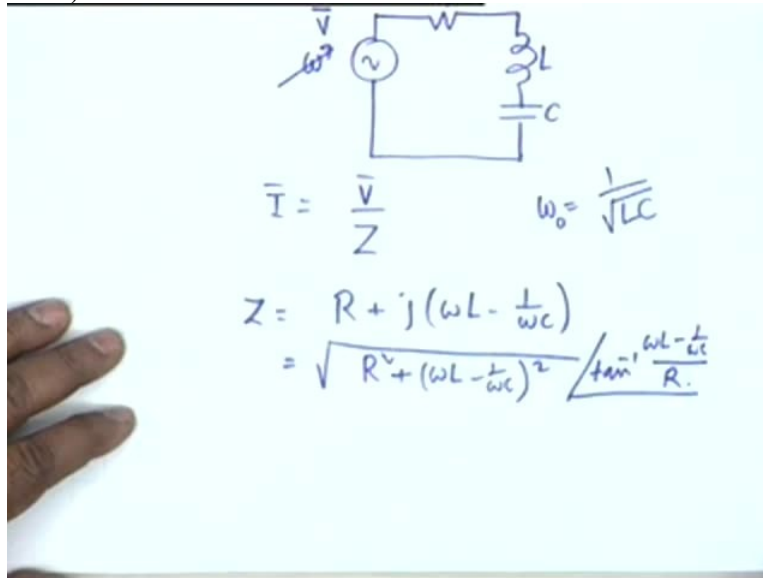


Introduction To Electronic Circuits
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Indian Institute of Technology Delhi
Module No 01
Lecture 22: Resonance (Contd.)

That is our reputation. 22nd lecture and the lecture is on resonance.

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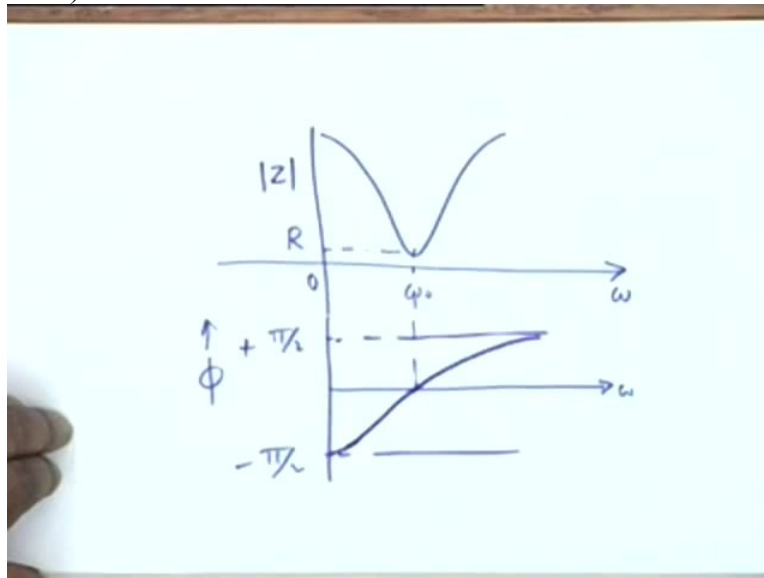


Last time I had started series resonance that is a series RLC circuit which was excited by an AC generator with the variable frequency and if I call this as the voltage phaser \bar{V} , then the current phase in the circuit is given by \bar{V} over the complex impedance and the impedance of the circuit is $R + j\omega L - \frac{1}{j\omega C}$ which has a magnitude and a phase. The magnitude is square root of $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$ and the angle is $\tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$.

And you see that if frequency is increased from 0 towards infinity, then the impedance starts from infinity, infinite value. At ω equal to 0, the impedance is infinite and the angle at ω equal to 0 is simply $\tan^{-1} -\infty$ which is $-\pi/2$. As ω is increased, there will be reached a point when ωL becomes equal to $\frac{1}{\omega C}$ and under that condition, the circuit shall be purely resistive, that is the imaginary part or the reactive part shall be 0. It will be purely resistive and the impedance obviously shall be a minimum.

The angle at that point and ωL is equal to 1 by ωC , obviously the angle is 0 . As ω increases beyond this frequency which we call the resonance frequency, ω_0 which is 1 over square root of LC , beyond this frequency, this term dominates. ωL becomes greater than 1 by ωC . So the circuit becomes inductive because its reactance is positive. The impedance again increases towards infinity. As ω goes to infinity, the impedance magnitude goes to infinity and the angle goes to $\tan^{-1} \infty$ which is $\pi/2$.

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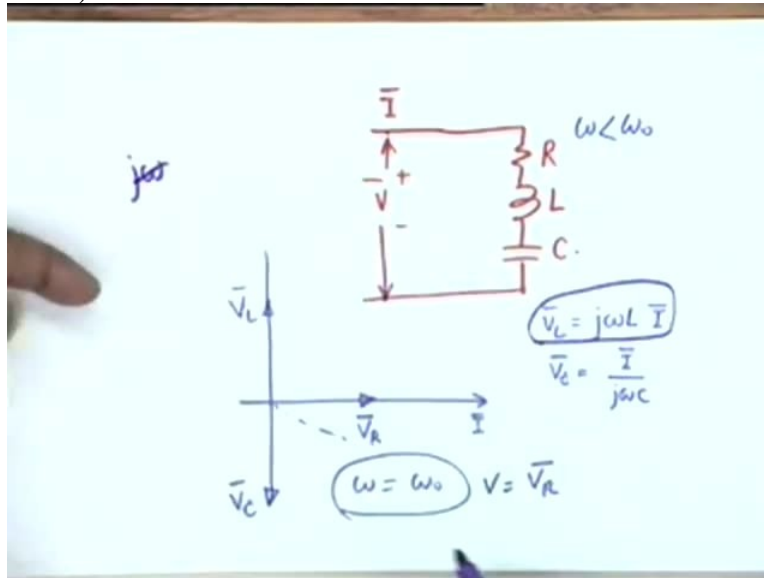
And therefore if I draw a plot of the magnitude and phase, the magnitude goes like this. It starts from infinity, goes towards infinity and attains a minimum value of capital R at ω equal to ω_0 . And at this frequency, if you plot the phase, $+\pi/2$ and $-\pi/2$, if you plot the phase, ϕ vs ω , then it starts from at ω equal to 0 , the angle is $-\pi/2$ and $-\pi/2$ and it goes through 0 at ω_0 and goes to $+\pi/2$ as ω goes to infinity.

This frequency ω_0 is called the frequency of resonance and resonance implies, the definition, formal definition of resonance is that the impedance or the admittance of the circuit becomes purely resistive means that the voltage and current at this frequency shall be in phase. If the circuit is purely resistive, the current and voltage shall have no phase difference between them. Another way of looking at resonance is that the admittance, admittance of the circuit, Y shall show a peaking response because admittance is the reciprocal of the impedance and the

maximum value of the admittance shall be equal to $1/R$ and this curve shall also change its sign that is it will start from $+\pi/2$ and go towards $-\pi/2$ as ω increases.

This phenomenon, the phenomenon of resonance under which the voltage and current in a circuit are in phase, this is the definition of resonance can also be expressed in terms of current and voltage.

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Suppose we have a current phaser \bar{I} which is fed to the series combination of R , L and C then the phaser diagram can be drawn with regard to, let the voltage be \bar{V} , voltage phaser be \bar{V} , then if this is the current phaser, if this is the current phaser \bar{I} , then the drop, voltage drops across the resistance shall be IR and this obviously is in phase with \bar{I} so if I call this \bar{V}_R then with regard to \bar{V}_L and \bar{V}_C , you notice that \bar{V}_L is $j\omega L \bar{I}$ and therefore it leads the current phaser by 90 degrees and therefore \bar{V}_L would be in this direction let us say. \bar{V}_L , it will lead \bar{I} by 90 degrees.

On the other hand, the capacitor voltage, voltage across the capacitor \bar{V}_C shall be \bar{I} divided by $j\omega C$, so it would be in the opposite direction, that means this will be the direction of \bar{V}_C and depending on the frequency, whether the frequency is below resonance, above resonance or at resonance, the total voltage or the the total voltage across the circuit, \bar{V} shall either lead the current lack the current or be in phase with the current. For example, the picture that I have shown here is almost true for ω equal to ω_0 .

That means V_L and V_C are equal in magnitude and opposite in direction. So they cancel each other and therefore the total voltage V under this condition shall be simply equal to V_R which is obviously in phase with the current phaser I . Suppose the situation was below resonance, that is ω less than ω_0 , if ω is less than ω_0 , then which of these 2 phasers will have a greater magnitude?

Student: V_C .

Professor: V_C .

And therefore, the resulting voltage would have been $V_C - V_L$ added to vectorially added to V_R and therefore the resulting voltage would have been would have some direction like this and it would have lagged the current I . Similarly if the frequency is above resonance, the voltage leads the current I and the angle of lead or lag depends on the angle of impedance, of the impedance Z , that is $\tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$. This is the story of series resonance. Now under series resonance, a very peculiar thing occurs that if you consider V_L equal to the V_C phaser as $j\omega L I$ and let me take another.

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$$\begin{aligned} \bar{V}_L &= j\omega L \bar{I} \\ \omega &= \omega_0 & \bar{I} &= \frac{\bar{V}}{R} \\ \bar{V}_L &= j\omega_0 L \frac{\bar{V}}{R} \\ V_L &= \left(\frac{\omega_0 L}{R} \right) V \\ V_L &= Q V \end{aligned}$$

V_L phaser is $j\omega L I$, this is in general true. Suppose we consider the situation at resonance, that is ω equal to ω_0 . Under this condition, you know I is in phase with the voltage and therefore I shall be equal to V by R . Under resonance, the impedance is purely resistive,

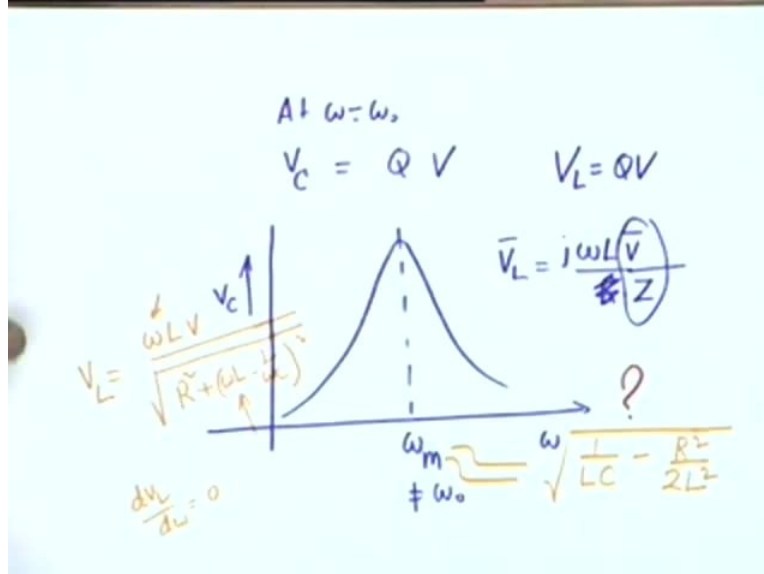
therefore the current phaser is simply given by the ratio of the voltage phaser to resistance. And therefore, V_L phaser would be $J \omega L$, we are considering conditions at resonance,

V divided by R , alright? Which means that the magnitude of the voltage across the inductance is equal to ωL by R , the magnitude of the voltage across the total circuit. All right? For example, if V is one volt, root mean square and value because in phasers, we use RMS values, then V_L can be one volt multiplied by ωL by R all right? And this quantity could be as large as infinity. Suppose we had no resistance in the circuit, suppose the resistance was 0, the resistance that we are talking of basically belongs to the inductance. You cannot make an inductance without a wire and a physical wire shall always have a resistance.

And therefore, this resistance that it is, usually is the resistance of the coil or this resistance determines the merit of the coil, a coil from which we want only an inductance, this is an inevitable nuisance, capital R . If capital, if we could make an ideal inductor, then obviously, this quantity could become infinite all right? And therefore, capital R is a figure of merit of the coil. But not by itself, it is a figure of merit with respect to ωL , that is the inductive reactance is and this quantity is usually denoted by Q , V_L is Q times V and Q can be very large.

Getting a Q of capital L at high frequency is absolutely no problem, which means that you can get a stepping up of voltage without using a transformer. And sometimes, this is also called a resonance transformer. That is you are using, you are using the phenomenon of resonance to get a stepped up voltage all right, Q times V .

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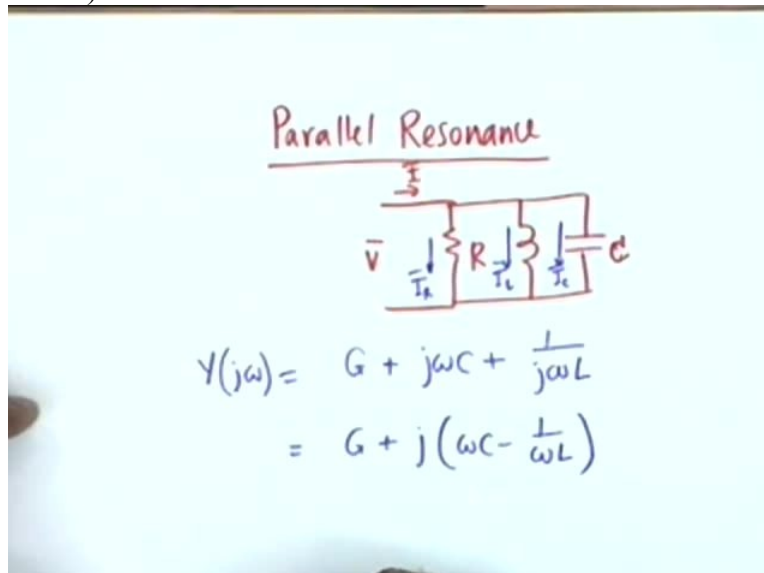
In a similar manner, if you had investigated the capacitor voltage, V sub C at ω equal to ω_0 , if you had investigated, you would have seen that V sub C is also equal to Q times V . Remember, we are considering only amplitudes, only magnitudes all right? So at resonance, V sub C and V sub L can be much larger than the applied voltage and so there is a stepping up and this stepping up is technically known as the resonance transformation. That is a transformer which uses the property of resonance. You must remember however that V_L and V_C that they are individually Q times V should not matter because one is in opposite phase to the other.

So the total voltage across the pure inductance and the capacitance is equal to 0. The total voltage is dropped only across the resistance and that is how the current in the circuit becomes capital V divided by R . Now does that mean that if I plot, let us say if I measure the voltage across the capacitor, V sub C by means of voltmeter, a VTVM and plot it was is ω , does it mean that it will show a response like this? Yes? The current is maximum at ω equal to ω_0 and therefore the capacitor voltage, the voltage across the capacitor should show a curve like this, should show a variation like this. Similarly, voltage across the inductor. However, it can be very easily shown that the frequency of maximum voltage is not equal to ω_0 .

Omega M is not equal to omega 0 and the reason is not difficult to find. What is the reason? Reason is VL or V sub C, let us consider VL, VL is omega L times V J omega L V divided by R. No, I beg your pardon. In general, it is Z, in general. All right? In general, the current phaser is V by Z, impedance and the voltage across the inductance is equal to J omega L, it is impedance multiplied by the current and therefore if you take its magnitude and I omit a couple of steps, you can see that the magnitude at the omega L V divided by Square root of R squared + omega L - 1 by omega C whole squared. Is that clear? I am taking the magnitude.

So J I omit, omega L, magnitude of V is V and magnitude of Z is this. And you see that omega occurs in the numerator and also in the denominator. And therefore, VL is not necessarily maximum when the denominator, this term becomes 0 because there is an omega here. In fact, to find out the frequency or maximum, what you have to do is DVL by D omega shall be put equal to 0 and you can show that omega M, this I leave it to you as an exercise, you can show that omega M is equal to 1 by Square root LC - a quantity which depends on capital R and I shall write down as far as I remember, the expression is this, - R squared by 2 L squared but I will put this with a question mark. You can verify or annul this claim, you can find out the current expression.

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Similarly for the voltage across the capacitor. Now the circuit that we had considered was a series circuit, that is a resistance, inductance and a capacitance, all connected in series and the resistance as I said came from the coil, came from the inductance, we can also consider the dual circuit in which the 3 elements are connected in parallel, R, L and C and then it would make sense to talk of a voltage excitation and find out the total current phaser in the circuit. And it will consist of 3 components, namely IR, IL and I sub C. And you can see, that a similar phenomenon in shall occur, a similar phenomenon unlike resonance shall occur under which condition, the reactance of the inductor shall be cancelled by the reactance of the capacitor.

They are not in series now and therefore what can be cancelled? Not reactance, but susceptance, susceptance of the inductor shall be cancelled by the susceptance of the capacitor. Well, this can be formally proceeded with exact the same manner as we did for the series resonant circuit but you can say that the most appropriate thing to computing here is the admittance, all right? So Y of J omega as you can say is G, G is 1 by R place the admittance of the capacitor J omega C and the admittance of the inductor is J omega, 1 by J omega L and therefore, L and C interchange their roles and G takes the part of resistance in a series resonant circuit, this I can write as J omega C - 1 by omega L and one can proceed exactly similarly.

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$$|Y| = \sqrt{G^2 + (\omega C - \frac{1}{\omega L})^2}$$

$$\angle Y = \tan^{-1} \frac{\omega C - \frac{1}{\omega L}}{G}$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega < \omega_0$$

$\bar{I}_L = \frac{V}{j\omega L}$
 $\bar{I}_C = j\omega C V$

That is, the magnitude of Y shall be $\sqrt{G^2 + (\omega C - 1/\omega L)^2}$ and the angle of Y shall be $\tan^{-1}(\omega C - 1/\omega L) / G$. And once again, you can see that when ωC is equal to $1/\omega L$, that is when ω equals to $\omega_0 = 1/\sqrt{LC}$, the susceptance of the capacitor which is ωC cancels the susceptance of the inductor which is $1/\omega L$ and the current and voltage while the admittance angle would be 0, so the current and voltage shall be in phase.

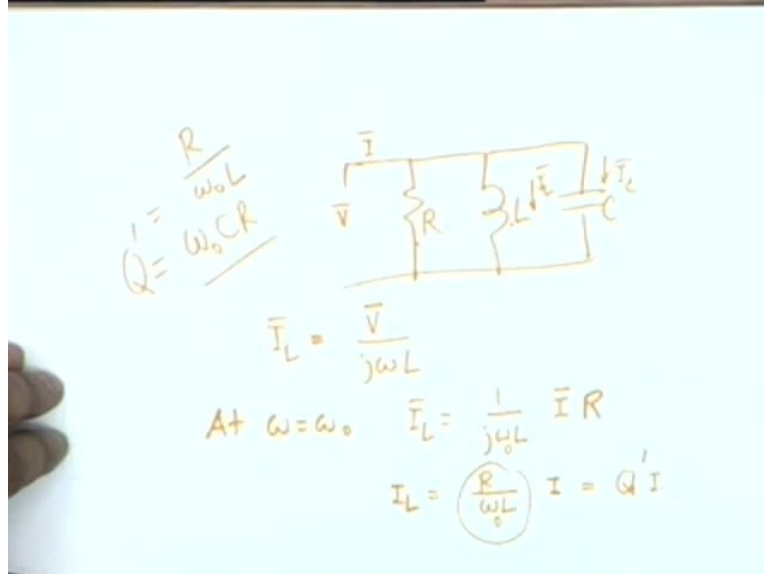
In the phaser diagram now, instead of taking I as the reference, we take V as the reference all right? Then we argue that I_R , the current in the resistance must be in phase with V , so this is $I_R = V/R$. I_L phaser shall be V divided by $j\omega L$. So it would lag it would be lagging. This would be the direction of I_L and I_C , the current phaser in the capacitor shall be $j\omega CV$, in other words, this should be leading leading the voltage or the current in the resistance by an angle 90, this will be I_C and the figure that I have drawn now is approximately it represents resonance condition in which I_L and I_C are equal and opposite to each other, they cancel each other.

And therefore the total current in the circuit is equal to the current in the resistor which is simply the ratio of the voltage phaser to the resistance. And this resonance, because of the parallel connection of the elements is called parallel resonance as opposed to a series resonance in a series connection of R , L and C . Can you tell me now what will happen if ω is less than ω_0 ? If ω is less than ω_0 , then which current shall be higher length?

Student: I_L .

Professor: I_L . And therefore the total current shall be lagging the voltage. Similarly, if ω is greater than ω_0 , then the total current shall lead the voltage okay? So series and parallel resonance are very similar phenomenon and one should not be surprised to see similarities because one circuit is the dual of the other circuit. The role of series connection is taken over by parallel connection, the role of voltage is taken over by current and the role of current is taken over by voltage. So however, there is an important distinction now.

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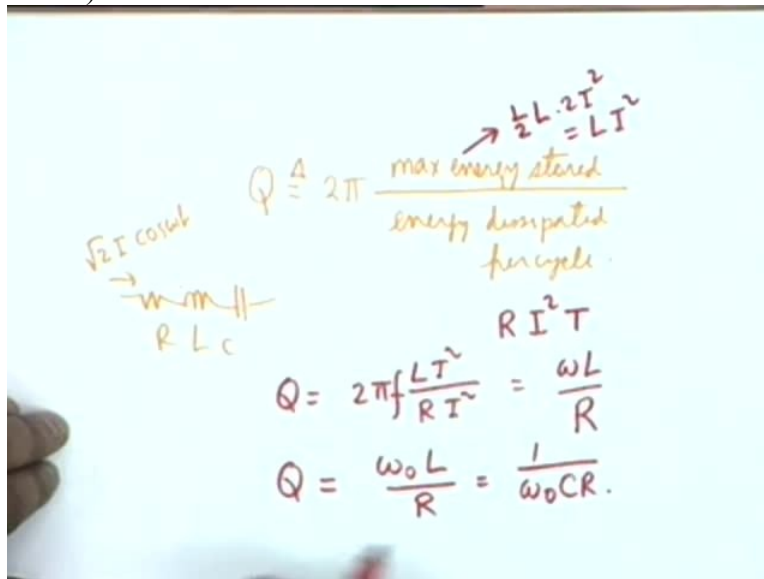
Namely that if you have let us say this parallel resonant circuit, in a very similar manner, if this is the voltage phaser, R, L and C, the current in let us say the inductor branch, I_L , the current in the inductor branch I_L or the current in the capacitor branch, you could take any but let us take I_L . The current in the inductor branch will be voltage phaser divided by $j\omega L$ all right? And at resonance at ω equal to ω_0 , the voltage phaser, the voltage is in phase with the total current I and you know that I_L shall be 1 by $j\omega L$, this will be simply I phaser multiplied by R because the circuit is at resonance all right?

As ω equal to ω_0 , the current and voltage are in phase and the voltage is simply current multiplied by the resistance because the 2 currents I_L and I_C cancel each other. There are 2 currents, I_L and I_C but one is equal and opposite to the other. So they cancel each other. Now you notice that the magnitude, if you are talking simply of the magnitude, then this is R by ωL multiplied by I . R by ω not L , yes, thank you. R by ω not L times I . And you notice that this quantity qualify, call this Q prime let us say, some other quantity, Q prime I , you see that at resonance, the current in the inductor in a similar manner, you can argue the current in the capacitor also can be multiplied by constant Q prime which could be as large as Infinity.

When is it Infinity? If capital R is infinite. That means there is no resistance here all right? The current can be infinitely large. And therefore I have denoted this by Q prime. However, this Q will not be same as the Q defined for series resonance. The two Qs are different and the difference comes because the resistance here is in parallel with the inductor whereas in the earlier case, the resistance was in series with the inductor. For a parallel combination like this, it is, the Q is resistance divided by reactance.

In a similar manner, one could show that Q prime is omega not CR, resistance divided by reactance all right? These are the same as you can easily see. R by omega 0L. The point that I am mentioning to you is that the definition of Q in the 2 cases are quite different. In one case, it is reactance divided by resistance, in this series okay. In the parallel case, it is resistance divided by reactance. And therefore, it appears that if we wish to use the parameter Q to characterise the merit of an inductor or the lack of it all right, then a more universal definition is needed.

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And this definition for any circuit, any inductor or capacitor is formally given in terms of energy, that is Q. Q for quality or figure of merit. The quality of any reactance or any storage element which is either an inductor or a capacitor is defined as twice pi multiplied by maximum energy stored maximum energy stored divided by energy dissipated per cycle what which means the Q, the quality has something to do with the energy storage capability of the element vis-a-vis the energy that is dissipated in a Freecycle all right?

And it is this ratio which is considered as the definition of Q, universal definition of Q. Now we can easily calculate this, for example, for a series RLC circuit, it is very easy to calculate. Suppose the current is $\sqrt{2} I \cos \omega T$, let us say $\cos \omega T$ suppose the current $\sqrt{2} I \cos \omega T$ flows, then what is the maximum energy stored? The maximum energy stored in the inductor if we consider, it is half L maximum energy, maximum current is $\sqrt{2} I$, therefore $2 I^2$ squared that is equal to $L I^2$ squared. Is that okay? Maximum energy stored will be at the instant when the current is maximum, half $L I^2$ squared and energy dissipated per cycle would be $R I^2$ squared multiplied by per cycle.

So $R I^2$ squared is the power energy per unit time, this should be multiplied by the time period, capital T all right? And therefore Q for this circuit shall be $2 \pi L I^2$ squared divided by $R I^2$ squared. Now what is capital P ? P is ω by frequency and therefore $2 \pi F$ which obviously is ωL divided by R . This is the definition that we gave arbitrarily, that is a figure of merit. By considering the ratio of voltage across inductance to the applied voltage in a series resonance circuit, it comes out to be the same. However, it is conventional it is conventional to define Q at the resonance frequency.

In other words, Q if you define at resonance frequency, it is $\omega_0 L$ by R . And from an exactly similar arguments, you could show that if C was associated with this resistance, then this is simply 1 by $\omega_0 C R$. This also follows from the fact that ω_0 is 1 by square root LC . And therefore Q of an element is a very useful nature of the quality or the energy storage capability vis-a-vis relative to the energy dissipation. Energy dissipation is a fact of life. It is an undesirable phenomenon but it happens because you cannot make an inductance without a series resistance.

Similarly as you will see, you cannot make a capacitor without losses. Whatever the capacitor is, you know capacitor is simply 2 parallel plates and in between, there is an dielectric or an insulator. No insulator is perfect but. Insulator is an insulator only approximately. It has a leakage, that is it has a parallel conductance which accounts for losses and therefore we talk of loss and therefore we talk of energy storage capability of a capacitor related to its dissipation and a Q factor characterises this. In practice, it is very easy to make very high-quality capacitors but it is very difficult high-quality inductors.

And therefore, resistance almost always in such circuits, series resonance or parallel resonance is usually associated with the inductor. And we assume that the capacitor is almost perfect all right? Now because of the similarity between parallel resonance and series resonance, it appears that we should be able to find some kind of a universal expression which determines the resonance properties, either series or in parallel.

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$$Z(j\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z(j\omega_0) = Z_0 = R$$

$$\frac{Z_0}{Z} = \frac{Y}{Y_0} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

And if we take the question of the question of series resonance, you see, Z equal to $R + j\omega L - 1/\omega C$, this is Z of $j\omega$ at any frequency and we know that at resonance, Z of $j\omega_0$, if I call this as Z_0 , is simply equal to R and therefore if I could express the phenomenon of resonance in terms of dimensionless quantities, it would have been very welcome. That is, quantities which are independent of the actual values of resistance and actual value of capacitor and inductance all right? So what we do is, we divide Z_0 by Z .

Obviously, then this ratio will be dimensionless. And you can see that this will be the same as admittance to the admittance at resonance and you can express this as R divided by $R + j\omega L - 1/\omega C$ all right?

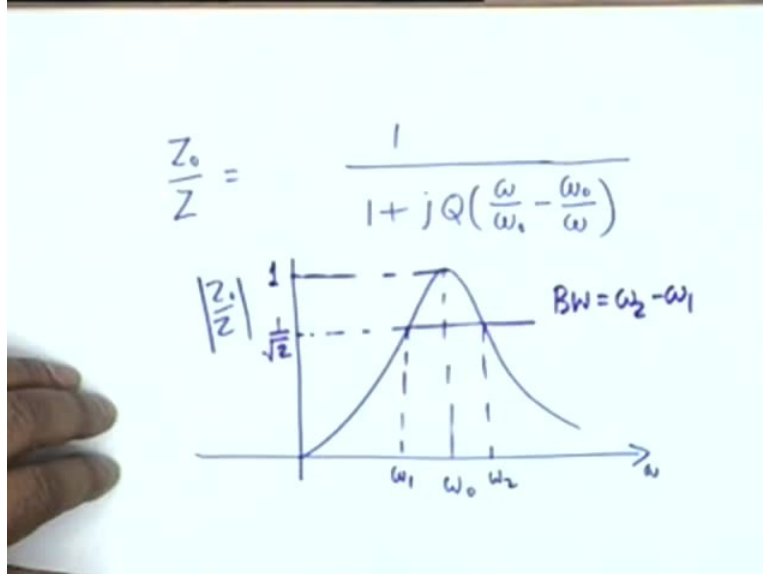
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$$\begin{aligned} Z_0 &= \frac{R}{R + j(\omega L - \frac{1}{\omega C})} \quad \frac{1}{\omega_0 C} = \omega_0 L \\ &= \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)} \\ \frac{\omega L}{R} &= \frac{\omega}{\omega_0} \frac{\omega_0 L}{R} = Q \frac{\omega}{\omega_0} \\ \frac{1}{\omega CR} &= \frac{\omega_0}{\omega} \cdot \frac{1}{\omega_0 CR} = Q \frac{\omega_0}{\omega} \end{aligned}$$

I shall repeat this expression here, Z_0 by Z equal to R divided by $R + j\omega L - 1$ by ωC which I can write as 1 now divide by R . 1 divide by $1 + j\omega L$ by $R - 1$ over ωCR all right? Now you see that ωL by R is also dimensionless. Is not that right? This is in ohms, this is in ohms. Similarly, 1 by ωCR 1 by ωC is in ohms, divided by R , that is also dimensionless. It has to be because you are adding one quantity to the other, they have to be of the same dimension.

However, if you look at ωL by R , you can write this as ω by ω not all right and multiply by ω not, ω not L by R . Agreed? I brought in, I have divided by ω_0 , multiplied by ω_0 . But we recognise that this quantity is simply the Q and therefore I can write this as Q times ω by ω_0 . In a similar manner, 1 by ωCR , I can write as 1 by well I can write as ω_0 divided by ω , then 1 by $\omega_0 CR$ and you notice that this is also equal to Q . 1 by $\omega_0 CR$ because 1 by $\omega_0 C$ is $\omega_0 L$. Is not that right? This is equal to $\omega_0 L$. So this is also Q , therefore this is Q times ω_0 by ω .

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And therefore we get a universal form of expression for the dimensionless ratio of impedances. This is $1 / (1 + jQ)$. From both of these terms, I can take Q out and then I write ω by ω / ω_0 and ω_0 by ω_0 / ω . All right? I can do that. Now obviously, this will have a maximum when ω / ω_0 equals to ω_0 / ω . That means, when ω equals to ω_0 , the maximum value is 1. This is independent of the actual values of resistance, inductance and capacitor.

And in that sense, it is a universal curve. The curve will go like this. When ω equals to 0, obviously the ratio would be 0 because ω_0 to 0, this makes it infinity, $1 / \text{infinity}$ is 0. When ω is infinite it is also 0 and in between, at ω equals to ω_0 , well it shows a maximum like this. This frequency is ω_0 , this is ω and what we are plotting is a dimensionless ratio of the 2 impedances all right? This now becomes unity all right? And it now makes sense, you see that this is the curve for a band pass filter. Is not it right? Bandpass filter. So it now makes sense to talk about the bandwidth or the frequencies at which the impedance falls down to $1 / \text{square root of } 2$.

All right? Let us call these frequencies as ω_1 and ω_2 . Then the difference between ω_2 and ω_1 is defined as the bandwidth of the circuit, width of the band at which the impedance magnitude is no less than 70.7 percent of its maximum value. All right? We can

actually find out ω_2 and ω_1 . When will this magnitude become equal to 1 by root 2? Obviously, this quantity will have to be equal to + 1 or - 1. In either case, it would be 1 by root 2.

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Handwritten notes on a whiteboard:

Tuned

$\omega_{1,2}$ satisfy

$$Q \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \pm 1$$

$$\omega_{2,1} = \omega_0 \sqrt{1 + \frac{1}{4Q^2}} \pm \frac{\omega_0}{2Q}$$

$BW = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$

Selectivity or circuit Q = $\frac{\omega_0}{\omega_2 - \omega_1}$

And therefore ω_1 and ω_2 satisfy the equation, $Q \omega$ by $\omega_0 - \omega_0$ by ω would be equal to either + 1 or - 1. In each case, it will give a quadratic equation. Is not it right? Whether it is + 1 or - 1, it does not matter. It will give a quadratic equation and therefore, there shall be how many solutions to this equation?

Student: 4.

Professor: 4 of them.

Student: 2 (())(36:27)

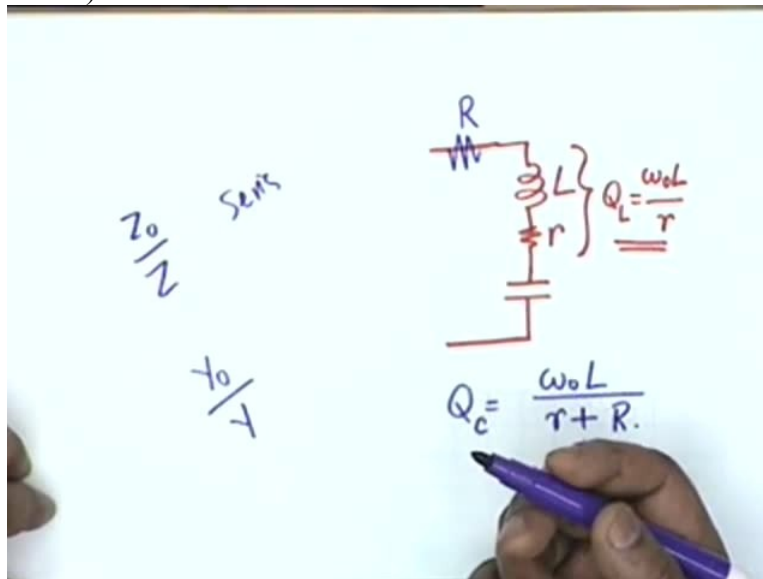
Professor: 2 will be negative and 2 will be positive.

We accept only the positive solutions and you can show that ω , you can show by actual calculation, I skip the algebra, you can show that $\omega_{2,1}$, the 1st we take ω to the higher frequency is equal to ω_0 not. ω_0 not we have already defined, the resonant frequency, 1 by square root LC. ω_0 not square root of $1 + 1$ by $4Q^2$. Q is ω_0 not L by R or 1 by ω_0 not CR . Q has already occurred, already occurred in this dimensionless so we find out in terms of Q , +/- ω_0 not divided by $2Q$. alright?

This is the expression and if you notice, if you notice the bandwidth, the bandwidth is equal to $\omega_2 - \omega_1$, therefore it is simply ω not by Q and therefore, the circuit Q can also be related to how good the circuit behaves as a bandpass filter. You can see the small Q is ω not divided by $\omega_2 - \omega_1$. All right? In other words, what does this mean? It means that higher the value of Q , the narrower will be the bandwidth. So if you want a sharp resonance, for example if you want to pick up one particular station from all possible stations which are radiating up on your antenna, when your tuned circuit or the resonant circuit, resonant circuit is also called tuned circuit, tuned, T U N E D.

Tuned circuit has to have a Q which is very large, a Q of the order of 10 in practice usually suffices. Now in this application, we give this Q a separate name. We call it selectivity or circuit Q as it is usually called. Selectivity or circuit Q and the reason that we have to distinguish between the 2 is as follows.

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You see, I can have an inductor let us say L and its associated resistance let us say small R all right? The Q of this, Q of this combination is Q_L let us say is $\omega_0 L$ divided by small R . Now I want to do this by using a capacitor and perhaps if I do that, then the selectivity of the circuit if I simply aim the circuit here, then the selectivity of the circuit is the same as Q value. All right? Suppose I want a greater bandwidth, instead of a smaller bandwidth, I want a larger

bandwidth, so what do I do? Connect a resistance. That is right. Connect an external resistance R.

This will reduce the selectivity of the circuit and reduction of selectivity means that I want a larger bandwidth all right? So the Q or selectivity of the circuit now Q_C will now be different from Q_L . Q_C will be less than Q_L . In fact $Q_C = \omega_0 L$ divided by $R + \text{small } R$, capital $R + \text{small } R$. Is the point clear? The selectivity of a circuit, selectivity of a resonant circuit is not necessarily the Q of the coil, is not necessarily the Q of the coil because there can be extra resistors in the circuit. All right? Yes?

Professor-student conversation begins

Student: (41:05) are these valid for any circuit?

Professor: any circuit, any circuit consisting of R, L and C, parallel as well as series. In the parallel case, you will have to take the admittance. That is, what did we take here? Z_0 by Z is is not that what we took in the in the case of series resonance. In the case of parallel resonance, we shall have to take Y_0 divided by Y .

Student: Sir but (41:36) not series or parallel.

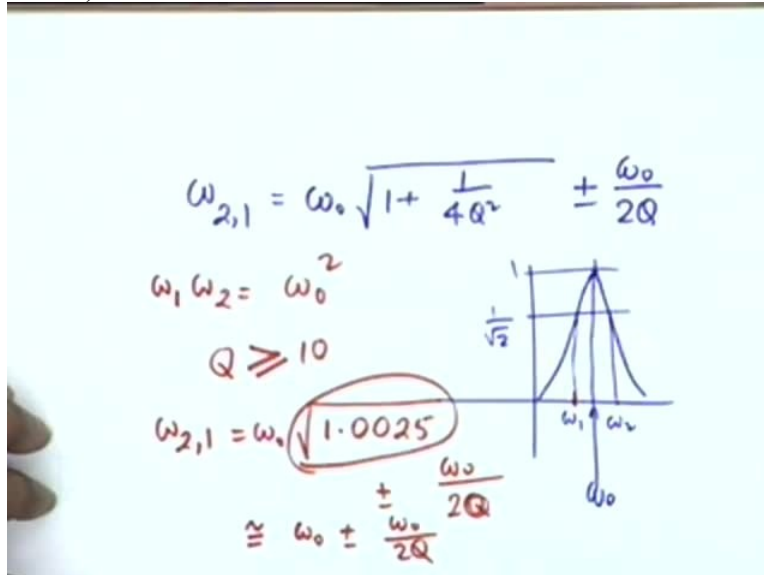
Professor: If they are not series or parallel, yes that is a good question. Then we shall have to start at initial, we will have to start from the roots. We cannot apply any of these formulas but nevertheless, it can always be shown that whatever the complication of the circuit, if there is resonance, then around resonance, it can always be described by a formula of the type 1 by $1 + jQ$, the one that is shown. It can always be described by a formula of this type. That is 1 by $1 + jQ$ ω by $\omega_0 - \omega_0$ by ω .

Student: Sir, what will be the basic condition for resonance?

Professor: Basic condition for resonance if the current and voltage are in phase. That is right. The phase difference between current and voltage as 0 , this is defined as the condition for resonance. All right? This is why the voltage maximum across an inductor is not necessarily at the resonance frequency. That is not a condition of resonance. Voltage maximum across a capacitor,

it occurs at a slightly different frequency, slightly lower frequency and that is not the condition of resonance. All right?

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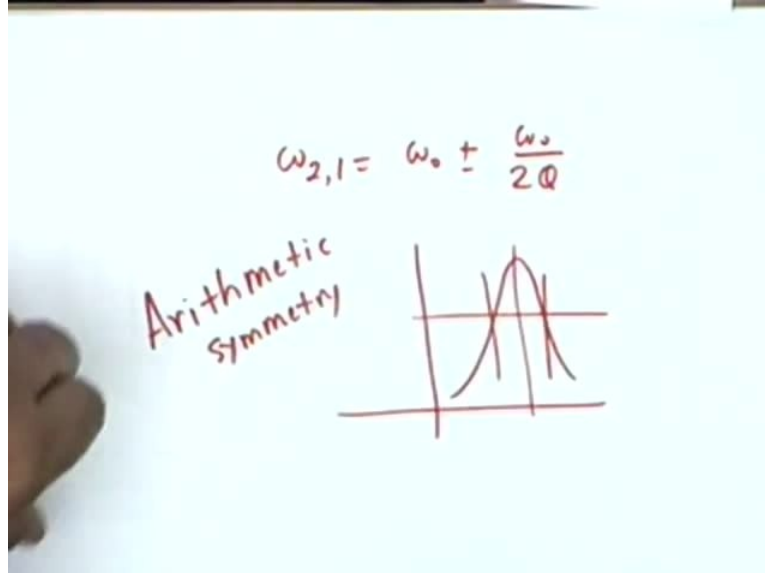
But before we go to complicated circuits, let me mention to you some of the interesting features of these 2 frequencies on either side of resonance at which the magnitude is $1/\sqrt{2}$ of the maximum value, $\omega_0 \pm \omega_0/2Q$. By definition, the maximum is 1 and at these frequencies, the value is $1/\sqrt{2}$. This frequency is ω_0 . All right? Now we notice a very interesting feature that the product of ω_1 and ω_2 , if you multiply this, it is of the form $A + B$ times $A - B$. So $A^2 - B^2$ and you can see that this is simply ω_0^2 .

What does this mean? It means that ω_1 and ω_2 have geometric symmetry with respect to ω_0 . Is not that right? If this is, if ω_2 is 10 times ω_0 , then ω_1 shall be one tenth of ω_0 . This is called geometric symmetry and therefore this curve shall also be geometrically symmetrical. It does not have arithmetic symmetry. Is that clear? This curve of bandpass filter does not have arithmetic symmetry. It is not that if you go equal distance on both sides, you will get equal amplitude. No, you have to go according to geometric symmetry.

If ω_2 is 10 times ω_0 , then ω_1 shall be $0.1 \omega_0$ all right? However, if Q is large, suppose Q is greater than equal to 10 let us say then $\omega_{2,1}$ would be equal to $\omega_0 \pm \omega_0/2Q$. square root of 1.0, this is 400, so 0.25, 0.0025, is that clear? $\pm \omega_0/20$. Agreed?

And you see that square root of 1.0025 which is approximately 1.00125. Did do you know these tricks? This is $1 + \Delta$ to the power half, approximately equal to $1 + \Delta$ by 2 and therefore, this is approximately equal to $\omega_0 \pm \omega_0$ by $2Q$ or 20 , whatever it was. ω_0 by $2Q$ all right. What does this mean now? It means that ω_2 and ω_1 are arithmetically symmetrical. Is not that right?

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ω_2 and ω_1 , they are at equal distances. In general, they have geometric symmetry but if Q is large, then they have approximately arithmetic symmetry. Another way of saying this is that in a high Q circuit, in a high Q circuit, if you confine your attention to the region of bandpass, then you can approximate it by arithmetic symmetry. Is that point clear? Arithmetic symmetry and geometric symmetry, they are facts of life and one has to recognise them, appreciate them and utilise them. We shall conclude this class with a couple of examples. One example okay and another a problem. One example and another a problem.

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Ex RLC ckt
 $\omega_0 = 200 \text{ Kr/s}$
 $\text{BW} = 5 \text{ Kr/s}$
 $L = 2.5 \text{ mH}, Q_L = 65$
 $\omega_0 = \frac{1}{\sqrt{LC}}$ $65 = \frac{\omega_0 L}{r}$
 $R = ?$ $40 = \frac{\omega_0 L}{R+r}$

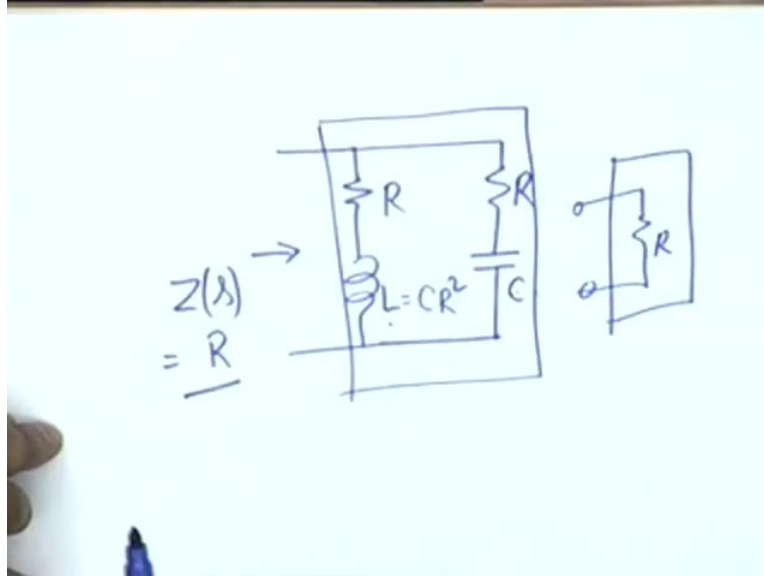
The example is, design an series RLC circuit for resonance at, series RLC circuit you have to design at, resonance is required at 200 kilo radians per second and a bandwidth of 5 kilo radians per second all right? Resonance you want at 200 kilo radians per second and the bandwidth, $\omega_2 - \omega_1$ has to be 5 kilo radians per second. The this an inductance of 2.5 millihenry and a Q, Q of the inductance of 65 is available all right? What is given to you with an inductance whose value is 2.5 million henry. It has certain resistance, the resistance is not expressed directly, it is expressed in terms of Q and Q is 65. Now do you see that in this circuit in this circuit, do you have to add a resistance?

Student: Yes.

Professor: Yes. Because the selectivity as you can see from here is 40 and therefore, it has to be reduced from 65. In other words, you have to add a resistance.

Now how do you design the circuit? Design means L is given. You have to find out what additional resistance is required and what capacitance is required. Well, capacitance is very easy. ω_0 is $1/\sqrt{LC}$, ω_0 is known, L is known, so you can find out C. How do you find the extra resistance? 65 is $\omega_0 L / R$ and 40 is $\omega_0 L / (R + r)$. And therefore from this, you will have to find out capital R. Okay? One problem and the other is a problem to be left with you.

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It is the problem that we had talked about in one of the classes when the lights went off. It is a complicated resonant circuit. It is a parallel resonant circuit in which L and C are in parallel but unfortunately both of them have a series resistance of capital R and in addition, the elements are so conspired that L becomes L is equal to CR^2 . You can see that the impedance if you calculate in terms of S , it is simply R . We had discussed this?

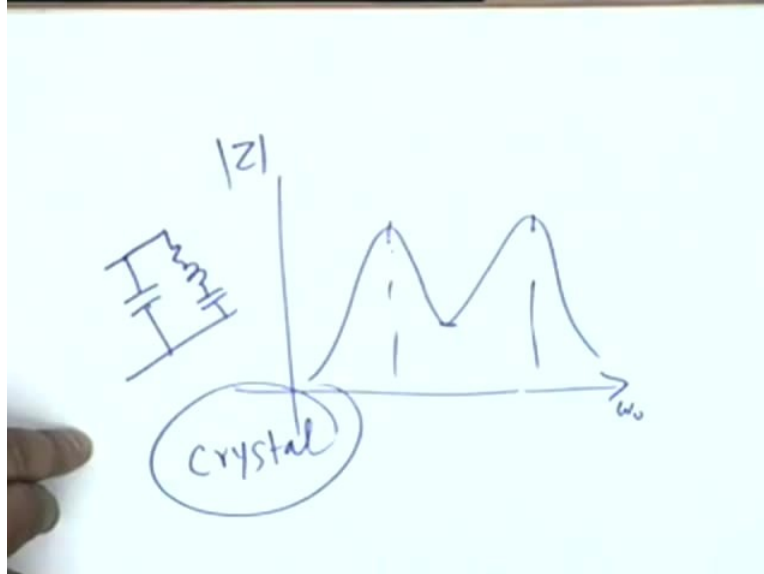
Student: yes.

Professor: Whether it is a transient, whether it is an exponential waveform, the battery, an AC or whatever it is, it is equal to R . And therefore, there is no way to distinguish between this and a pure resistance R if they are enclosed within black boxes, black within inverted commas of electrical engineering all right in electrical engineering terminology. And this is one of the manifestations of resonance. Is this a resonance? Is this a case of resonance? Yes because there is an there is a reactive element, there is another reactive element and the conspiracy is such

Student: Pure resistance.

Professor: pure resist but it is not at a single frequency. It is at all frequencies and therefore this is all frequency resonance. This is not a single frequency resonance. Well, which means that in theory, we can expect that there will be circuits which resonated more than one frequency.

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For example, we can have circuits like this. All right? Resonating at 2 different frequencies. This is also resonance. A minimum in the characteristic is also resonance. For example, it could be the plot of the impedance vs frequency. If it is the plot of the impedance vs frequency, then what will you call these 2 frequencies? The impedance is a maximum, these 2 must be corresponding to parallel or series? That is the question I am asking.

Student: Parallel.

Professor: Parallel. In parallel resonance, the impedance maximises, admittance minimises. And this can be considered as D2, a series resonance. All right? And a simple circuit that one can think about is the following. A capacitor shunting a series resonance circuit, RLC. There will be a series resonance in this and there will be a parallel resonance also because the capacitor is shunting a reactive element. This is the equivalent circuit of, have you heard of a crystal, crystal oscillator? A crystal is the standard for frequency, that you see at MPL or any other standard. So crystal oscillator crystal equivalent circuit is this which shows series resonance as well as parallel resonance. All right? Let us leave it at that. On Sunday, we will talk about general network analysis.

Student: Sunday?

Professor: Monday. I beg your pardon.