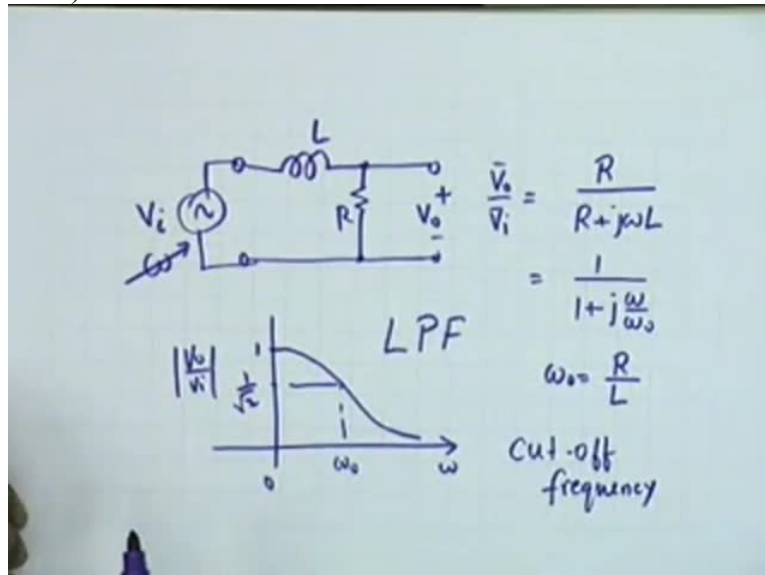


**Introduction To Electronic Circuits**  
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**Department of Electrical Engineering**  
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**Module No 01**  
**Lecture 21: Filter Circuits and Resonance**

But no acceleration. 21<sup>st</sup> lecture on filter circuits and resonance. I had introduced the concept of filter circuits in the previous picture.

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And we saw that this circuit with an inductance in series and a resistance in shunt if you excite with an AC  $V_i$  whose frequency is variable and you take the output across the resistance then you can work out that  $V_o$  by  $V_i$  the phasors is simply equal to  $R$  divided by  $R + j\omega L$ , this is the division, potential division and then if you take the magnitude, well it can be put in the form  $1$  by  $1 + j\omega$  by  $\omega_0$  and where  $\omega_0$  is equal to yes? What is the value of  $\omega_0$ ?

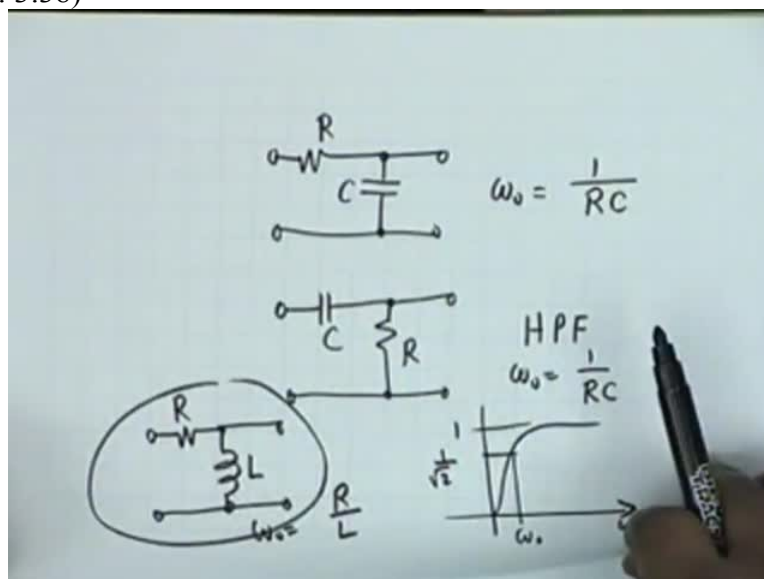
Student:  $R$  by  $L$ .

Professor:  $R$  by  $L$  or  $L$  by  $R$ ?

Student:  $R$  by  $L$ .

Professor: It is R by L. Right? And the magnitude plot, if you plot the magnitude  $V_0$  by  $V_I$ , our magnitude plot starts at 1 at  $\Omega$  equal to 0 and it goes to 0 at infinity with half power point at  $\Omega$  equal to  $\Omega_0$  where the magnitude drops down to  $1/\sqrt{2}$  and therefore the power in the resistor which is  $I^2 R$  or  $V^2/R$ , the power gets reduced by a factor of half. So  $\Omega_0$  is also called the half power point. It is also named as the cut-off frequency, cut-off frequency. I also mentioned that this, this is a filter because it discriminates against some frequencies and allows others to pass. And it is a low pass filter because it favours low frequencies. So it is called an LPF, lowpass filter.

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Another possible candidate for lowpass filtering is one in which there is a resistance in series and a capacitance in shunt. This is also a low pass filter of the same type of frequency response except that  $\Omega_0$  in this case is given by  $1/RC$ . Now by a similar reasoning, one can say that if the capacitance and resistance are interchanged, that is we have  $C$  and then an  $R$ , then this circuit shall act as a high pass filter with a half power or cut-off frequency given by the same quantity  $\Omega_0$  equal to  $1/RC$ . Its response shall be of this form where at  $\Omega_0$ , the response is  $1/\sqrt{2}$  times its maximum value.

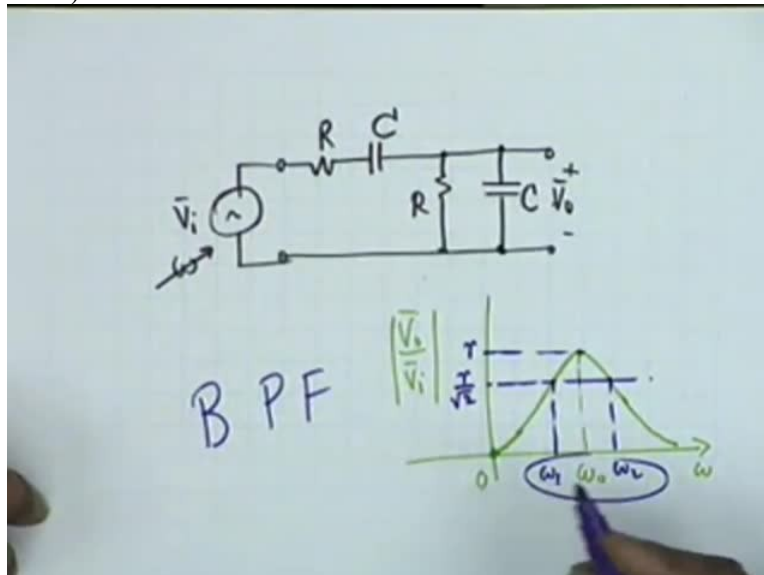
The maximum response occurs at  $\Omega$  equal to infinity, alright? So it favours high-frequency is and therefore this is a high pass filter. In a similar manner, one can argue that if I have a resistance in series and an inductance in shunt, this also acts as high pass filter with the cut-off

frequency which is given by  $R$  by  $L$ . Now this is one should not look at this only as mathematical confidences or mathematical elegances. There is a physical meaning. There is a physical meaning. Given a circuit, you can always argue what kind of filtering will this circuit perform?

Please remember that filtering can be done only when there is at least one energy storage element because it is only energy storage elements which are sensitive to frequency, resistance is not sensitive to frequency. So you require at least one inductance in this circuit. There can be more. You require at least one inductance or at least one capacitance all right. For example, if you wish to look at this, this circuit for example, you can see that at DC, inductance acts as a short. Therefore whatever voltage apply you here, nothing shall appear at the output and therefore, it discriminates against DC.

As the frequency rises, the impedance of the inductor also increases. The output voltage is a potential division of the input voltage between  $R$  and  $L$  and therefore as the impedance of  $L$  increases, the voltage at the output also increases and therefore it should act as a high pass filter. In a similar manner, any given circuit can be, you can physically reason out whether it shall be a lowpass or a high pass or an any other kind of filtering.

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For example, let us consider an example of some other kind of filtering, a circuit like this. Let us say for simplicity, let us say the resistances are equal and so are the capacitances. This is my

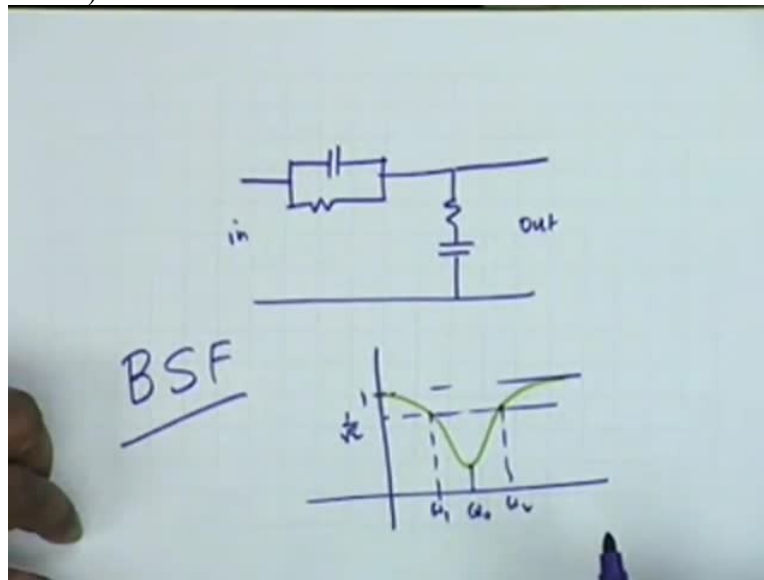
input and this is my output  $V_0$  okay. This is a phasor, this is a phasor and the frequency of this source is variable. We wish to find out what kind of filtering will this achieve. The argument is very simple. It goes like this, physical argument. Then we can do it mathematically.

The physical argument is that, at low frequencies, at DC for example, at DC, this is open, the capacitor is open. So at DC, this impedance  $R$  and  $C$  in parallel behaves simply as an  $R$ . Whereas the series combination at DC behaves as open. And therefore at DC, nothing shall appear across the output. Therefore the DC response if you plot  $V_0$  by  $V_I$ , this ratio vs frequency, then at DC, it starts from 0. All right? Now let us look at the other extreme, in finite frequency. At infinite frequency, this capacitor acts as a short and therefore the series arm consists of a pure resistance  $R$ .

But this capacitance also acts as short and therefore the shunt arm acts as a short, nothing shall appear here and therefore the ratio shall again be 0 and therefore it must come like this. Now in between obviously there must be a maximum. The curve rises from here, the curve goes down at infinity and therefore in between there must be somewhere a maximum and therefore it acts as a bandpass filter. That is, if this frequency is  $\Omega_0$ , then and this response is let us say  $R$ , then you find out the find out where the response is  $R$  by root 2. This is not necessarily 1. At some frequency, there will be a maximum.

Let us call this maximum value as small  $R$ . Find out at what frequencies, there shall now be two frequencies at which the response shall be 70.7 percent of the maximum response. That is we have find out these 2 frequencies,  $\Omega_1$  and  $\Omega_2$ . This band of frequencies centred around  $\Omega_0$ , between  $\Omega_1$  and  $\Omega_2$ , we say the filter passes these frequencies. Therefore it passes a band of frequencies and therefore it is called a bandpass filter, BPF. Bandpass filter. And this, the qualitative nature of this filter, we derived simply by physical reasoning and it is possible do so in most of the cases. All right? Now obviously, it discriminates against low frequencies, it also discriminates against high frequencies and it only passes a small band of frequencies. In a similar manner, if you had interchanged these series and short arms, if you had interchanged then you could argue out that it would have been a band stop filter, not bandpass.

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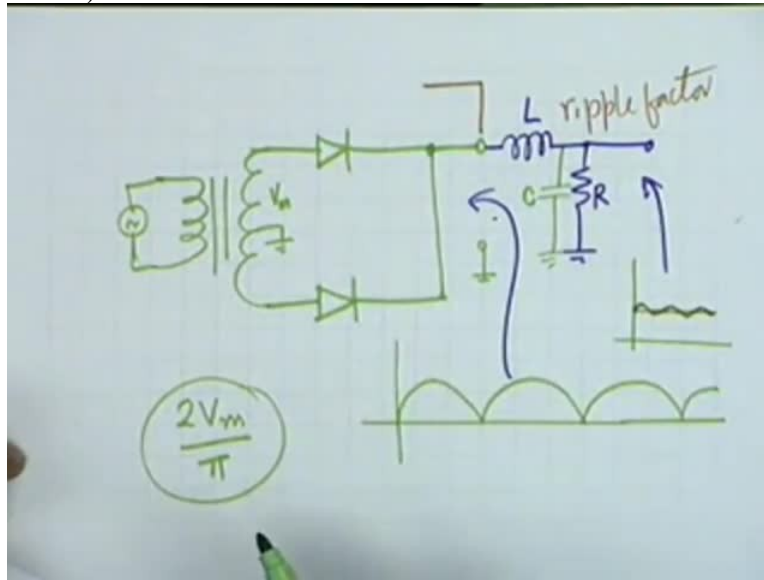


If you had interchanged the series and the shunt arms, that is if our series arm was like this and shunt arm was like this and this is input and this is output, alright, then you see at DC, this is a resistance. At DC, I am sorry. At DC, this will be open and therefore this is open and therefore whatever you connect here, voltage shall appear across the output. And therefore the DC response is 1 the infinite frequency, at infinite frequencies, this acts as a short. And therefore, anything you connect here, shall appear at the output. So at infinite frequencies also, the response shall be one.

And in between therefore, there must be a minimum, something like this. This minimum may be 0, may not be 0 but you find out the frequencies at which the response is  $1/\sqrt{2}$  and you find out these frequencies  $\Omega_1$  and  $\Omega_2$ , denote the minimum frequency by  $\Omega_0$ . It is as if the filter rejects or discriminates against this band of frequencies centred around  $\Omega_0$  and therefore this band is said to be stopped or it is not, it is non-pass, it does not pass which means it stops and therefore this is called a band stop filter, BSF.

Obviously, it passes low frequencies, it passes high frequencies, it only rejects a band of frequencies centred around  $\Omega_0$  and therefore it is called the band stop filter. All right? Now this is fine. Let us see one of the practical applications of a filter, one of the practical applications.

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You recall that our rectifier circuit, a full wave rectifier circuit if you recall, it had a centre tapped transformer and 2 diodes, 2 diodes like this. The diodes are connected together and then you have, you have, you can connect the load here. From this point to ground, let us say from this point to ground, the voltage that you get is full wave rectified voltage. In other words, its waveform should be like this. Each half of the sine wave that you feed here, each half of the sine wave that you feed here gets stepped up or stepped down by the transformer. It is centre tapped to there are 2 diodes, one to take care of the positive half cycle, the other to take care of the negative half cycle and this is the full wave rectified waveform.

Now obviously, there is an average value, there is a DC value which is twice if this is  $V_m$ , if the maximum value here is  $V_m$ , then what is the average value of this? Twice  $V_m$  divided by  $\pi$ , we had done this earlier. Now we want only this DC. We do not want this fluctuating waveform and that we must get rid of this fluctuating waveform which is an AC. The whole waveform can be thought of as a DC of twice  $V_m$  by  $\pi$  upon which is superimposed an AC all right? So we want to get rid of the AC.

Therefore we can use a filtering. Obviously, the filter that we should use is a lowpass filter and a simple filter could be like this. A lowpass filter has an inductance in series and a resistance in shunt all right? The voltage across this resistance, if inductance and resistance are properly chosen, then the filter should attenuate or should reduce the AC component. In other words, if

this is the input, then the output waveform should be a DC upon which is superimposed maybe a small AC. Maybe what you get is something like this. The AC component is greatly reduced.

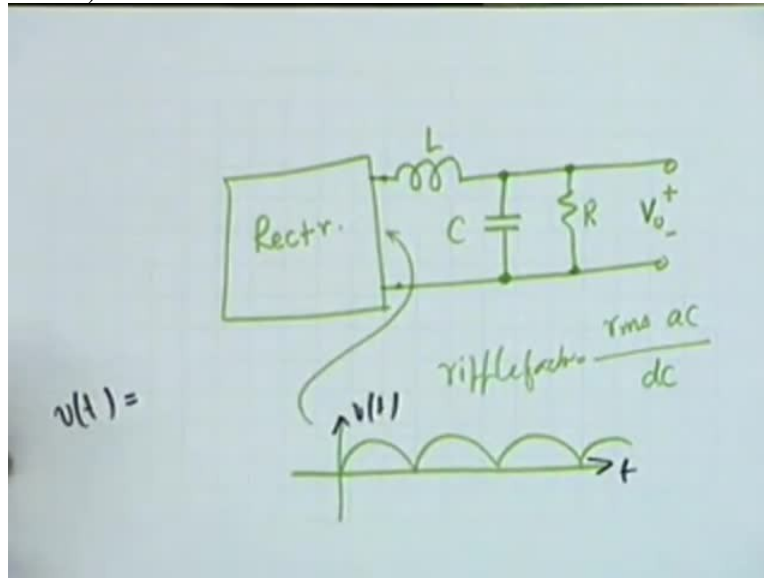
And the order of reduction, the order of reduction is denoted by a term called ripple factor. Ripple factor is not quite, it is the ripple factor reduction capability of a filter that is the ripple factor at this point shall be much less than the ripple factor at this point because of filtering by the inductance resistance circuit. Now we would like to make a little more in depth study of this filtering effect but let me mention to you 1<sup>st</sup> that if you want to do this kind of filter, the inductance that is usually needed, if you want a simple filter like an LR filter, did I tell you that this is an L type filter because it resembles the letter L?

Student: You did.

Professor: I did, okay.

If you want to use a simple L type filter like this, now there is a problem. The problem is that the inductance that is required is usually a very large value because power frequency is small, 50 hertz only.  $\omega$  is  $100\pi$  all right and therefore in that sense that is required is usually large and becomes expensive. So one prefers instead of an LR, one prefers to use an LC. That is, what you do is, you use two reactive elements. One, a capacitor and the other, an inductance. Then the cost of the filter comes down. Although we are increasing one element, the value of L that is needed is much smaller. An inductance accounts for most of the cost for the filter. We shall look at this circuit a little more cautiously all right.

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Let us consider the LC filter. L, C and R. This R is the load. This resistance R is the load. It could be a transistor circuit for example for which you have the power supply. So what we have is the rectifier, transformer rectifier combination. We call this a rectifier and then at this point at this point, the waveform that I have a this is L, this is C, that is R okay. And what I want this, this is my V<sub>0</sub>. This is the output voltage. What I want is that V<sub>0</sub> should be purely DC. Unfortunately, the filter cannot reduce it to 0, cannot reduce the ripple for the AC component to 0 but it reduces substantially and the nature of performance is, ripple factor at the output and ripple factor at the input. The ripple factor at the output should be much less than the ripple factor at the input. By definition, ripple factor is what? The root mean square value of the AC component divided by the DC, the average value. Ripple factor is the ratio of RMS value, RMS AC components divided by the DC or the average value all right? So the ripple factor is a measure of the effectiveness of the filter. We measure the ripple factor at the output and ripple factor at the input, these 2 should differ greatly. The output ripple factor should be much less compared to the input ripple factor, then the filter has become, has been effective. Now to analyse this circuit, we require an analytical expression for this input waveform which is of this form. Okay. And it can be shown by Fourier analysis. Are you acquainted in Fourier analysis?

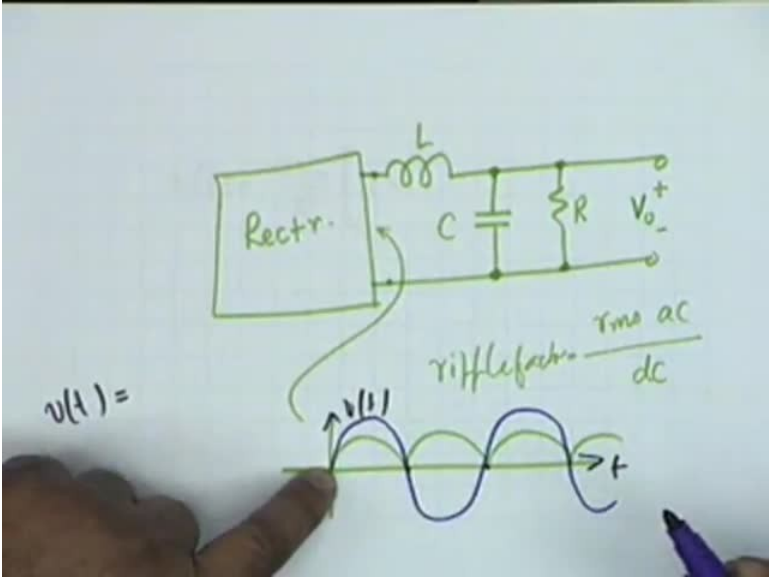
Student: No.

Professor: Not yet. Anyway.



Any periodic waveform, any periodic waveform whether sinusoidal or non-sinusoidal can always be decomposed into sinusoidal components. Can always be decomposed into sinusoidal components and you will learn this later, you will learn it later that such a waveform can be decomposed into the following. If I call this as  $V_T$  vs  $T$ , then  $V_T$  can be written as, let me use some other page.

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$$v(t) = \frac{2V_m}{\pi} \left[ 1 - \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t - \dots \right]$$

$$v(t) \cong \frac{2V_m}{\pi} - \frac{4V_m}{3\pi} \cos 2\omega t$$

This full rectified waveform can be written as  $\frac{2V_M}{\pi}$ , the DC value which we have already derived. In addition, there are alternating components. Now if you look at this waveform, the input waveform was something like this, input waveform was something like this and the output waveform is like this. So the input waveform, the period was this much whereas the period of the output is this much. Obviously, the output waveform's frequency is double that of the frequency of the input. And therefore if the input waveform is a frequency  $\Omega$ , the output waveform has a frequency, twice  $\Omega$  and what we have is, by Fourier analysis one can show that this is  $\frac{1}{3} \cos \Omega T + \frac{2}{15} \cos 3\Omega T + \dots$ , this is the frequency.

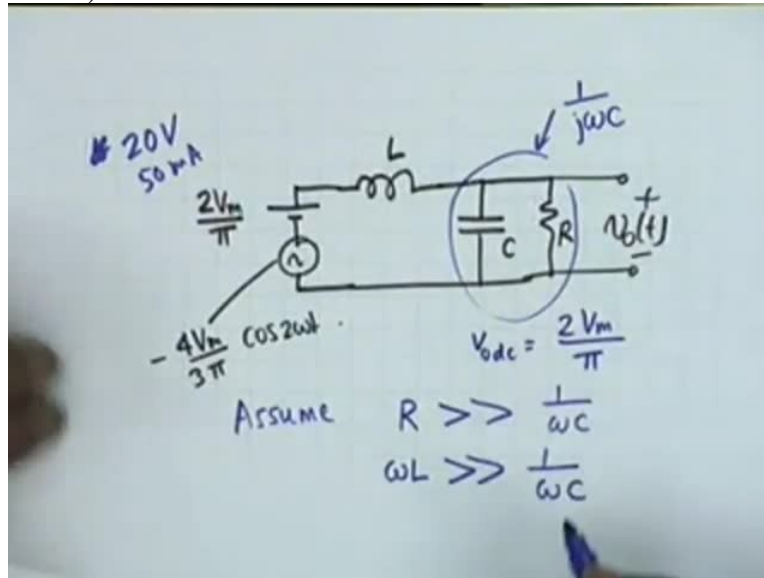
This is the 2<sup>nd</sup> harmonic because the frequency of this is harmonically related to the input frequency to the transformer. Input frequency is  $\Omega$  and this is  $2\Omega$ . Any frequency which is an integer positive multiple of another frequency is called a harmonic.  $5\Omega$  is the 5<sup>th</sup> harmonic,  $10\Omega$  is the 10<sup>th</sup> harmonic,  $2\Omega$  is the 2<sup>nd</sup> harmonic and what is the frequency of the 1<sup>st</sup> harmonic?

Student: Fundamental.

Professor: 1<sup>st</sup> harmonic and the fundamental are the same, that is  $\Omega$ . All right. Then in this waveform, in the full wave rectified waveform, it is only even frequencies which are there because the fundamental of the rectified waveform is  $2\Omega$  and that is why,  $4\Omega$ ,  $6\Omega$ ,  $8\Omega$  and so on. You cannot have a  $3\Omega$  component all right? So you have  $\frac{2}{15} \cos \Omega T$  and so on. The number of frequencies in theory is infinitely large. However, the amplitude these components, the 2<sup>nd</sup> harmonic is two third, the 4<sup>th</sup> harmonic is  $\frac{2}{15}$  and it diminishes as the order of the harmonic increases. All right?

And therefore in practice if we can take care of the 2<sup>nd</sup> harmonic, if we can reduce this substantially, obviously all other harmonics shall become negligible. If we can make the 2<sup>nd</sup> harmonic negligible, the 4<sup>th</sup> harmonic shall also be further negligible all right? So it suffices in practice to approximate this by only 2 terms, the 1<sup>st</sup> 2 terms, let us say the DC component,  $\frac{2V_M}{\pi}$  and the AC component in this which is the 2<sup>nd</sup> harmonic, that will be  $\frac{4V_M}{3\pi} \cos 2\Omega T$ . Is that correct? Have I done it correctly?  $\frac{4V_M}{3\pi}$ . So let us see how we can reduce the 2<sup>nd</sup> harmonic by means of an LC filter, LC filter of the architecture L, inverted L okay.

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So what we wish to investigate is the following. We have an inductor, a capacitor and a resistor  $R$  and our input consists of, consists of a DC whose value is twice  $V_M$  divided by  $\pi$  and an AC and an AC which is  $-4 V_M$  divided by  $3 \pi$  cosine  $2 \Omega T$ . This is my input, I have shown this as two sources. One is a battery, the DC of value  $2 V_M$  by  $\pi$  in series added to an AC component whose value is  $-4 V_M$  by  $3 \pi$  cosine  $2 \Omega T$ . We wish to find out the output voltage okay. If I call this as  $V_0$  of  $T$  let us say, the output voltage  $V_0$ .

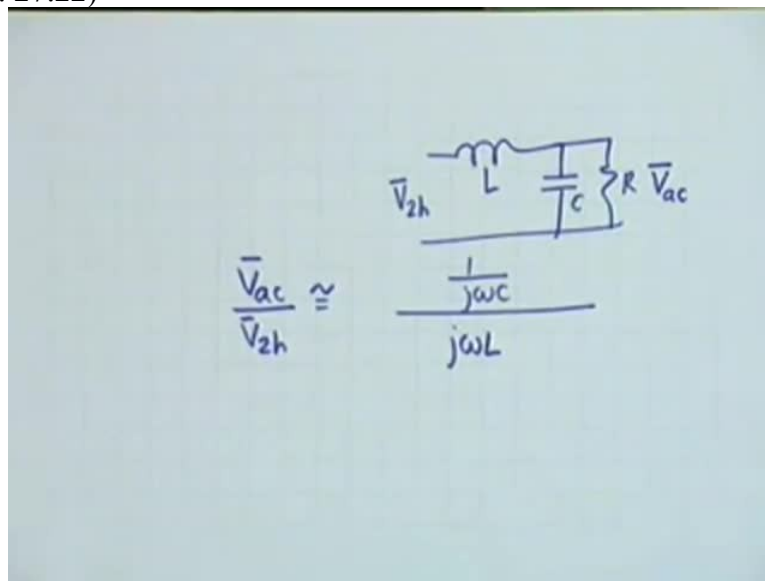
Obviously, obviously for DC because this is a linear circuit, we can find out the output by superposition all right? Now by superposition means, we consider one source at a time. So if we kill this source, then for DC, the conductor is short and the capacitor is open and therefore the DC component at the output, DC component at the output shall be  $V_0$  DC shall be the same,  $2 V_M$  by  $\pi$ . It is the AC component that we are concerned with. Now for AC analysis, for AC analysis, we make several assumptions which are true in practice, several simplifying assumptions.

One is that capital  $R$ , the load is much larger assumed that capital  $R$  is much larger than  $1$  by  $\Omega C$ . What does it mean? It means that the effective impedance of this country is determined by  $C$  only and therefore this impedance is  $1$  by  $J \Omega C$  all right? Also assume, this is true in practice. You have to choose your  $C$  such that its impedance is at least 10 times

less than capital R. Capital R if it is let us say 400 ohm, 400 ohms how do you determine this? Suppose we want 20 volt output and the output should be 15 milliampere, then you know what your load is. All right?

You want 20 volt across a load and if 15 milliampere passes, obviously the resistance must be 400 ohms. Then your capacitor must be for those that its impedance  $1/\omega C$  is at the most 40 ohms, not more than that. 40 ohms at 50 hertz all right? Then, also assume that  $\omega L$ , there is impedance of the inductor at the given frequency, at the power frequency, is much greater than  $1/\omega C$  all right? This is also to be satisfied in practice. For good filtering, this is to be satisfied. That is, the impedance of the inductor should dominate over the impedance of the capacitor. All right?

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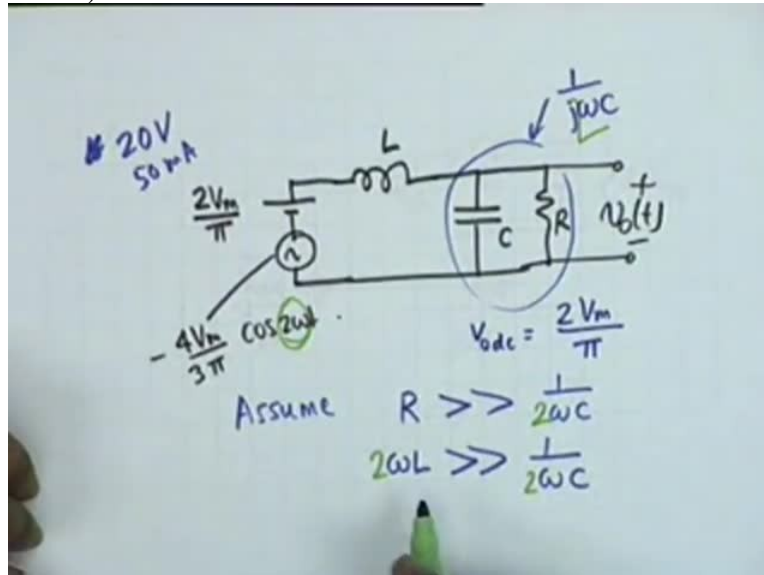
Under these conditions, you see, the thephaser, the volt is phaser for the output, the AC phaser for the output, that shall be simply equal to the I draw the circuit again, L, then C, parallel R. Suppose this phaser, 2<sup>nd</sup> harmonic phaser,  $V_{2H}$ ,  $V_{2H}$  2<sup>nd</sup> harmonic phaser and suppose this phaser is let us say  $V_{AC}$  at the output, then obviously, the ratio of the 2,  $V_{AC}$ , over  $V_{2H}$  phaser, that shall be equal to the impedance at this which is approximately  $1/\omega C$ . Why? Because capital R was much larger. Divided by the total impedance. Total impedance is  $j\omega L + 1/j\omega C$  but we have assumed that  $\omega L$  is much larger than the impedance of the capacitor.

So this must be the story where obviously we had made a mistake. Can you tell me what the mistake is? Pardon me. What is the mistake?

Student: Negative.

Professor: No, that is not it.

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Professor: We made the mistake right here.

Student: Sir, is it (( ))(28:41))

Professor: No,  $R$  is much greater than  $1$  by  $\Omega C$ . That is not the mistake. Mistake is the frequency is not  $\Omega$ , frequency is  $2\Omega$ . And therefore everywhere we must substitute  $2$ .

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$$\frac{\bar{V}_{ac}}{\bar{V}_{2h}} \approx \frac{\frac{1}{j\omega C}}{j\omega L}$$

$$= -\frac{1}{4\omega^2 LC}$$

$$\bar{V}_{ac} = -\frac{1}{4\omega^2 LC} \cdot \frac{4V_m}{3\pi} \cdot \frac{1}{\sqrt{2}} = -\frac{1}{6}$$

The frequency of the AC input is 2 Omega and therefore this is 2 Omega which means that our relation should be  $j 2 \text{ Omega } C$  divided by  $j 2 \text{ Omega } L$ . And obviously, you see this is  $1$  by  $- 4 \text{ Omega squared } LC$ . This is what it is and therefore,  $V_{AC}$  phaser would be equal to  $- 1$  by  $4 \text{ Omega squared } LC$  times  $V_{2H}$  phaser which is given by  $4 V_M$  divided by  $3 \pi$ , this is the maximum value. This is to be divided by  $\sqrt{2}$ ,  $1$  by  $\sqrt{2}$  and you can show very easily that this is  $1$  over  $6$ , no, I am sorry.

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$$\bar{V}_{ac} = -\frac{V_m}{3\sqrt{2}\pi\omega^2 LC}$$

$$Y_{in} = \frac{\frac{4V_m}{3\sqrt{2}}}{\frac{2V_m}{\pi}} = \frac{2}{3\sqrt{2}}$$

$$Y_{out} = \frac{|\bar{V}_{ac}|}{V_{dc}} = \frac{V_m}{3\sqrt{2}\omega^2 LC} \cdot \frac{\pi}{2V_m} = \frac{1}{6\sqrt{2}\omega^2 LC}$$

VAC is equal to VM divided by 3 root 2 pi Omega squared LC. It is a phaser. All right? We have not, no I think we must use a negative sign - this, all right?

Student: Sir, VM is a phaser?

Professor: VM is not a phaser. VM is the maximum value. Is that okay?

Student: Yes.

Professor: This is VAC.

Therefore the ripple factor R at the output, ripple factor at the output which is VAC magnitude divided by VDC would be equal to VM divided by 3 root 2 pi Omega squared LC and this will be twice VM and here pi. VDC is twice VM over pi. And now we can sell out things, VM, VM, pi and pi. And therefore this becomes 1 over 6 root 2 Omega square LC. This is the ripple factor at the output. What is the ripple factor at the input? What is the ripple factor at the input? If you recall, 4 divided by 3 pi VM root 2, this is the VAC right?

4 VM by 3 pi root 2 divided by twice VM divided by pi. What is this equal to? VM, VM, 2M to 2, pi pi cancels. So 2 by 3 root 2 all right? 2 by 3 root 2. No compared to this, let us find out the improvement factor all right?

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$$\frac{r_{out}}{r_{in}} = \frac{1}{6\sqrt{2}\omega LC} = \frac{1}{4\omega LC}$$

$L = 20H, \quad C = 40\mu F$   
 $R = 400\Omega$

$$\frac{2}{3\sqrt{2}} = 0.47$$

$$\frac{0.47}{3} = 0.15\%$$

$$r_{out} \approx 0.0015 = 0.15\%$$

R out divided by R in, the ratio of the 2 ripple factors. Pardon me. Yes? Say it loud.

Student: Sir.

Professor: Yes.

Student: Sir please solve last step.

Professor: last step?

Student: Yes.

Professor: The input ripple factor.

Input ripple factor is  $\frac{V_{2H}}{V_{DC}}$  phaser which is  $\frac{4V_M}{3\pi \sqrt{2}}$  divided by the DC value,  $\frac{2V_M}{\pi}$  which is  $\frac{2}{3\sqrt{2}}$ . What I want to do now is to compare the output ripple factor to the input ripple factor to find out

Student:  $\frac{1}{4\Omega^2 LC}$

Professor: 1 by?

Student:  $4\Omega^2 LC$ .

Professor:  $4\Omega^2 LC$ . That is quite correct. As an example, as an example, suppose our L, suppose our L was 20 henry and C, 40 microfarad, typical values, capital R as I said, 20 volts at 50 milliamperes which means 400 ohms, then one can show that the ripple factor at the output is approximately equal to 0.0015 which means 0.15 percent.

Student: (0)(33:49)

Professor: Of?

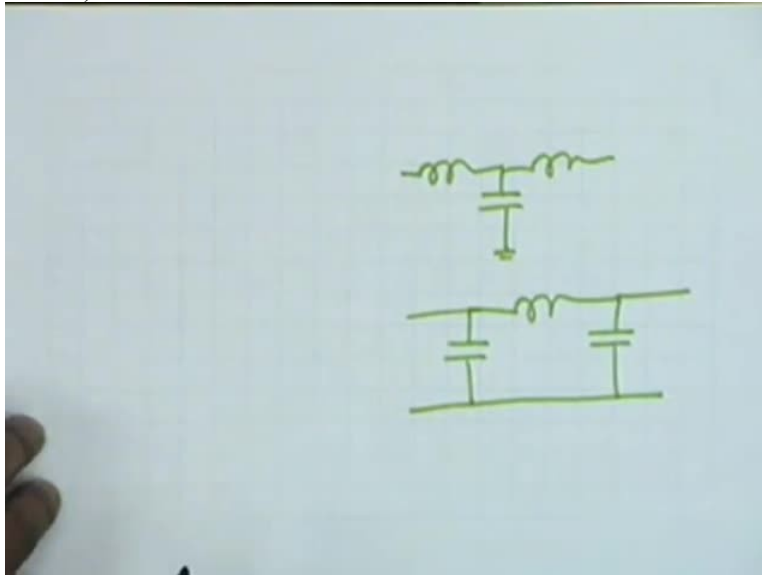
Student: R out by R in. This is R out by R in.

Professor: Or is it R out?



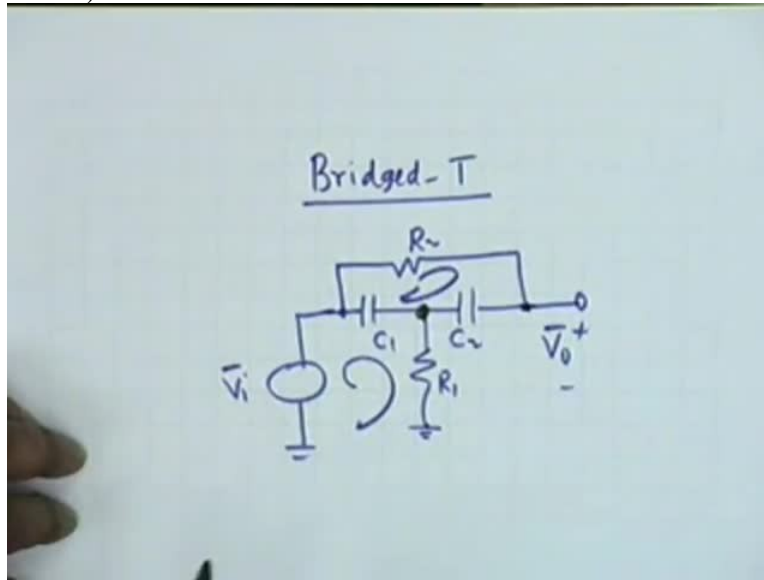
It is  $R_{out}$ , not  $R_{out}$  by  $R_{in}$ . Both are dimensionless. The input ripple factor is simply  $2$  by  $3$  root  $2$  which is how much? That is root  $2$  by  $3$  equals to point how much?  $0.47$  and which means  $47$  percent is that right? The input ripple factor is  $0.47$ , the output is  $0.0015$  and you can see your order of reduction that has been obtained. All right? And therefore whenever you use a rectifier, you always use a filter to reduce the ripple to a quantity which is negligible, to reduce the AC component and thereby you get pure DC. So this is one example of filter. There are many other kinds of filter.

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I have told you one could instead of a L filter, one could use a T, for example, 2 inductors and one capacitor, this does look like a T or one could use a pi, this looks like a pi. The left-hand side is the rectifier, the right-hand side is the load and all of them act as lowpass filters to reduce the harmonic components present in the output.

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There are many other kinds of filters and one of the examples that one comes across in instrumentation and measurement very often is the so-called bridge T filter, bridged T. This means, at T, this is that has to be architecture of the circuit. That is, how does it look like? Well, it is a T. There are 2 capacitors,  $C_1$  and  $C_2$ , then that is a resistance here, let us say  $R_1$ . This looks like a T and then you have a bridge across the T. It is a flyover, a resistance  $R_2$  all right? It is a flyover. So your input is here,  $V_i$  and your output is here,  $V_o$ . Now if you wish to analyse this circuit, you have to assume phasors here,  $V_i$ ,  $V_o$  and then work in terms of the impedances, you know KVL, KCL?

Both are valid for AC circuits and therefore, we replace  $C_1$  by  $1/j\omega C_1$ ,  $C_2$  by  $1/j\omega C_2$ ,  $R_1$  and  $R_2$ . Or you could work in terms of  $S$  instead of  $j\omega$ . Finally, we replace  $S$  by  $j\omega$ . It will give the same result. All right? It is more convenient to work in terms of  $S$  because you do not have to handle the quantity  $j$ . Wherever there is an application of  $j$  by  $j$ , a negative sign is up. If you miss that negative sign, then you are done with, okay? So it is better to work in terms of  $S$ . Now how would you analyse the circuit? What is the best method? I can do it by loop. How many loops loop equations shall I have to write in this particular case?

Student:3.

Student: 2.

Professor: 3 over 2?

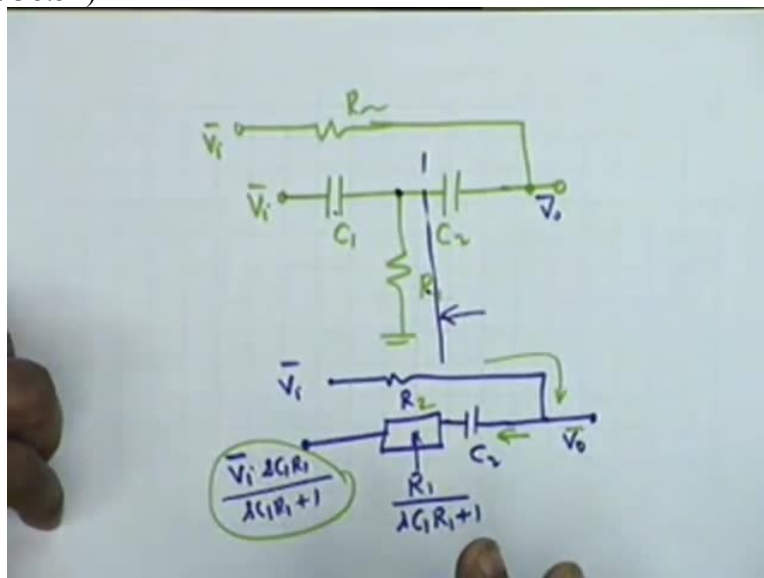
Student: 3.

Professor: if this is left open, then only 2. If there is a load here to ground, then 3, agreed. But now, there are only 2 loops, one is this and the other is this. This is not a loop. Is not it right? If I know both these currents, then of course I shall know the voltage, this voltage will be drop across  $C_2$  + drop across  $R_1$ . This current is  $I_2$  and this current is  $I_1$ , ya this current is  $I_1$ , ya, okay?

Student: Node analysis.

Professor: Node analysis, all right? For node analysis, how many nodes? How many equations? 2 nodes. One is  $V_0$  and the other is this intermediate node. So there is nothing much to choose between. It is almost the same kind of thing but let me quickly show you another kind of analysis which sometimes is more advantages where the chance of making a mistake is less all right? Let us let us look at this quickly. This is somewhat innovative and interesting.

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You have  $C_1$ ,  $C_2$ , a resistance  $R_1$ . You see, if you look at the circuit,  $V_i$ , this input, the potential at this point is  $V_i$  whether you consider this branch or this branch, it does not matter. Is not it? So if I separate this out, if I separate the 2 wires and connect 2 sources,  $V_i$  each, shall it make a difference?

Student: No.

Professor: it should not.

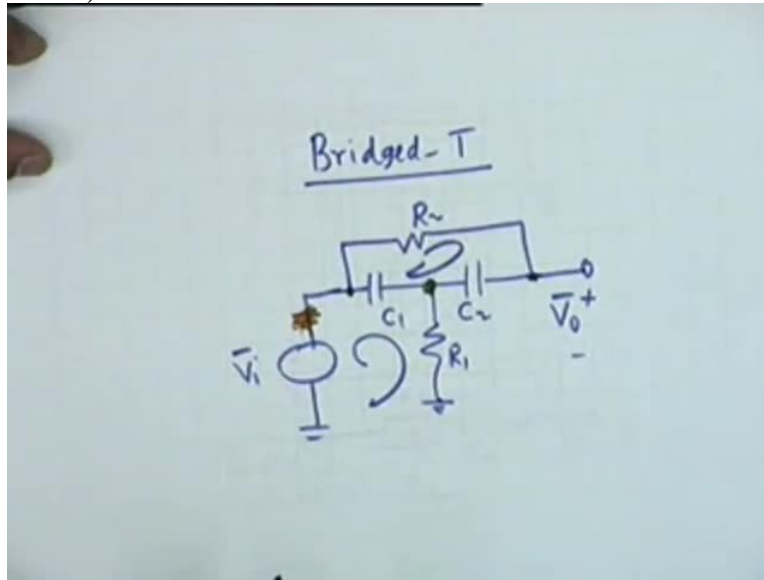
Let us see what it gives in terms of, this is  $V_I$  and the other branches, you have a resistance  $R$  and this voltage is also  $V_I$ . I have separated the 2 input branches all right? Now and this is  $V_0$ . Now I can find out the voltage at this node independently of what happens to the other. Can I? I can find out the voltage at this node by simply one node equation. I can find this in terms of  $V_I$  and  $V_0$  all right and if I know this node voltage, how does that help me? No, it does not. That is not the way one should think. Let me not mislead you further.

You will end up in absurdity because you are not solving anything. You are finding an unknown in terms of another unknown.  $V_0$  is the one that we want to find out. No, the simplification comes by applying Thevenin's theorem. Is KCL and KVL are valid, Thevenin's theorem should also be valid for AC circuit all right? So we apply Thevenin's theorem. What do you get? You get a  $C_2$ ,  $V_0$ ,  $R$ , this is  $V_I$  and then you get  $N$  impedance. What is the value of this impedance?  $R_1$  and  $C_1$  in parallel. And therefore  $R_1$  by  $S$ ,  $C_1 R_1 + 1$ , I am working in terms of  $S$ .

You must be able to write this expression without doing any calculation.  $R$  and  $C$  in parallel gives  $R$  over  $SCR + 1$  and the voltage source here would now be not  $V_I$  but  $V_I SC_1 R_1$  divided by  $SC_1 R_1 + 1$ , that is the potential division between  $C_1$  and  $R_1$ . So  $V_I R_1$  divided by  $R_1 + 1$  over  $SC_1 R_1$  which simplifies to this. All right? Now the circuit is extremely simple. All you have to do is to apply one KCL, that is this current which is  $V_I - V_0$  divided by  $R_2$ . Where is this  $R_2$ ? Yes.  $V_I - V_0$  over  $R_2$  should be equal to should be equal to be  $V_0$ . It must be equal to this current. KCL at this node, the current that comes must leave and therefore, that this current is  $V_0 -$  this voltage divided by this total impedance. All right? Just one equation.

The other, you did not have to write an equation. It is by inspection and therefore just one equation and you solve for  $V_0$  for  $V_0$  over  $V_I$  which is the transfer function. Is this point clear? I will skip the algebra all right? Is the point clear, the analysis method? One has to decide what will be the best? I am sure, you had not come across this type of analysis. If a voltage source the branches can be torn off and connect two voltage sources, identical voltage sources. This does not apply, let me caution you, let me caution you because you might fall in a trap.

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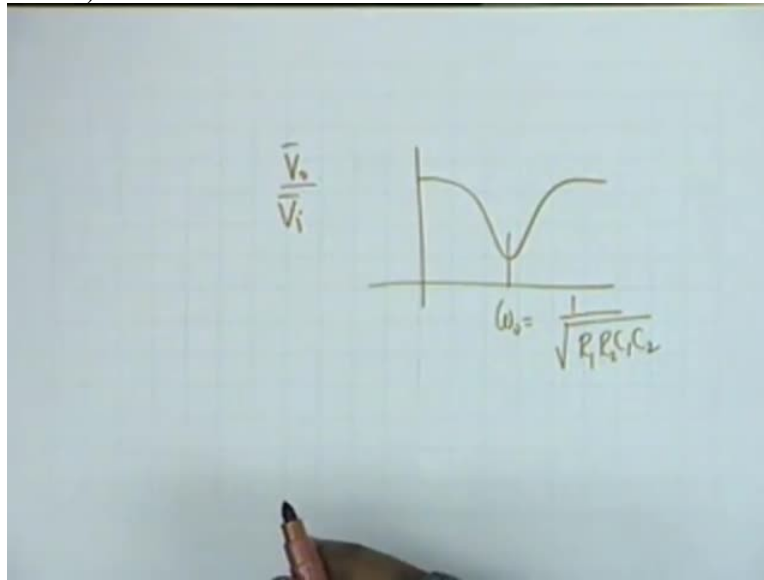


This does not apply if you are S you are S in internal resistance here. If there is an internal resistance here, then this method of analysis fails. Not only that, if instead of a voltage source, it is a current source, it does not apply. Only in terms of voltage source, ideal voltage source, you can separate the 2 branches and connect 2 identical voltage sources. All right? Wherever a situation like this occurs, this allows immense simplification. Yes?

Student: In case, we have an internal resistance, sir even then, sir the potential between these 2 branches, it is same, the potential difference is same across these 2 branches.

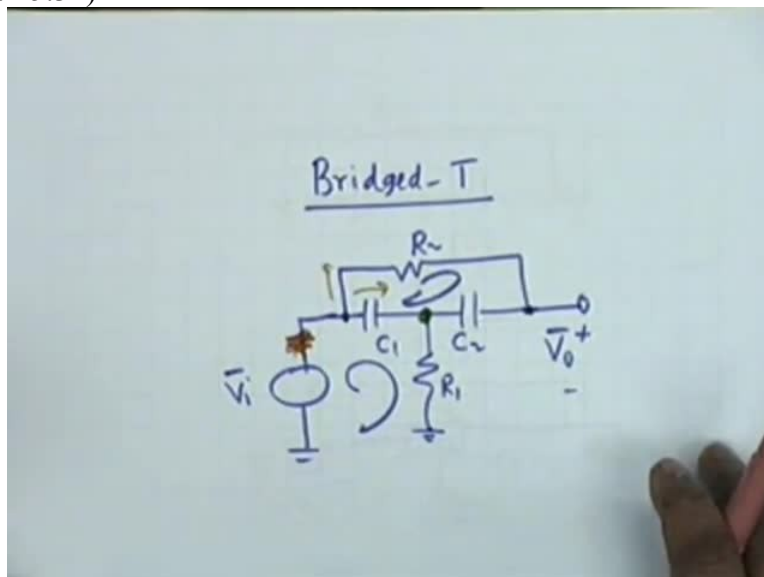
Professor: Correct but we do not know, we do not know what this potential else. We do not know because it depends on what current goes in this branch, what current goes in this branch, a problem to think about, all right? How to tackle the situation where there is an internal resistance all right? Now, I will give this to you as an exercise.

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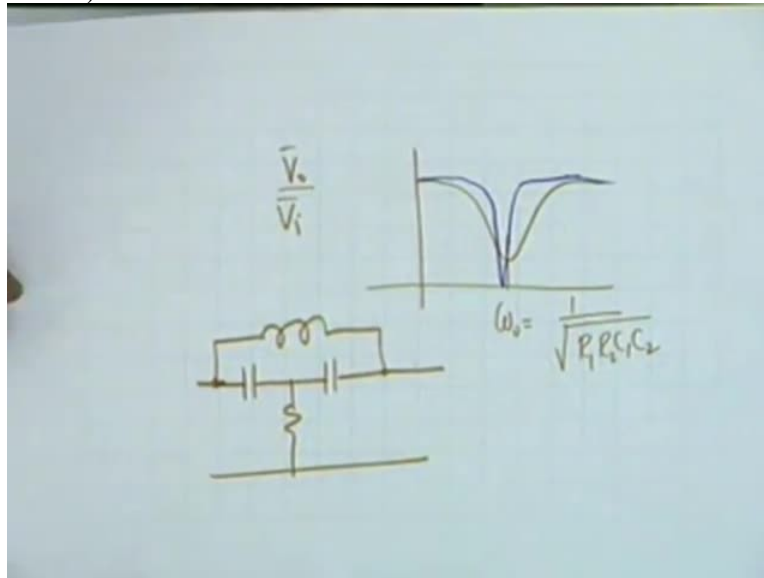
The answer to  $V_0$  by  $V_i$ , that is this ratio, this is given in the textbook and it can be shown that this indeed acts as a band stop filter like this. It acts as a band stop filter with a frequency, with a centre frequency of the stop band which is given by  $1$  by square root of  $R_1 R_2 C_1 C_2$ . All right? And the width of this will depend on the relative values that is  $\Omega$ ,  $\Omega_1$  and  $\Omega_2$ , it depends on the relative values of the various components. But I will skip that algebra. Now this is one way of getting a band stop.

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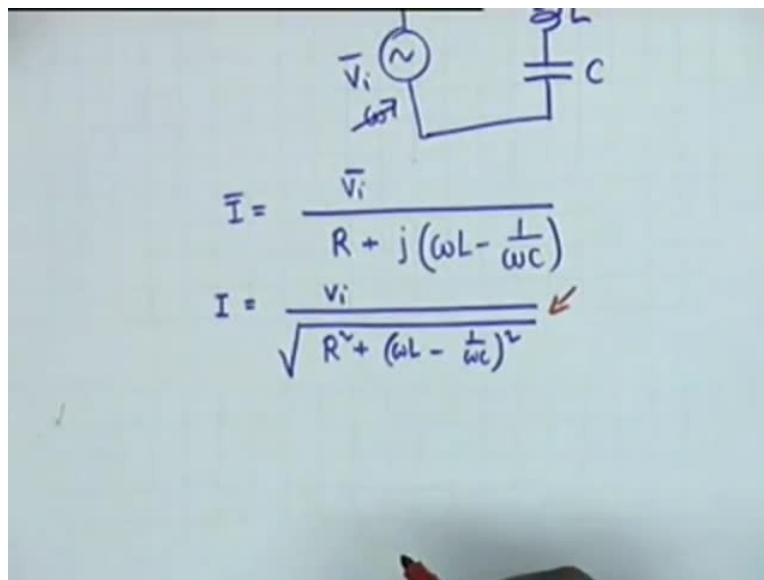
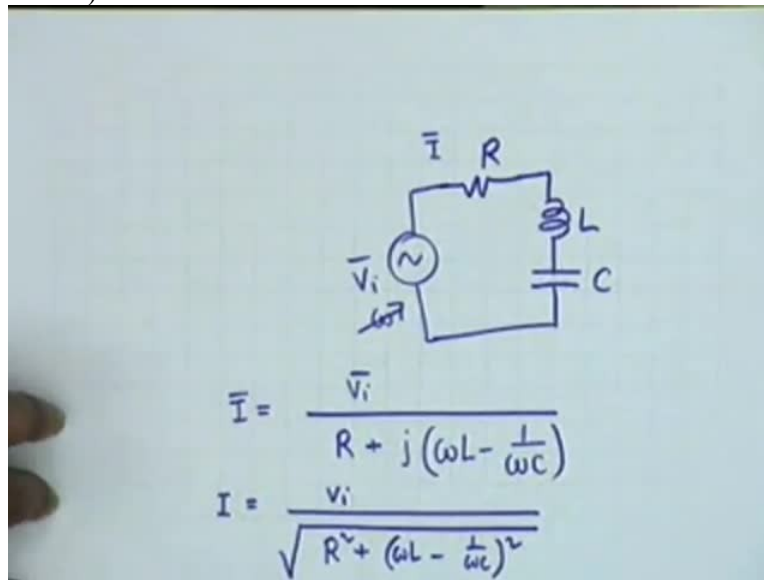
If you recall, if you recall the circuit is, the architecture is that of a bridged T. There is a T and then there is a flyover but it contains only resistances and capacitances. Much better performance is obtained if you mixed the 2 kinds of energy storage elements, that is if you use an inductor and a capacitor, one or more capacitors.

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For example, one of the circuits that you shall encounter in practice if you measure, try to measure something, one of the circuits is this. 2 capacitors, one resistor, the same configuration but instead of resistance, if you use an inductance here, it can be shown that this gives a much sharper band stop, much sharper band stop, perhaps something like this as compared to an RC, bridged T if you introduce both kinds of energy storage elements. And one of the simple, simplest examples of 2 kinds of energy storage elements in a circuit is the so-called resonant circuit.

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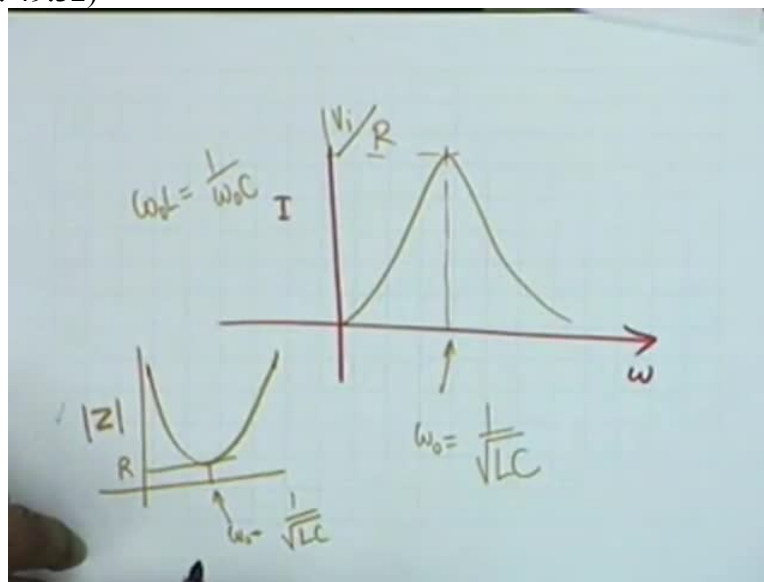
The simplest of which is this circuit, this series circuit. Suppose you have 3 elements in series, resistance, inductance and capacitance. We have already analysed this circuit, this is a 2<sup>nd</sup> order circuit. It can be under damped, it can be critically damped, it can be over damped all right? Now let us look at this circuit, the steady-state response of the circuit to let us say a sinusoidal source. Suppose you have a source  $V_i$  whose frequency is variable all right? We wish to find out how does this circuit respond to various frequencies. And suppose, what we wish to find out is the current response. All right?



Then you know the current response is the voltage phasor divided by the impedance and the impedance for sinusoidal frequencies is  $R + j\omega L + \frac{1}{j\omega C}$  which I can write as, if I skip one of the algebraic steps,  $j\omega L - \frac{1}{\omega C}$ . And we see that the magnitude of the current phasor  $I$  shall be given by the magnitude of the voltage phasor  $V_i$  divided by the magnitude of the impedance which is  $\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$ . All right? As frequency is varied, let us look at this, let us look at this expression a little more carefully. As frequency is varied at, when  $\omega L$  is greater than  $\frac{1}{\omega C}$  all right?

When  $\omega L$  is greater than  $\frac{1}{\omega C}$ , obviously the angle of the current shall be negative. Is not it right? Angle of the current shall be negative. That is why the current shall lag the voltage. On the other hand, when  $\frac{1}{\omega C}$  is greater than  $\omega L$ , the angle of the current shall be positive, that is the current shall lead the voltage. When these 2 are equal, then the current shall be in phase. Not only that, if you look at the magnitude, see that the current will be maximum when  $\omega L$  is equal to  $\frac{1}{\omega C}$ .

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And therefore, if you vary the frequency of the circuit and you measure the current phasor, all right, the current obviously for  $\omega$  equal to 0, for  $\omega$  equal to infinity, what will be the current in the circuit? 0. Why? Because there is a capacitor and at infinite frequency, inductance acts as open. So again, the current shall be 0. And in between, you have a maximum and you can see, you can see that this is indeed the circuit of a bandpass filter. The current shows a bandpass

response and the frequency at which the current is a maximum is given by  $\omega_0 L$  should be equal to  $1/\omega_0 C$ , that is the impedance of the inductor and the impedance of the capacitor, they should cancel each other and therefore  $\omega_0$  should be equal to  $1/\sqrt{LC}$ .

And the maximum current, then shall be given by  $V/R$ . This is the maximum current all right? Now this is obviously a reflection of the fact that the impedance of the RLC circuit, impedance of the RLC circuit if you plot that, the magnitude of the impedance obviously it goes through a maximum or minimum?

Student: minimum.

Professor: minimum.

It is infinitely large at DC, infinitely large at infinite frequency and in between, it goes through a minimum where the value is capital R. And this happens at the frequency  $1/\sqrt{LC}$ . At this frequency,  $1/\sqrt{LC}$ , the impedance is purely resistive which means that its angle is equal to 0 and this phenomenon is called the phenomenon of resonance. Resonance is a phenomenon in which at a certain frequency, the angle of the impedance becomes 0. That is, the current and the voltage are in phase. And you see, the resonance effect is accompanied by either a bandpass filtering effect or a band stop.

Well, if you take the the voltage as a response, current is held constant, then obviously, it will be a band stop type of thing whereas if you take the voltage as the excitation and the current as the response, that is a band stop type. Not only that, on Friday we shall see that it is possible to multiply the input voltage by any number. For example, for 1 volt, without using a transformer, you could get if you so desire, 100 volts. 1 volt AC could be transformed into 100 volt without using a transformer by the using the phenomenon of resonance. That is resonance can, you can be multiply voltages all right, this is what we shall see on Friday.