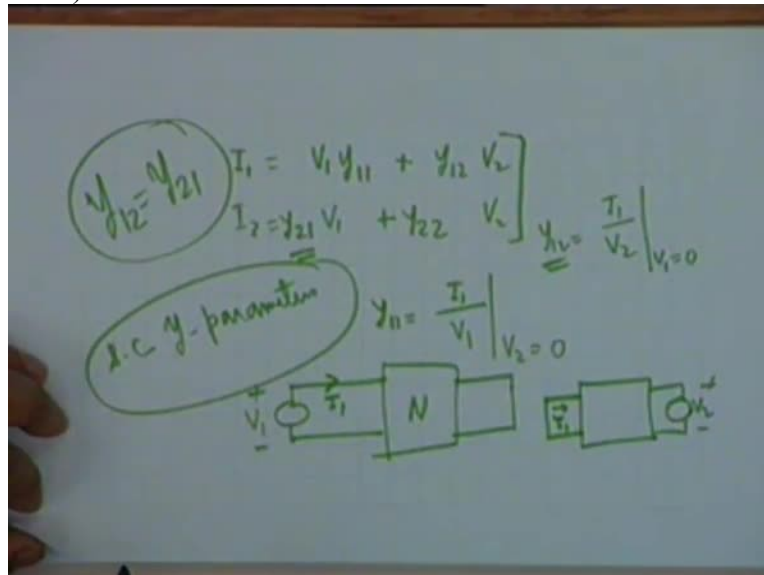
You can easily show that these 2 relationships can be represented by 3 elements like this 3 elements like this where the shunt element is simply  $Z_{12}$  and the series elements are, this is  $Z_{22}$  minus  $Z_{12}$ , this is  $Z_{11}$  minus  $Z_{12}$ , this one. Then this simple network represents the 2 equations. I am not saying this network represents the original N because the original N is 4 terminal whereas the model is 3 terminal. So one, this cannot be equivalent to this. It is equivalent, it is equivalent to the 2 equations representing the network.

And therefore, if this network was 3 terminal, if these 2 terminals were common, then it would have been, this would have been a model of the network. It must be remembered all right? If it is a truly 4 terminal network, you cannot replace N by this. But as far as the mathematical relationship is concerned, yes, this network represents. This can be very easily shown. You write  $V_1$ ,  $V_2$  here and the 2 currents are  $I_1$  and  $I_2$  and you can easily see that  $V_1$  is equal to  $I_1 Z_{11}$  minus  $Z_{12}$  plus  $I_2$ . The current  $I_2$  is  $I_1$  plus  $I_2$ ,  $Z_{12}$ . And if you add them up, you get precisely the 1<sup>st</sup> relation.

Similarly, for the 2<sup>nd</sup> relation. So this is the shape of the network is that of a T and therefore, it is called T network model of the 2 equations. T network model, capital T. You can also if you pull this thing, this terminal up and this terminal up then you can also show it like this. If you pull it a little bit right, if you pull these 2 terminals up, then it becomes of the shape of a Y and it is sometimes called an Y equivalent model of the mathematical equations. So it is either called a

T network or a Y network. Now as I said, there is nothing sacred about the currents being the independent variables.

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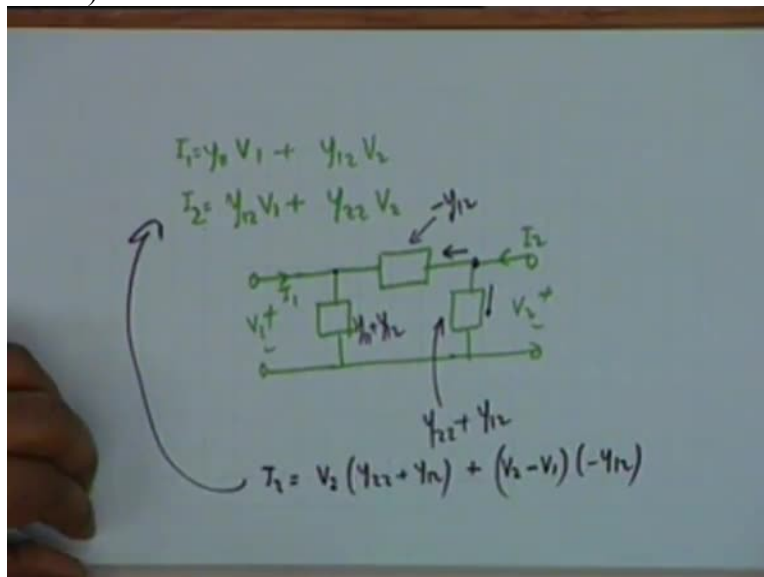


One could also have the voltages as independent variables and under that condition, the relationship shall be  $V_1, V_2$ .  $I_2$  shall also be expressed as a linear combination of  $V_1$  and  $V_2$ . The coefficients now shall be of the dimensions of admittances and therefore, we call this as  $y_{11}$ , small  $y$ , we call this as  $y_{12}$ , we call this as  $y_{21}$  plus we call this as  $y_{22}$ . The dimensions have to be admittances and you see that the coefficient of  $V_1$  is simply the driving point admittance at port number 1 all right?  $y_{11}$  for example you can see that this is  $I_1$  by  $V_1$  under the condition  $V_2$  equal to 0. In other words, what you do is, you apply a voltage generator  $V_1$  at terminal 1 okay?

Then you short-circuit terminal 2,  $V_2$  equal to 0 and measure current  $I_1$  all right? That is how it becomes a driving point admittance under the condition that the other port is short-circuited all right? Similarly, when you measure  $y_{12}$ , what is  $y_{12}$ ?  $y_{12}$  is  $I_1$  over  $V_2$  under the condition that  $V_1$  equal to 0. That means, you apply a voltage generator, let me represent it, you apply a voltage generator here,  $V_2$ . Then you measure, you short-circuit port 1.  $V_1$  equal to 0 and measure this current  $I_1$ . The ratio  $I_1$  to  $V_2$  shall be  $y_{12}$ .  $y_{12}$  is a transfer admittance, similarly,  $y_{21}$  is also a transfer admittance.

And you can see that all these parameters are measured with one port short-circuited and therefore these are known as SC Y parameters, that is short-circuit admittance parameters all right? In similarity with the open circuit Z parameters, one can argue here that if the network N is reciprocal, then you do not require 4 coefficients to characterise the network. You require only 3. And the reason is that  $y_{12}$  shall be equal to  $y_{21}$  and under this condition, one can draw a very simple equivalent model for the 2 equations and that model by duality shall now look like a pi instead of a T, by duality.

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If I write these equations again,  $I_1$  equal to  $y_{11}V_1$  plus  $y_{12} V_2$  and  $I_2$  is equal to  $y_{21}$  is equal to  $y_{12}$ , so let me write  $y_{12} V_1$  plus  $y_{22}V_2$ . It is very easy to show that these 2 equations can be very easily represented by a network looking like a pi all right?  $V_1$ ,  $I_1$ ,  $V_2$ ,  $I_2$ . I am omitting the bars because it is implied that in the context, they are all phasers. And these various parameters are, this is  $y_{11}$  plus  $y_{12}$ , this admittance is minus  $y_{12}$  and this admittance is  $y_{22}$  plus  $y_{12}$ . For example, to verify that this is indeed so, you notice that  $I_2$ , if I write an expression for  $I_2$ ,  $I_2$  is  $V_2$  multiplied by this admittance.  $I_2$  is  $V_2$ .  $V_2$  is the voltage across this.

So  $I_2$  is the current through this network, this element and the current through this element, the sum of the 2.  $I_2$  is  $V_2 y_{22}$  plus  $y_{12}$  plus  $V_2$  minus  $V_1$ , this voltage is  $V_2$ , this voltage is  $V_1$ , multiplied by minus  $y_{12}$ . And you see that this is precisely the 2<sup>nd</sup> relationship. And in a similar manner, you can show that the 1<sup>st</sup> relationship is also valid. So this is an equivalent model for the

mathematical equations but not of the original network. The reason is that this is 3 terminal and the original network was 4 terminal. If the original network was also 3 terminal, then this would have been truly an equivalent model of the original circuit.

Is the distinction clear? I have repeated this. It must be understood carefully because otherwise you can make a mess of the whole thing. If the 2 lower terminals do not have the same potential and you substitute this, you might short-circuit something. The circuit might burn. So these are the 2 kinds of parameters which we are presenting at the present moment. We shall be concerned with other ways when we talk of transistor circuits and that is the other sets of parameters.

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$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

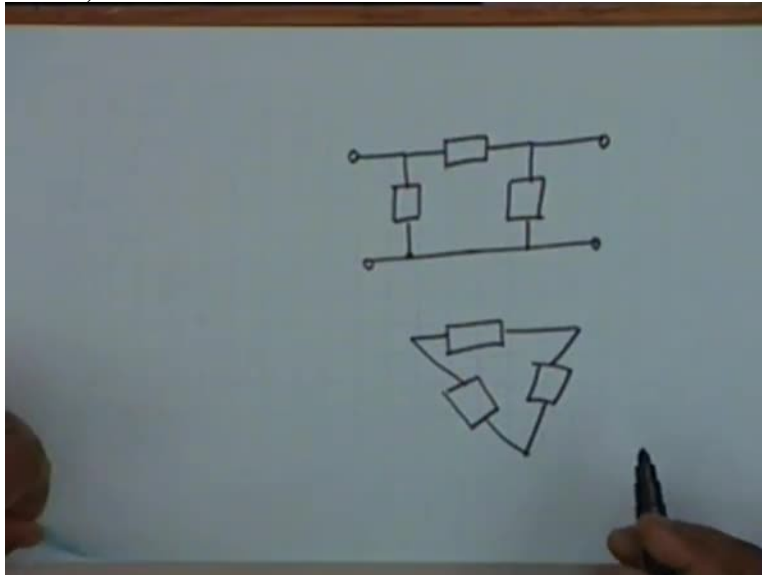
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

But if you notice, if you notice this carefully, the 2 sets of parameters, you see if I take  $V_1, V_2$ , the open circuit impedance parameters and I write this as a matrix of single column, a single column matrix is also known as a vector. Then you notice that the coefficient matrix is simply this.  $Z_{11}, Z_{12}, Z_{21}, Z_{22}$  all right? Whereas this is for the open circuit that parameters whereas for the short-circuit  $y$  parameters, it is the other way round.  $y_{11}, y_{12}, y_{21}, y_{22}$ . We are assuming that the networks are reciprocal. That is why we are writing  $y_{12}$  and  $y_{21}$  are equal. Multiplied by  $V_1, V_2$ . And if you are acquainted with a matrix operations, you notice that one coefficient matrix is simply the inverse of the other.

All right? For example,  $y_{11}$ ,  $y_{12}$ ,  $y_{21}$ ,  $y_{22}$  is simply the inverse of the other provided the Z the determinant of this matrix is not 0. That means this matrix is not singular. Inversion can be carried out only when the determinant  $Z_{11}Z_{22}$  minus  $Z_{12}$  square is not equal to 0. All right? If this fact is recognised, then you can also find out, these parameters,  $y_{11}$ ,  $y_{12}$ ,  $y_{21}$ , and  $y_{22}$  in terms of the Z parameters. Is not that right? You can find for example,  $y_{12}$  as you can see is simply minus  $Z_{12}$  divided by the determining of Z. Is that okay? This is obvious by inspection all right? And I had to do all this to convince you that the transfer admittance is not the reciprocal of the transfer impedance. They are quite different. One is not the reciprocal of the other. There is a negative sign and the denominator is the determinant of the matrix. Okay? Similarly you can find out  $Z_{12}$  in terms of  $y_{12}$ . One is not the reciprocal of the other.

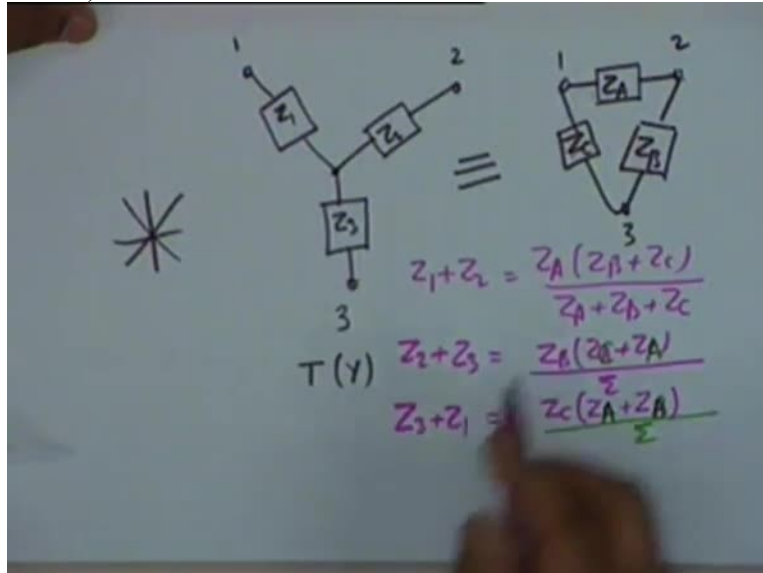
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This pi network which we have just drawn which is the short-circuit y parameter equivalent model of the equations, well if you pull, if you pull these 2 terminals close together then another way of representing this network is a delta. This looks like a Delta and therefore this is also called the Delta network model. Instead of pi, it is also called Delta network model of the 2 equations, 2 simultaneous equations. In the context of network simplification, you have seen that if 2 elements are in series, you can find out the equivalent, if 2 elements are in parallel, you can find out the equivalent.

Now in general in a complicated network, series parallel reduction can proceed up to a certain extent but not beyond it and the two port theory then comes in to help.

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For example, if I have a T network like this, T or a Y, all right, let us mark the terminals 1, 2, 3. A 3 terminal network which is of the shape of a T or a Y, T or Y and I wish to convert it to a delta. That is, I want to find out the equivalent, I wish to convert this into an equivalent 3 terminal network such that the parameters are 1, 2,  $Z_A$ ,  $Z_B$ ,  $Z_C$ , yes, another name for this they start. This also looks like a star although star has many others, this is only 3. All right. I want to find out the conditions under which this network would be equivalent to this. This is a 3 terminal network, this also is a 3 terminal network.

To determine the equivalence of one network to the other, what we do is, we make either open circuit or short-circuit measurements for both the networks and equate the results all right? The algebra is quite simple once you understand what is it that we are doing. For example, let us let us leave 3 open and measure the impedance between 1 and 2. Obviously then  $Z_1$  plus  $Z_2$ , this is what you will measure. On the other hand, if you leave this open and measure the impedance between 1 and 2, what you will measure  $Z_A Z_B$  plus  $Z_C$  divided by  $Z_A$  plus  $Z_B$  plus  $Z_C$ . Is that okay? So this gives you one condition. Since you have to find out 3 of the elements, 3 elements, you require 3 such measurements.

So the other measurement can be  $Z_2$  plus  $Z_3$ , that is between 2 and 3, leaving 1 open. And this shall always be between 2 and 3. So  $Z_B$   $Z_A$  plus  $Z_C$  divided by the sum of the 3. Let us put it at  $\Sigma$ . All right? The 3<sup>rd</sup> equation shall be, measure between 1 and 3. So  $Z_3$  plus  $Z_1$ , you must notice that there is a certain kind of circular symmetry that I am following. 12, 23, 31, it always helps nature as symmetries, beauties of symmetry and it helps to preserve those symmetry.  $Z_3$  plus  $Z_1$  shall be between 1 and 3. So I write  $Z_C$ , then  $Z_B$  plus  $Z_A$ , no, that does not retain the symmetry. I should have written pardon me?

Student: 2<sup>nd</sup> equation.

Professor: 2<sup>nd</sup> equation. No, it is not wrong. I did not maintain the symmetry. I should have written like this.  $C$  plus  $A$ . And then this should have been written as  $A$  plus  $B$  divided by  $\Sigma$ . These are 3 equations now which have to be manipulated to find either  $Z_1$ ,  $Z_2$ ,  $Z_3$  in terms of  $Z_A$ ,  $Z_B$ ,  $Z_C$  or vice versa. All right? And I will simply point out how to do this. The algebra shall be left to you. You see if I look at these 2 relations,  $Z_2$  plus  $Z_3$  and  $Z_3$  plus  $Z_1$ . Suppose I subtract this equation from this then I shall get  $Z_2$  minus  $Z_1$ . The 1<sup>st</sup> equation gives me  $Z_2$  plus  $Z_1$  and therefore by adding and subtracting, I can find out  $Z_1$  and  $Z_2$ . Similarly for the 3<sup>rd</sup>. And the results is this clear, the manipulation?

Student: (())(21:26)

Professor: all right, we will do that also. Let us 1<sup>st</sup> find out  $Z_1$ ,  $Z_2$ ,  $Z_3$  in terms of  $Z_A$ ,  $Z_B$ , and  $Z_C$  all right? That is we are going from  $\pi$  to  $T$ .  $\pi$  to  $T$  or  $\Delta$  to  $\star$ . All right? The results are, I will I will come back to this.

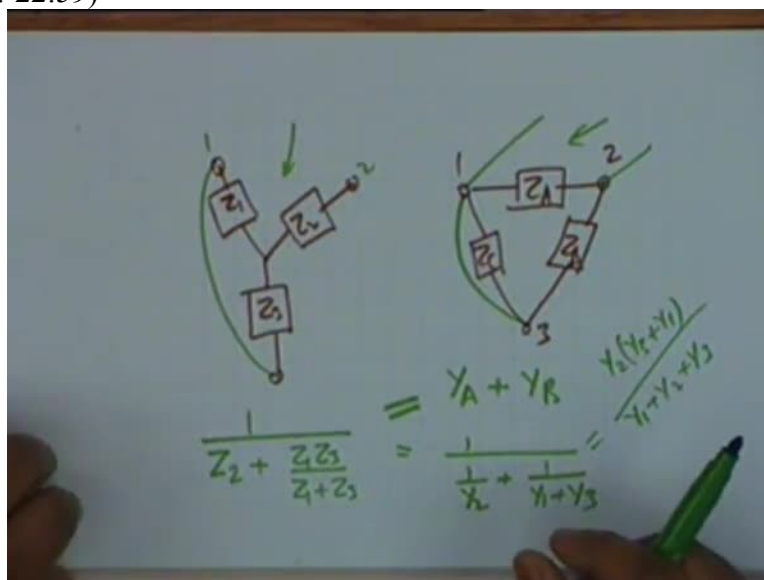


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$$Z_1 = \frac{Z_C Z_A}{\Sigma}, Z_2 = \frac{Z_A Z_B}{\Sigma}$$
$$Z_3 = \frac{Z_B Z_C}{\Sigma}$$
$$\Sigma = Z_A + Z_B + Z_C.$$
$$\pi \rightarrow T$$
$$\Delta \rightarrow * (Y)$$

The results are  $Z_1$  equal to  $Z_C Z_A$  divided by  $\Sigma$ , then this is correct,  $Z_2$  is  $Z_A Z_B$  over  $\Sigma$  and  $Z_3$  is  $Z_B Z_C$  over  $\Sigma$  where  $\Sigma$  is the sum of the 3, that is  $\Sigma$  equal to  $Z_A$  plus  $Z_B$  plus  $Z_C$ . This gives you conversion from  $\pi$  to  $T$  or  $\Delta$  to star.  $\Delta$  to star is also  $Y$ . No, I beg your, yes, okay.  $\Delta$  to star all right? If you want the other way, that is you want to go from  $T$  to  $\pi$ . In other words, you want to find out  $Z_A, Z_B, Z_C$  in terms of  $Z_1, Z_2$  and  $Z_3$ , let us see what should we do.

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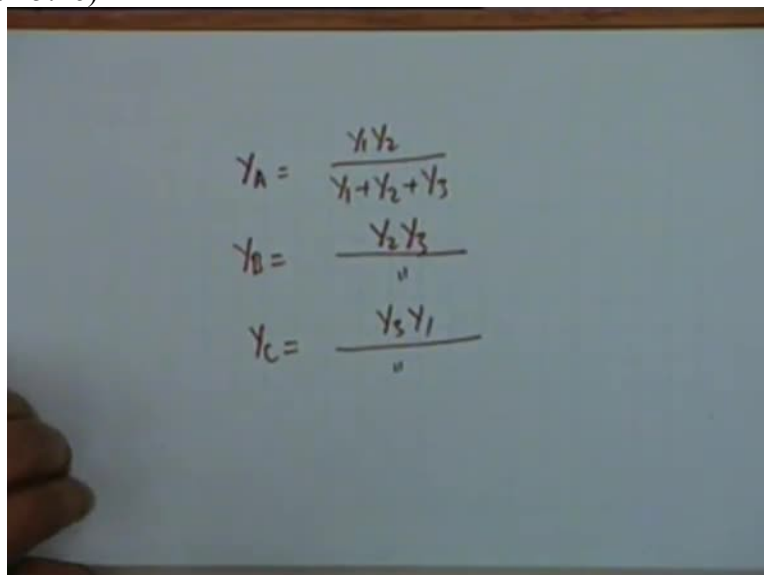


Let us draw the network again. You have  $Z_1$ ,  $Z_2$  and  $Z_3$  and then you have here  $Z_A$ , again I will only indicate the steps, I will not carry out. 1, 2, 3. In the previous case, the duality of the situation suggests that we should measure admittances under short-circuit conditions. So what we do is, suppose we short-circuit 1 to 3 and measure the admittance between 2 and 1 all right? Suppose we short-circuit 1 to 3 and measure the admittance between these 2 points, what shall we get?

We shall get  $y_a$  plus  $y_b$ , the admittance shall be  $y_a$  plus  $y_b$ . Now we short-circuit 1 to 3 and measure the admittance between 1 and 2, what shall we get? We shall get  $1$  over the impedance. Impedance will be  $Z_2$  plus  $Z_1 Z_3$  divided by  $Z_1$  plus  $Z_3$ . Agreed? These 2 should be equal. This is okay? I am finding out by inspection.  $Z_2$  plus the parallel combination of these 2. Well, I can write this as  $1$  over  $y_2$  plus  $1$  over  $y_1$  plus  $y_3$  all right which I can simplify as  $y_2 y_3$  plus  $y_1$ , I am writing in a circular fashion, circular symmetry, divided by  $y_1$  plus  $y_2$  plus  $y_3$ .

Now similarly, you see what I have done is, I have expressed the sum of admittances in terms of the admittances of the T network and I can now write  $y_b$  plus  $y_c$ , another equation,  $y_c$  plus  $y_a$ , a 3<sup>rd</sup> equation and then eliminate one by one to find out the values of  $y_a$ ,  $y_b$  and  $y_c$ . From symmetry, the end results do I have to explain further? I have to obtain 3 equations. One I have shown how to obtain. You obtain the other 2, then you make systematic eliminations.

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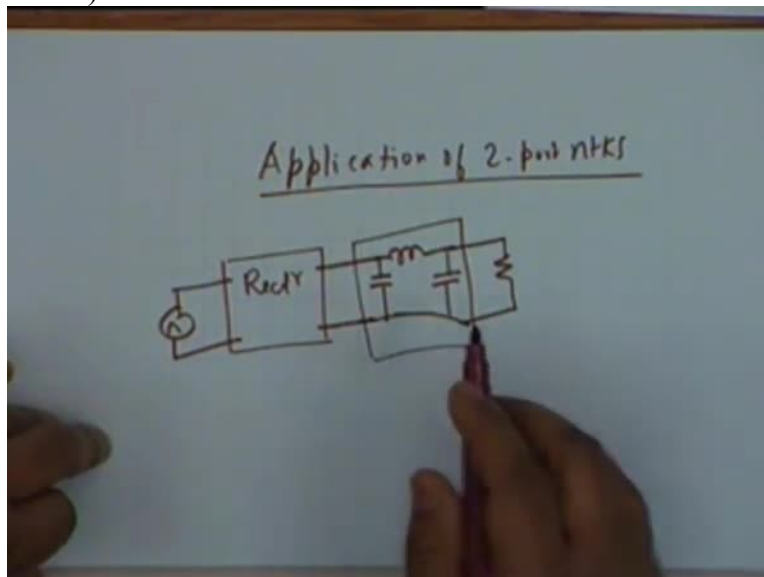


The image shows three handwritten equations for admittance in a T-network. The first equation is  $y_A = \frac{y_1 y_2}{y_1 + y_2 + y_3}$ . The second equation is  $y_B = \frac{y_2 y_3}{y_1 + y_2 + y_3}$ . The third equation is  $y_C = \frac{y_3 y_1}{y_1 + y_2 + y_3}$ . The denominator for all three equations is  $y_1 + y_2 + y_3$ .

The final results are as follows.  $y_a$  becomes equal to  $y_1 y_2$  divided by  $y_1 + y_2 + y_3$ ,  $y_b$  now I can write this from symmetry yes  $y_2 y_3$  and  $y_c$  shall be  $y_3 y_1$  divided by the same summation. These formulas do not have to be remembered if you recall the Z and y parameters but I leave it to you what you find most convenient. In any case, you have to find out. If you want to remember, then you have to find out some convenient way of remembering and that also, yes of course.

The question was barred. I do not know why you ask the question again and again. As soon as long as I sit here is teaching a course, there cannot be a closed book examination. All right?

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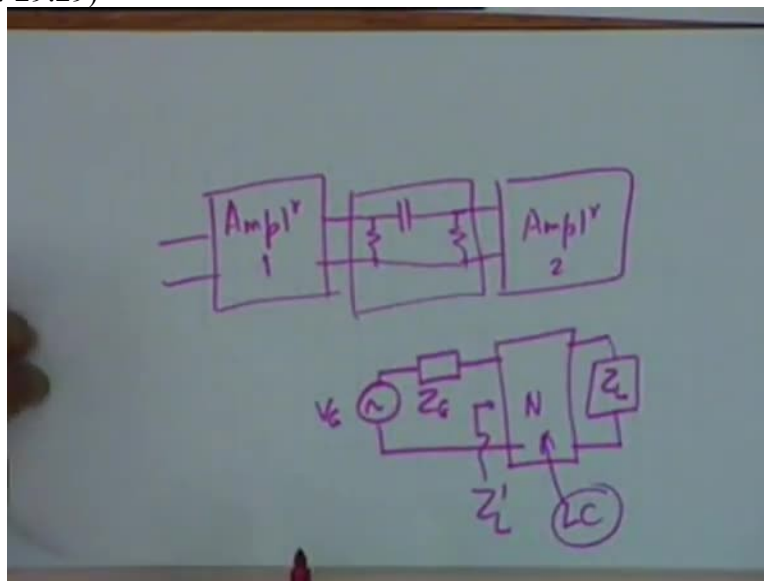
Application of 2 port networks. Where are 2 port networks used? Almost all networks that you see are 2 ports all right? For example, a communication receiver. Where is the input? Input comes from the antenna? And Where is the output? Output is the loudspeaker. A CRO, there is an input, input signal. Where is the output? Output is the visual output. It is a picture all right. Almost all networks are 2 ports. 2 ports of linear bilateral elements are also used to couple one system to another, to couple. So they are used as coupling network.

For example, you have the rectifier. You have let us say full wave rectifier containing a transformer and let us say a bridge rectifier, for diodes all right? You have to supply, what does a rectifier do? Rectifier is also a 2 port. Is not it right? It takes an AC at the input and produce a

DC. It does not. It produces a DC on which is superimposed a ripple or an AC. You wish to to drive a load with this VC and if you want to improve the DC content or reduce the AC content, or reduce the ripple content of the output, what do you do? You apply a filter in between.

And we have seen simple filters. For example, you could have a pi filter here okay? You could have a pi filter or you could have an L filter all right? Now the filter obviously, this is obviously a 2 port network. All right? So a 2 port is used to couple the system which is the rectifier system to the load. This is the load and this is the rectifier. So rectifier to load is being the filter or the 2 port acts as a coupling network all right? It couples one system to another. Similarly, when you want to amplify a given signal, and one stage of amplification does not suffice, then you have to couple one amplifier to another all right?

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Let us take that example. I have two amplifiers, amplifier 1 and I want to present output of amplifier 1 to amplifier 2. Now the output invariably as we shall see later contains DC as well as the signal and one should get rid of the DC. So what one does is one uses a capacitor and this is the amplifier output and this is the amplifier input. So this CRC network or RCR network is being used as a coupling network. All right? There are also situations where Jack you use a matching network, an impedance matching network.

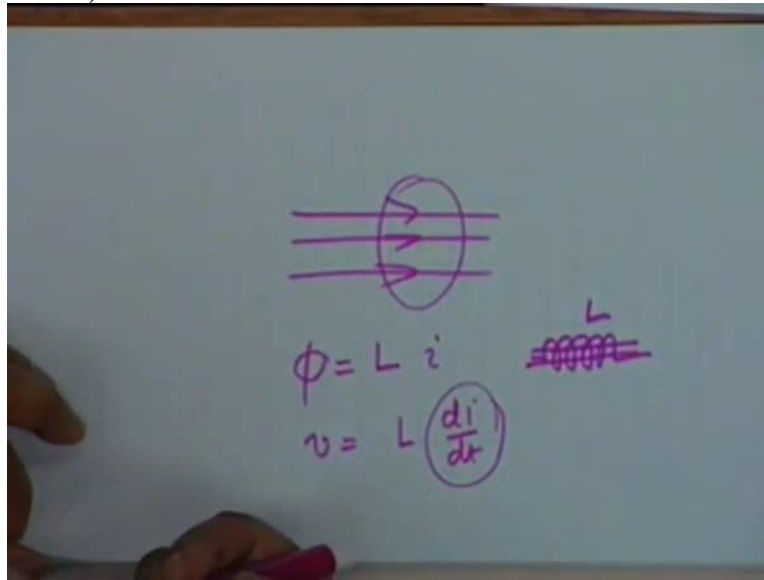
That is, you have a source  $V_G$  in series with let us say an impedance  $Z_G$ , this is a phaser. I am not using the bar because it is clear in the context. And then you have the load  $Z_L$ . You wish to transfer the maximum amount of power to  $Z_L$ , maximum available power from the source to  $Z_L$ . So what you do is if  $Z_G$  is not the complex conjugate of  $Z_L$ , you know this is the condition for maximum power transfer. If this is not true, then what you do is, you insert a network  $N$ , a 2 port  $N$  to achieve this, to achieve the effective impedance  $Z_L$  which you call  $Z_L$  prime, you have to achieve prime equal to the complex conjugate of  $Z_G$ .

Then you make sure that  $Z_L$  shall receive the maximum available power. A question. Can  $N$ , what should be the components of  $N$ ? What kind of components should it contain? Inductance and capacitance only, no resistance. Why not? Because then resistance itself will absorb power and the maximum available power will not be transferred to the load. So this  $N$  must be an LC network. These are the various applications of a 2 port network but in all these, 3 examples that we have cited, the coupling is through actual contact, that is actual wires leading current from one place to another.

The coupling can also occur through indirect means like magnetic field or electrostatic field also. If you bring a capacitor, if you bring a capacitor in an electrostatic field, a potential difference shall be developed between the 2. If you place two conductors in an electrostatic field, then the 2 conductors depending on the location of the 2 conductors, the potential shall be different and therefore, it has acquired, it has it acts like a capacitor with a charge. On the other hand, if there is a magnetic field, if there is a magnetic field, magnetic lines of force and you put a coil in this, and if the field changes with time, then you know the coil shall have an induced voltage.

This is not a coupling by conduction or by actual contact. This is a coupling through invisible means, that means the magnetic field. A coupling can occur through electrostatic field or magnetic field. In particular, the magnetic field coupling is of great interest to electrical engineers because it does not disturb the original circuit to the extent that it does if it was taken by direct conduction. Coupling can be magnetic coupling and magnetic coupling is of great interest.

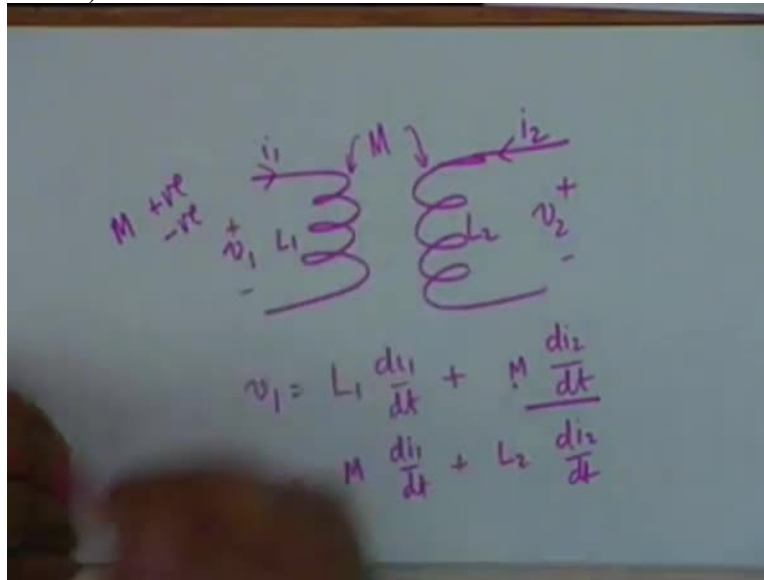
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Now to understand magnetic coupling and to be able to characterise in terms of, in terms of mathematical equation, you recall that if I have a coil with an inductance  $L$  which carries a current  $I$  then the flux associated with the inductance shall be the product of  $L$  and  $I$ .  $L$  by definition is induct, the flux per unit current. So  $\phi$ , the flux is equal to  $LI$ . And if  $I$  varies, if  $\phi$  varies, the magnetic flux varies, then you know that the inductance develops a voltage across it which is  $L \frac{di}{dt}$  which is  $\frac{d\phi}{dt}$  and it is equal to  $L \frac{di}{dt}$ .

Now this is a case in which a coil of inductance  $L$  is in a magnetic field and links a current, the magnetic field is created by its own current all right? The magnetic field is created by its own current and if the field changes, that means the current changes, then the voltage across it is developed which is equal to  $\frac{d\phi}{dt}$ . On the other hand, if the flux is a constant, then  $\frac{d\phi}{dt}$  is 0 and the voltage across this is 0. This is why we said inductor acts as a short-circuit to direct current.

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Now, if this magnetic flux is generated by another coil, another coil carrying a current let us say  $I_2$  and we have this magnetic flux also links the 2<sup>nd</sup> coil and if this flux  $\phi$  changes with time, then obviously, the 2<sup>nd</sup> coil we shall have if the inductance is  $L_2$ , well let us say this is  $I_1$ ,  $L_2 \frac{di_1}{dt}$ , some constant multiplied by the rate of change of current all right? To distinguish between self induction and induction from another source, we use the symbol instead of  $L$ , we use the symbol  $M$  and  $M$  stands for mutual inductance, that is the inductance associated with 2 coils all right?

The symbol  $L$  is used for self inductance that is flux linkage due to its own current whereas the symbol capital  $M$  stands for mutual inductance and it characterises flux linkage due to current in another coil. All right? Now one should understand that if there are 2 coils interacting with each other, that is one generates an electric field and magnetic field which cuts across the 2<sup>nd</sup> coil, similarly the magnetic field emitted by the 2<sup>nd</sup> coil cuts across the 1<sup>st</sup> coil, and if these currents are changing, then obviously, the voltages across the 2 coils shall have 2 components.

One would be due to its own current variation and the other components would be due to the current variation in the 2<sup>nd</sup> coil. And to denote, to characterise the 2<sup>nd</sup> component, we require a constant called mutual inductance and we denote this as capital  $M$ . For example,  $V_1$  shall be, if this current is  $I_1$  and if this current is  $I_2$ , then  $V_1$  shall be  $L_1$ , let the self inductances be  $L_1$  and

$L_2 \frac{di_1}{dt}$ , this is due to its own current and the 2<sup>nd</sup> component shall be due to  $M \frac{di_2}{dt}$  and the coefficient is capital M, the mutual inductance.

What you should do actually is to have  $M_{12}$  and  $M_{21}$ , that is the voltage induced in the 1<sup>st</sup> coil due to current in the secondary, if you call this  $M_{12}$ , then the other one, you should call  $M_{21}$  and you should write  $M_{12} \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$ , this would be the voltage in the 2<sup>nd</sup> coil. However due to reciprocity theorem, the 2 constants are equal, that is  $M_{12}$  and  $M_{21}$ . Does it require further explanation to convince you that they are indeed equal? Because the cause and response, the excitation and the response are interchanged. Therefore the coefficient or the ratio of the 2 must be the same.

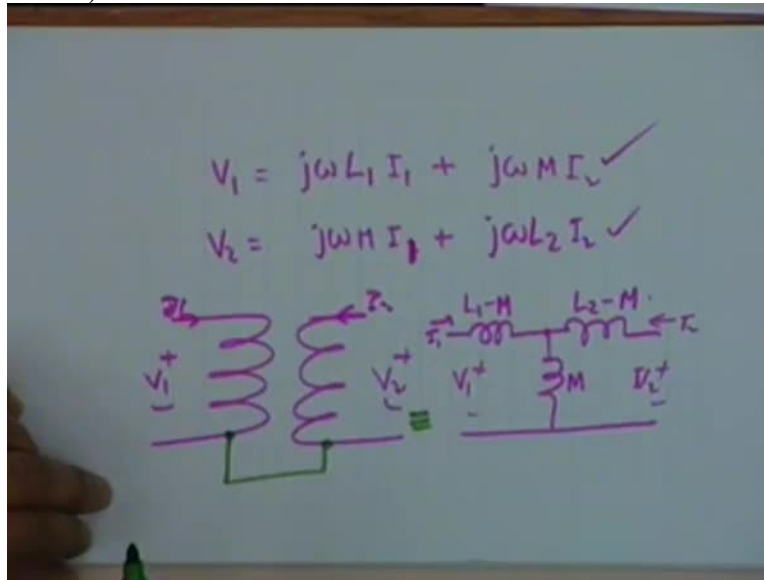
That is why we have used 2, the same usual inductance. Now the voltages do not necessarily add. It depends on the orientation of the coil. For example, a flux in one coil might try to cancel the flux in the other coil. Under that condition, this voltage shall subtract from the self induced voltage. Is the point clear? Depending on the orientation of the currents, for example, you can have let us say clockwise turns, like this. One current can be clockwise, the other can be anticlockwise.

And then one of the coils produces a flux which tends to cancel the flux produced by the other coil. And therefore, this voltage may try to destroy the self induced voltage. Under that condition, this sign shall be negative and we say that under that condition, not the voltage, we say the mutual inductance is negative. So M, mutual inductance can be either positive or negative. Do you understand this situation when they are positive and when it is negative?

It is positive if the mutual induction produces a voltage which is of the same polarity as due to self induction, then M is positive. On the other hand, if mutual induction produces a voltage which tries to annul the voltage due to self induction, we say capital M is negative. We will have more to do with this later but at the present moment, you look at these 2 relationships and tell me if the excitation was sinusoidal, then how would these relationships exchange?



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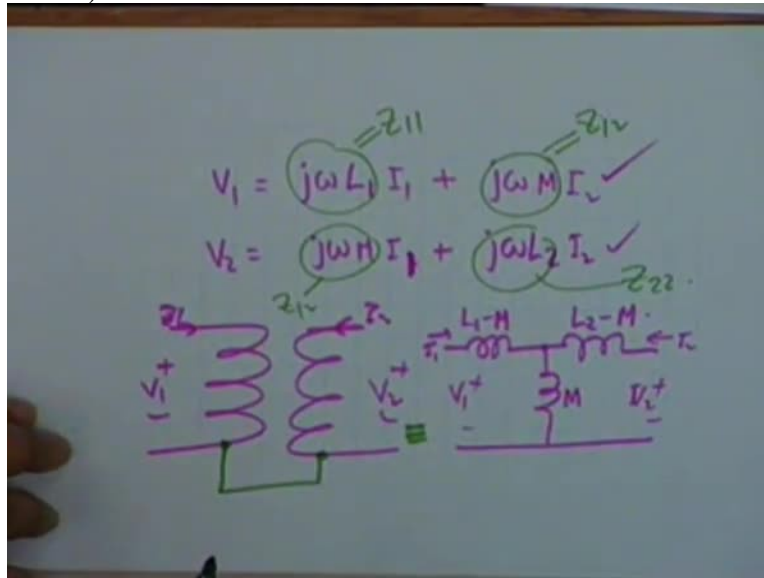


If the excitation would be sinusoidal, then we could express in terms of phasors. That is we would have simply return,  $V_1$  equal to  $J \omega L_1 I_1$ , this is impedance of the inductor, self inductance multiplied by  $I_1$  plus  $J \omega M I_2$  and  $V_2$  would be written as  $J \omega M I_1$  plus  $J \omega L_2 I_2$ . All right? And it takes very little thinking to show that this is equivalent to, this circuit which has no physical contact, the coupling is through magnetic field, that this is equivalent to simply a T network like this where this inductance is  $M$ , this inductance is  $L_1$  minus  $M$  and this inductance is  $L_2$  minus  $M$ .

Can we say, is this okay? This is  $V_1$ ,  $I_1$ ,  $I_2$ ,  $V_2$ . Similarly here,  $V_1$ ,  $V_2$ ,  $I_2$ ,  $I_1$ . You can easily show by writing the loop equation that this 1<sup>st</sup> equation is satisfied, the 2<sup>nd</sup> equation is also satisfied. Now can we say that this transformer is equivalent to this? No. I am going to test this again and again so that you never make such a mistake. This is not equivalent to this. It is only equivalent to the 2 mathematical equations. On the other hand if we had this, then they would have been exactly equivalent.

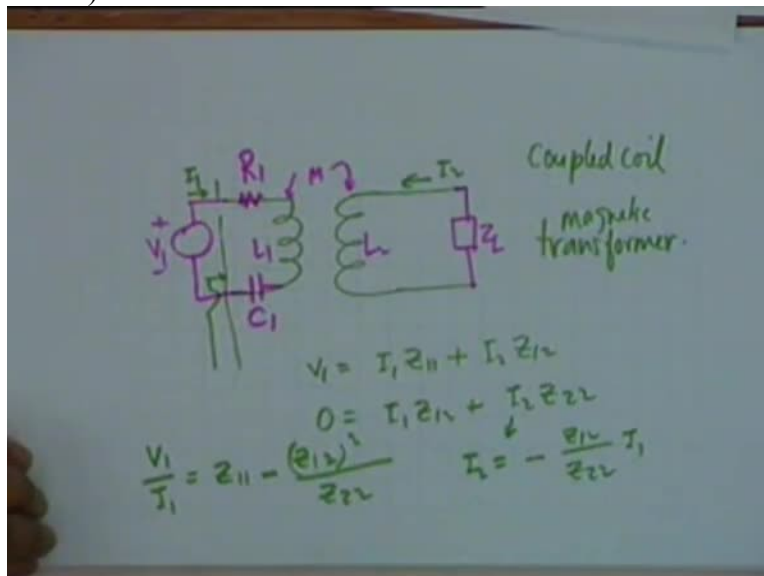
That is why, I have shown this in green and equivalence is also in green. Only when this line is there, the Green line, there shall be a green to this equivalence. Is that okay? All right.

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Now suppose I write  $J \omega L_1$  as  $Z_{11}$ ,  $J \omega M$  as  $Z_{12}$ , is as  $Z_{12}$  and this as  $Z_{22}$ , then do not you see that this is simply the same T model, that is  $Z_{11}$  minus  $Z_{12}$ ,  $Z_{22}$  minus  $Z_{12}$  and this is simply  $Z_{12}$ . Is that okay? So instead of looking at it from the impedance parameters, OC impedance parameters, we look at it directly in terms of the components. Well, there are 2 things that one should notice.

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One of them is that if you have let us say incidentally this is called, this circuit, this two port where the flux is generated in 2 coils interact with each other, is called a coupled coil system. At

least 2 are needed. They are coupled to each other. Or it is also called simply a transformer and to indicate that the coupling is magnetic, sometimes one says, this is a magnetic transformer. All right? Suppose that we connect a transformer like this. All right? And we have a voltage source here,  $V_1$ . The mutual inductance is  $M$ , this is  $L_1$ , this is  $L_2$ . This could be capacitance, resistance or whatever it is, let us call it  $Z_L$  all right?

There may be other elements here. For example, one could have a resistance  $R_1$ . Let us also introduce a capacitor  $C_1$ . What we want to find out is, what impedance does this voltage generator see? What impedance does this voltage or in other words, we want to find out what current  $V_1$  drives into this system of coupled coils. What current? Well, all we do is we write the 2 equations. If this is the  $V_1$  and this is, this current is  $I_2$ , if this current is  $I_1$ , then you know you can write  $V_1$  equal to  $I_1 Z_{11}$  plus  $I_2 Z_{12}$  all right? And the 2<sup>nd</sup> loop, there is no driving voltage. So 0 shall be equal to  $I_1 Z_{12}$  plus  $I_2 Z_{22}$  all right?

Now from the 2<sup>nd</sup> equation, you can write  $I_2$  as equal to minus  $Z_{12}$  by  $Z_{22}$   $I_1$ . Okay? You substitute this here, then what do you get?  $V_1$  by  $I_1$ . If you substitute this relation in the 1<sup>st</sup> equation, what do you get? You get  $Z_{11}$  minus  $Z_{12}$  Square divided by  $Z_{22}$ . So what does this do? What does the couple coil do? Not only the 1<sup>st</sup> coil interacts with the 2<sup>nd</sup> because the 1<sup>st</sup> coil induces a current to flow in the 2<sup>nd</sup> circuit. But the 2<sup>nd</sup> coil also has an effect on the 1<sup>st</sup> coil. How? If the 2<sup>nd</sup> coil was not there, if there was no current here, if this was kept open, then my input impedance should have been simply  $Z_{11}$ . By definition,  $V_1$  by  $I_1$  under the condition that the output, the 2<sup>nd</sup> port is open. Now because of a current flow in the 2<sup>nd</sup> coil, the impedance is modified by this.

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$$\begin{aligned}\frac{V_1}{I_1} = Z_{in} &= Z_{11} + \frac{\omega^2 M^2}{Z_{22}} \\ &= R_1 + j\omega L_1 + \frac{1}{j\omega C_1} + \frac{\omega^2 M^2}{j\omega L_2 + Z_L}\end{aligned}$$

Can you tell me about this relationship shall be in this particular case? We call this  $Z_{in}$ .  $Z_{11}$  minus what is  $Z_{12}$ ? What is  $Z_{12}$  in this case?  $j\omega M$  and therefore minus  $j^2 \omega^2 M^2$  will be simply  $\omega^2 M^2$  divided by  $Z_{22}$ . Whatever is included here, for example I had included  $R_1$  plus  $j\omega L_1$  plus  $1$  over  $j\omega C_1$ , you recall we had a resistance, inductance and a capacitance. So this is  $Z_{11}$  minus  $\omega^2 M^2$  divided by  $j\omega L_2$  plus  $Z_L$ . Pardon me? It is plus, thank you so much.

Well, it is plus but this is not resistive. Jack It has the resistive component, a reactive component also. And one can work wonders with this reflection of impedance. You can see what is happening is the secondary impedance, the secondary, the 2<sup>nd</sup> coil impedance is being reflected onto the 1<sup>st</sup> as an addition of term  $\omega^2 M^2$  by  $Z_{22}$ . And this allows us to use the transformer as a matching device. What is a matching device? A matching device is one which allows maximum power from the source to be transferred to the load. For example, in a stereo amplifier, the output impedance of the amplifier can be as large as the order of let us say K, 1K, 1000 ohms whereas a speaker, what is the impedance?

8 ohms and 8 ohms has to be matched with 1000 ohms. If you connect directly the speaker, you will not be able to hear. The the whole voltage, the power shall be killed because 8 ohms cannot take maximum power from 1000 ohms. So what you do is, before you connect the speaker, you have an output transformer. What does the output transformer do? It converts that 8 ohms into

1000 ohms. So what the amplifier sees is 1000 ohms and what the speaker sees, what does the speaker see? 8 ohms. And therefore maximum power is delivered. Well we shall work out problems on this and other aspects of a transformer after the minor.

Student: ( ) (50:03)

Professor: I have told you, till what is done today.

Student: ( ) (50:10)

Professor: Work it out. I have given you the problems in the tutorial set.