Introduction to Electronic Circuits
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Lecture no 20
Module no 01
AC Circuit Analysis

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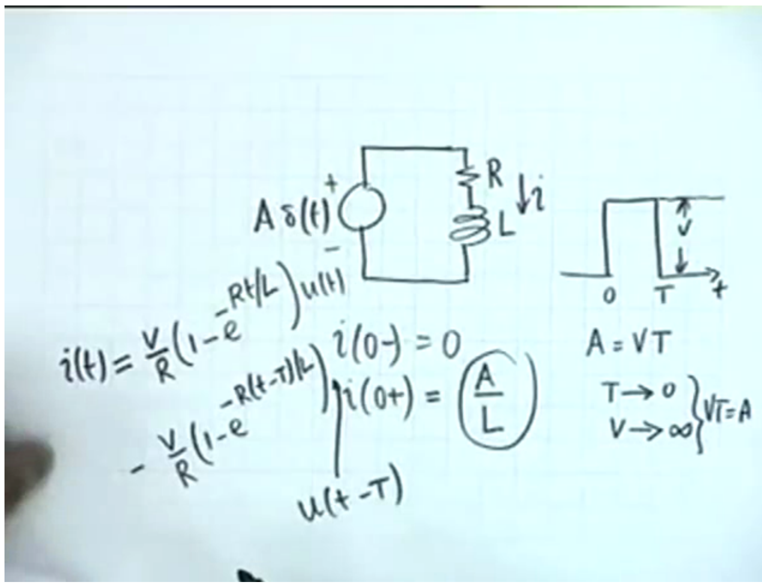

The 20th lecture being delivered on 22nd February 1994 and the topic is AC circuit analysis. Before we start the topic proper, I want to give a bit more physical interpretation of the impulse function. Recall that yesterday we were trying to find out the current response in this circuit when the applied voltage is an impulse, let this impulse be capital A Delta t. Capital A Delta t is the limit of the pulse of height capital V and duration capital T so that capital A = V T and the excitation is A Delta t where capital t tends to 0 and capital V tends to infinity in such a manner that V T is always equal to A V T is always equal to A. I want to give a physical interpretation, we argued that we argued that i of 0 – = 0 but i of 0 + is not equal to 0 and we argued that it must be equal to A by L.

This was done from the argument that the total drop occurs across capital L and therefore for 0 – to 0 + the current in the circuit current in the inductance shall be 1 by L integral of the voltage with respect to time and integral of A Delta t from 0 – to 0 + is simply equal to A and therefore the current was capital A by L. I want to give you bit of physical intimidation of this by approaching the problem from that of a pulse response and allowing capital T to go to 0. You see, if I apply a step like this a step which rises from 0 to capital V at t = 0 and which remains a constant, the current i of t in the circuit shall be simply V by R then the initial current is 0 and therefore it would be 1 – e to the – R t by L get me, this will be the current when a step is applied, the current gradually builds up from the value 0 to the value V by R.

Now if we apply the pulse that is another negative step at small t = capital T, the response to that shall be – V by R 1 – e to the power – R t – capital T divided by L alright, and the total current response would be equal to this multiplied by u of t because it occurs for t greater than equal to 0 and this quantity shall be multiplied by u of t – capital T agreed, this is by inspection that the pulse has been resolved into 2 steps, one positive, one negative, positive one occurring at small t = 0, negative one occurring at small t = capital T and therefore it is the difference it is the superposition of these 2 responses.

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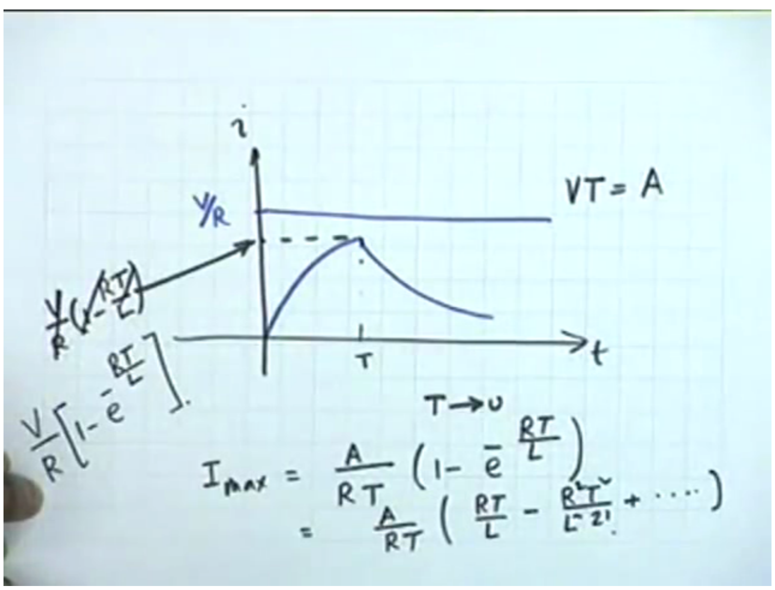

Now if I plot current i versus t then the current rises like this and would have gone to let us say V by R would have gone to V by R if it was allowed to, but at small t = capital T the negative pulse negative step comes into effect and therefore the current gradually diminishes it diminishes like this alright. And you see the maximum response would be simply given by V by R 1 – R capital T divided by L is that okay? I have (())(6:16) V by R 1 – e – R capital T by L alright. The point is, now when capital T goes to 0 tends to 0, obviously what will happen is that this rise shall be limited from 0 – to 0 + and then it will decay, what we used to find out is what will be the height under this condition.

There is a small trick involved here, it is a slightly tricky problem because capital V goes to infinity capital T goes to 0, so instead of capital V let us use the quantity which remains a constant, which is capital V times T which is equal to capital A and therefore the maximum current if I called that I max when capital T tends to 0, well 1st let us do it for general capital T therefore capital V, I write as A divided by R T A divided by R T when 1 – e to the power – R capital T divided by L alright. Now I expand this expand the exponential then I get A by R T alright, what do I get inside the brackets, R T by L – R square T square by L square 2 factorial + and so on to infinity okay, this current the maximum current can therefore we simplify the following.

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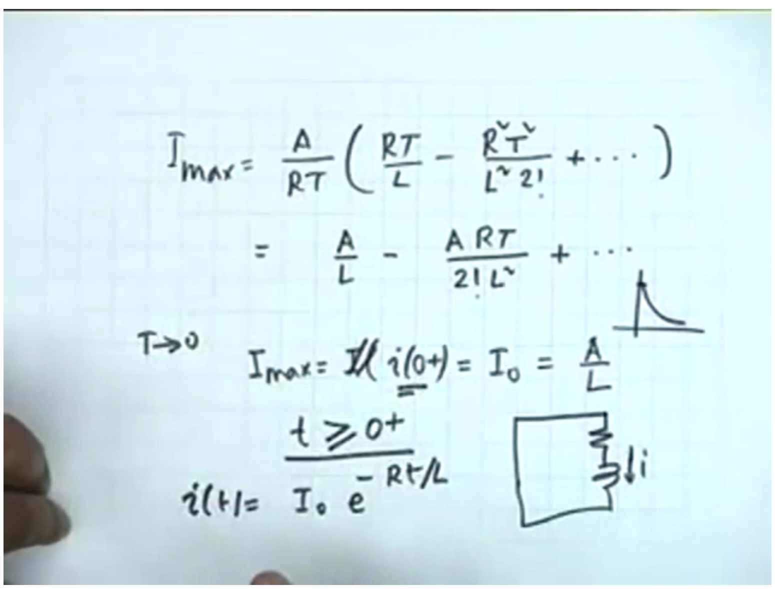

I max is equal to if I do away with the brackets what I had was A by R T, R T by L – R square T square by L square 2 factorial + et cetera, this I can write as A by L the first-term – A R T divided by factorial 2 L square + et cetera. And you see when capital T goes to 0, the first-term is unaffected, all others become 0 and therefore I max which I shall callsmall i of 0 + which is equal to let us say I 0 is given by simply A by L and this is what was obtained earlier on the basis of the property of a Delta function.

Here we obtain this is as a limiting case of pulse response and therefore I of 0 + is A by L and when t greater than equal to 0 + the circuit goes back to its natural mode in other words we have R and L where this current i at 0 + = i 0 and therefore for time greater than 0 the current response shall be simply equal to i 0 e to the power – R t by L, which means that current response would be it rises to I 0 and then simply goes to 0 exponentially, is this point clear? This gives you physical interpretation of the impulse function. What we did earlier was to obtain the impulse function as the limit of pulse, what we have done now is to obtain the response to a pulse and then allow pulse to become an impulse and we see that the 2 responses are the same. This gives a physical interpretation to the question of impulse response, now let us go to AC circuit analysis and we shall start with the consideration of power.

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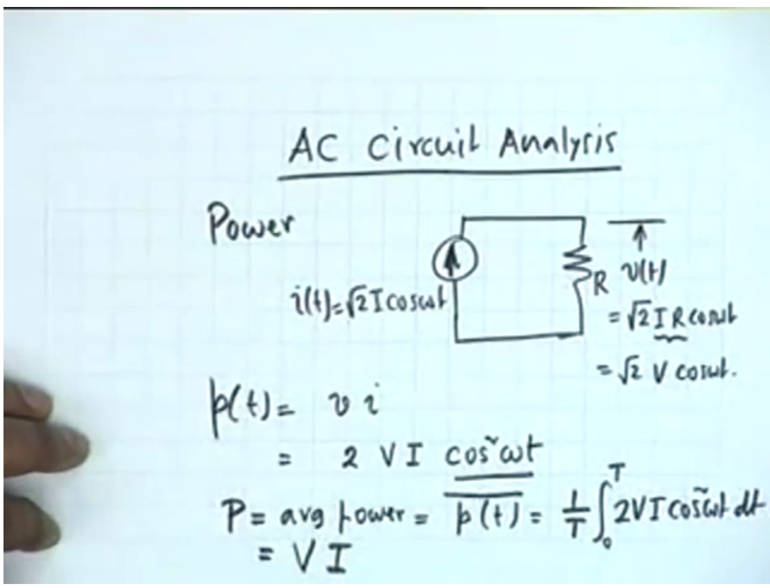

As you know that if we have an AC, let us take a current generator let us take a current generator i of t = root 2 I cosine Omega t and let this current AC pass through a resistance R, then the voltage across the resistance v t shall be simply equal to root 2 I R cosine Omega t and if we call I R, capital I is the root mean square value of the effective value, if I call I R as we then root 2 V cosine Omega T, the corresponding phasors are I 0 degree and V 0 degree alright. The power the power that is dissipated in the resistance obviously is a function of time because both current and voltage are functions of time so the instantaneous power p of t shall be equal to v times i and you see that this is twice V I cosine square Omega t alright cosine square Omega t, in other words the power is always positive.

The power is always positive because it is square of cosine and the negative halves are also flipped into squares of the cosine and therefore it makes sense to talk about the average power capital P the average power averaged over a complete cycle, another notation for this p of t average this average is over a cycle in other words, it is 1 by t integral 0 to capital T or any other instant of time t 1 small t 1 to t 1 + T the only thing is, it has to be one complete cycle twice V I cosine square Omega t dt and you know the integral the average value of cosine square or sin square over 1 period is half and therefore the average power is simply equal to the product of the root mean square values of the voltage and current and you know the unit for this is Watt, V I volts into amperes it is watt.

And therefore even AC circuit if there is a resistance and the current and voltage amplitudes are identified as root 2 V and root 2 I, the average power dissipated in the circuit is the product of the root mean square values of voltage and current. And if you wish to find the total energy dissipated over a certain instant of time that you have to do is to multiply V I by the time factor and then we have what is known as watt second the energy unit is watt second, the commercial unit is KWh kilowatt-hour.

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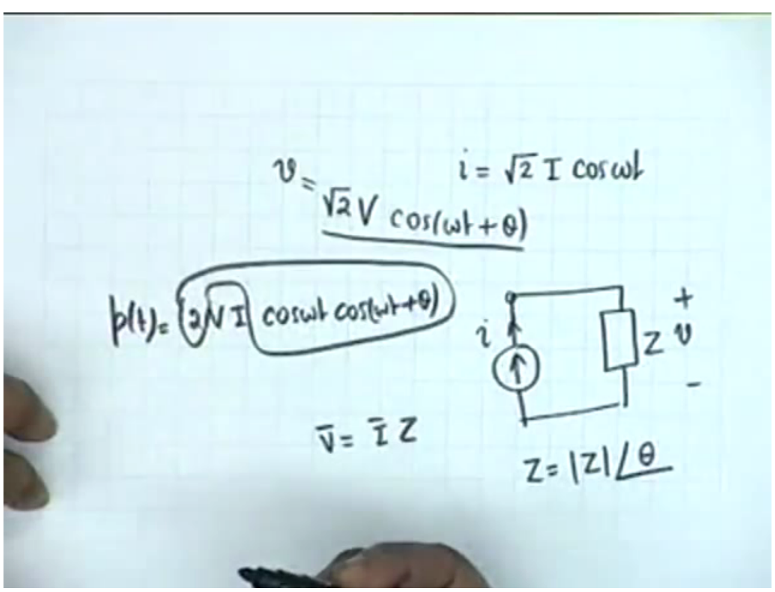

Now the situation changes if the voltage v and current i are not in phase, suppose i is root 2 I cosine of Omega t and suppose small v = root 2 V cosine of Omega t + Theta, which may occur typically let us say in a situation in which the current i is allowed to pass through an impedance Z where Z is magnitude Z angle theta it could be an inductive circuit for example, resistance, inductance or a capacitive circuit, resistance, capacitance and so on. Then the voltage developed across the impedance shall either lead or lag the current i by some angle Theta which is indeed the angle of impedance Z okay. You see the current and voltage phasors are I bar Z and therefore the phase difference between V and I shall be the angle of Z the impedance.

And typically my voltage expression in the time domain shall look like this, root 2 V where V capital V obviously is the product of capital I and magnitude of the impedance okay. Now if this is the situation then what is the power? The power obviously if you multiply V by I, small p t shall be equal to twice V I cosine Omega t and cosine Omega t + Theta alright. And if you wish to find out the average value of this then you have to integrate this over 1 period then divide by capital T. Before that you recognize that twice cosine A cosine B can be written as cosine A – B + cosine A + B.

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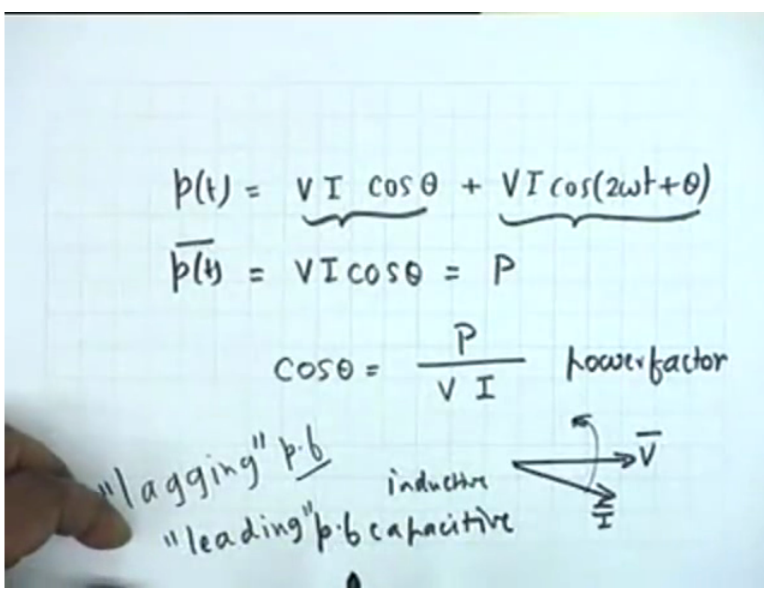

In other words, the instantaneous power in a situation where the circuit is not resistive is not purely resistive shall be given by V I cosine of Theta the difference between the 2 then + V I cosine of 2 Omega t + Theta alright cosine of A – B whether you take Omega T – Omega T –

Theta or the other way round it does not matter because cosine of – Theta is equal to cosine of Theta. Now if I take the average value of this over 1 period, you see the 1st is a constant so it does not matter you are simply multiplying this by capital T and dividing by capital T so this would be V I cosine Theta. On the other hand, the average value of cosine over 1 period is equal to 0 and therefore the average value of small P is simply V I cosine Theta and this represents the power dissipated in the circuit.

You notice that the modification that compared to a resistive circuit, it is still the product of V and I volts into amperes but it is also multiplied by cosine of the angle between the 2 phasors, the current and voltage. And this quantity cosine Theta which is the ratio of P to the volt-ampere product is called the power factor of the circuit alright, the ratio of the power absorbed to the product of voltage and current is the power factor. Naturally the power factor for a resistance for a purely resistive circuit is 1, power factor for a purely inductive circuit is 0 because cosine 90 is 0, the angle difference between current and voltage phasor for a pure inductance is 90 and similarly if a circuit is purely capacitive then again 0 and therefore power factor is nonzero only for a circuit which has a resistive component, if it is pure reactive it is 0.

Two kinds of power factors are distinguished, one is if the circuit is inductive if the circuit is inductive then who leads whom if the circuit is inductive, voltage leads the current and therefore the current lags the voltage alright, the current lags the voltage this is the positive direction, this is the positive direction the current goes in this direction compared to V phasor and therefore the current lags the voltage in an inductive circuit. And in such situation wells whether Theta is positive or negative, power factor makes no difference because cosine of – Theta is cosine Theta, but in such situation we say it is a lagging power factor it is the word lagging.

It does not relate to power factor, it relates to the angle of the current with respect to the voltage but the word the objective "lagging" power factor is added if the circuit is inductive. And conversely the adjective "leading" is used for a capacitive circuit where the current leads the voltage alright for a capacitive circuit the current leads the voltage and we say in a capacitive circuit the power factor is leading alright.

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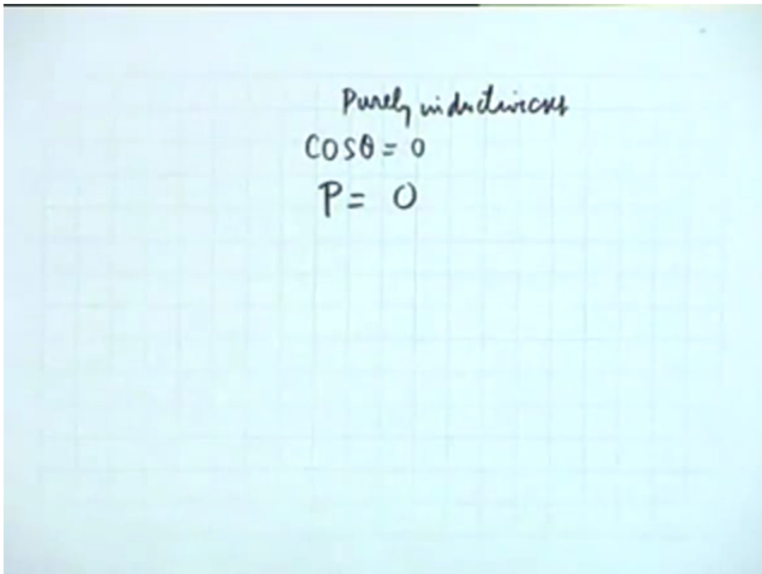

Now if we consider a purely inductive circuit if you consider a purely inductive circuit, cosine Theta is 0 purely inductive circuit cosine Theta is 0 and therefore, power absorbed is 0. This does not however mean that energy for all times to come is 0, no, what it means will be shown by means of this this figure.

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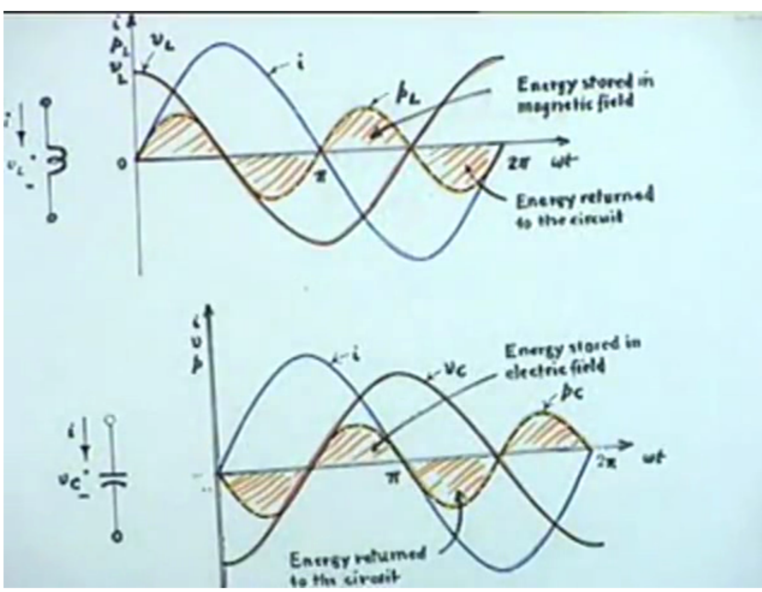

You do not have to draw it, it is there in the textbook, I have enlarged it to make things clear. The 1st we consider an inductive an inductance a pure inductive and suppose the current is as shown by this blue line, this is the current i = root 2 I Sin Omega t because it starts from 0 alright. The voltage across inductance which is v L equal to L di dt does it lead or lag? It leads and therefore its maximum is reached one quarter of a period earlier that is 90 degrees earlier alright. When Omega t is less than the maximum of this by 90 the voltage reaches its maximum because the voltage is 90 degrees ahead of the current and therefore this curve, what color is it, red the red color curve is that of the voltage across the inductance.

And the instantaneous power is the product of the 2 and therefore whenever the current and voltage are both positive or both negative, you see this portion, current and voltage are both negative, the power instantaneous power is positive, so it is positive here one quarter of a cycle then next quarter the current is positive, voltage is negative and therefore the power is negative, similarly the next quarter the power is positive and so on alright. Now what it means is you see the power is also sinusoidal, the power is also sinusoidal with a frequency which is equal to twice the frequency of the current or voltage, is that clear, the frequency is twice of that current or voltage which means that if I take average over one cycle of either current or voltage or even half cycle, the average power is equal to 0.

And that is all that it means by saying that capital P = 0, the average over one cycle or even half a cycle is 0 alright. On the other hand well, then we must have an interpretation before we conclude, we must have an integration of what it means when P L is positive, P L positive means energy is being stored in the magnetic field, P L positive means that inductor takes power, it does not absorb though, it does not dissipate, it stores it in the magnetic field. On the other hand in the next quarter of a cycle, this quarter means refers to the current or the voltage cycle, not the power cycle, power cycle is half of the voltage cycle alright.

So in the next quarter of a cycle the power is negative, which means that the magnetic field is returning energy to the circuit that is the inductance no longer stores magnetic field stores energy no longer takes energy into its magnetic field, this energy that was stored is being given out so that at the end of this again the energy becomes 0 and again it stores again it returns and this goes on, so the average power is equal to 0.

The same story holds for a capacitance except that the roles of current and voltage are reversed. In a capacitance, the voltage leads the current so if this is the current the blue curve, the red one is that of the voltage you see the maximum occurs is that correct, the voltage across a capacitor voltage across the capacitor lags the current? Okay so the current leads the voltage yes that is true, the maximum current occurs before the maximum of the voltage alright 90 degrees. And therefore once again the same phenomena occurs that is when the current and voltage are negative when the current and voltage are of opposite polarity, the power is negative, when the current and voltage are of the same polarity as here and also here the power is positive.

And when the power is positive, energy is being stored in electric field not in magnetic field, when the power is negative energy is being returned to the circuit and the sum total over an integral number of cycles is that the average power is equal to 0.

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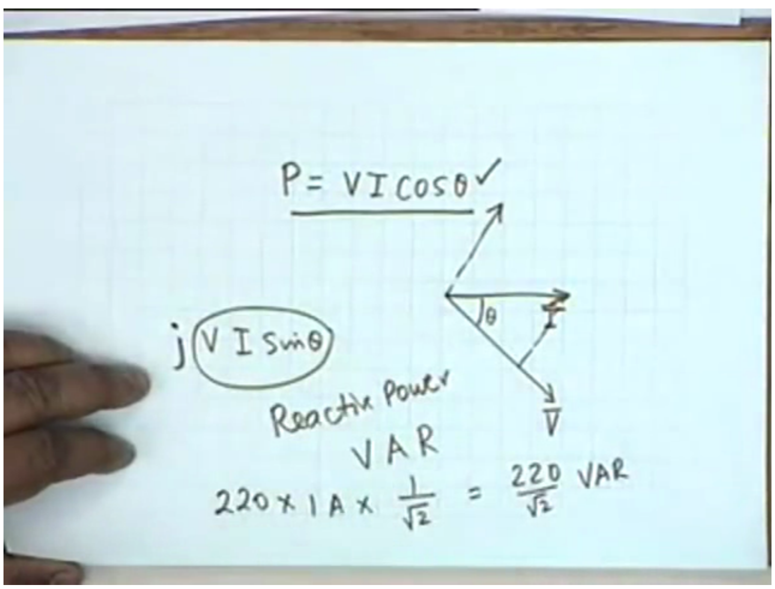

If you interpret this in terms of this quantity V I cosine Theta, which is the power absorbed by a circuit purely inductive circuit, purely capacitive circuit, the power absorbed is 0 but you see the general expression for power absorbed is obtained by drawing the phasor diagram of voltage and current, let us say we have arbitrarily let us say this is the current phasor and let us say this is the voltage phasor then the power is obtained by projecting 1 vector on the other, this angle is Theta it would be in this case the current is leading or lagging? Leading, so power factor shall be leading power sector this angle Theta and you see the power absorbed by the circuit is given by the product of one of the phasors and the projection of the other phasor on the same line. That is if you project I on to the direction of V then the current in the direction of V is I cosine Theta and therefore V I cosine Theta is the power absorbed by the circuit.

It could also be looked at from another point of view that it is I multiplied by projection of V on the line on the direction of I, it is the same thing because the projection is simply the magnitude of the quantity multiplied by cosine Theta. And therefore one argue what about the projection the orthogonal projection that is what about the product V I sin Theta, if this is I cosine Theta obviously there must be an I sin Theta in this direction alright, what about the product V and I Sin Theta? Well, because this is orthogonal it can be represented by multiplication by j this is 90 degrees away from V I cosine Theta and this quantity V I Sin Theta called the reactive power because it does not cause any absorption or dissipation of energy.

In a reactance average power is equal to 0 and this quantity is called the Reactive power , sometimes it is called VAR volt ampere reactive V A R alright. For example, if it is 220 volt, 1 ampere and Theta is let us say 45 degrees, then you say 1 by root 2 so this is 220 divide by root 2 and the unit which do not say it is watts, actually the unit should be watts but we do not say watts because watts is reserved for the energy dissipated or absorbed or energy which cannot be recovered, which is lost. So this is not called watt, this is called 220 by root 2 VAR, volt ampere product reactive but multiplied by Sin Theta.

And if you consider power as a complex quantity now, if you consider power as a phasor, voltage is a phasor, current is a phasor, if you consider power as a phasor then obviously then obviously V I cosine Theta is its real part and the real part is the part that is dissipated that is in recoverable from the circuit and it is imaginary part shall therefore be the VAR alright and therefore power can also be considered as a complex quantity and this is called complex power.

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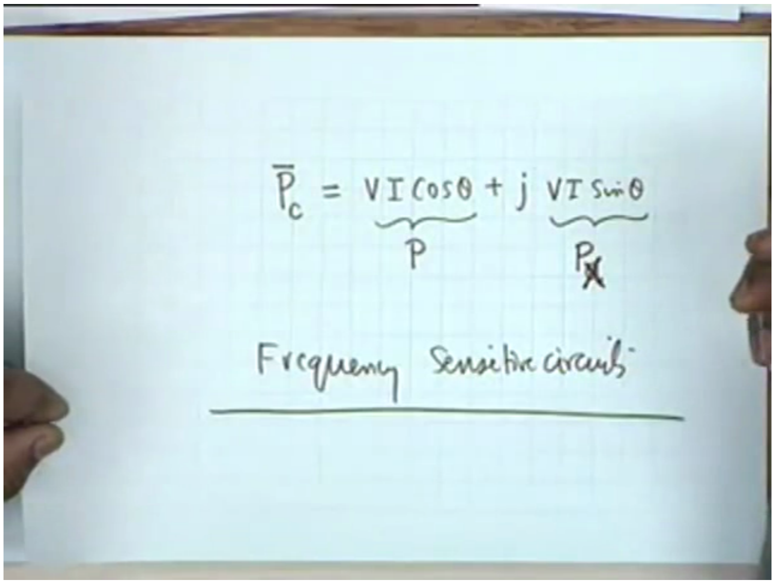

Let us call this as what do we say P c let us say complex power, it is a phasor and its real part is we shall reserve the symbol capital P for this + j V I sin Theta, and this power as you as you know it is called VAR its unit is VAR but it is also called an apparent power or reactive power, I think that is better let us call this reactive power P x okay. You will see that reactive power is of great importance in power, introduction to electrical machines if you take that course or power calculation that is large power consumption and so on. However, in the case in the case of electronic circuits it is basically the power that we shall be concerned with and therefore from now onwards we shall not make a reference to reactive power or complex power unless the the context of the situation demands so.

We next consider some circuits, which are frequency sensitive, is there any question on power? Any equation on power? Okay, we next consider frequency sensitive circuits. When you put up an aerial on a simple 2 in 1 radio set to pick up the FM signal for example, there are many signals which are being transmitted which are in the air, many signals and you tune a certain knob to pick up let us say daily FM or AM for that matter alright, now what you basically do is different transmitting stations are transmitting at different frequencies.

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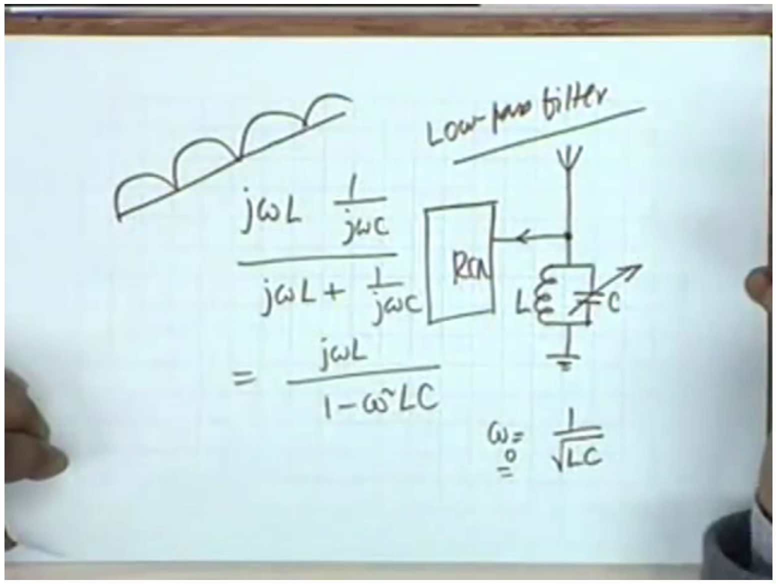

And what you basically do is, this is the aerial basically do is you have an inductance-capacitance circuit in parallel L and C and tuning what you are doing essentially is rotating a capacitor knob so you are varying the capacitance. Now you see that the impedance of the circuit impedance of the circuit is j Omega L multiplied by 1 by j Omega C divided by j Omega L + 1 over j Omega C, which is equal to j Omega L divided by 1 – Omega square L C. And you see that at the frequency Omega 0 = 1 over square root L C, the impedance becomes infinite. Now the impedance becomes infinite means what? That this particular frequency shall be picked up by the circuit, this impedance is infinite and therefore any small current actually in practice it cannot be infinite it cannot be infinite, it is very high.

So any small current flowing at that frequency shall be dropped across here and all other frequencies shall find an easy path to ground, it is only at this frequency that the impedance has become very large, all other frequencies find an easy path to ground and therefore the current that is picked up, well we can consider this to be infinity no problem, the current that is picked up at this frequency cannot pass to ground, it must pass to the to the receiver alright, this is the simple tuning circuit of a radio, is that clear? So what does this L C these 2 parameters do? They perform a frequency sensitive job, there is a pickup of certain frequency and the combination of resistance-inductance, the combination of resistance-capacitance or the combination of inductance- capacitance or a combination of these 3 elements can perform frequency sensitive jobs.

For example, if you wish to pass the low frequency in a circuit and cut off high frequencies for example, in-based control what you do is basically allow the high frequencies to be rejected, you admit only the low frequencies. Or in the case of a power supply, you have 2 diodes, a transformer and a rectifier circuit which produces the full wave rectifier produces something like this, what we used to do is to retain the DC and reject all the alternative components and therefore you require what is known as a low pass filter low pass filter, low pass filter is one which passes low frequencies and rejects high frequencies.

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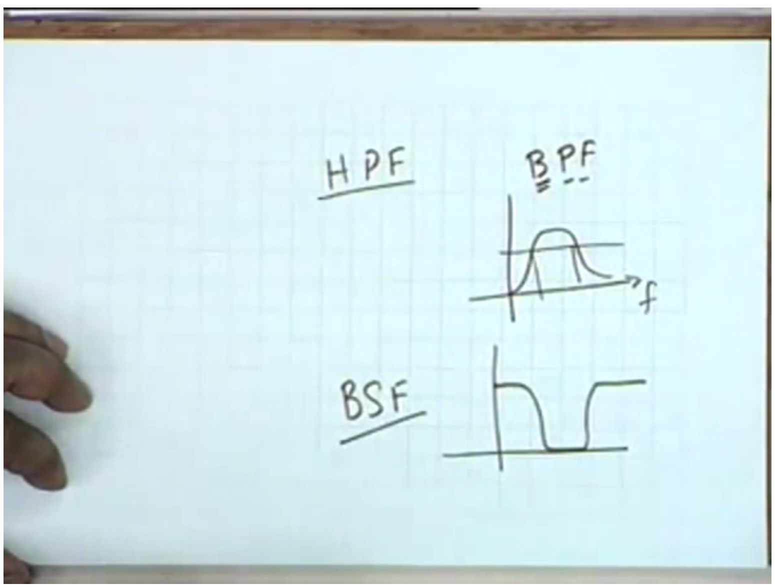

In a similar manner the high pass filter HPF is one which passes high frequencies but rejects low frequencies alright High pass filter. A Band pass filter is one which rejects low as well as high frequencies, BPF is one which rejects both low and high frequencies, if I plot its transfer characteristics it will be something like this, it is rejects both if this is frequency, it rejects both low frequencies and high frequencies and passes a certain brand of frequency that is why it is called the band pass filter. Similarly you can have band stop filter like this, it passes low frequencies, it rejects a band of frequencies and then it passes high frequencies, so this is called a band stop filter BSF and you shall look at what some of the elementary filters that one can think of with a very specific purpose, we shall consider a few practical examples of such filters.

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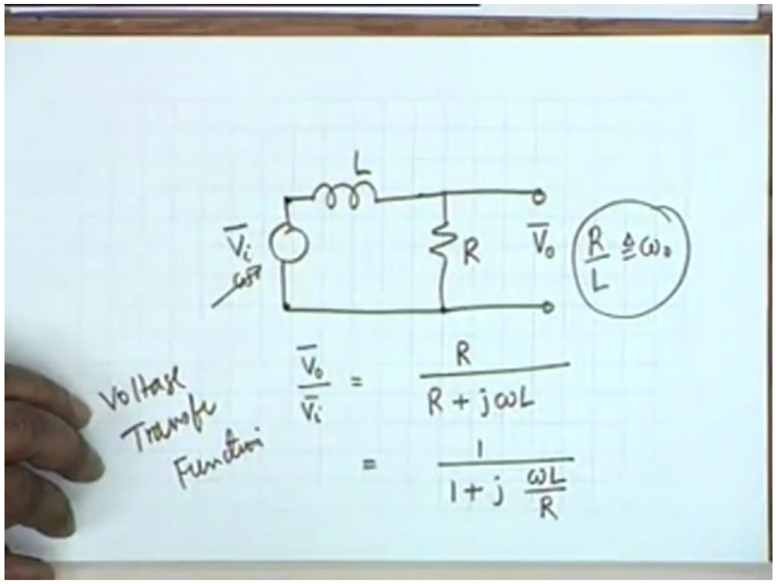

Let us look at 2 very simple circuits, one is let us say an inductance in series with a resistance, please be with me and let us say this is a voltage phasor V whose frequency Omega is variable, frequency Omega for example this could be typical signal generator that is available in the laboratory, its frequency is variable. I want to find out let us say this is V i phasor input voltage, this is sinusoidal of a frequency Omega which you keep up varying and the output is V 0 and let the phasor be V 0. Then you see V 0 over V i would be simply phasors can be treated in exactly the same manner as DC voltages and therefore this would be R divided by R + j Omega L alright, this is the transfer function.

A transfer function is the ratio of the desired quantity to the source quantity of the excitation, it could be current to voltage ratio or voltage to voltage ratio or voltage to current ratio or current to current ratio alright, so this is the expression for the voltage transfer function because voltage is being transferred from this terminal to these 2 terminals alright so it is the voltage transfer function is this term clear, Voltage transfer function is being transferred from one point to another. And what you want to do in the at the output, we used to retain certain frequencies and reject others, now if you look at this expression this is equal to I can write this as 1 + j Omega L by R alright, let R by L let R by L be equal to Omega 0.

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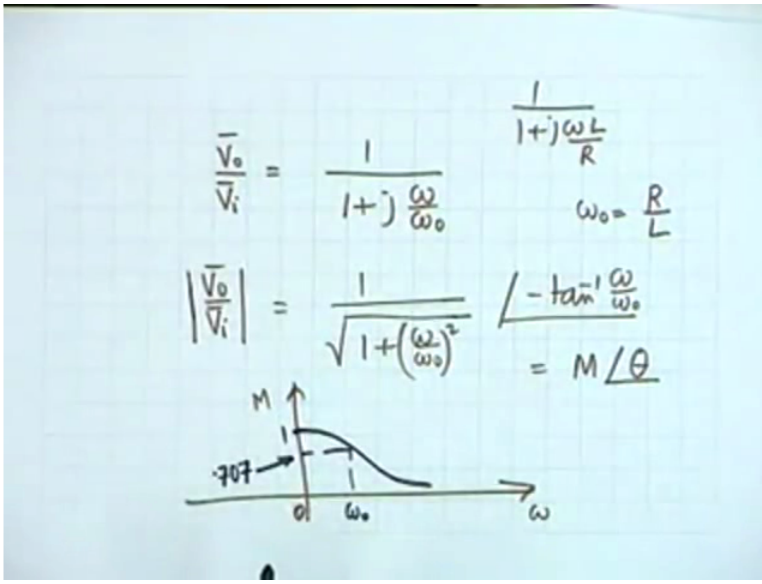

Define the quantity R by L as Omega 0 then you can write for the circuit V 0 by V i the ratio of the 2 phasors equal to 1 by 1 + j Omega by Omega 0 alright, is that clear. What we have was 1 by 1 + j Omega L by R, and R by L has been designated as Omega 0 so I can write this as Omega by Omega 0. It is as if the frequency is being normalized with respect to the fixed frequency Omega 0, Omega 0 is fixed because Omega 0 has been defined as R by L which are the parameters of the circuit okay, and therefore the magnitude of this phasor ratio is given by 1 by square root of 1 + Omega by Omega 0 square alright and the angle is – Tan inverse Omega by Omega 0, is this clear is this okay or I have gone too fast? It is okay.

Now if I plot this, if I plot the magnitude, I can write this as M angle Theta. If I plot M the magnitude versus Omega, you see at Omega equal to 0 it would be 1 and then as Omega increases it would gradually fall like this, is that clear okay. As Omega increases the denominator increases so the curve gradually falls like this, what happens when Omega equal to Omega 0? What would be the magnitude? 1 by root 2 which is 0.707 alright 1 by root 2, so the voltage output let us look at the circuit now.

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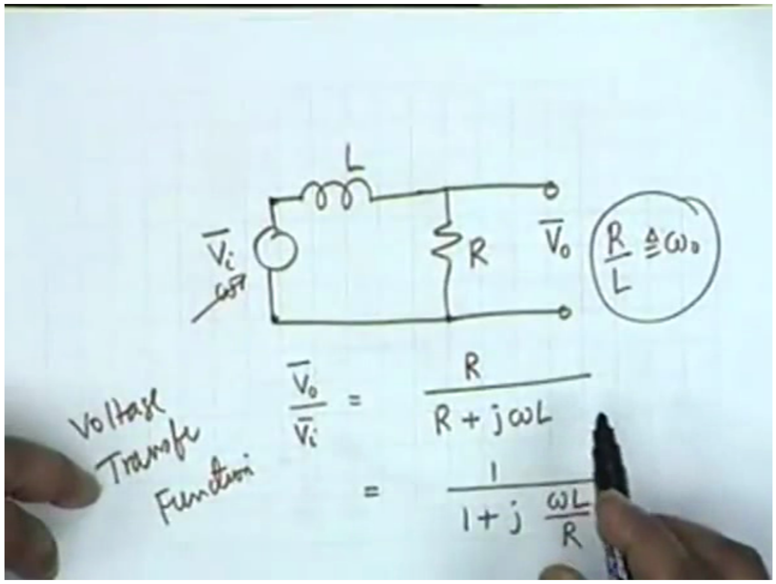

At DC at Omega equal to 0 the ratio of the 2 quantities magnitude is one, which is quite (()) (42:15) reason because at DC this L behaves as a short-circuit so the total voltage is dropped across R agreed, so the voltage across R is the same as the input voltage and the ratio is 1. Now, when the ratio is 1 what is the power? V square by R alright, so input voltage magnitude square divided by R. On the other hand, when the output voltage falls to 0.707 of the input voltage, what is the power in the resistor? Half of the power at DC that is the power in the resistor, power in the resistor is magnitude V 0 square divided by R alright, the phasor V 0 you take the magnitude, square it and divide by R.

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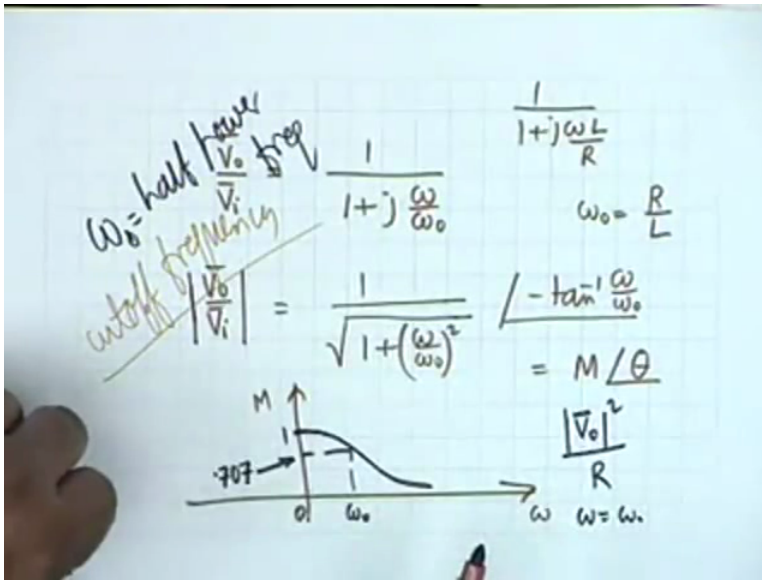

Now this was equal to input voltage at Omega equal to 0, at Omega equal to Omega 0 the voltage magnitude has fallen by 1 by root 2 so the power now has become half of the power at DC and therefore Omega 0 is given the name of half-power frequency or in popular (())(43:45) it is called the Cut-off frequency cut-off frequency, it is as if the circuit behaves as a filter retaining frequencies from 0 to Omega 0 and rejecting from Omega 0 to infinity therefore this circuit is a low pass filter because it favours low frequencies as compared to high frequencies so it is a low pass filter, is that clear?

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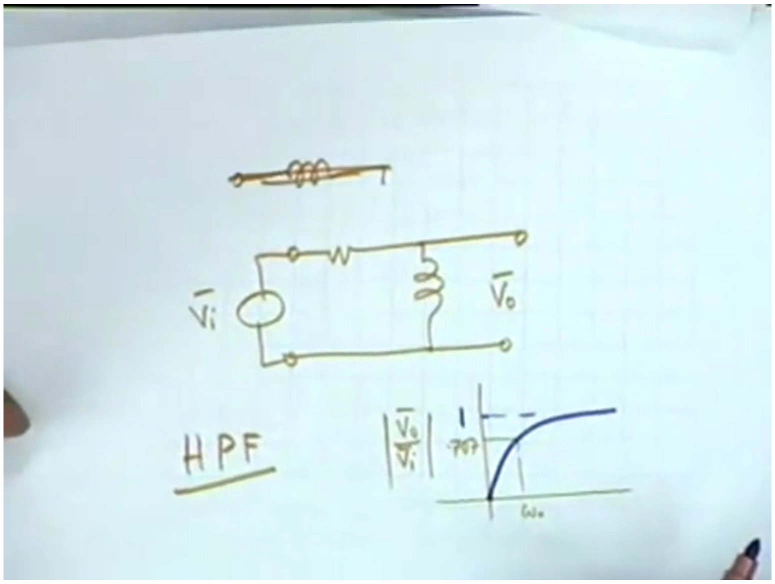

Suppose I interchange the two suppose I interchange the resistance and inductance that is I take no, it is the other way round I take the resistance and inductance and this is my V i, this is my V 0, what kind of filter do you expect? DC at DC this is short so if you plot V 0 by V i mod it will start from 0. At DC it is 0 and when the frequency increases the impedance of this increases and therefore the ratio of the voltage increases and at infinite frequency when this becomes Z of s at s = infinity, it becomes open and therefore I shall get this will rise to 1 at infinite frequency alright, and once again you can define a half-power frequency as the frequency at which the ratio of the 2 voltages becomes 0.7, this would be Omega 0 and we say this is a high pass filter with a power frequency of Omega 0 or cut-off frequency of Omega 0.

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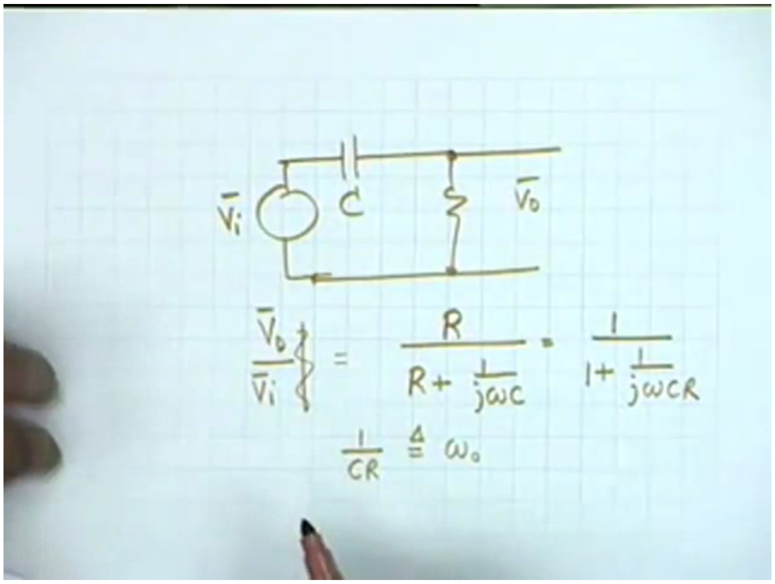

It does not take very long to show that the circuit containing a capacitor and a resistance what kind of circuit is this? It is also a high pass filter and I can write down the ratio of V 0 to V i as equal to yes R divided by R + 1 over j Omega C and this can be written as 1 divided by 1 + 1 over j Omega C R and if I write 1 by C R, 1 by C R is the dimension of frequency.

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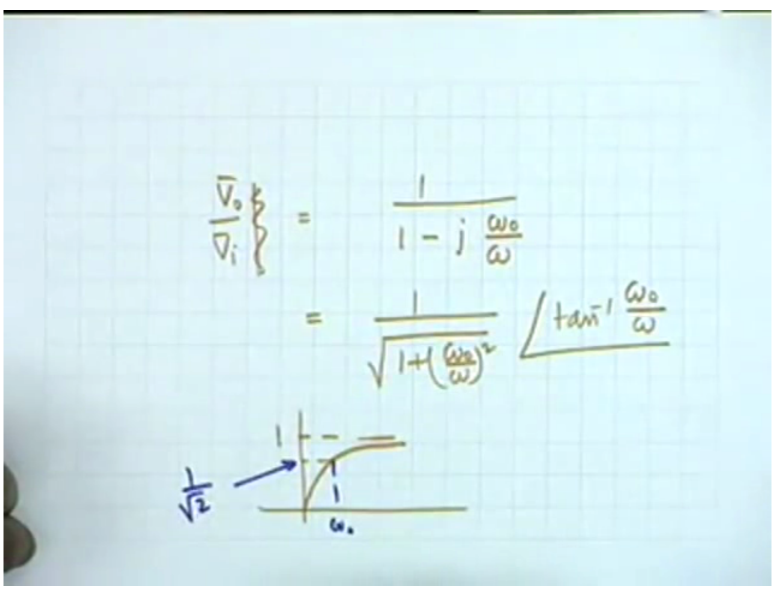

If I define this as Omega 0 then my ratio of the 2 voltages V 0 to V i becomes equal to 1 by 1 – j Omega 0 divided by Omega and so the magnitude becomes square root of 1 + Omega 0 by

Omega whole square and the angle becomes Tan inverse + Omega 0 divided by Omega. Now you see that Omega is 0 the magnitude is 0, when Omega 0 this becomes infinity, 1 by infinity becomes 0 and the angle becomes Pie by 2. On the other hand when Omega increases, the magnitude increases and reaches the value 1 at infinite frequency, when small Omega is infinity the value is 1 and therefore this also behave as a high pass filter. Now if I take the frequency Omega 0, at Omega 0 the value of the magnitude is 1 by root 2 and therefore the power in the resistor becomes half of the power at which frequency? Infinity not Omega = 0, Omega equal to 0 Power is 0. So the power at Omega equal to Omega 0 becomes half of the power at infinite frequency alright.

And the angle what is the angle at this point at this frequency Pie by 4, therefore this is high pass filter with a cut-off frequency of Omega 0 equal to 1 by C R alright, we shall look into some applications of low pass and high pass filter next time. Is the next time Friday? Friday is a holiday so Monday, Monday is 28 Monday is 28^{th} okay. I may not be able to take my class on Monday but I shall notify this on my door by day after tomorrow. When does the mid semester break starts? Is it mid semester break? No... Student's week when does it starts? This Friday... Thursday okay, Thursday morning I will notify on my door, will the class will be held or not okay.