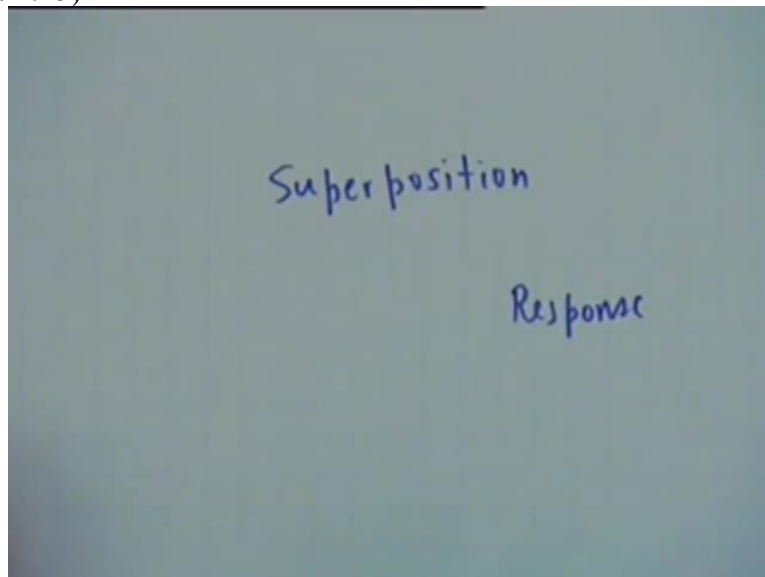


**Introduction To Electronic Circuits**  
**Professor S.C. Dutta Roy**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Delhi**  
**Module No 01**  
**Lecture 23: General Network Analysis**

This is the 23<sup>rd</sup> lecture on general network analysis and this I had commented earlier, shall mark the conclusion of the part on circuits of this course. And today's and tomorrow's lecture, we shall concentrate on this and then the 3<sup>rd</sup> lecture from today shall be on electronic circuits. So far, we have only considered very simple networks in which levels are connected in series or in parallel or in series parallel. It is time now to extend this concept to general networks in which the connections are more complicated than is or parallel.

If there are 2 elements in series, you can always combine them, if there are 2 resistance in series, you can combine them into a single resistance. If there is an inductor and a capacitor in series, you can find a single impedance by combining, by adding the 2 impedances. Similarly for parallel networks. Now in general, in making these simplifications for analysing complicated circuits which cannot be done by inspection, several theorems are useful.

(Refer Slide Time: 2:25)

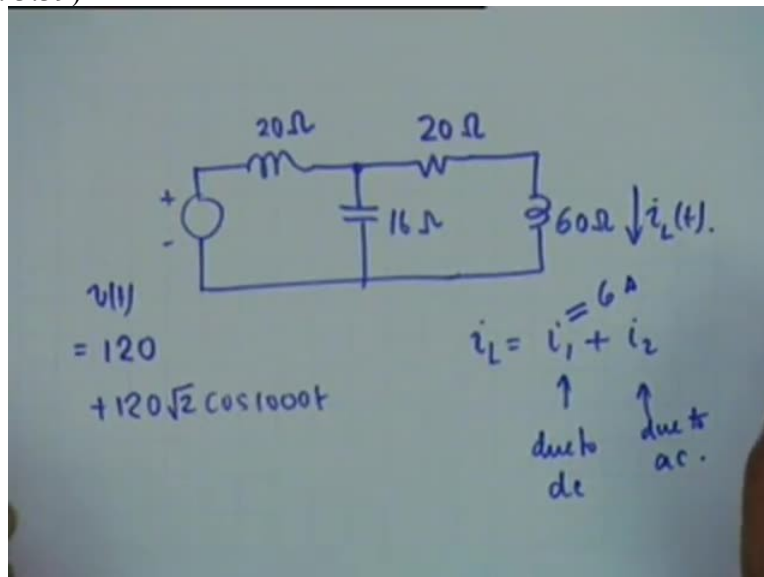


And one of them is the so-called superposition theorem. Superposition theorem when it was stated, we had not made any constraint on the nature of the elements whether they are resistive,

inductive or capacitive or even with active elements, superposition always holds. In other words, the response due to a number of sources, response due to a number of sources is equal to the superposition or the algebraic addition of the responses due to individual sources with all other sources killed. Killed means that if it is a voltage source, ideal voltage source, it should be short-circuited.

If it is an ideal current source, it should be open circuited but there is one class of sources which you must leave intact. What are they? The dependent sources. Very good. We shall illustrate, we know that superposition holds for all kinds of sources but we shall illustrate this with the help of a simple example.

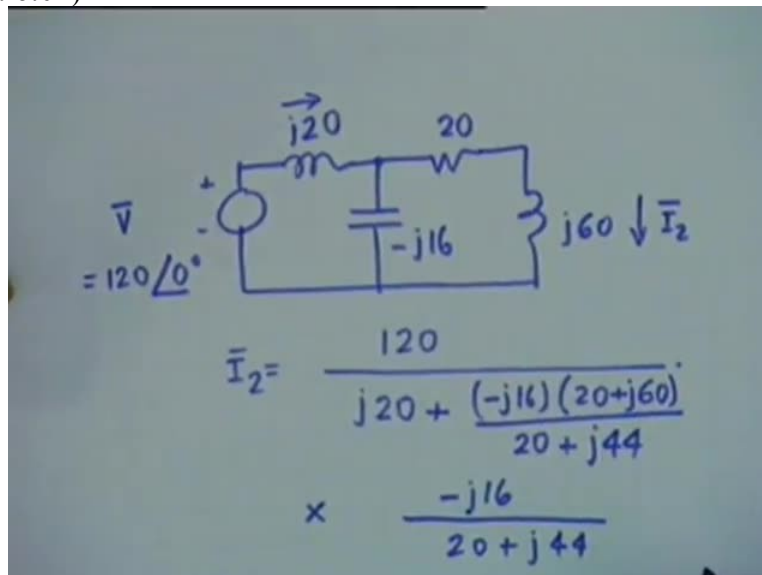
(Refer Slide Time: 3:39)



This example I have taken it from the book. We have a voltage source  $V$  of  $T$  which consists of a DC voltage, 120 + an AC  $120\sqrt{2} \cos$  of  $1000 T$ , this is one of the problems in the tutorial sheet in fact. And then, we have an inductance whose reactance is specified and therefore its impedance shall be, if it is inductance, then the impedance shall be  $J$  times 20. And obviously, this impedance is valid only at the given frequency of interest, that is 1000 radians per second. And then we have a 16 ohms, whenever a capacitance and its reactance is specified, you have to, it is your responsibility to insert the sign.

The impedance of this obviously shall be  $-j16$  and then you have in  $20$  ohm resistance and a  $60$  ohm reactance induct and it is this current  $I_L$  of  $T$  which is of interest all right? Since the source and we want to do this by calculating by utilizing the theorem of superposition or the superposition theorem. Since the source consists of 2 sources, one is  $120$ , a DC and the other an AC, we can find out the response  $I_L$  in two steps. That is, I write  $I_L$  as  $I_1 + I_2$ .  $I_1$  due to the DC source and  $I_2$  due to the AC source and we can combine the 2. As far as DC is concerned you notice that this is a short-circuit, inductor. The capacitor is an open circuit and this inductor is a short-circuit and therefore  $I_1$ , you can very easily see is  $120$  divided by  $20$  ohms that is  $I_1$  is  $6$  amperes. To find out  $I_2$  now, you forget about the DC part and because it is a pure AC, you can apply the phaser relationship.

(Refer Slide Time: 6:02)



And if I draw the phaser equivalent circuit, what I shall have is a voltage phaser which is  $120$ , the RMS value, angle  $0$  degree, then we have now you talk in terms of impedance because this is what you shall have to manipulate.  $j$  times  $20$  and then  $-j$  times  $16$  and now it is  $20$  and  $+j$  times  $60$  and this phaser is  $I_2$  all right? And  $I_2$  can now be written down by inspection.  $I_2$  can be written down by inspection, it is the voltage  $120$  phaser divided by the total impedance looking from here which is  $j20 +$  the parallel combination of  $-j16$  and  $20 + j60$ . And therefore  $-j16$  multiplied by  $20 + j60$  divided by  $20 + j60 - j16$ , so  $20 + j44$  multiplied by this is the total current phaser, then it makes a division between 2 branches and therefore this shall be multiplied

by - the impedance of the other branch divided by total of this impedances, which is  $20 + j44$  or all right? This you can write down by inspection.

(Refer Slide Time: 7:54)

$$\bar{I}_2 = -12(1+j) \checkmark$$
$$i_2 = 24 \cos(1000t - 135^\circ) \checkmark$$

Now the next task is to simplify and if I leave the algebra to you, the final result is well,  $I_2$  bar becomes equal to  $-12\sqrt{2}(1+j)$ . I hope I am right. Yes,  $-12\sqrt{2}(1+j)$ . And therefore, I can write  $i_2$  as a function of time as  $24$ .

Student: Sir  $24\sqrt{2}$ .

Student: Sir  $\sqrt{2}$  comes from answers here.

Student:  $24\sqrt{2}$ .

Professor:  $\sqrt{2}$  will come from  $1+j$  and therefore, I think it is simply  $-12(1+j)$ . This is the correct way. I am sorry, I made a mistake. And therefore, the peak value, the root mean square value will be  $12\sqrt{2}$ . Then you multiply by  $\sqrt{2}$  to find out the maximum value. So it is  $24$ . Then you have cosine of  $1000T$  + now the angle.

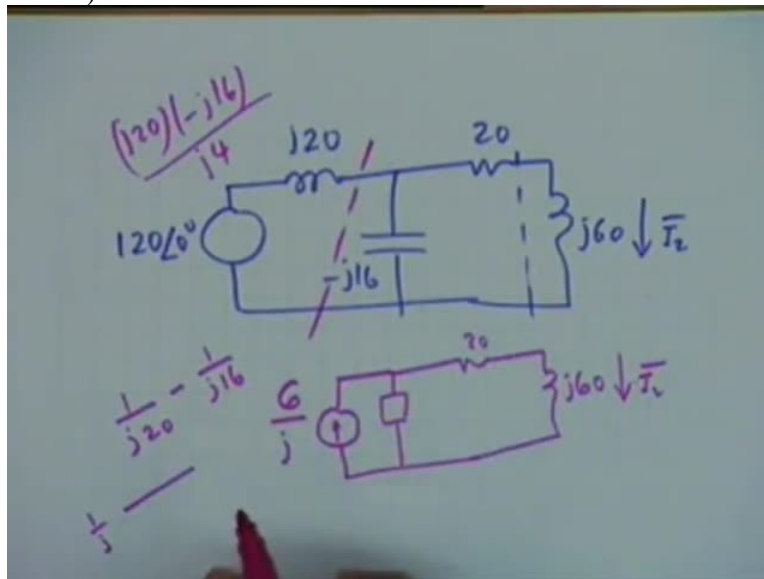
Student: 45

Student: 45

Student: 135.

Professor: It is - 135 degrees. Explained, let me explain why it is so. The negative sign gives rise to an angle of  $\pi$  and  $1 + j$  gives rise to an angle of 45. So it is either  $180 + 45$  or  $-180 + 45$ . The angles are the same, + 225 degree and - 135 degrees all right? This angle is the same as this angles.  $180 + 45$  or  $-180 + 45$  which means - 135. And this is the correct answer. Well the answer, as far as the mathematics is concerned, both answers are correct because the angle is the same, + 225 and - 135 but one shows that the current is leading the voltage and the other shows that the current is lagging the voltage and in this particular case, the current is indeed lagging. And therefore, this is the correct answer. I write that from physical considerations. This is the point that I wished to make in this exercise.

(Refer Slide Time: 10:36)



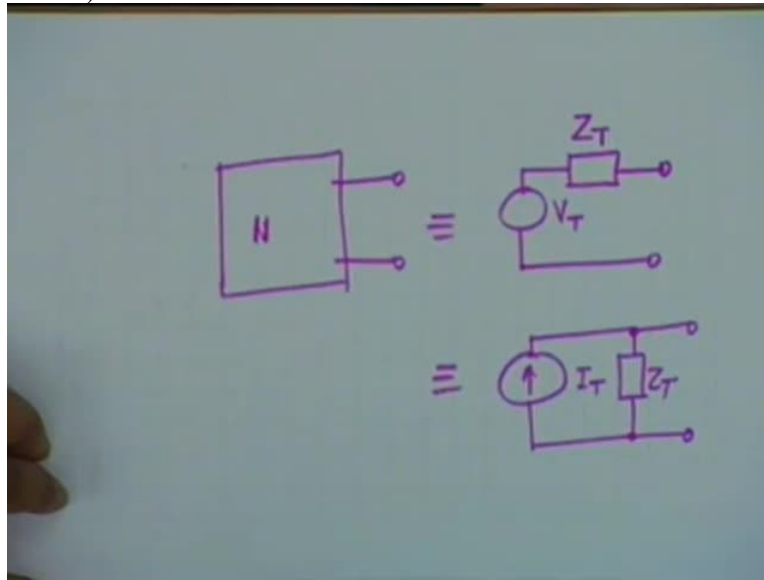
If I take the same problem namely to find out the current, let me draw it again,  $120$  zero degree,  $j20$ ,  $-j16$ ,  $20$  and  $j60$  and the problem is to find out this current,  $I_2$ . I could also have done this by the application of either Thevenin's or Norton's theorem. Thevenin's and Norton's theorems are valid for AC phasors also and these are also very important tools for determining the for solving a complicated network. For example, if I apply let us say Norton's theorem, Norton's theorem to find out this current, all right?

Then you see, you short-circuit this, find out the short-circuit current and then you find the equivalent Thevenin impedance all right? This can be done, can you suggest a better procedure? Source transformation. What we do is, we 1<sup>st</sup> apply in Norton's theorem to the left of these 2 lines. Then we get 120 zero degree divided by  $j20$  and therefore, it would be 6 divided by  $j$ , this is a current source all right? If I apply in Norton's to the left of this, current source in parallel to  $j20$ . This comes in parallel -  $j16$  and therefore the equivalent admittance,  $j20$  and not the sum of the 2. We have to take the admittance and then take the sum.

Therefore,  $1$  by  $j20 - j16$  which is  $1$  by  $j$ , yes you can simplify this. Whatever the result is, that is what you have to put here, a single admittance. Is this clear?  $j20$  impedance comes in parallel with -  $j16$  whose equivalent impedance would be, you can either calculate by admittances or by impedance. For example,  $j20$  multiplied by -  $j16$  divided by  $j4$ , this is the equivalent impedance all right? Then you have the 20 ohms and  $j60$  all right? You could now apply the formula for division of current directly is not it right? There is a current source which divides into 2 parts, this one and this one and therefore you could write the current  $I_2$  phaser directly from the circuit.

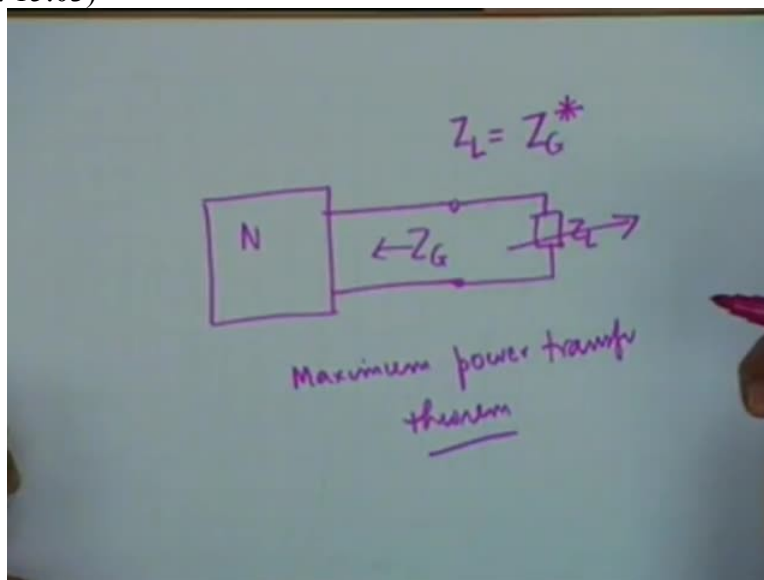
By inspection, nothing is to be solved. All that we are doing now is almost by inspection. We are not solving any differential equation, we are not making even large-scale algebraic simplifications. We are going almost by inspection. This is the beauty of the phaser analysis that it affords network analysis almost by inspection.

(Refer Slide Time: 14:00)



So the end result is that given any network  $N$  which consists of sources and linear elements, well Thevenin's theorem says that it can be reduced to a voltage source  $V_T$  in series with not a resistance now because it could contain inductance, capacitance and resistance and therefore, some impedance  $Z_T$ . This is equivalent to this or you could also work in terms of  $I_T$ , the Norton equivalence in parallel with the same impedance  $Z_T$  all right? And as I said this affords a simplification, tremendous simplification. And whether it is DC or AC, it does not matter. If it is a combination, well you can work in terms of superposition. All right?

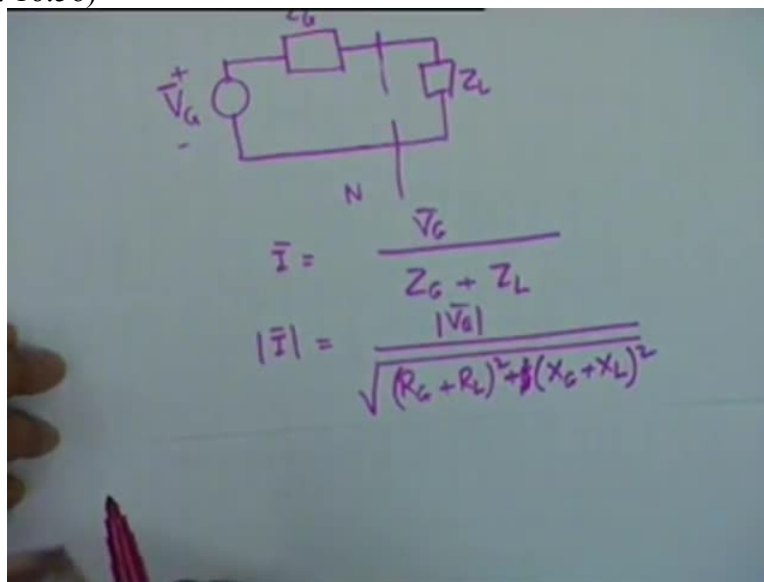
(Refer Slide Time: 15:03)



The point is, if you have a load to be connected to the network N, if you have a load let us say  $Z_L$ , we ask the question, under what condition if you have a load  $Z_L$  which is fixed, now if you have a load  $Z_L$  which let us say varies. The network is fixed, the network contains sources and inductance, capacitances all right? Under what condition will the load absorb the maximum possible power from the network? And this is in the literature known as the maximum power transfer theorem or simply, maximum power theorem all right?

It is very easy, it is very easy to show that if the network was resistive, then the load should also be purely resistive and should be equal to the Thevenin equivalent resistance of the network. This is the condition for maximum power transfer. If capital N is resistive all right? If capital N contains resistances as well as reactance is, then the condition is that the load  $Z_L$  if the Thevenin equivalent resistance is  $Z_{th}$ , then  $Z_L$  should be the complex conjugate of the equivalent Thevenin impedance. And this is what we are going to prove in a few lines. We will take a shortcut to the proof and these are some tricks of the trade.

(Refer Slide Time: 16:56)



And let us replace the network N by a voltage source  $V_G$  and then impedance  $Z_{th}$ . Well, this is a phasor. And we connect it to the load  $Z_L$ , then the current in the circuit  $I$ , this represents the network N, this is the Thevenin equivalent. The current in this, current phasor in the circuit is  $V_G$  phasor divided by  $Z_{th} + Z_L$  all right? So that the magnitude of the current, magnitude of the phasor is given by magnitude of the voltage phasor divided by square root of  $R_G + R_L$ , now we



are breaking up the impedances into their real and imaginary parts, that is resistive and reactive parts. + J times XG + XL. I beg your pardon, square of this because we are finding out the current. All right? Let me write it down again.

(Refer Slide Time: 18:10)

$$P_L = |I|^2 R_L = \frac{|V_0|^2 R_L}{(R_G + R_L)^2 + (X_G + X_L)^2}$$

$$\text{Max } P_L \quad R_L \rightarrow \quad X_L \rightarrow$$

$$P_L = \frac{|V_0|^2 R_L}{(R_G + R_L)^2 + (X_L - X_L)^2}$$

$X_G = -X_L$

The current phasor magnitude is equal to the voltage phasor magnitude divided by square root of  $R_G + R_L$  whole square +  $X_G + X_L$  whole square. So if I wish to find out the current, the power absorbed by the load, well the power would be absorbed by the resistive component of the load. Power would be absorbed by the resistive component of the load, power is stored in the reactive components of the load. So the power absorbed by the , if I call this  $P_L$  would be simply this squared, current squared multiplied by the resistive component of the load which is  $R_L$  and therefore what will happen is, this will be squared, it will be multiplied by  $R_L$  and this square root shall go.

This will be the expression for power all right? And we wish to maximise this power. The problem now is, maximise  $P_L$  by varying the load, that is  $R_L$  and  $X_L$ . Let me write it down clearly.  $V_G$  Square  $R_L$  divided by  $R_G + R_L$  whole square +  $X_G + X_L$  whole square. Since there are 2 variables, namely the resistive part of the load and the reactive part of the load ,  $P_L$  is a multi-variable function. It is a function of 2 variables all right? And therefore any maximisation that I do shall require partial differentiation but I want to avoid all this by applying a little bit of trick. The trick is, I argue let us 1<sup>st</sup> look at  $X_L$  let's keep  $R_L$  constant, let us vary  $X_L$ . This is what

we mean by partial differentiation. In partial differentiation, one of the variables is assumed constant, you differentiate with respect to the other. So if I concentrate on my attention on  $X_L$ , you notice that this  $P_L$  is a monotonically decreasing function of  $X_L$ , that is you increase  $X_L$ ,  $P_L$  decreases. And therefore  $P_L$  will be maximized if  $X_L$  is the variable,  $P_L$  will be maximised when  $X_G$  is equal to  $-X_L$ , that is the minimum value that this expression can take is 0 and it happens when  $X_G$  is equal to  $-X_L$ . That is, if the source is inductive, then the load should be capacitive, the reactances should be equal and opposite of each other. On the other hand, if the source is a capacitive, then the load should be inductive.

This is the only way that they can cancel each other and therefore one of the conditions is therefore that  $X_G$  is equal to  $-X_L$ . And then, under this condition, we have not got the maximum power but a sub-optimum because we have attacked only one variable, we have attacked with only one weapon whereas we have 2 in our hands. All right?

(Refer Slide Time: 21:34)

$$P_L = \frac{|V_G|^2 R_L}{(R_G + R_L)^2} \quad (X_G = -X_L)$$

$$= \frac{|V_G|^2}{\frac{R_G^2 + R_L^2 + 2R_G R_L}{R_L}}$$

$$1 - \frac{R_G^2}{R_L^2} = 0 \quad \frac{a}{x} + \frac{x}{b}$$

One weapon has given us the intermediate expression which is  $V_G$  Square  $R_L$  divided by  $R_G + R_L$  whole square. This has been obtained under the condition that  $X_G$  equal to  $-X_L$ . Now we argue, how do I maximise with respect to  $R_L$ ? I can again differentiate and put the result equal to 0 but a shortcut is this and I want you to remember this. I can write this as  $V_G$  Squared, now I expand this and divide by  $R_L$  so that the variable is brought completely into the dominator. So I

get  $R_G$  squared by  $R_L + R_L$  squared divided by  $R_L$ . So it would be simply  $R_L +$  twice  $R_G R_L$  divided by  $R_L$ .

So twice  $R_G$  of which you see this quantity is a constant this and this is the thing that is variable. Whenever you have an expression of this form,  $A$  by  $X + X$  by  $B$ , that is whenever, this you should remember throughout your life that whenever you have an expression in which the variable, one of the components increases with the variable, the other component decreases, then the optimum, that means the minimum is obtained when the 2 are equal. You can verify this by differentiation. That is  $\frac{d}{dR_L}$  of this expression would be  $1 - R_G$  squared divided by  $R_L$  Square and if you put this equal to 0, obviously what you get is  $R_G$  equal to  $R_L$ . Is that all right?

No differentiation is needed. If you can put it in this form, then the minimum value of this, not maximum mind you. Maximum obviously is infinity, when  $R_L$  goes to infinity, the maximum is infinity. Or  $R_L$  goes to 0. Maxima occurs at both extremes,  $R_L$  equal to 0,  $R_L$  to infinity, in either case the power absorbed is 0 however. Is that is that clear why? If the load is short-circuit, it cannot absorb any power. If the load is open, no current can flow. So no power can and therefore, this quantity is a minimum when  $R_G$  equal to  $R_L$  and therefore, the power is the maximum.

(Refer Slide Time: 24:02)

$$P_L = P_{L \max} = \frac{|V_G|^2}{4 R_G}$$
 M.A.P.

Impedance Matching

$$R_L = R_G$$

$$X_L = -X_G$$

$$Z_L = R_L + jX_L = R_G - jX_G = Z_G^*$$

And therefore we can write  $P_L \max$ ,  $P_L$  equal to  $P_L \max$  equal to  $V_G$  squared. If you put  $R_G$  equal to  $R_L$ , then you get  $V_G$  squared by  $4 R_G$  why? Because  $R_G$  equal to  $R_L$ . So twice  $R_G$

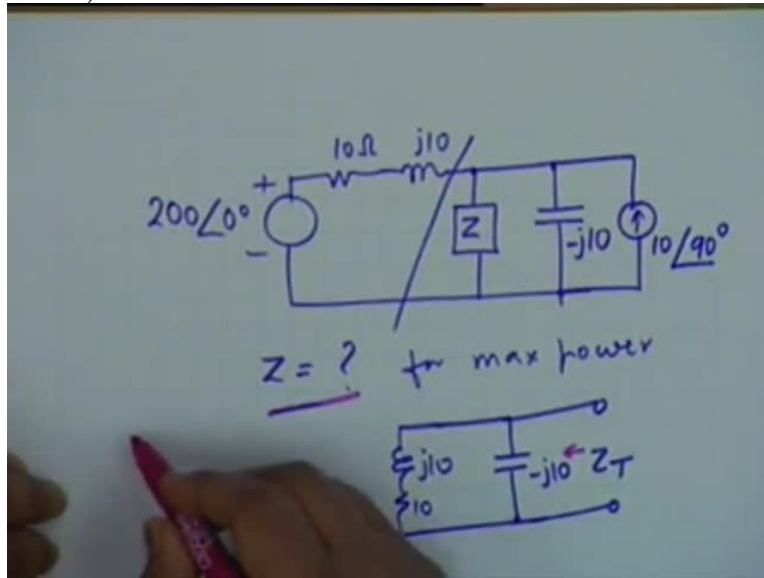
whole square is  $4 R_G$  squared and cancels with  $R_L$ . So  $4R_G$  and this is called the M A P, the maximum available power from the source. You cannot draw more than this. Under no circumstances can you draw more power than this and this happens when  $R_L$  equals to  $R_G$  and  $X_L$  equal to  $-X_G$ . In other words,  $Z_L$  which is  $R_L + jX_L$ .  $R_L + jX_L$  is simply equal to  $R_G - jX_G$  which is equal to the complex conjugate of the source impedance and this is one of the most important theorems in electrical engineering, maximum power transfer.

Wherever the power is limited to, wherever the power is limited, for example the exacta signal received through communication receiver, whenever the power is limited, you should try to draw the maximum available power by matching this is called impedance matching that is by taking a load, by designing the receiver in such a manner that it matches the antenna through which the signal is received. Well, the antenna can be considered as a signal source because it absorbs the radiation from the atmosphere and then pass it to your receiver and this process is known as impedance matching.

Impedance matching is not of importance in for an example, lighting of this room or in drawing power for the refrigerator, you do not match the impedances. Why? Because what is of concern there is efficiency. In such cases where maximum power is required, we do not care about efficiency. What is the efficiency under this condition when maximum power is being drawn? The maximum available power is this and equivalent amount of power is being absorbed in the source. Is not that right. You see the source delivers through a resistance to a load  $R_L$  and when the load and source are matched, that is one is the complex conjugate of the other,  $R_L$  absorbs this amount of power.

This amount of power must then be absorbed by  $R_G$  also. So half of the power is wasted in the source. We do not want this for an ordinary domestic supply for example. We do not want this. We want efficiency and therefore where efficiency is not important, impedance matching becomes extremely important. And this is one of the most important theorems of electrical engineering and finds application almost everywhere in all fields of electrical engineering.

(Refer Slide Time: 27:26)



Let me illustrate this with the help of a problem. I have a source  $200 \angle 0^\circ$  and an impedance consisting of  $10 \text{ ohms}$  and  $j10$  which means an inductive reactance and my load is here. My load is this  $Z$  which is shunted by a reactance of, an impedance of  $-j10$  which is that of a capacitor and a current source which is  $10 \angle 90^\circ$  all right? The question is, what should this impedance be so that it absorbs the maximum available power? Now you notice that here, there are 2 sources, not one. One fine point is that these both of these sources, when I write sources like this, what is the information that is hidden in this?

Student: (())(28:32)

Professor: Pardon me.

Student: Source transformation.

Professor: They can become?

Student: They can be combined.

Professor: They can be combined.

Student: They have the same omega.

Professor: They have the same?

Student: omega.

Professor: That is correct. This is the information that is hidden. Well, you can combine sources in many ways but whenever we write like this, it means that they are at the same frequency. Otherwise the phase shall be different. Not only that, if there are 2 sources of different frequencies, they cannot be combined. Rajat's answer is also correct to some extent that they can be combined but they can be combined, one of the necessary conditions not sufficient, one of the necessary conditions is that they must be of the same frequency, all right.

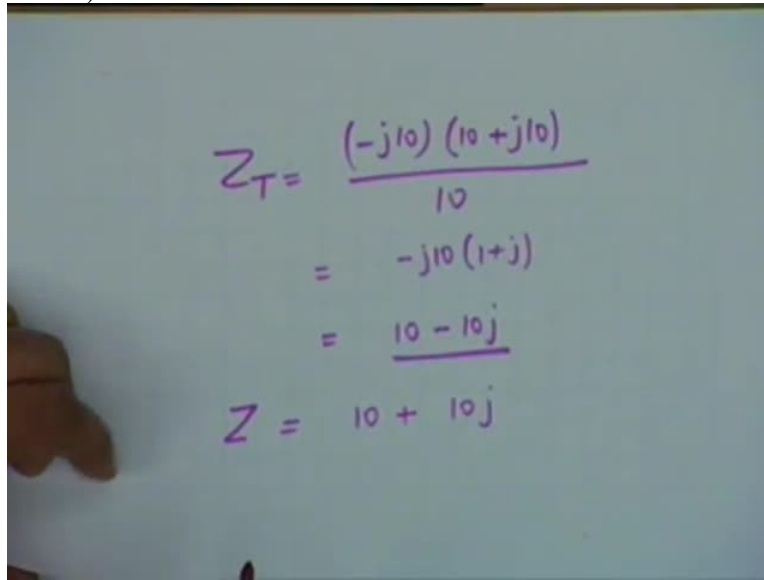
Student: Can we combine in this case?

Professor: Yes, of course.

Of course we will apply Norton's to the left of this and I shall combine that current generator with this current generator surely, no problem. But the question here is different. Do I have to combine? No. The question is what is  $Z$  for maximum power absorption by  $Z$ , for maximum power in the load? All that we require is to find out the Thevenin equivalent impedance which means, what is this impedance? It is simply  $J 10$ , then  $10$  ohms, the source is to be killed in parallel with  $- J 10$ .

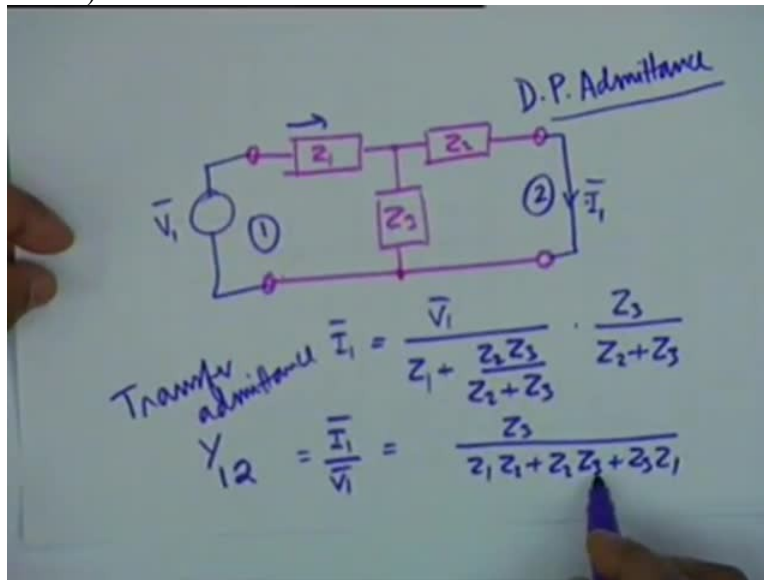
That is all. Now if you apply, if you think blindly, now one can think while one closes his eyes. So thinking blindly is not an impossibility but do not do so. Look at the problem carefully. You do not need to find, you do not need to combine sources because all that is asked is, what should be the value of  $Z$  so that maximum power is absorbed?

(Refer Slide Time: 30:33)


$$\begin{aligned}Z_T &= \frac{(-j10)(10 + j10)}{10} \\&= -j10(1 + j) \\&= \frac{10 - 10j}{1} \\Z &= 10 + 10j\end{aligned}$$

So you find out  $Z_T$  obviously your  $Z_T$  is  $-j10$  multiplied by  $10 + j10$  divided by  $10$  all right?  $10 + j10, -j10$ , they cancel each other. So you get  $-j10(1 + j)$  which is equal to  $10 - 10j$ . Is that okay? Therefore all that you need is  $Z$  should be equal to  $10 + 10j$ . In other words, the source is now capacitive when the reactance is negative. And therefore your load should be inductive and they should match each other. The resistive components should be equal, the reactive components should be equal and opposite of each other. Let us look at something else. No we are going to the enunciation of another theorem which we have not been familiar with so far.

(Refer Slide Time: 31:36)



~~Impedance Admittance~~

Let us look at a typical problem. Let us say we have a network like this. what would you call this network? It is a two port. How many ports? 1 and 2 but it is also a 3 terminal network because there are only 3 terminals, 1, 2 and 3. This terminal is common between input and output all right? Now let us apply a voltage source here,  $V_1$  and let us find out the short circuit current  $I_1$  here all right? Obviously I can find out  $I_1$  by inspection. It is  $V_1$  divided by  $Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$ , this is the total current. Then I take current division between  $Z_3$  and  $Z_2$  and therefore this is  $Z_3$  divided by  $Z_2 + Z_3$  agree? Is that okay? All right.



So therefore  $I_1$  by  $V_1$  is equal to well you can see that it is  $Z_3$  divided by  $Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$ . Is that okay? Is this algebra all right?. Now let me give a name to this quantity. That is why the current, the current is obtained here at a port which is different from the exciting work. The excitation is at Port 1 and the current, the response is obtained at Port 2. The ratio of the response to the excitation is now an admittance. Is it that right? The response is a current and the excitation is a voltage and therefore current to voltage has the dimension of admittance and this is called a transfer admittance, transfer because the current response is obtained at a different port and the symbolic that is used for it is  $Y_{12}$ .

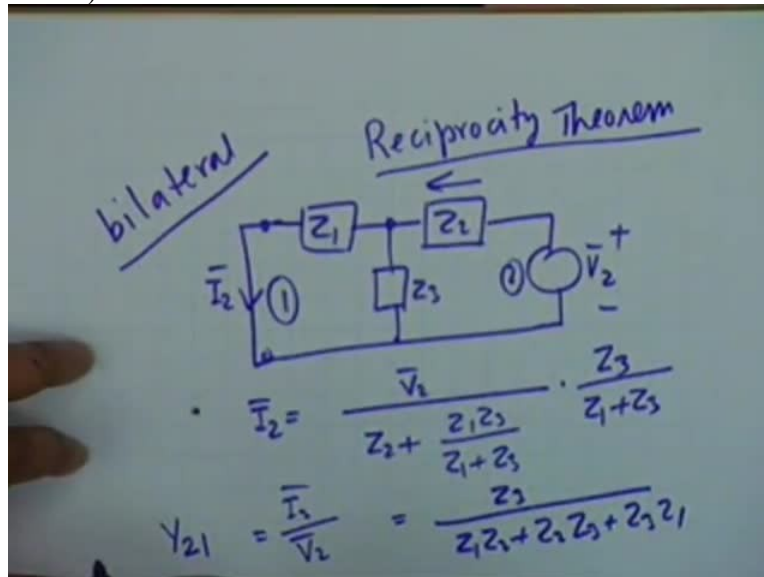
That is, this is the exciting port, port 1 and the current is obtained at Port 2. There are different conventions.  $Y_{12}$  is the response obtained, the current response at Port 2 divided by the voltage excitation at Port 1. And it is called transfer admittance. You understand the meaning of transfer. If it was not transfer, if you had applied the voltage at a port and measured the current at the same port, then you simply call this an admittance or sometimes to distinguish between transfer admittance and simple admittance, we may also call the latter as driving point admittance, that means admittance is measured at the port at which you are driving all right?

If the excitation and response are at the same point, then the ratio of the response to excitation, there is nothing sacred about voltage excitation. You could also have a current excitation and measure the voltage all right? Then the response to excitation would have been an impedance all right? So these are called driving point impedance or driving point admittance in contrast to transfer impedance. Here also I could have a voltage generator, I could have a current generator here and I could have measured the open circuit voltage. So in that case, we have been transferred impedance all right?

Transfer impedance and driving point impedance, these are 2 different kinds of quantities. Although dimensionally, they are the same. If it is impedance, then it is measured in ohms, if it is admittance, then it is measured in mhos but but physically, they are different quantities because the response is measured at a different port in the case of transfer, impedance or admittance. Did I tell you the common name for impedance and admittance? Immitance and this is derived like this, impedance, admittance.

What we do is we eat away a part of it. So this part is eaten away. So if we mean, if we can if we want to say transfer impedance or admittance, we will say transfer admittance all right? Or driving point admittance. Now let me go back to the circuit. I found out here that the transfer admittance as obtained as  $Z_3$  divided by  $Z_1Z_2 + Z_2Z_3 + Z_3Z_1$ . Now let us interchange the source, the excitation in the response ports.

(Refer Slide Time: 37:00)



Let us interchange, that means I have the same network,  $Z_1$ ,  $Z_2$  and  $Z_3$ , I apply the voltage source here. Let us say, a different voltage source,  $V_2$  and the response is obtained at Port 1, let us call this as current  $I_2$ . Do you see what I have done? I have simply taken, I simply interchanged the the excitation and the response points. Initially, the source was a voltage source was here and this was short-circuited. Now after I transferred the voltage source to port 2 and I am measuring the current at Port 1. Now can you find out  $I_2$  bar? In the same way, it is  $V_2$  bar divided by  $Z_2 + Z_1Z_3$  divided by  $Z_1 + Z_3$ . This is the current through  $Z_2$ . It will come to be the same, that is correct.

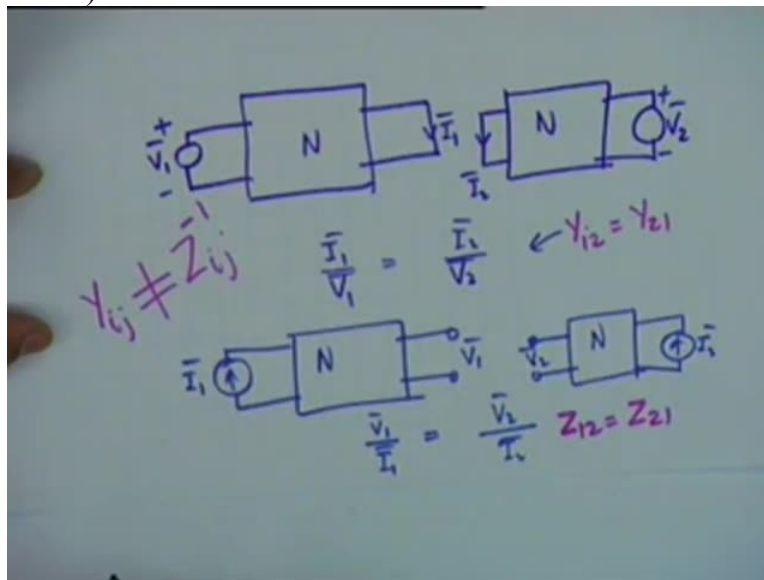
This will be  $Z_3$  divided by  $Z_1 + Z_3$  and you see that  $I_2$  divided by  $V_2$ , what should WE call this now?  $Y_{21}$ . Is that correct? Yes, yes. The source at Port 2 and the response at Port 1 and you see that surprisingly, this is the same. This is the same as in the previous case. All right? And this is what is generalised to a theorem called the theorem of reciprocity or reciprocity

theorem. Reciprocity theorem. It, in general terms, it simply means the following. That if there is a network if there is a network which contains bilateral elements, let me introduce this term, bilateral, that is elements which conduct equally well in both directions.

A resistance, it does not matter whether one end is positive or one end is negative. If the same voltage is applied, the same currents would flow. And a resistance conducts equally in both directions. Similarly, an inductance, similarly a capacitance. But you know at least one case, one element which defies this, that is a diode. So if a network does not contain unilateral elements like a diode, if it contains only bilateral elements, then the network is said to be reciprocal. That is if the excitation and the response are interchanged, then the ratio remains the same. All right?

This is called the theorem of reciprocity or the reciprocity theorem and in the context, one must remember that the network shall should not contain any element which is partial to polarity which is partial to polarity. For example, a diode if it is forward biased, it conducts easily. If it is reverse biased, it does not conduct.

(Refer Slide Time: 40:34)



And in general, the reciprocity theorem is stated like this. You have a network  $N$  which is a reciprocal network. Any network which obeys reciprocity theorem is called a reciprocal network but then you cannot define reciprocity theorem in terms of reciprocal network. So you say  $N$  contains linear elements, linear and bilateral elements.  $N$  contains only linear and bilateral

elements, it is a 2 port network and therefore if you apply a voltage source, I am going to ask you a question.

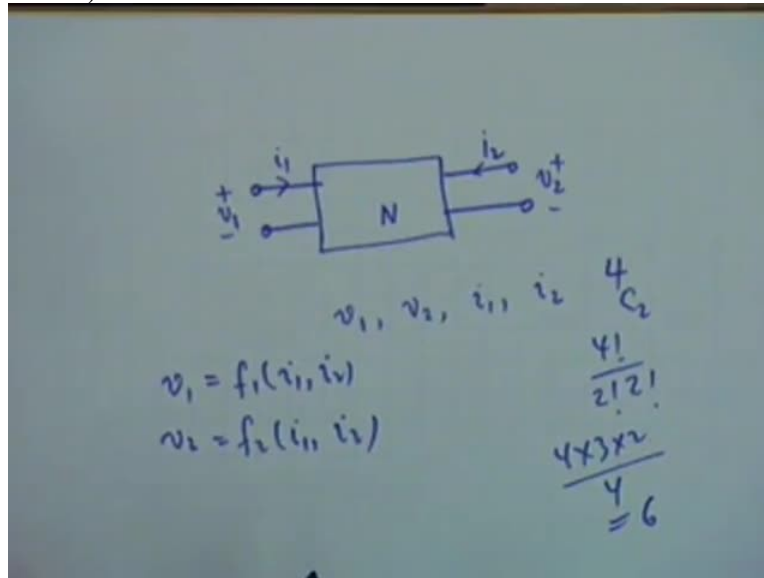
Now if you apply the voltage source here and measure the current, the same network if you measure the current here, due to a voltage source here, well the ratio is the same, that is  $I_1$  by  $V_1$  is equal to  $I_2$  by  $V_2$ , that is the transfer admittances are the same, then the network  $N$ , well this is a statement of the reciprocity theorem that if the excitation and the response are interchanged, there is no change in the ratio. All right? If this is one volt and this is one ampere, this is 50 ampere, then this should be 50 ampere. The ratio shall be the same. Now I am asking you a question. Look at the question carefully.

Suppose, well under this condition, suppose I apply a current generator all right? Let us say  $I_1$  and I measure the voltage  $V_1$  here, similarly I apply a current generator  $I_2$  here and measure the voltage response  $V_2$ . By reciprocity theorem, the ratio from these the same. That is, response to excitation should be equal to the response to excitation. If  $N$  is a linear network with bilateral elements, then the ratio of the response to excitation should be the same as the ratio of the response to excitation. If I call this, if I characterise this in terms of transfer admittances, response is a current, excitation is a voltage. Current to voltage is admittance and current and voltage are at different ports.

And therefore we say  $Y_{12}$  is equal to  $Y_{21}$ . Then what should we call this? Transfer impedances,  $Z_{12}$  should be equal to  $Z_{21}$ . The question that I ask is, is there a relationship between  $Y_{12}$  and  $Z_{12}$ . They are reciprocals of each other? Much to your frustration, they are not. They are not and this is something which we wish to investigate. What is the relationship between them? They are not simply the reciprocals of each other. Why? What is the reason? The reason is that the conditions here are quite different from conditions here. There is a short-circuit and there is a open circuit.

They are dual situations all right but they are not, the impedance and admittance are not reciprocals of each other. One must remember this that  $Y_{IJ}$  is not equal to  $Z_{IJ}$ . Transfer impedance okay. All right? This must be remembered because this is a mistake very often committed even by graduate electrical engineers. So you should be particularly careful about this.

(Refer Slide Time: 44:40)



To investigate this point, relationship between the transfer admittances, we consider a two port in general. Let us say, capital N is a 2 port, a general 2 port where what you can do is, you can apply a voltage. What are the things that you can do? You can apply two voltage sources and measure the currents. You cannot reach inside the network. You have only two ports all right? So what you can do is, you can apply two voltage sources, measure the current. Or you can apply two current sources and measure the voltages. Or you can apply a combination, one of them can be a voltage source, the other can be a current source and you can measure one voltage and one current.

The point, the fact of the matter is that there are 4 quantities,  $V_1, V_2, I_1$  and  $I_2$  of which 2 can be specified independently. The other 2 then shall be determined from the characteristic of the network N. Is this clear? There are 4 quantities, 4 parameters, 4 variables of which 2 can be independently specified. Other 2 will be determined by the characteristics of the network. In how many ways can you specify the Independent and the dependent variables? Obviously, there are 6 ways.  $4C_2$ , 2 taken out of 4.

Factorial 4 divided by factorial 2 factorial 2 which is 4 times 3 times 2 divide by 4, so 6 ways all right? Now 6 ways means what? You can say  $V_1$  and  $V_2$  can be a set of dependent variables.  $V_1$  can be let us say  $F_1(I_1, I_2)$ ,  $V_2$  can be  $F_2(I_1, I_2)$ . These are independent variables,  $I_1$  and  $I_2$  and  $v_1$  and  $V_2$  are dependent variables or you can write  $I_1$  and  $I_2$  in terms of  $V_1$  and  $V_2$ , two voltage

sources and then you find the dependent currents or you make a combination is  $V_{1I1}$ ,  $V_{2I2}$ , you can make  $V_{1I2}$ ,  $V_{2I1}$  or their permutations. You can have  $V_{1I1}$  or you can have  $I_1V_1$  all right? That is also a possibility.

(Refer Slide Time: 47:32)

$$\begin{aligned}
 v_1 &= k_1 i_1 + k_2 i_2 \\
 v_2 &= k_3 i_1 + k_4 i_2 \\
 \hline
 i_1 &= k_5 v_1 + k_6 v_2 \\
 i_2 &= k_7 v_1 + k_8 v_2
 \end{aligned}$$

There are in all 6 ways. But at the present time at the present time we shall concern ourselves with only 2 of these representations, that is  $V_1V_2$  in terms of  $I_1I_2$ . Or the other way round, that is  $I_1I_2$  in terms of  $V_1 V_2$  all right? And since the network what we mean is,  $V_1$  is expressed as a function of  $I_1$  and  $I_2$ ,  $V_2$  is expressed as a function of  $I_1$  and  $I_2$  or  $I_1$  is expressed in terms of  $V_1$  and  $V_2$ ,  $I_2$  expressed in terms of  $V_1$  and  $V_2$  all right? Now the network is linear. The network is linear and therefore  $V_1$  shall be of the form  $K_1I_1 + K_2I_2$  and  $V_2$  shall be of the form  $K_3I_1 + K_4I_2$ . All right?

It is a linear network and therefore the voltage and the current, the voltage which is a dependent variable, must be a linear combination of  $I_1$  and  $I_2$ . That is if pardon me? Superposition. That is it.  $I_1$  and  $I_2$  are Independent variables, they are sources and therefore you can determine the responses to each independently and add them up. Similarly, you can write  $I_1$  equal to  $K_5V_1 + K_6V_2$  and  $I_2$  as  $K_7V_1 + K_8V_2$  all right? Now if the network is excited by sinusoids that is AC

sources, then a special form of these relation relationships are valid and these special forms can be obtained by a very simple consideration. What is the of K1? It must be impedance.

Student: Impedance

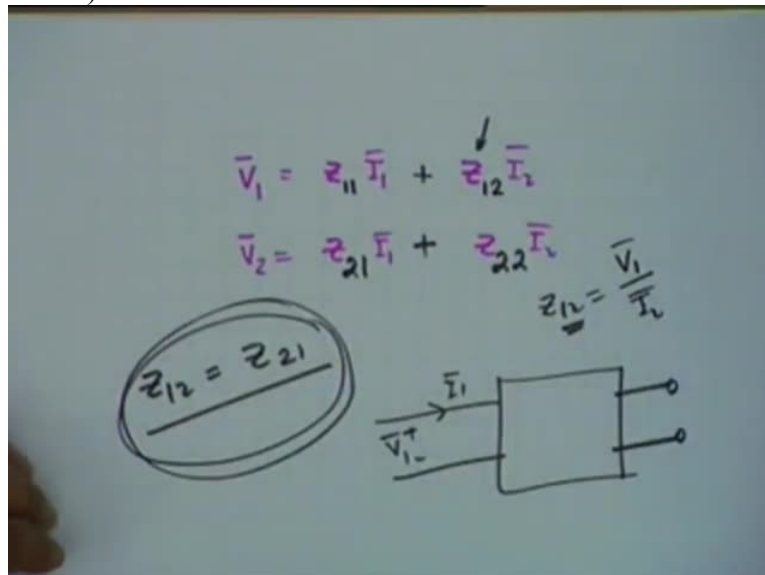
Student: Impedance

Student: Impedance

Professor: Impedance.

Similarly K2, K3, K4 all of them should be impedance. Agreed?

(Refer Slide Time: 49:50)



So what I do is if we take this, if we take the 1<sup>st</sup> set of relationships where  $I_1$  and  $I_2$  are dependent variables and  $V_1$  and  $V_2$  are dependent variables, then we say, the voltage phaser  $V_1$  and the voltage phaser  $V_2$  can be written as in terms of  $I_1$  and  $I_2$ , similarly  $I_1$  and  $I_2$  and the coefficients are impedances. So we shall have  $Z, Z, Z, Z$ . We must now specify all all  $Z$ s we must now distinguish between the 4  $Z$ s all right? Now what shall we call this? what shall we call this  $Z$  for example? Well, we call this  $Z_{11}$  and the reason is the following. The reason is that, what is  $Z_{11}$ ? It is  $V_1$  divided by  $I_1$  with  $I_2$  equal to 0.

Now  $V_1$  by  $I_1$  means that you are measuring the response at the same port as the excitation. Is not that right? If you recall, this is port 1, this is  $V_1$  and this is  $I_1$ . What you are doing is you are making  $I_2$  equal to 0.  $I_2$  equal to 0 means these are left open circuit and therefore, if you are applying a current source here and measuring the voltage. Is not this a driving point impedance? Driving point impedance. And since it is measured at Port 1, we call it  $Z_{11}$ . Question is, why two subscripts? Why two subscripts are needed?

Because the transfer impedances required two subscripts. So to keep uniformity, we maintain 2 subscripts for the driving point impedances also. And from the same consideration, this will be called  $Z_{12}$  the current is measured at port 2. Okay. Intentionally, it is okay. If you agree, then I will not explain further. It is a matter of convention. This impedance is called  $Z_{12}$ . If I had introduced a different convention, go back and change it. All right. What should we call this? No. Before we go to the other one,

Student: Sir but  $I_2$  is the independent one.

Student: Yes sir, so it should be  $Z_2$ .

Professor: Okay,  $I_2$  is the dependent variable and I am measuring  $V_1$ .

Under what conditions?  $I_1$  equal to 0. So  $Z_{12}$  is equal to  $V_1$  divided by  $I_2$  all right? And we call this  $Z_{12}$ , that is from response at Port 1 and excitation at Port 2. If I had used a different nomenclature earlier, go ahead and change it all right? This is the IEEE accepted terminology, universal terminology. Similarly, this impedance we shall call  $Z_{21}$  and this impedance we shall call  $Z_{22}$ . You understand why double subscripts?  $Z$  We could have called this  $Z_1$ , we could have called this  $Z_2$  but since we we have to use double subscripts for the transfer impedances, we maintain uniformity.

And you say that if the network is reciprocal, if the network is reciprocal, then what is the condition?  $Z_{12}$  should be equal to  $Z_{21}$  all right? If the network is reciprocal, if the reciprocity theorem is obeyed, then this should be satisfied. What does this mean? That in order to describe a 2 port, we do not require 4 parameters. We require only 3 provided the network is reciprocal. Suppose this network has a transistor. Transistor can be considered as a combination of diodes + an amplifying element which are unilateral elements and therefore, it will not be valid there.



In the case of a transistor amplifier,  $Z_{12}$  equal to  $Z_{21}$  is not valid. They are quite different. One is much larger than the other. But in the case of a passive reciprocal network, that is a network containing linear and bilateral elements, this shall always be valid. Tomorrow we shall consider the other kind of description, that is considering voltages as independent variables and currents as dependent variables and then establish a relation between  $Y_{12}$  and  $Z_{12}$ .