

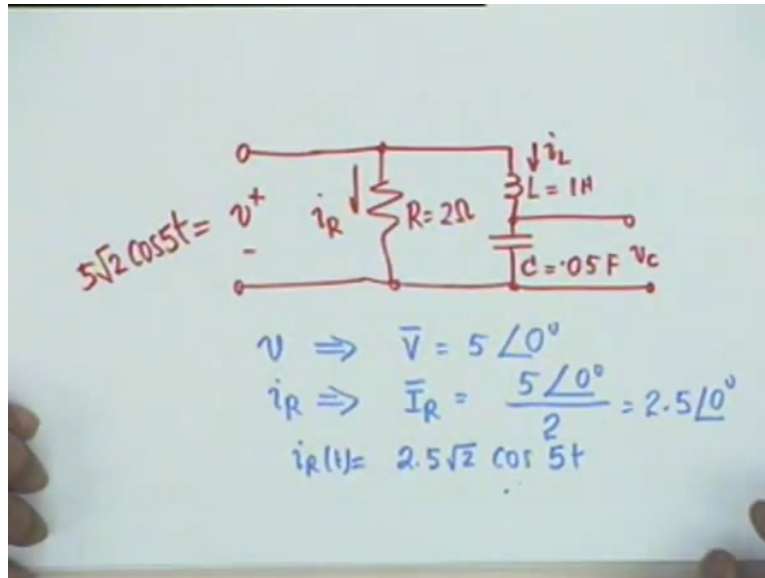
Introduction to Electronic Circuits
Professor S. C. Dutta Roy
Department of Electrical Engineering
Indian Institute of Technology Delhi

Lecture no 18

Module no 01

More about Phasors and Introduction to Complete Response

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This is the 18th lecture and our topic today is More about phasors and an Introduction to complete Response of an electrical circuit. Last time we had started with an example which we could not finish and the example is that sinusoid v which is equal to $5\sqrt{2} \cos 5t$ is applied to a circuit consisting of resistance $R = 2$ ohms and this current is i_R , then an inductance L and a capacitor C , $L = 1$ Henry and $C = 0.05$ farad and this voltage is v_{sbc} which is our concern, our concern is also the current i_L . And to analyse the circuit for various voltages and currents our starting point was that we represent v as a phasor.

The phasors corresponding to v is \bar{V} which is $5 \angle 0^\circ$ degrees alright, and therefore corresponding to i_R the phasor will be \bar{I}_R phasor would be equal to simply the voltage divided by the resistance, the angle of resistance is 0° degrees and therefore this is $2.5 \angle 0^\circ$ degrees, which really means that i_R as a function of t will be given by $2.5\sqrt{2} \cos 5t$, angle is 0° so it is cosine of $5t$ alright, next let us find out i_L and to that end let us find out the impedance of the L C branch that is L in series with C .

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The image shows a whiteboard with handwritten mathematical work. On the left, there is a circuit diagram of a series combination of an inductor (L) and a capacitor (C). To the right of the diagram, the following equations are written:

$$Z(j\omega) = j\omega L + \frac{1}{j\omega C}$$

$$= j(\omega L - \frac{1}{\omega C})$$

$$= j(5 \times 1 - \frac{1}{5 \times 0.05})$$

$$= j(5 - 4) = j$$

Below this, the current phasor is calculated:

$$i_L \Rightarrow \bar{I}_L = \frac{\bar{V}}{j} = \frac{5 \angle 0^\circ}{j}$$

$$= 5 \angle -\pi/2$$

On the left side of the whiteboard, the current in the time domain is given as:

$$i_L(t) = 5\sqrt{2} \cos(5t - \pi/2)$$

$$= 5\sqrt{2} \sin 5t$$

This impedance would be equal to $Z = j\omega L + \frac{1}{j\omega C}$, which I can write as $j\omega L - \frac{1}{\omega C}$. And if I substitute the values ω is 5, L is 1 Henry $5 \times 1 - \frac{1}{5 \times 0.05}$ Farad, so this is equal to $j5 - \frac{1}{0.25}$ is 4 so this is simply equal to j . The impedance of this branch is equal to j which means that the branch is inductive because the reactance is positive reactance is j times the constant and that is positive and therefore this is inductive.

In other words, if I now want to find out the phasor responding to i_L that is the current in that particular branch, the current in this branch i_L the corresponding phasor shall be equal to the voltage phasor that is V divided by the impedance that is j simply j , and V is $5 \angle 0^\circ$ divided by j and therefore the current phasor is simply 5 then $-\pi/2$, which simply means in terms of time domain quantities that i_L of t shall be equal to $5\sqrt{2} \cos(5t - \pi/2)$, which means that the current will be $5\sqrt{2} \sin(5t)$, this is the time domain corresponding time domain quality, but we did not work in terms of time domain we work in terms of phasor alright.

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$$\begin{aligned}
 \bar{I} &= \bar{I}_R + \bar{I}_L \\
 &= 2.5 \angle 0^\circ + \frac{5}{j} \\
 &= 2.5 - j5 \\
 &= \sqrt{(2.5)^2 + 5^2} \angle -\tan^{-1} 2 \\
 &= \sqrt{31.25} \angle -\tan^{-1} 2 \\
 i(t) &= \sqrt{62.5} \cos(5t - \tan^{-1} 2)
 \end{aligned}$$

If we wish to find out the total current now okay, we have found out the 2 currents i_R and i_L , there are 2 branches one of them has i_R and the other is i_L and therefore the total current i think this is the current that because i the total current i is by KCL the sum of i_R and i_L and I showed do you the last day that KCL is valid for phasors also because all that we take out from time domain representation is root 2 times e to the power $j \Omega t$ and the real part, which is common to all quantities.

And therefore the current phasor the current phasor I the total current shall be the sum of the 2 phasors I_R and I_L which is 5 what was I_R , $2.5 \angle 0^\circ$ and I_L is $5 \angle -\frac{\pi}{2}$, since I have 2 add them I show this in terms of j representation 5 by j , which means that this is $2.5 - j5$ or in terms of its magnitude angle, it is square root of real square $2.5^2 + 5^2$ and angle would be \tan^{-1} imaginary part divided by real part so -5 by 2.5 so it is $\tan^{-1} 2$ with a negative sign here. This is equal to square root of 31.25 ; $2.5^2 + 25$ and then angle $-\tan^{-1} 2$ alright.

If I want to show this in the form of a phasor diagram then I have a phasor V and the phasor I_R which is in phase with V , the phasor let us say V_R which is also in phase with V , the phasor I_L is 5 divided by j so it is $5 \angle -\frac{\pi}{2}$, so it would be something like this, this would be I_L where this length is 5 and the angle is $-\frac{\pi}{2}$ and the total phasor the total current shall be represented by a phasor which is the vector sum of the 2 and therefore the total current shall be

represented by this and this is I, the magnitude the length of the vector shall be square root of 31.25 and this angle is Tan inverse of 2 alright.

If I want the total current as a function of time all that I have to do is to write is to multiply this by root 2 so where root of 62.5 right and cosine of 5 t – Tan inverse 2, this is the current phasor in the time domain. You must be conversant and confident with transformation from one domain to other that is from time domain to the complex number and from complex number to the time domain easily back and forth otherwise, the phasor analysis loses its meaning.

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$$\begin{aligned} \bar{V}_c &= \bar{I}_L \frac{1}{j\omega C} \\ &= \frac{5}{j} \frac{1}{j \times 5 \times 0.05} \end{aligned}$$

$\begin{array}{c} \bar{I}_L \\ \downarrow \\ \text{---} L \text{---} \\ \downarrow \\ \text{---} C \text{---} \\ \downarrow \\ \bar{V}_c \end{array}$

$$= -20 = 20 \angle \pi$$

$$v_c(t) = -20\sqrt{2} \cos 5t$$

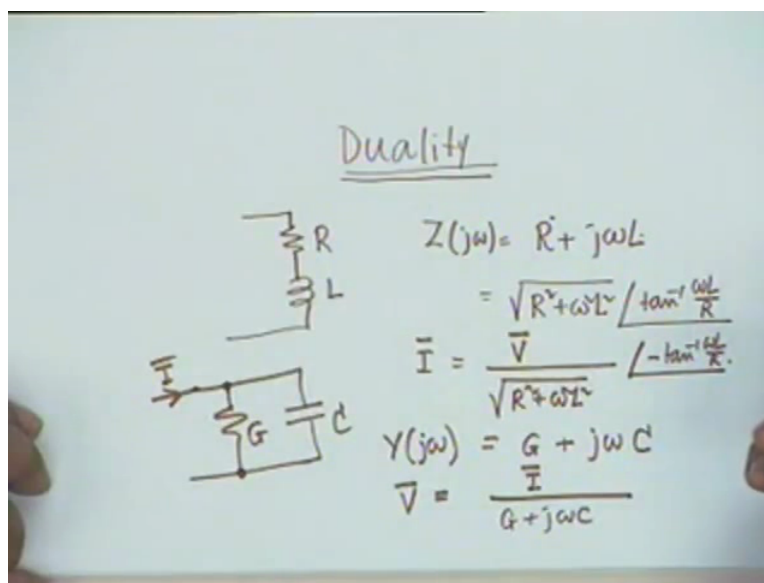
$\leftarrow \bar{V}_c \qquad \bar{V} \rightarrow$

If I now want, you remember I had this L and C series combination and I also said that there is the voltage C is also of our concern, all that is require to do is to take the phasor I L and multiply this by 1 over j Omega C that is the impedance of the capacitor, this is the current I L phasor and therefore V C phasor is equal to I phasor over j Omega C and I L was 5 by J, I can write in terms of angles on I can write in terms of complex representation and 1 by J, Omega is 5 and C is 0.05 and therefore 5 and 5 cancel. This is simply equal to – 20 means 20 angle Pie which means that if this is my V if this is the voltage phasor the exciting quantity then V C shall be in this direction V C phasor shall be opposite in phase to the exciting voltage phasor.

And in terms of current in terms of actual time domain quantity, v sub c t shall be equal to yes – 20 root 2, I can either write – 20 or cosine of 5 t + Pie it is the same thing so it is cosine 5 t if I

take the – sign here so this is what it means in terms of time domain quantity and this is the beauty of phasor analysis that as long as the excitation is sinusoidal, all quantities shall be all currents and voltages shall be sinusoidal and we can reverse the Omega t component that is the time dependent, we can work only in terms of numbers. There is a small modification which shall be needed if we are interested not in the forced response only but in the complete response that is that is the complete response shall be the forced response + the natural response and therefore you might add another term.

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But before we start talking about that there is a concept called Duality, which sometimes is important in electrical circuits and also in many other systems whether mechanical, civil or textile or whatever system is. The Duality simply refers to systems which are described by the same type of mathematical equation for example, for example the series R L circuit for example, series R L circuit the impedance $Z j \Omega = R + j \Omega L$ alright and its magnitude is square root of R square + Omega square L square and the angle is Tan inverse Omega L divided by R that is the angle.

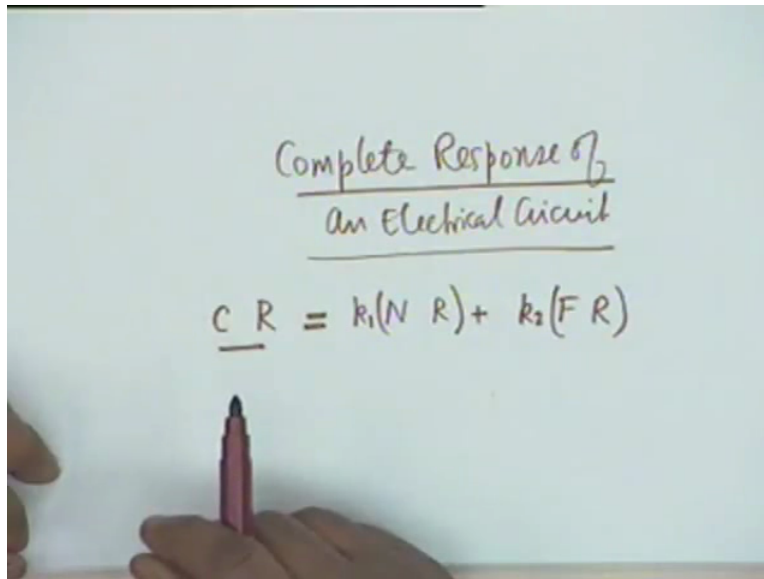
So if I if I want to find out the current phasor in the series circuit due to voltage excitation, all I do is V divided by square root of R square + Omega square L square and angle would be – the angle of the denominator shall be negative – Tan inverse Omega L by R. On the other hand, if I had let us say a conductance in parallel with a capacitance G and C then the appropriate

quantities because the components are in parallel appropriate quantity that we should consider is the admittance and you see that the admittance is exactly similar the place of R is taken by conductance, the place of L is taken by capacitance Ω is the same and instead of impedance we are talking of admittance.

Now, as you see here the current and voltage they interchange their roles. In other words if you wish to find out the voltage due to current excitation due to current excitation I then the voltage would be equal to current divided by current multiplied by impedance which means that current divided by admittance and if you take the magnitude and angle they are exactly similar. So without going further we can say that the series circuit series resistance reactive circuit series R L circuit is exactly similar exactly Dual of parallel RC circuit or parallel GC circuit because the mathematical equations are exactly similar except that physical quantities interchange their roles, such Duality concepts are extremely useful in advance level circuit analysis, in advance treatment of machines and many other systems including mechanical systems.

However, for the purpose of this course the concept the concept will not be utilised to the extent that it will in the future advance courses in electrical engineering so we shall leave the concept of Duality at this stage. The only thing which you shall remember is that there are circuits in which if you interchange the current and voltage, the relationships are exactly similar alright that is all that one wishes to remember. Now let us talk about the Complete response of an electrical circuit.

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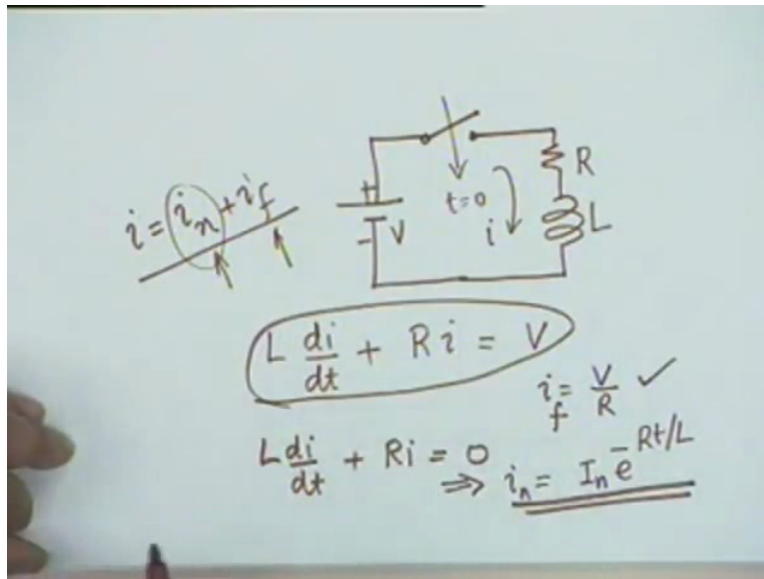


Complete Response of
An Electrical Circuit

$$\underline{C R} = k_1(N R) + k_2(F R)$$

As I had told you the complete response of an electrical circuit either voltage or current shall consist of 2 components, one is the natural response and the other is the forced response and it is a the complete response shall be the linear combination of the natural response and the forced response, a linear combination provided the circuit itself is linear provided we do not take we do not bring in nonlinear circuit elements into the picture the complete response shall be a linear combination of the natural response and forced response. And this is why we always write the complete response in terms of the natural response and forced response with some arbitrary constant and it is those constants that we evaluate from the initial condition. We shall approach this problem of finding out the complete response purely 1st purely from a mathematical point of view and then from physical point of view.

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To introduce the concept of complete response has considerable circuit like this, let us say an R L circuit to which we are considering an example only to illustrate this to which is applied let us say a voltage V a DC voltage V and a switch is time bound at $t = 0$, let us say to be specific that well the inductor may carry initial current or may not carry an initial current alright, it might have some initial energy it may not have some initial energy alright. Then what we say is that the obviously if the current in the circuit is i then the the differential equation obeyed by the circuit is $L \frac{di}{dt} + Ri = V$, we have been used to the right-hand side being equal to 0 to describe natural response.

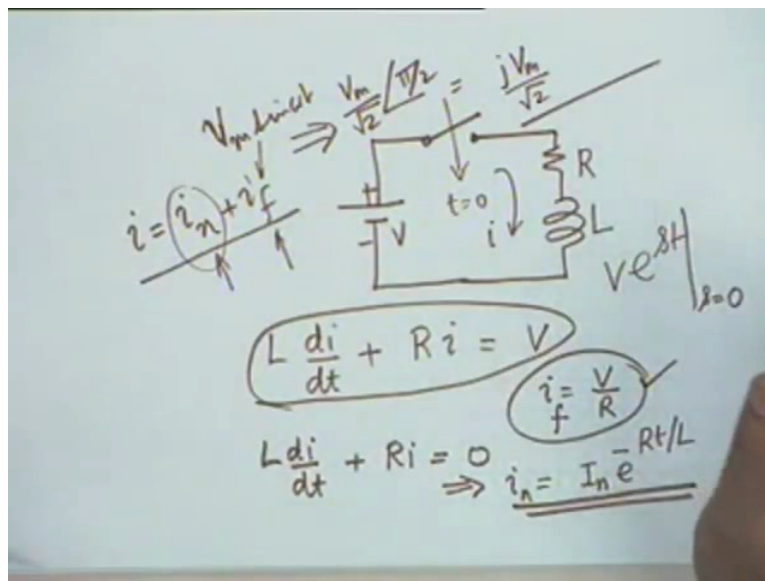
Now we have a forcing term, now mathematically as you know any solution any value of i , which makes left hand side equal to right-hand side is a solution alright. For example, $i = \frac{V}{R}$ here is a solution because $\frac{di}{dt}$ is 0 and you can see from physical point of view from circuit point of view that after a long time has passed indeed the inductance will behave as a short-circuit because its impedance Z of s is s times L and $s = 0$ and therefore after a very long time the current in the circuit shall reach this value and this is the steady-state value and this is the value that is obtained because of the forcing function capital V and therefore this is the forced response.

On the other hand, if you put the right-hand side equal to 0 that is you consider the homogeneous equation $L \frac{di}{dt} + Ri = 0$ and solve this equation then you know that the response the solution

would be of the form $I_n e^{-Rt/L}$ you know that the response would be of this form, so what does this quantity if this expression is substituted in the left-hand side of the homogeneous equation, what will be the value of the left-hand side, it has to be 0 and therefore if I put $i = i_n$ I called this $i_n + i_f$ purely from mathematical point of view, this equation will be satisfied because what i_n does is makes the left-hand side 0, what i_f does is makes the left-hand side equal to v and $0 + V = V$ and therefore we have 2 parts of the solution of the differential equation.

In terms of in terms of electrical circuit or physical concepts you see that this part is the forced response and this part is the natural response alright so we are adding the 2 to get the complete response. But mathematically this is justified because what i_n does is makes the left-hand side equal to 0, $0 +$ any quantity is equal to that quantity and therefore this is the complete solution. Its style of working shall be that we shall determine the forced response from impedance concept or e to the s t concept for example, the forced response if the right-hand side here is a constant then you know the forced response shall also be a constant and that constant can be determined from the fact that the exciting function here is of the form $V e$ to the s t which s equal to 0 and therefore I can work in terms of impedance, admittance by putting $s = 0$.

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For example here I argue that the impedance of the inductor shall be 0 because s equal to 0 and that helped us determine this V the response the forced response. On the other hand if this

exciting function was a sinusoid for example if this was $V_m \sin \Omega t$ then I know I can work in terms of phasors, what would have been the phasor corresponding to this? Angle $\frac{\pi}{2}$; $+\frac{\pi}{2}$ or $-\frac{\pi}{2}$? $+\frac{\pi}{2}$ alright, which means that I could represent this as $j V_m$ divided by $\sqrt{2}$ is that okay. Then to find out the current all that I have to do was to divide the voltage phasor by the complex impedance that is $R + j \Omega L$ and from that I could go back to the time domain equation, all I do is multiply the magnitude by $\sqrt{2}$ and take the angle inside the argument of cosine $\Omega t + \text{angle whatever the angle is}$.

So I can determine the forced response from the nature of the forcing function. On the other hand if the forcing function was let us say a decaying sinusoid decaying exponential like $e^{-\alpha t}$ then also I can work in terms of the the impedance function, all that I do is $s = -\alpha$ alright. And therefore the forced response I can determine from observation of the circuit, for the natural response I again apply the impedance concept if it is to be a current then I measure the impedance then I break the circuit and I measure the impedance between the 2 breakpoints.

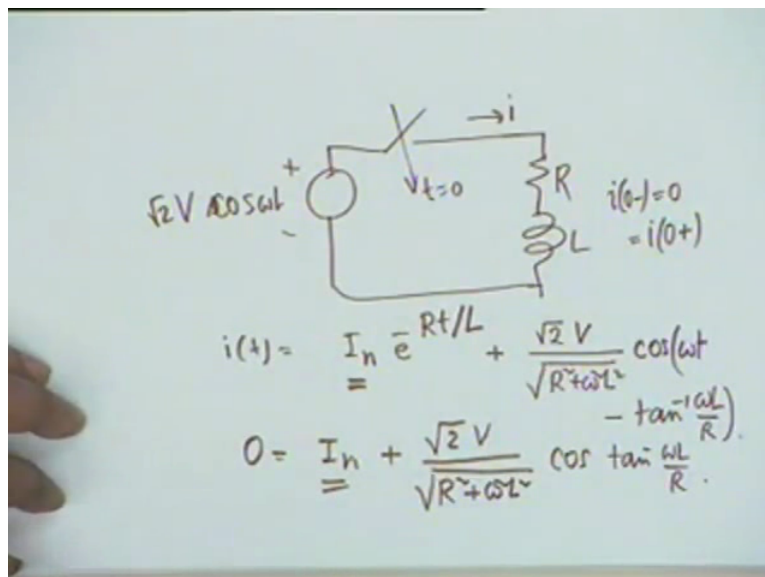
The zeros, I determine the poles and zeros, the 0 shall determine the frequency of natural response of current natural response and I write the solution with 1, this is the 1st order circuit so one undetermined constant, if it is a 2nd order circuit and I introduced 2 constants there will be 2 natural frequencies and I add the natural response and the forced response. The forced response is completely known, it is the constants in the natural response that we have to find out and we find them out from the initial conditions alright. Let us take an example as an illustration of the style of working yes.

Student: Sir what is the () (24:42)

Professor: That is what the fact of line phase, whenever you switch on excitation to an electrical circuit for example, whenever you switch on the light or the light itself represents a RLC circuit for example, there could be capacitance in parallel, resistance and inductance in series, so the light does not glow immediately, there is a certain amount of time before the current builds up till the final value alright and what we get is the final value is generally the forced response or the steady-state response, in between there are transients.

There are situations when the motor when switched ON suddenly it burns off, on the other hand the same voltage if it was in the steady-state the motor runs perfectly alright, it is because the transient current or the natural response current + the forced response current at the time of switching $t = 0 +$ is much more than what the motor can stand and therefore it burns away alright so that is the significance of complete response. Complete response is what happens in practice, it is what our convenience that we break up into 2 parts; natural response and the forced response, it is by adding these 2 that we can determine total response and we shall take an example to illustrate this a little later.

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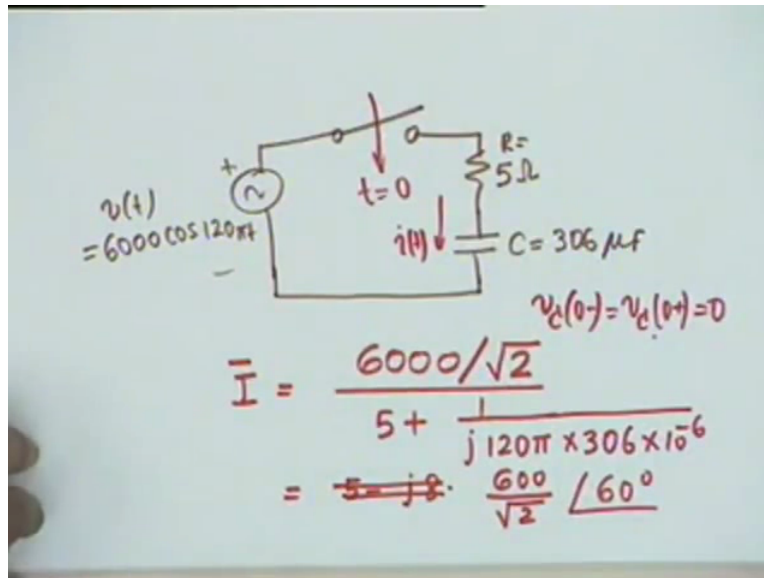


The same circuit to be specific the same R L circuit let us say the same R L circuit if it is supplied from a sinusoidal source $\sqrt{2} V \cos \Omega t$ and the switch is put ON at t equal to 0, the current in the circuit from the discussion that we have done so far will be of the form $I_n e^{-Rt/L}$ to the $-Rt/L$ this is the natural response and can you tell me what the forced response shall be in the time domain? Yes it will be $\sqrt{2} V$ divided by $\sqrt{R^2 + \Omega^2 L^2}$ cosine of $\Omega t - \tan^{-1} \frac{\Omega L}{R}$. Yes I can write it down by inspection, square root of $R^2 + \Omega^2 L^2$ cosine of $\Omega t - \tan^{-1} \frac{\Omega L}{R}$. This angle is in the denominator, $-\tan^{-1} \frac{\Omega L}{R}$.

And all that we have to do now is to find out I_n and you know in the end of the name there is no initial current then $i(0^-)$ shall be equal to 0, which is also to be equal to $i(0^+)$ and therefore all you have to do is put $t = 0$ here then equate 0 to $I_n + \sqrt{2} V$ divided by square root of R

square + Omega square L square then cosine of - Tan inverse, cosine of negative angle is same as cosine of the corresponding positive angle so Tan inverse Omega L by R and from which you can find out I n the constant the unknown constant. This is the style of working so what we will do, we will take a fairly complicated example and illustrate okay and illustrate we will take a series of examples now to illustrate forced response.

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Let us take a tough one 1st, we have a voltage $v t = 6000$ fairly high-voltage 6 kilovolts Cosine of let us say $120 \text{ Pie } t$, do not worry about 120, it is the United States frequency 60 hertz so 2 Pie times 60 is 120, I took this example from an American book and that is why it figures here, I could change to $100 \text{ Pie } t$ 50 hertz but I did not feel like it does not illustrate anything new alright. There is a switch and there is an RC circuit 5 Ohms R and the capacitor C is 306 microfarad, the problem is as simple as that and this is closed at $t = 0$, you have to find out the total current response that is $i t$.

If the capacitor was initially uncharged then $v \text{ sub } c \ 0 - = v \text{ sub } c \ 0 +$ would be equal to 0. I do not need to indicate the polarity of $v c$ because I have shown the direction of the current, it means that $v c$ is positive up and negative down alright. Although the circuit looks very simple, the solution as you will see illustrates a very large number of important points. Now the forced response and let us 1st indicate the forced response, for the forced response obviously the forced response because it is sinusoidal the forced response will also be sinusoidal and all that I have to

find out is the current phasor, the current phasor would be the voltage phasor which is 6000 angle 0 cosine 120 Pie t divided by R + the impedance that is 5 + 1 over J, Omega is 120 Pie times 306 times 10 to the - 6, this is a current phasor whatever root 2 yes this would be divided by root 2 thank you for pointing this out.

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The image shows handwritten mathematical derivations on a whiteboard. The equations are as follows:

$$i_f(t) = 600 \cos(120\pi t + \frac{\pi}{3})$$

$$Z(s) = R + \frac{1}{sC} = \frac{sCR + 1}{sC}$$

$$= R \frac{s + \frac{1}{CR}}{s}$$

$$i_n(t) = I_n e^{-\frac{t}{CR}} = I_n e^{-654t}$$

$$i(t) = I_n e^{-654t} + 600 \cos(120\pi t + \frac{\pi}{3})$$

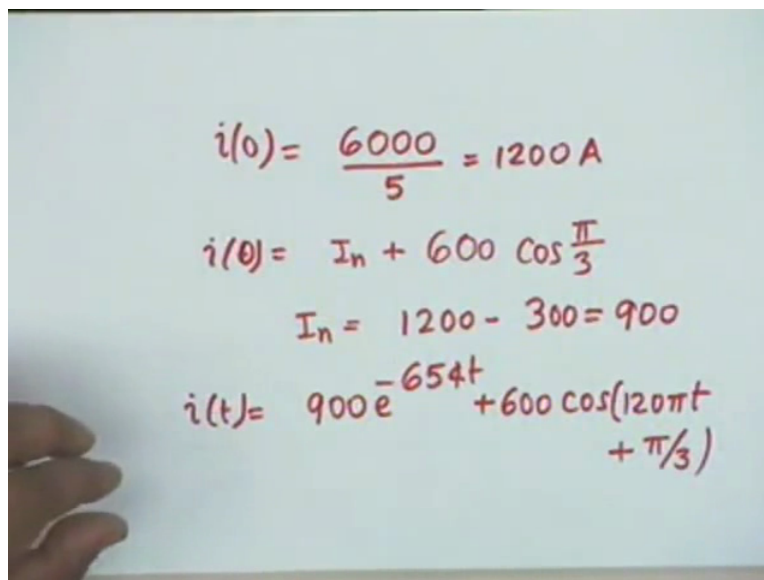
Well if you do this calculation well I skip the algebra the complex algebra, the calculation gives you simply $5 - j 8$ point no it gives you 600 divided by root 2 and angle is 60 alright this is what the current phasor is and therefore the current the forced response component of the current is given by $i_f(t) = 600$ by root 2 you multiply by root 2 that will repeat value cosine of $120 \text{ Pie } t + \text{Pie by } 3$ I must write it in radianse okay, now the natural response. For the natural response I again find the impedance Z of s and the impedance is, since I am finding a current and I already have a switch so all you do is between these 2 points you find the impedance and impedance is simply $R + 1$ over $s C$ alright.

The impedance is $R + 1$ over $s C$, which is $s C$ divided by $s C R + 1$ and there is a certain discipline that we had incorporated, we wanted to write the numerator and denominator with leading coefficients = unity and therefore I have $R s + 1$ over $C R$ divided by s alright. Since we are trying to find the current response natural current response, it is the 0 which shall determinate and therefore i_n of t the natural response will be equal to capital I_n some constant e to the $- t$ by $C R$ and this happens to be if you include the values, if you insert the values you get e to the $-$

654 t alright and therefore the total current, i by C R is 654 therefore the total current i of $t = I_n e^{-654t} + 600 \cos(120\pi t + \pi/3)$, this is the total response of the circuit.

The question, well you still have not found the I_n so you have to find I_n and I_n we shall find out from the initial condition, what was initial condition that $v_c(0^-) = v_c(0^+) = 0$ therefore, $i(0^+)$ that is when the switch is put ON shall be simply this voltage by R alright and therefore $i(0)$ would be equal to well the voltage is 6000 cosine $120\pi t$, if $t = 0$ it is simply 6000 divided by 5 which is 1200 amperes alright.

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$$i(0) = \frac{6000}{5} = 1200 \text{ A}$$

$$i(0) = I_n + 600 \cos \frac{\pi}{3}$$

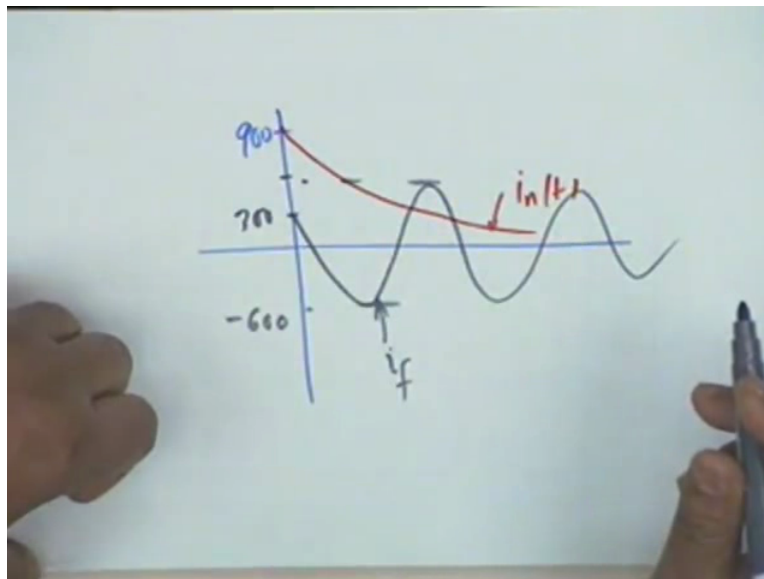
$$I_n = 1200 - 300 = 900$$

$$i(t) = 900 e^{-654t} + 600 \cos(120\pi t + \pi/3)$$

No this is the current, the voltage it in the time domain at $t = 0$ so (35:05) it is not the phasor, 1200 amperes. And therefore your i of t well i of 0 also is $I_n e^{-654t} + 600 \cos(120\pi t + \pi/3)$ and therefore $I_n = 1200 - \cos(120\pi t + \pi/3)$ is half so 300 that is equal to 900. In other words the total current response i of $t = 900 e^{-654t} + 600 \cos(120\pi t + \pi/3)$. I want you to look at this expression very carefully and you notice that when t tends to infinity that is under the steady-state when a long amount of time has passed, the current will be totally sinusoidal because this term shall become 0 and totally sinusoidal with a maximum or peak value of 600 amperes and a minimum of -600 amperes alright, so the current fluctuates between $+600$ and -600 in a sinusoidal manner with the frequency of 60 hertz 60 cycles per second.

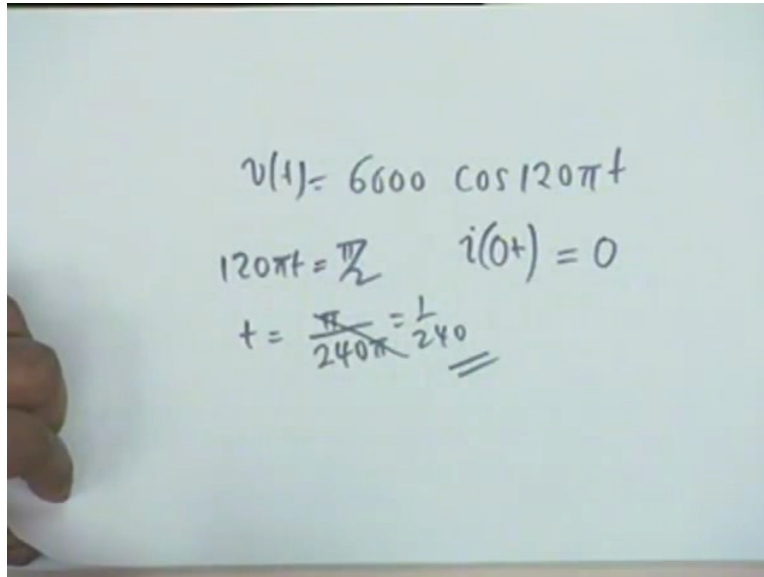
However, at this instant of switching the current is double of what the maximum or minimum under steady-state is, the current is 1200 amperes and if R C that combination represents an electrical equipment which is not able to stand 1200 amperes will it will burn alright so this is what the natural response does in the transient state that is the state before it before the current is stabilised in the circuit, the current may exceed the limit that is within the tolerance of the electrical circuit or the equipment at hand.

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The question now is, well to complete this this exercise you notice that $i_n(t)$ decays exponentially like this, this is i_n starts from 900 with time constant of what is the time constant? $1/654$ that is correct that is the time constant. And $i_f(t)$ varies like this, it starts from what value? 300 because at $t = 0$ $600 \cos(\pi/3)$ so it starts at 300 and then goes very sinusoidal like this alright this is my i_f , this is 600 and this minimum is -600 alright. And it is the sum of these 2 that is the total current so the total current initially is not purely sinusoidal, it has an exponentially decaying component and as time proceeds after sufficiently long time it would become purely sinusoidal and in between $+600$ and -600 but at the time of starting and for quite some time the effect of the exponentially decaying waveform or the natural response component shall show its state, it might injure the equipment.

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Handwritten mathematical equations on a whiteboard:

$$v(t) = 6000 \cos 120\pi t$$
$$120\pi t = \frac{\pi}{2} \quad i(0^+) = 0$$
$$t = \frac{\pi}{240\pi} = \frac{1}{240}$$

On the other hand, if you notice the condition of the problem, the switch ON at $t = 0$. At $t = 0$ the voltage that we had applied what was the voltage, $6000 \cos$ of $120 \text{ Pie } t$, at $t = 0$ the voltage is maximum and that is perhaps why the starting current was so large 1200. Suppose we had switched ON when this voltage was passing through 0 then what would have been i of 0^+ , it would have been 0. Suppose this taken point was when $120 \text{ Pie } t = \text{Pie by } 2$, I could have waited till the sinusoid reached its Zero crossing alright first Zero crossing and that happens at $\text{Pie by } 2$. In other words $t = \text{Pie by } 240 \text{ Pie}$ in other words $1 \text{ by } 240$.

If I had waited if the switch was not put at $t = 0$ that it was put at $t = 1 \text{ by } 240$ then the initial current would have been 0 is not it right? Why 0? Because v of t is 0 and v c 0^+ is 0 and therefore the drop in the 5 Ohms resistance would have been 0 and the current would have been 0, does this guarantee then that there will be no natural response or no transient part in the solution? No it does not it does not.

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$$i(t) = 600 \cos(120\pi t + \frac{\pi}{3}) + I_n e^{-654t}$$

$$t = t_1$$

$$600 \cos(120\pi t_1 + \frac{\pi}{3}) + I_n e^{-654t_1} = \frac{6000 \cos(120\pi t_1)}{5}$$

$$t_1 = \frac{1}{360} \text{ s}$$

If you look at let's look at the total solution 600 cosine of 120 Pie t + 60 I am sorry I should put Pie by 3 + I n was how much, no it was 900, 900 e to the - 654 t this is the total current response alright. The question now being asked him, now listen to the question carefully, is there an instant of time t at which if we switch there will be no transient? How did we evaluate this? This was evaluated by assuming that the switching ON was done at t = 0 so if I want to change the starting point the starting time I should not use 900 I will use I n alright because I n varies with the instant at which it was switched ON agreed alright. Example if the switching ON was at t = 1 by 240 second which we have calculated what would have been I n? Pardon me... What would have been I n?

Student: - 300.

Professor: Yes, it would have been - 300 because of t = 0 then the current will be 0 agreed.

So I n depends on the instant of switching, the question that is being asked listen to me carefully, is there a switching instant of time let us say t = t 1 such that there is no transient in this circuit.

Student: 300 ampere (42:53) .

Professor: I am asking about time, the answer cannot be in volts for amperes.

Is there an instant of time t_1 , this is quite a tricky question. Is there an instant of time t_1 such that when I switch on at t equal to t_1 there are no transient in the circuit? Well the condition would be this, look at the condition $600 \cos(120\pi t_1) + \frac{1}{3} e^{-654 t_1}$, this should be equal to steady-state current which is $6000 \cos(120\pi t_1) / 5$, t_1 is the initial instant at which it is switched on at which it is switched on when the current would be $6000 \cos(120\pi t_1) / 5$, we have already seen if t_1 is 0 then this is 1200 amperes. If t_1 is $1/240$ seconds then it is 0 alright. So if the total response if the total response is equal to this then obviously we can find out the value of t_1 if I put if I put what?

“Professor–student conversation starts”

Student: Sir in the left-hand side of the equation we should put t_1 we should put 0.

Professor: No.

Student: (())(44:53) because when we are switching on it should be 0.

Student: Yes sir.

Student: Yes sir.

Professor: What should be 0?

Student: I n.

Professor: I n should be 0 that is correct. It is that condition under which there is no transient and you can solve for t_1 from here. My solution is that t_1 is $1/360$ seconds. The total response is this, at $t = t_1$ if I stipulate the condition that at $t = t_1$ we do not know what t_1 is, the switch is put on and there are no transients in this circuit, the current is the steady-state current so it should be at $t = t_1$ also and at $t = t_1$ the current through the circuit steady-state current is this $6000 \cos(120\pi t_1) / 5$ because $v_c(t_1) - v_s(t_1) = 0$, $v_c(t_1)$ should also be equal to 0.

Student: (())(46:03)

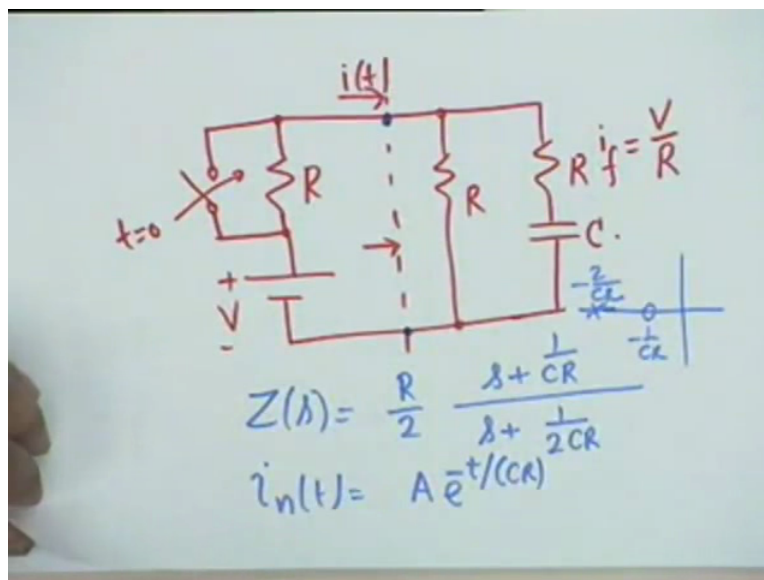
Professor: Yes after a long time what would be the value?

Student: Sir but (())(46:13) steady state.

Professor: Because I have added the steady-state current along with the transient current okay that is in this form and I stipulate the natural response I want it to be 0. What is the unknown? Unknown is (())(46:31) this is the initial current and you are taking this to the steady-state current, this is the steady-state current, steady-state current expression is this so I am equating the steady-state current to the initial current in the resistance R and I am saying that I want that I n should be equal to 0. Now what I want you to do is to find out the total response if the switch in ON is at t 1 = 1 by 360 and show that indeed this is true alright.

“Professor–student conversation ends”

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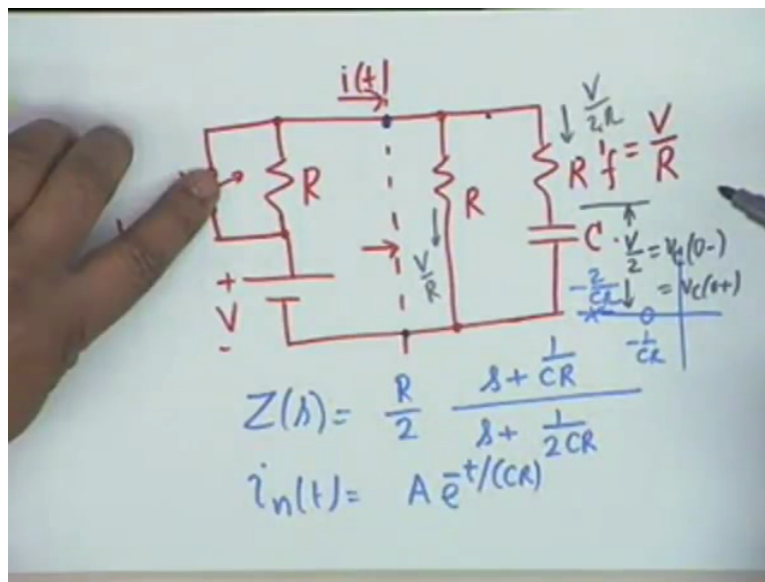


Let us take another example, this is not the run-of-the-mill problem, there is no solution given in the text so you will have to work it out. We look at the problem quickly and this time it is not sinusoidal, it is a simple battery type of circuit battery excitation. There is a switch which closes at $t = 0$, across the resistance R there is a battery $V + -$ and there is a resistance R and there is a resistance and capacitance $R C$. What we are asked is 1st to find the pole-zero diagram of the impedance between these 2 points pole-zero diagram, to determine an expression for $i t$ and third to sketch $i t$ during the transient period that is to make the sketch of $i t$ the current in the circuit.

Well if you solve this, I will skip the algebra, if you determine the impedance between these 2 points okay, the expression becomes simply R by 2 you might check this, $s + 1$ by CR $s + 1$ by 2 CR where I follow the same kind of convention, there is a constant term, leading coefficient is unity and therefore the natural response of the circuit natural response would be determined by this frequency $s = -1$ by CR and therefore this current i in the natural response would be some constant $A e^{-t/CR}$ alright. Poles and zeros you can show at -1 by CR there is the 0 and -2 by CR there is the pole alright, this is the pole-zero diagram.

To find $i(t)$ we have to add the natural response to the steady-state or the forced response, now what is the forced response? Because this exciting voltage is a DC, the forced response will also be DC and you can see that if $s = 0$ then this acts as open and therefore there is no current through this path agreed, only current shall exist here and when this which is closed, the voltage existing across the resistance is capital V and therefore your immediate conclusion is i must be equal to V by R alright.

(Refer Slide Time: 50:25)



Therefore, the total current $i(t)$ shall be equal to V by $R + A e^{-t/CR}$ divided by CR . The question now is, how do I determine the constant A ? Obviously, I shall have to take i of 0 into account, the initial condition however what is the initial condition?

Student: V by $2R$.

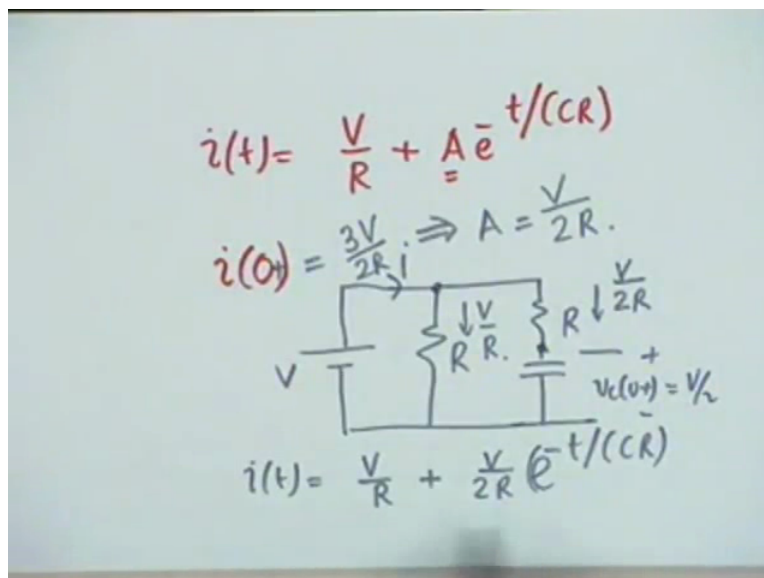
Professor: V by $2R$, how is that? How is that determined? What is this voltage?

Student: V .

Professor: No, at 0^+ ? Pardon me...

At 0^- , this was off this was open so there was a voltage division between R and R and therefore this voltage is V by 2 , there was no current in R therefore this must have been V by 2 at V c 0^- or V c 0^+ is that clear? At $t = 0^-$ the switch was open and therefore this battery would see a resistance R , another resistance R , this is open alright therefore the voltage across R is V by 2 and since there is no current the voltage across C shall also be V by 2 . At t equal to 0^+ this switch is closed, this current at $t = 0^+$ is V by R and this current at t equal to 0^+ is $V - V$ by 2 divided by R so V by $2R$ is that clear? You have to think about it, let me draw the circuit once more.

(Refer Slide Time: 52:27)



Capacitor R , RC , we are looking at the condition of $t = 0^+$, at 0^+ the battery comes across here, this is my i , this is R , this is R and this $v_c 0^+ = V$ by 2 and therefore the current through this R voltage is V , this voltage is V by 2 so this current is V by $2R$ and this current is V by 2 therefore i of $0^+ = 3V$ by $2R$ from which you get A as equal to V by $2R$ is that correct? And therefore, + or -? + okay and therefore the total current response is V by R + V by $2R$ e to the power $-t$ by CR , does it satisfy the initial conditions of the problem? At $t = 0$ $t = 0$ the current is V by R + V by

$2R$, which means $3V$ by $2R$, at $t = \text{infinity}$ current is simply V by R , so it does satisfy and this is the total response of the circuit.

Next time Monday we will take a couple of more examples to illustrate complete response.