

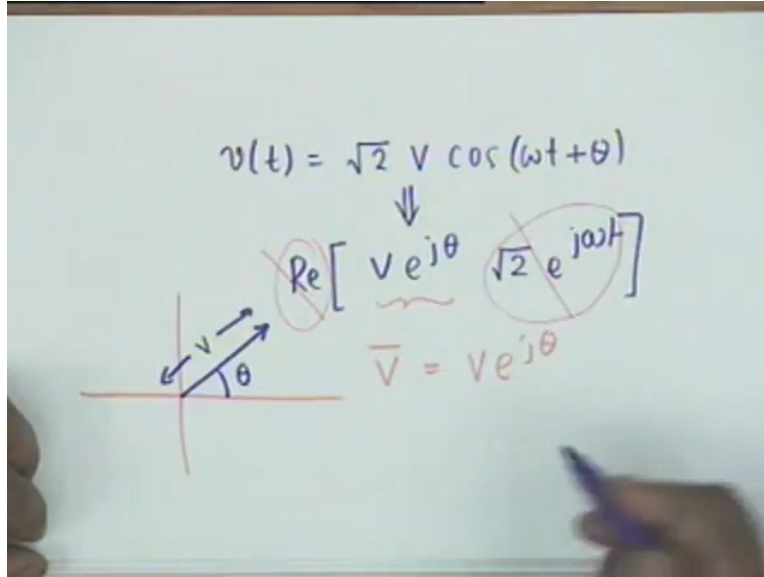
Introduction to Electronic Circuits
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Lecture no 17

Module no 01

Phasors and their Applications in AC Circuits Analysis

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This is the 17th lecture on Phasors and their application in AC Circuits analysis. As we recall in the last lecture the 16th, we introduced an artificial concept of phasors, which basically transforms a voltage or current which is of the form root 2 V cosine of Omega t + Theta into a phasor which is represented as $V e^{j\theta}$ and then the cosine Omega t and root 2 is taken care by root 2 e to the power j Omega t and you take the real part of this, this is obviously equal to this real part of this. And here since all currents and voltages in the circuit shall contain this factor, we decided to ignore this and we also decided to ignore the real part.

In other words, if these 2 are cut out, this will be implied for all voltages and currents then the quantity that is left is represented by a phasor \bar{V} with a bar above it and we call this $V e^{j\theta}$ and in the complex plane we represent this by means of vector whose magnitude is V and whose angle from the real axis is Theta. If I do this then obviously this is a transformation from the time domain into the complex number domain complex number domain.

And it has been shown that with the help of phasors the circuit analysis, steady-state circuit analysis or forced response of a circuit to a sinusoidal excitation becomes extremely simple.

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$$\begin{aligned}
 & \text{Circuit diagram: } i \rightarrow \text{---} \underset{R}{\text{---}} \\
 i &= \sqrt{2} I \cos \omega t \\
 \bar{I} &= I e^{j0} = I \angle 0 \\
 \bar{V}_R &= \bar{I} R = V_R \angle 0 \\
 V_R &= \sqrt{2} I R \cos \omega t \\
 \frac{\bar{V}_R}{\bar{I}} &= R = R \angle 0^\circ
 \end{aligned}$$

Let us consider the 3 basic elements namely a resistance, an inductance and a capacitance. Resistance suppose I excite this by means of a current i which is $\sqrt{2} I \cos \omega t$ let us say cosine of Ωt then obviously the corresponding phasor is \bar{I} , which is equal to $I e^{j0}$ or angle, I also write this times as $I \angle 0$ alright. Now if this passes through a resistance R , the corresponding voltage phasor \bar{V}_R shall be simply \bar{I} times R alright, in other words the voltage the actual voltage you must be able to go back and forth between the time domain and the complex number domain.

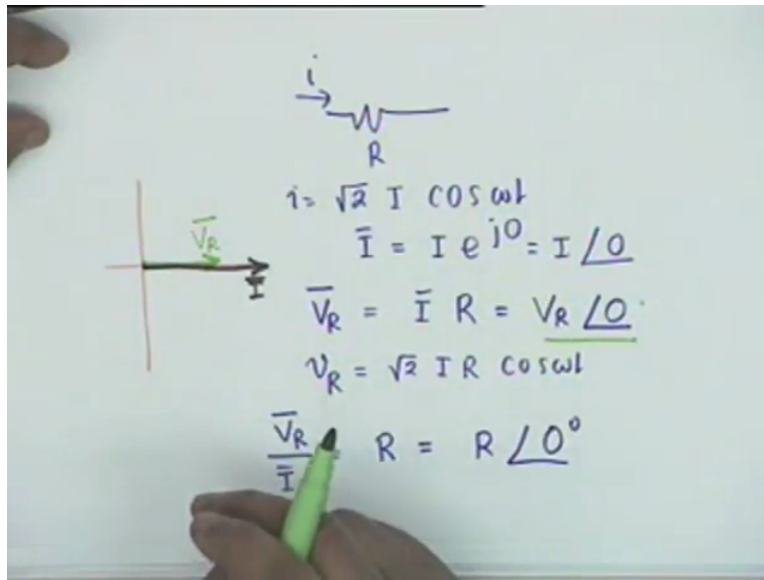
The actual voltage shall be equal to $\sqrt{2} I R \cos \omega t$, this is equal to obviously you see the phase is 0 , so there is no phase change between the current and voltage, it is true in the time domain it is also true in the transform domain and the ratio of the voltage phasor to the current phasor is simply equal to R , which you can write as $R \angle 0^\circ$ alright.

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The image shows a whiteboard with handwritten mathematical derivations and a circuit diagram. At the top, a circuit diagram shows an inductor with inductance L . An arrow labeled i points to the right above the inductor. Below the inductor, the voltage is labeled $+ v -$. To the right of the diagram, the current is given as $i = \sqrt{2} I \cos \omega t$. Below this, a downward arrow points to the phasor $\bar{I} = I \angle 0^\circ$. The voltage is then derived as $v = L \frac{di}{dt} = -\omega L \sqrt{2} I \sin \omega t = \sqrt{2} \omega L I \cos(\omega t + \pi/2)$. Finally, the voltage phasor is given as $\bar{V}_L = \omega L I \angle \pi/2 = V_L \angle \pi/2$.

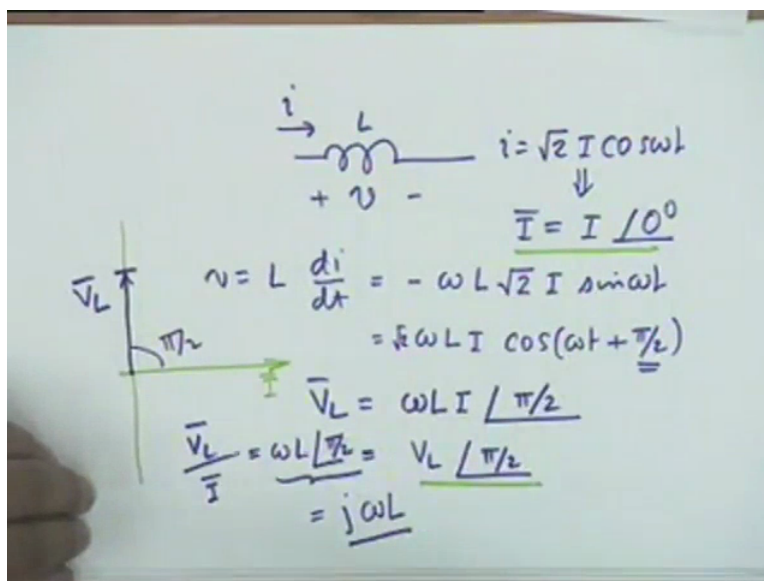
Now this is a trivial case, if we take an inductance now and cause the same current i to flow that is $i = \sqrt{2} I \cos$ of Ωt which corresponds to the phasor I equal to I angle 0 degree then you know the voltage across this is given by $L \frac{di}{dt}$ the actual voltage, this will be $-\Omega L \sqrt{2} I \sin \Omega t$ and $\sin \Omega t$ can be written as $\Omega \sqrt{2}$ times $\Omega L I$ can be written as \cos of $\Omega t + \frac{\pi}{2}$ agreed. So if I write this as a phasor obviously the phasor shall be $\Omega L I$, we L phasor shall be $\Omega L I$ this would be the magnitude and the angle would be $\frac{\pi}{2}$, this is the angle $\cos \Omega t + \theta$ alright. I can write this as V_L its magnitude angle is $\frac{\pi}{2}$.

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So in the case of the resistor if we go back, if we wish to draw a diagram that is the currents or voltages then you see the current is in this direction let us say this represents the current phasor \bar{I} then the voltage phasor is in phase with the current phasor that is the voltage phasor also has an angle 0 to the voltage phasor maybe up to this on different scales, current and voltage cannot be represented on the same scale for this would be \bar{V}_R phasor.

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On the other hand, if I represent the current voltage relationship in an inductor, the current phasor is this and the voltage phasor is this and therefore, if the current phasor is along the real axis I , then the voltage phasor shall be along the imaginary axis shall be along the perpendicular axis because it leads by an angle of $\text{Pie by } 2$, so the voltage across an inductor voltage phasor across an inductor leads the current by an angle of $\text{Pie by } 2$ this may be represented as the voltage phasor, where this angle is $\text{Pie by } 2$ alright. With respect to I as the reference the current as the reference, the voltage phasor across an inductance leads it by an angle of $\text{Pie by } 2$.

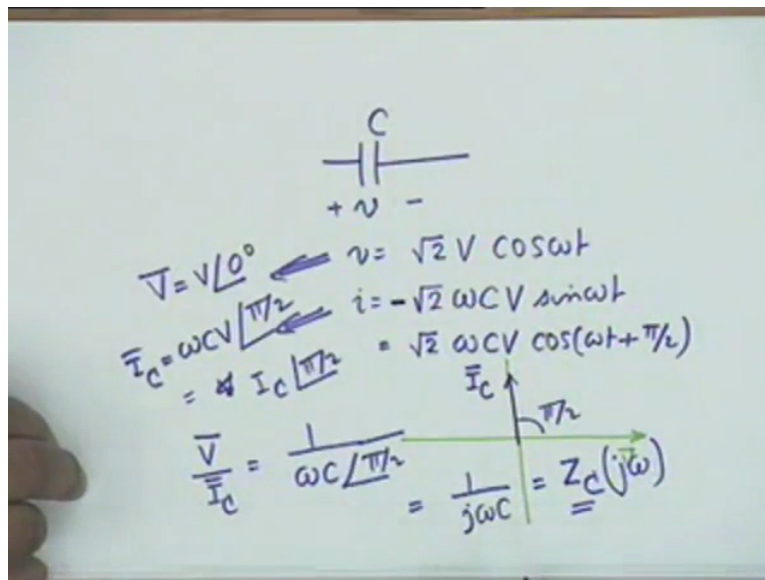
We also notice that the ratio of $V L$ phasor to I phasor the ratio of the 2 is simply given by $\text{Omega Omega } L \text{ angle Pie by } 2$, now is that a phasor? No. Is this a phasor? It is not a phasor. A phasor is used to transform a time domain quantity into the complex number domain. Now corresponding to $\text{Omega } L$ there is no time domain quantity so this is simply a complex quantity whose magnitude is $\text{Omega } L$ and angle is $\text{Pie by } 2$, what is such a complex quantity? Obviously it is j times $\text{Omega } L$, the magnitude is $\text{Omega } L$ and angle is 90 degree that is why it is purely imaginary j times $\text{Omega } L$ alright.

And you notice that for an inductor, the voltage and the current if one of them is sinusoidal if the current is sinusoidal I am sorry, if the current is exponential e to the $s t$ then the voltage is $s L$ times e to the $s t$ and $s L$ was defined as the impedance of the inductor, and you see that this quantity of voltage phasor to current phasor when the excitation is sinusoidal, it is simply given by $s L$ with s replaced by $j \text{ Omega}$ and therefore this can be called as the impedance $Z L$ of the inductor to a sinusoidal excitation of frequency Omega that is instead of e to the $s t$ what we have is e to the $j \text{ Omega } t$ and therefore s is replaced by $j \text{ Omega}$, and $j \text{ Omega } L$ is the impedance of the inductor.

To make this explicitly say $Z L$ of $j \text{ Omega}$ that is impedance of the inductor at a sinusoidal frequency of Omega that is s is replaced by $j \text{ Omega}$ alright. In that sense what is the impedance of resistor? Irrespective of the value of s whether it is DC or an exponential truly exponential quantity or a sinusoidal quantity, impedance is always equal to R and angle is 0 degree. However, I must caution you $j \text{ Omega } L$ is not a phasor, it is a complex quantity $j \text{ Omega } L$

alright. It is not a phasor, a phasor is a current or voltages which is the presentation of the corresponding quantity in the time domain, it is the transform of current or voltage.

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If I know now take a capacitor and allow the voltage across the capacitor C to be let us say root 2 $V \cos \omega t$ alright then the corresponding current is $C \frac{dv}{dt}$ and therefore root 2 $\omega C V \sin \omega t$ alright, the current is $C \frac{dv}{dt}$ is equal to this which I can write as root 2 $\omega C V \cos(\omega t + \pi/2)$.

Student: $\pi/2 - \omega t$ (11:46)

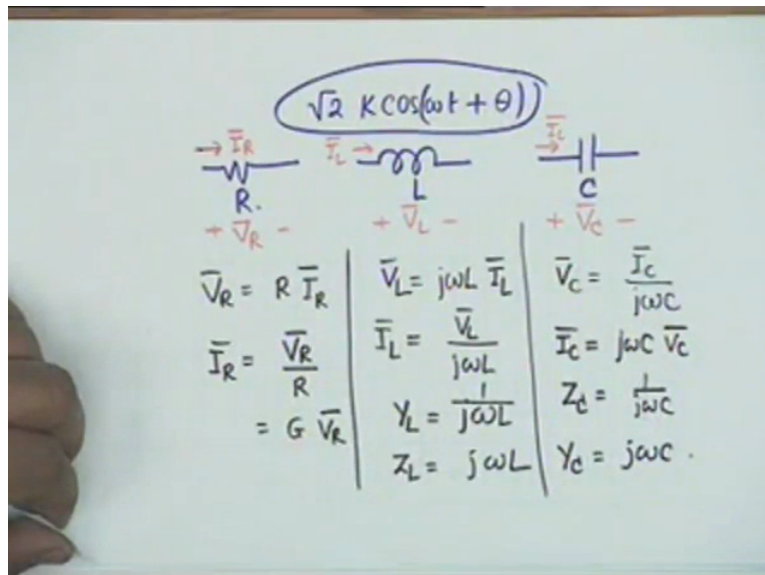
This is minus, $-\omega t$ can be written as $\cos(\omega t + \pi/2)$ yes, is that okay? I do not want $\cos(90 - \theta)$ that is $\sin \theta$ there is no sign change there is a sign change here, from cosine you go to sine and therefore $\cos(\omega t + \pi/2)$. And therefore if the corresponding phasor quantities are denoted by V then V phasor is simply $V \angle 0^\circ$ and I_c phasor the current in the capacitor is simply $\omega C V \angle \pi/2$, which indicates that is V sub no, this is I_c magnitude angle $\pi/2$ and therefore, in the phasor diagram on the real-imaginary or the complex plane diagram if this is V if this is the phasor V alright then the current leads the voltage phasor by an angle of 90 degrees.

And therefore the current would be like this I_c phasor and this angle is $\pi/2$ alright. So we need a capacitor, the current leads the voltage by an angle of 90 degrees, we always take

rotation in the anticlockwise direction as positive so we say the current leads the voltage by an angle of $\pi/2$. Correspondingly in the case of inductor, the voltage leads the current by an angle of $\pi/2$, if I take the ratio of voltage phasor to the current phasor you see that the ratio is simply $1/\omega C$ and the angle is $\pi/2$. Now complex quantity which is ωC magnitude ωC and the angle is $\pi/2$ obviously is a purely imaginary quantity and therefore, its actual representation will be $j\omega C$.

And since it is the ratio of a voltage phasor to a current phasor, this is the impedance of the capacitor Z_C alright the impedance of the capacitor is Z_C is that okay? Impedance Z_C is not a phasor alright, this is simply a complex quantity and we notice that it is quite in tune with the definition of the impedance for e to the s t excitation, e to the s t excitation the impedance was $1/sC$ and here s is replaced by $j\omega$ and therefore you say $Z_C = j\omega C$ alright impedance of the capacitor.

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If I summarise this relationship for a resistance, inductance and a capacitance $L C R$ then the relationship between the current and voltage phasors it be it is implied that all excitations all currents and voltages are of the form $\cosine \omega t + \text{some constant } \theta$ root 2 times a constant, all currents and voltages are of this form. If that is so then the current in the resistance where you can now represent only in terms of phasors, let us do that, this is I_R phasor and the voltage phasor is V_R , this is I_L phasor and the voltage phasor is V_L , I_C and the voltage

phasor is V_C with this polarity. Then the relationships are that V_R is simply equal to R times I or the other way round $I_R = V_R$ divided by R which I can write as G times V_R , where G is the conductance alright G is the conductance of the resistor.

Similarly, for a volt for an inductor we have $V_L = j\omega L I_L$, $j\omega L$ is the impedance of the inductor or I_L is equal to V_L divided by $j\omega L$ alright, which indicates that the admittance of an inductor Y_L is simply equal to j by $j\omega L$ that is correct, and its impedance $Z_L = j\omega L$ alright. For a capacitor for a capacitor, $V_C = I_C Z_C$ would be equal to I_C multiplied by its impedance that is by $j\omega C$, is that okay? Or $I_C = V_C / Z_C = V_C / (1 / j\omega C)$, the impedance of the capacitor is $1 / j\omega C$ and the admittance of the capacitor is $j\omega C$.

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Handwritten notes on a whiteboard showing formulas for impedance and admittance of inductors and capacitors:

$$Z = R + jX$$

$$Z_L = j\omega L = jX_L$$

$X_L = \omega L = \text{reactance of } L$

$$Z = R + jX$$

$$Y_C = j\omega C = jB_C$$

$B_C = \omega C = \text{susceptance of } C$

$$Z_C = \frac{1}{j\omega C} = j\left(-\frac{1}{\omega C}\right) = jX_C$$

$X_C = -\frac{1}{\omega C}$

Whenever a quantity, impedance or admittance is purely imaginary, the real part of that impedance or admittance goes by a special name and you should introduce the name here itself, the name is if there is an impedance which is purely imaginary let us say let us consider the inductors for example, if $Z_L = j\omega L$ then the real part of this is known as reactance that is what we do is we write this as jX_L , where $X_L = \omega L$ and it is called the reactance of inductance L . Similarly, when we write Y_C as equal to $j\omega C$, I write this as jB_C okay, j times real quantity where $B_C = \omega C$ and this is called the susceptance susceptance of C susceptance alright.

It is not that we could not write impedance also, Z of a capacitor impedance of a capacitor is 1 over j ΩC , which I can write as j times -1 by ΩC . When we write this as j times $X C$ then $X C$ is called the reactance of the capacitor C , reactance of a capacitor is negative, the reactance of an inductor is positive, as you see the reactance a capacitor is -1 by ΩC . We can, after you define the impedances for sinusoidal expectations we can combine them exactly in the same manner that we combine impedances for e to the s t excitation or combined resistances. For example...

“Professor–student conversation starts”

Student: Excuse me sir.

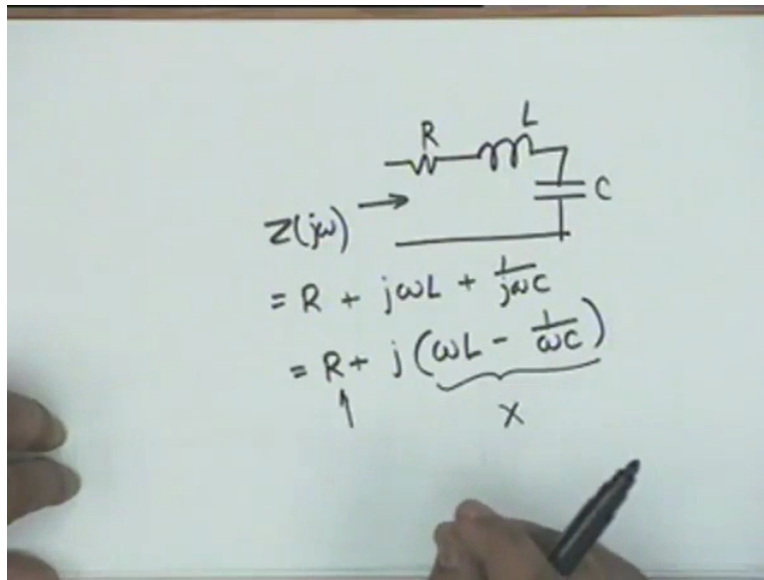
Professor: Yes.

Student: Suppose we have impedance of the form $A + j B$ then even in this case we deal with this reactance?

Professor: That is right, we reserve the symbol B for susceptance, for impedance we will use the symbol capital X alright and the real part of the impedance. And impedance can be of the form where R is the real part corresponds to a resistance, capital X is a reactance is a reactance, which corresponds to either an inductor or a capacitor or both combination of them that could also form a reactance, we shall come across this in a minute.

“Professor–student conversation ends”

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The image shows a handwritten diagram of an electrical circuit and its corresponding impedance equations. The circuit consists of a resistor (R), an inductor (L), and a capacitor (C) connected in series. An arrow labeled $Z(j\omega)$ points to the circuit. Below the circuit, the following equations are written:

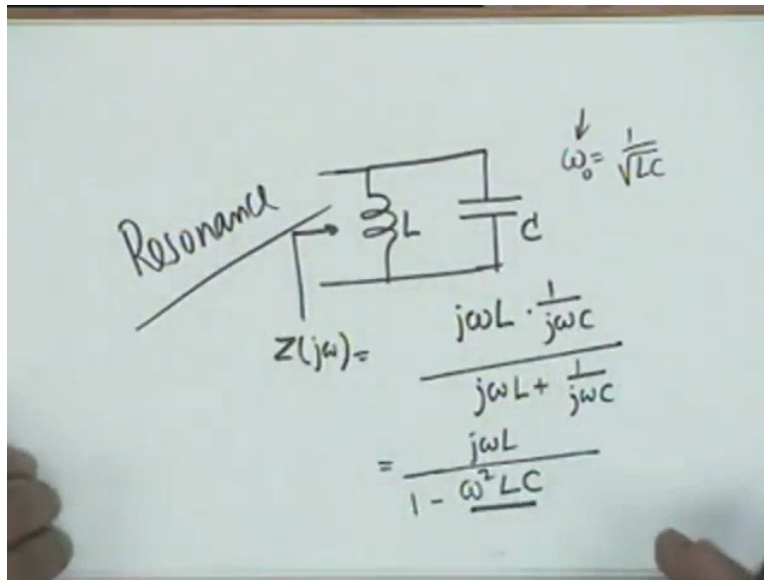
$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$
$$= R + j(\omega L - \frac{1}{\omega C})$$

An upward arrow points to the R term, and an 'X' is placed below the bracketed term $(\omega L - \frac{1}{\omega C})$.

Okay now as I said once you define impedance for sinusoidal excitation, you can combine impedances exactly in the same manner that you did for e to the s t excitation. For example, for impedance of this combination Z of j Omega would be equal to $R + j$ Omega $L + 1$ over j Omega C , so you simply sum up the impedances sum up the impedances, this you can write as j Omega $L - 1$ over Omega C and then you shall call this as the resistive part and this as the reactive part and you see that the reactive part will now consist of the combination of an inductor and a capacitor.

And because we also notice that the reactance of an inductor is an opposite sign to that of a capacitor and therefore depending on which is greater, reactance could be positive, if a reactance is positive we call it an inductive reactance. On the other hand if 1 by Omega C exceeds Omega L then the reactors will be negative and then you shall call it a capacitive reactance alright, so reactances, impedances, all of them can be combined in exactly the same manner that we combine resistances or impedances to e to the power s t excitation.

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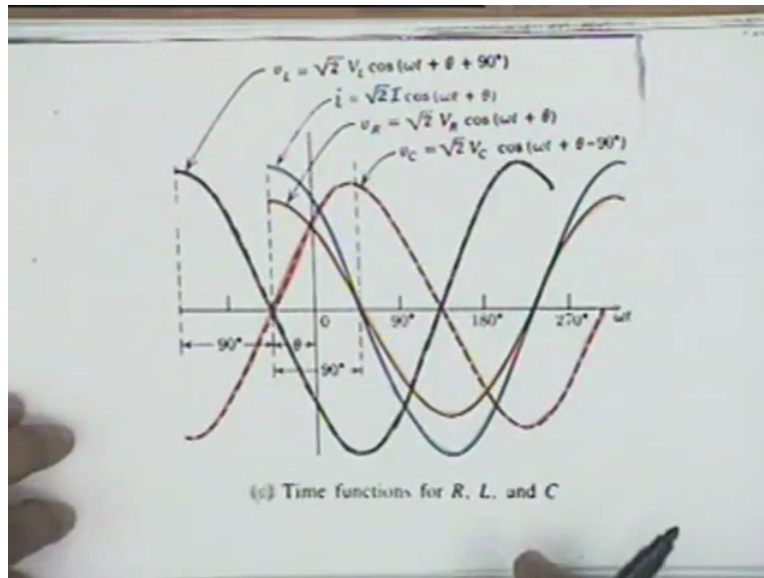


To take another example suppose we have an inductor L in parallel with a capacitor C then I can either find the impedance or I can find the admittance. Suppose you find the impedance then Z of j Ω shall be equal to j Ω L multiplied by 1 by j Ω C exactly like resistances, j Ω $L + 1$ over j Ω C , which is equal to j Ω L divided by Ω Square well $1 - \Omega$ square $L C$ is that okay? And you see that depending on Ω when Ω square $L C$ is less than 1 , the impedance is inductive that is the reactance is positive, when Ω square $L C$ is less than 1 then Z j Ω is equal to j times positive quantity, so it behaves like an inductance.

On the other hand, for frequencies Ω square $L C$ is greater than 1 and the reactance is negative and therefore the impedance behaves like that of a capacitor. And when Ω square $L C$ is equal to 1 , the 2 reactance is cancels each other and the impedance becomes infinite what does it mean? It means that no current can flow at that particular frequency that is Ω equal to 1 by square root $L C$ Ω_0 $L C$ equal to 1 , at this particular frequency this circuit behaves as an open circuit and this is called the condition of resonance as we shall see later. But let us enough of words, let us see from a picture as to what this lead and lag means in terms of actual sine waves alright.

We recall that the current and voltage in a resistor they are in phase, there is no phase difference. The current in an inductor leads or lags the voltage? Lags the voltage; current lags, the voltage across inductor leads the current by an angle of 90 degree. And in the capacitor the posted happens that is the current the current leads the voltage or voltage lags the current, let us see what it means in terms of a picture.

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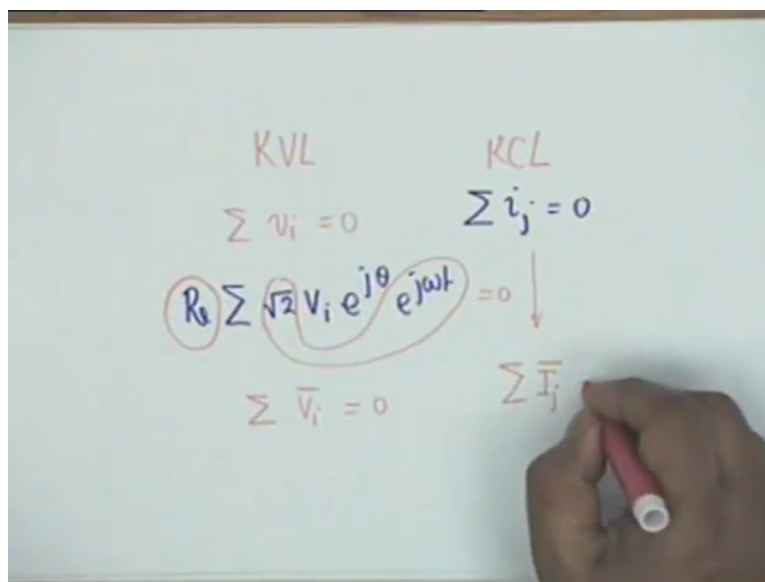
Is this visible on the monitor? This curve, I will explain one by one, this curve that is the blue colour curve this is the current I equal to $\sqrt{2} I \cos(\Omega t + \theta)$ and the corresponding drop in resistor V_R is simply this current multiplied by R which I have called I times R I have called V_R , $\sqrt{2} V_R \cos(\Omega t + \theta)$ and it is this curve. Obviously this and this are in phase that is the maxima and minima occurs together, whenever the current is a maximum the voltage is a maximum, whenever the current is minimum the voltage is minimum and so on alright.

On the other hand this curve the black one represents the voltage across an inductor when this current flows through this and you can see that the voltage leads the current by an angle 90 degrees, which means that on the X axis here it is Ωt , but when Ωt is lagging 90 degree ahead of the current, the voltage arrives at a maxima, the voltage reaches maximum half a period, is it half or one quarter? One quarter period. One quarter period earlier which corresponds to voltage phase advance or phase lead by 90 degrees, so this is what it is. On the

other hand the voltage across the capacitor is the same current passes shall lag behind in other words, it will reach its maximum quarter of a period later, this is what it means in terms of the actual voltages and currents, is this clear?

Omega t is represented in terms of degrees, you see because root 2 I cosine Omega t + Theta, we did not take Omega t, we take Omega t + Theta to the completely general, which means that the maximum to reach that Omega t equal to - Theta and that is what happens, the maximum the current reach that Omega t equal to - Theta this difference is Theta. And the maximum the voltage across the inductor has reached 90 degrees earlier, the maximum of the voltage is reached voltage across the capacitor is reached 90 degrees later, is this picture clear okay or there are there any questions? I am going to do an example now alright, to illustrate this point but before the example how do KVL and KCL translates in terms of phasors.

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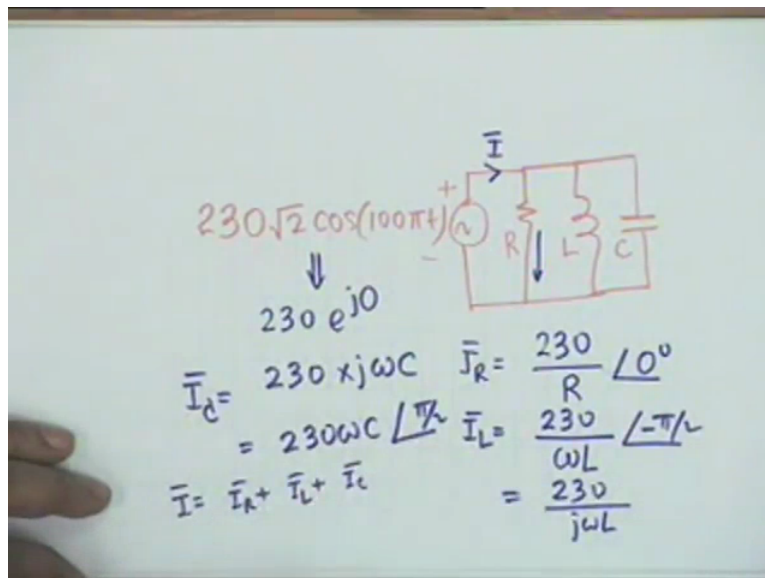


You know that KVL in general is submission $\sum V_i = 0$ around a loop and KCL is submission $\sum I_j = 0$ at a particular node. Now if each of these voltages is of the same frequency as it would in a linear circuit, it cannot change frequency alright then each of these voltages can be represented in terms of its phasor notations, that is each of these could be represented as real part I can take the real part outside root 2 V_i let us say e to the power $j\theta$ e to the power $j\Omega t$, now this quantity root 2 e to the $j\Omega t$ would be common to each of these voltages and the real part

would also be common to each of these voltages and therefore I can get rid of this and I can simply write V i phasor = 0, is this okay?

A very simple argument says that KVL should be valid if you replace the time varying quantities by their transforms by their transforms that is that means phasors. In an exactly similar manner, the KVL KCL translates into I j phasor submission I j phasor = 0 alright, so KVL and KCL can now be translated in terms of phasors, phasor quantities also obey KVL and KCL.

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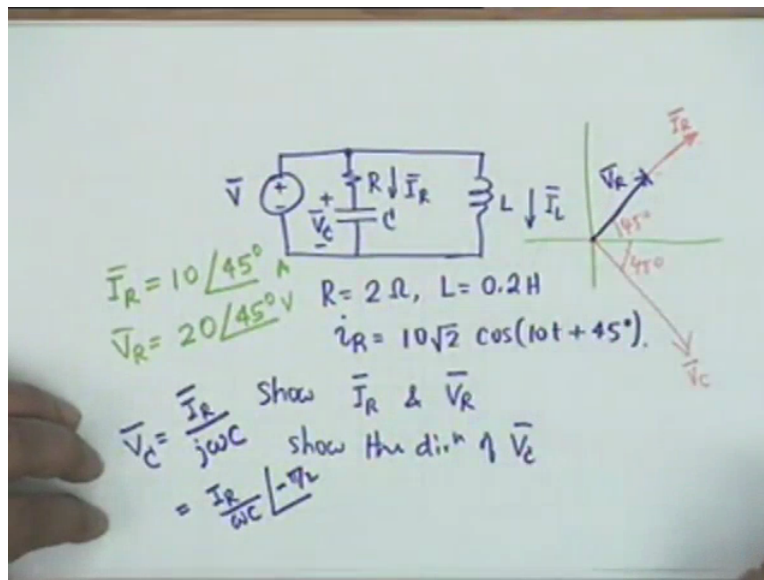


Let us take an example, suppose we have the line voltage $230\sqrt{2}\cos$ of let us say $100\pi t$, let this line voltage be applied to a parallel combination of let us say resistance and inductance and a capacitance then and let us say the values are R , L and C . The current in the 3 branches when added together shall obviously be the current that is supplied by the source. If I would represent the currents and voltages all of them by phasors then the corresponding phasor of this would be $230e^{j0}$ and therefore the current I_R phasor representation will be $230/R$ angle 0 , the current through inductor will be $230/\omega L$ and the angle would be $-\pi/2$ can also write this as $230/j\omega L$ that angle is taken care of automatically.

In a similar manner, the current I_c phasor shall be equal to $230 \times j\omega C$ that means $230\omega C$ angle $+\pi/2$. And therefore the total current phasor I shall be $I_R + I_L + I_c$, I can manipulate these quantities algebraically rather than writing out

differential equations and solving differential equations, if the excitation is sinusoidal I can work solely in terms of phasors and this offers tremendous simplification in AC circuit analysis.

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Let us take another example, I have a voltage now represented by phasor, frequency is unimportant, I represent it by phasor, voltage source, I have a resistance and capacitance, I do this very slowly, resistance and the capacitance and I have an inductor L, the current phasor in this is I L, the current phasor in this is I R, obviously I R and I sub C shall be the same right, the same current flows through resistance and capacitance so it suffices to find I R, it is given that R equal to 2 ohms, L = 0.2 henry.

And it is also given that I R that is the actual current through the resistance is given by 10 root 2 cosine of 10 t + 45 this is what is given alright is that clear, a voltage source is connected across parallel combination of 2 branches, one consists of inductor L value 0.2 henry and the other consist of a resistance and a capacitor in the series, resistance 2 ohms and capacitance is not known alright. The question is on a phasor diagram show I R and V R on a phasor diagram show I R and V R and draw an arrow to show the direction to show the direction of of the voltage across the capacitor that is V sub c alright, this is 1st part of the question, now that is obviously very simple.

I R as you can see the current phasor is simply equal to $10 \angle 45$ and the voltage phasor V R shall be equal to 10×2 that is $20 \angle 45$, this is volts and this is ampere alright, so the phasor diagram phasor diagram would be take a vector 45 degree at an angle of 45 degree and represent let us say this length by for the vector I R you take different scales and you represent a vector in the same direction as V R as the voltage across the resistance alright. Now this length is 20 and this length is 10 , this is in volts that is in amperes, we are showing the direction as the same but we have different scales okay. The next question is, show the direction of V C, and obviously V C...

“Professor–student conversation starts”

Student: Excuse me Sir.

Professor: Yes

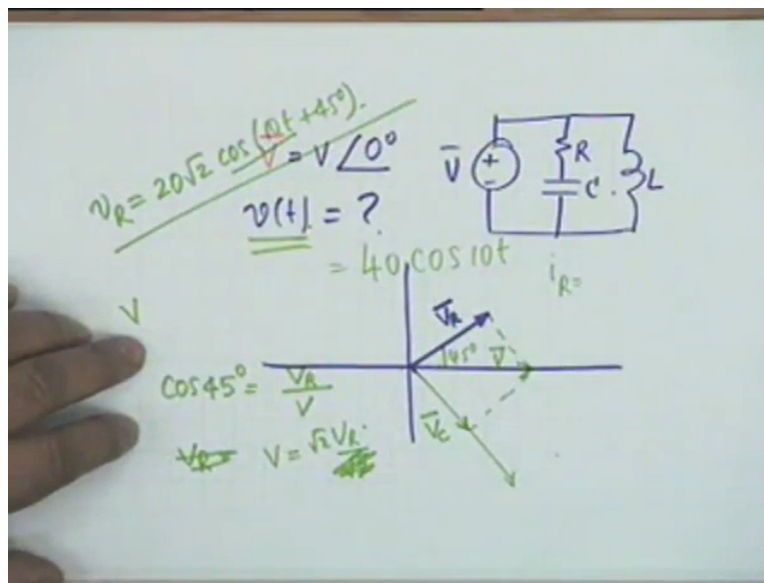
Student: V R will be smaller than I R.

Professor: They are different scales okay, we have choose to have a smaller scale for V R alright.

“Professor–student conversation ends”

Now the current the voltage phasor across the capacitor obviously this will be I R divided by $j \Omega C$, which means that it is I R by ΩC and angle is $-\frac{\pi}{2}$, $-\frac{\pi}{2}$ it refers to whom? It refers to the I R vector and therefore it goes like this, this is the direction of V C, is that clear? The capacitor voltage shall lag the current by 90 degrees and therefore this angle must be 45 , is that okay? This angle is -45 okay, the absolute value of the angle is 45 , the point is V C lags I C by 90 degrees, so this part of the question is fine.

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The next part says, the next part is quite interesting, if the voltage phasor if the voltage phasor that is V , recall V is the exciting voltage, let me draw the circuit again okay, if V is $V \angle 0^\circ$ then show V and V_C on the diagram and determine V of t alright, let us pose the question like this that you know our original vector diagram was like this. I had V_R in this direction and V_C was perpendicular to it, V_C direction was this, let me show the direction, this is the direction of V_C alright. Now it is said that V which is the sum of V_R and V_C has an angle of 0 and therefore V is in this direction, is that clear? V is in this direction and then what does it specify about V_C ?

Student: (0)(39:29)

V_C is equal to... Say it again the magnitude of the... Magnitude shall be equal and therefore if I measure this, this would be my V_C such that when I complete the parallelogram, the parallelogram will now be a rectangle is not that right or a square? It would be a square because this is 45 , this is 45 and therefore this is my voltage phasor, you are now required to find out V t , how do you find V t ? Obviously this angle is 45 so cosine 45 this is an interesting question, cosine 45 would be base divided by hypotenuse so V_R divided by V alright, therefore $V_R =$ well $v = V_R$ divided by root 2 or is the other way round, V would be equal to root 2 V_R that is right and therefore what is V t then? V t is simply yes I R ...

Student: 40 Cosine 10 t.

Professor: 40 cosine 10 t that is all where the angle is 0 alright.

You see why did this come, because small V R was 20 root 2 cosine of 10 t + 45 that is what it was alright, the magnitude of the phasor V the magnitude of the phasor V is root 2 times V R, so root 2 times this would be 20 root 2 multiplied by root 2 which is 40, then the angle is 0 so it is simply cosine 10, is that clear? Yeah okay.

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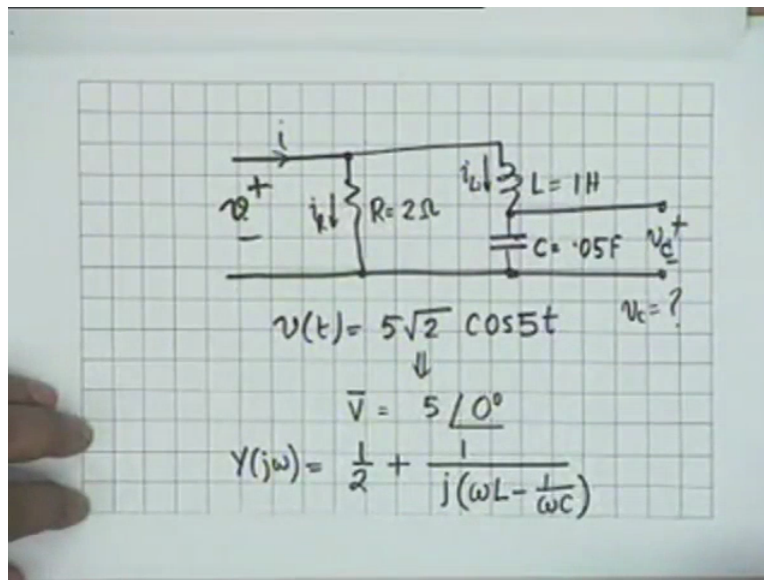
$$\begin{aligned} & \sqrt{2} I \cos(\omega t + \theta) \rightarrow I / \theta \\ & i_R = 10\sqrt{2} \cos(10t + 45^\circ) \\ & v_R = 20\sqrt{2} \\ & \bar{v}_R = 20 \angle 45^\circ \\ & v = \sqrt{2} v_R = 20\sqrt{2} \quad \bar{v} = v \angle 0^\circ \\ & v(t) = 40 \cos \omega t \quad v(t) = \sqrt{2} \times 20\sqrt{2} \cos(\omega t + 0) \end{aligned}$$

What we have seen is that our original equations let us say 10 root 2 cosine of 10 t + 45, so what is V R then? V R is 20 root 2 because R is 2 Ohms; 20 root 2 multiplied by cosine 10 t + 45 and therefore the phasor V R = 20 angle 45. We have seen that the magnitude of V, I want to find the phasor V, the magnitude of V = root 2 times V R, which means that magnitude of V is 20 root 2 and angle of V is 0 degree, therefore corresponding V t must be root 2 times the magnitude the RMS value which is 20 root 2 multiplied by cosine of Omega t + whatever the angle is, the angle is 0, is that clear, and therefore this is what we get.

You recall that root 2 i cosine Omega t + Theta corresponds to a phasor which is i angle Theta. It is the other way round, now we have to find V angle 0 degree, what is the corresponding V t, so we multiply V by root 2 to get the amplitude and cosine of Omega t + the angle which is 0 and

Omega is 10 and therefore this is the result, where did I write it, this is 40 cosine of Omega t alright, we will carry 2 more examples to illustrate this.

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Take another example, let us say we have a view, we shall make a change over from Time domain quantities to complex domain quantities and vice versa again and again. You must be noticing that small case letters I reserved for time and capital letters for phasors okay. So there is a current I, there is voltage V and the circuit consists of R = 2 Ohms and then an inductance L = 1 Henry and a capacitance C = 0.05 Farad and the output voltage is taken here V sub c, these are the time domain quantities. Now I specify that v of t let us say 5 root 2 cosine of cosine of 5 t alright, we are required to find out V sub c.

And in the process we are also required to find out a complete phasor diagram that is I want to know what the current current through this resistance is, what is its phasor, I want to know what is this current lets say i R and i L is equal to i C then I want to know what is the relationship between i L and V C in terms of phasors and finally I want to know what V sub C is alright. Now I also want to know what this current small i is, what is the corresponding phasor and what relationship does it bear to the phasor small v, let us proceed systematically.

1st of all the phasor representation of this would be V = 5 angle 0 okay, this is the phasor representation. If you want to find out i if that is my only concern then what I do is I find out the

impedance alright now instead of impedance obviously it is easier to find the admittance because things are in parallel so let us find out Y j Ω and you will get very interesting results. Y j Ω obviously is half + admittance of this which is 1 by $R + 1$ by impedance of this + this, so it would be j Ω $L - 1$ by Ω c is that okay? j Ω L is the impedance of the inductor in in series with the impedance of capacitor $-j$ by Ω C alright, let us substitute the values and see what happens.

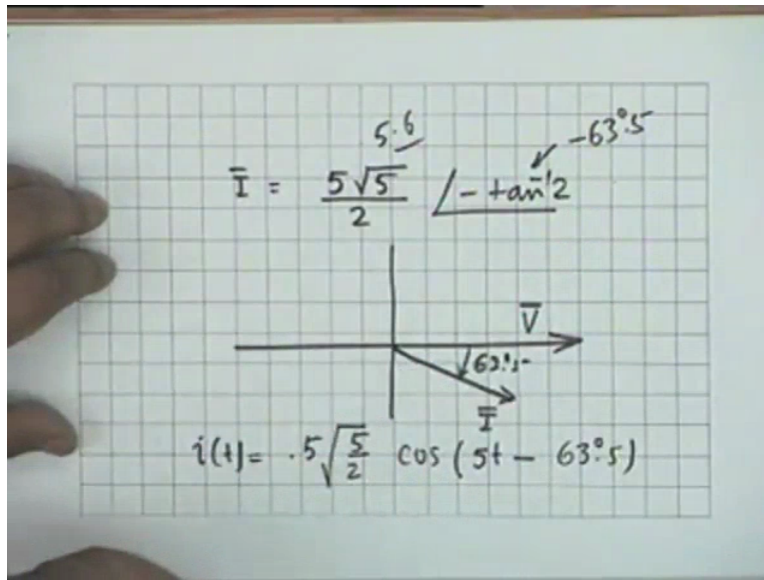
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$$\begin{aligned}
 Y(j\omega) &= \frac{1}{2} + \frac{1}{j(5 \times 1 - \frac{1}{5 \times 0.05})} \\
 &= \frac{1}{2} + \frac{1}{j(5 - 4)} = \underline{\underline{\frac{1}{2} - j}} \\
 \bar{I} &= Y(j\omega) \bar{V} = (\frac{1}{2} - j) 5 \angle 0^\circ \\
 &= \frac{5}{2} (\frac{1}{2} - j) \\
 &= 5 \sqrt{\frac{1}{4} + 1} \angle -\tan^{-1} 2
 \end{aligned}$$

If Ω is 5 alright and L and C are given and therefore Y of j Ω is equal to half + 1 over J , Ω is 5 and L is 1 so 5 into $1 - 1$ over 5 into 0.05 so this is equal to half + 1 over j $5 - 4$ so this is simply half - j alright 5 into 0.05 is 0.25 , 1 by 0.25 is 4 okay this is the admittance. Then what is this current phasor i ? This would be the admittance Y j Ω multiplied by the voltage phasor V alright, the current phasor is admittance multiplied by the voltage phasor V . So this is equal to half - j times what is V , 5 angle 0 degree and therefore what is the magnitude of this.

I can write this as 5 by 2 or I can simply write this as 5 half $2 - J$, angle 0 degree does not contribute anything and therefore what is its magnitude, it is 5 square root of half + 1 , is that okay, one quarter + 1 alright and the angle is $- \tan^{-1}$ not $\text{Pie by } 2 \tan^{-1}$ imaginary by real, so $- \tan^{-1} 2$ is that okay.

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Now if I simplify this the current phasor is equal to 5 square root 5 divide by 2 is that okay? Okay and then – Tan inverse 2, I do not know how much – Tan inverse 2 is. – Tan inverse – 45...

“Professor–student conversation starts”

Student: It must be around 65.

Professor: Well, whatever it is whatever it is we will make a phasor diagram now, this my view.

Student: Sir 63.5.

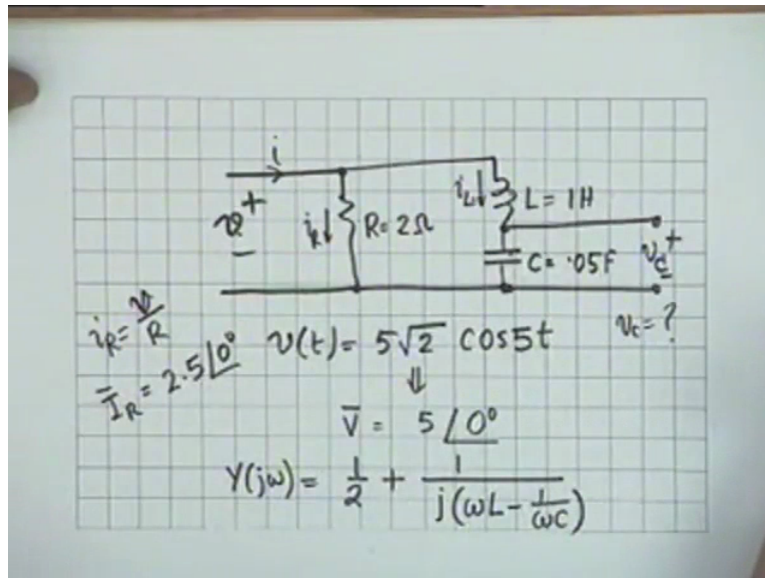
Professor: 63.5 alright – 63.5.

“Professor–student conversation ends”

And what is 5 root 5 by 2? 2.5 it is about 31 no, it is less than this, how much? Around 5.6 okay alright so the current phasor therefore lags the voltage by an angle of it is not to scale, 63.5 and the current phasor is here, it is that I want, it is that the only thing that I want alright, and the corresponding current quantity I of t, the corresponding current will be given by, Yes can you tell me what will be the current? Root 2 times that is 5 square root of 5 by 2 is that okay, root 2 times this alright then cosine of 5 t – 63.5 this will be the current, I did not manipulate anything I

simply manipulated some complex quantities alright. Now suppose instead of this I want to find out the current through the resistance, let us get back to the circuit.

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This is the circuit, what is i_R ? i_R is simply v small v by R and therefore i_R phasor would be equal to $2.5 \angle 0^\circ$ alright. Suppose I want to find out i_L that would be more interesting, now look at yeah... Phasor i_L , what do you think its relationship shall be with the voltage phasor v , I shall be happy with you if you can say....

Student: V by j ...

Professor: V by $jX_L + X_C$ which can be either positive or negative. How does this mean? It means that the current lags by 90° or by 270° , is not write? $780 + 90$, we shall continue this next time.