Introduction to Electronic Circuits
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Lecture no 14
Module no 01
Natural Response of 2nd Order Circuits (Contd)

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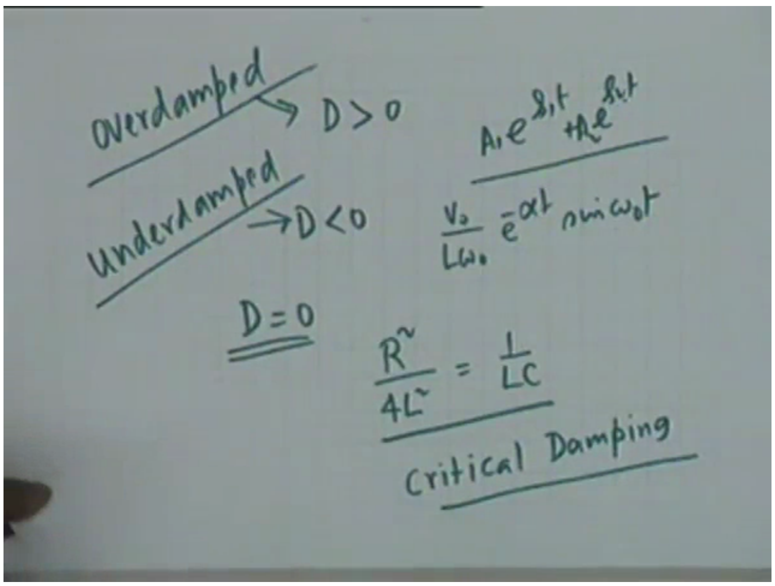

14th lecture on Natural response of 2nd order Circuits continued. We have been discussing 2 cases that is capital D greater than 0, in this case there are no oscillations, the solution consists of the sum of 2 exponentials e the – s 1 t and e to the – s 2 t where s 1 and s 2 are both real and negative alright, e to the s 1 t – e to the s 2 t A 1 + A 2 this is the solution, where s 1 and s 2 are both negative, there are no oscillations so the current are starts from 0, attains a maximum and then goes to 0. On the other hand, if D is less than 0 then we have oscillations, we have we have V 0 by L Omega 0 e to the – Alpha t Sin of Omega 0 t alright.

The border line between these 2 or the transition case, which is which happens D = 0 occurs obviously when R square by 4 L square = 1 over L C alright. This is a very special condition and the condition on the resistance R to satisfy this is called R critical, it is the critical value of resistance for which this shall be true alright. If they if this equality becomes an inequality there shall be either oscillations or no oscillations and therefore this is a critical case and this case is called the case of critical damping. Critical damping that is if the damping is slightly less than there shall be what?

There shall be oscillations. If the damping is slightly less there shall be oscillations, Alpha is the damping coefficient, if the damping is slightly more there should be no oscillations and therefore this case D greater than 0 is called an over damped case, over damped means no oscillations in the natural response and naturally D less than 0 is called an underdamped case right underdamped case and D = 0 is the so-called critical damping case critical damping, it determines the boundary between oscillations and no oscillations. Let us see what is the solution to this case is.

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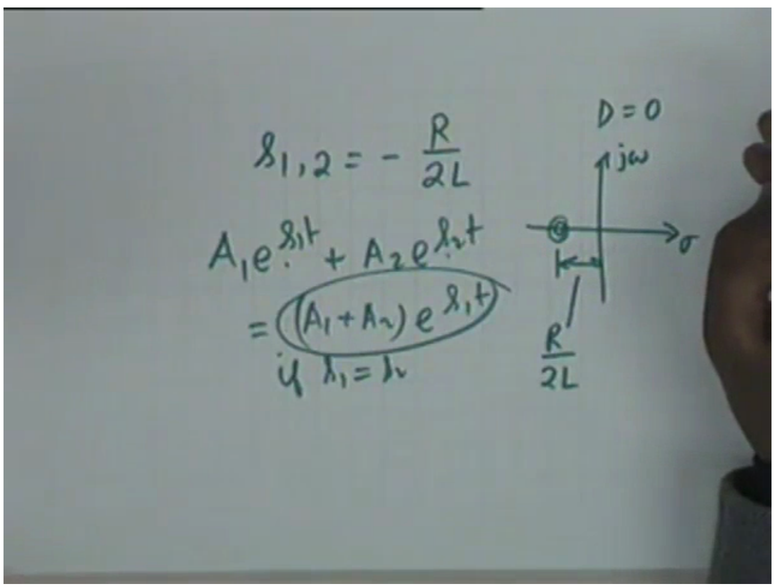

Our s 1 2 was equal to – R by 2 L and since D equal to 0 there is nothing else, + – 0 and therefore the 2 roots are now coincident, coincident on if you take the Sigma j Omega s plane coincident where, on the negative real axis, there are 2 roots and this distance is equal to R by 2 L, the 2 roots are no longer complex, they are both negative real and they are coincident. Now what is the solution under this condition? Obviously, if you take a general solution if you recall e to the s 1 t + A 2 e to the s 2 t and if s 1 is equal to s t obviously the 2 solutions are no longer independent of each other, is not that right they are identical solutions in fact except for the constants A 1 and A 2.

When s 1 and s 2 are equal I can always write this A1 + A 2 e to the e 1 t if s 1 = s 2 and therefore this cannot represent the general solution, general solution must have 2 arbitrary constants because it is a 2nd order system, it must have 2 arbitrary constants and the 2 solutions should be independent of each other they should be independent solution. And as you know as you know from the solution of differential equations, the solution in this case shall be given by A 1 + A 2 times t e to the power s 1 t alright, this is the general form of the solution.

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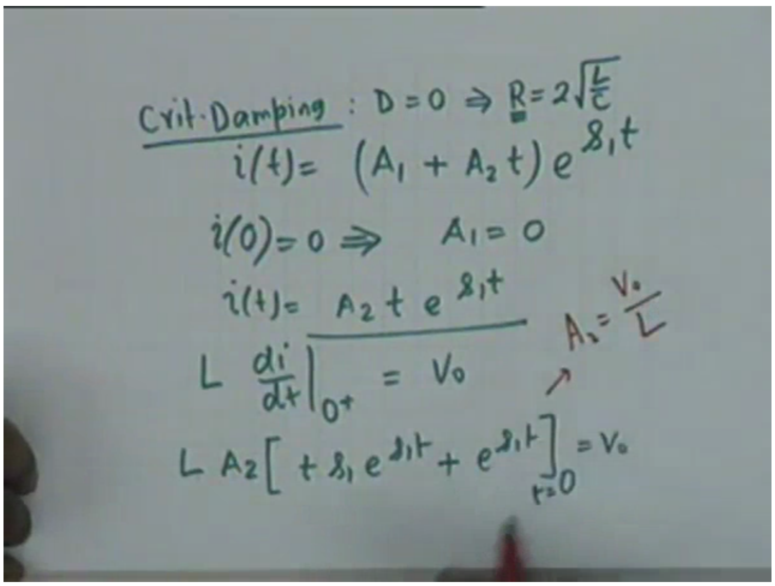

This is the case for critical damping that is that is capital D = 0 or which means that R square equal to well R will be equal to R square by 4 L square or it is equal to twice square root of L by C, is that correct? Alright. So this value of resistance is called critical value of resistance it is R critical and I know this condition this will be the general solution. Now if I apply the initial condition that is i 0 equal to 0 then obviously you see A 1 shall be equal to 0, is that right. If I put t equal to 0 here then obviously A 1 is equal to 0 and therefore my i of t shall be equal to A 2 t e to the power s 1 t. The 2nd boundary condition the 2nd initial condition is that L di dt L di dt at t = 0 should be equal to v 0 should be equal to v 0.

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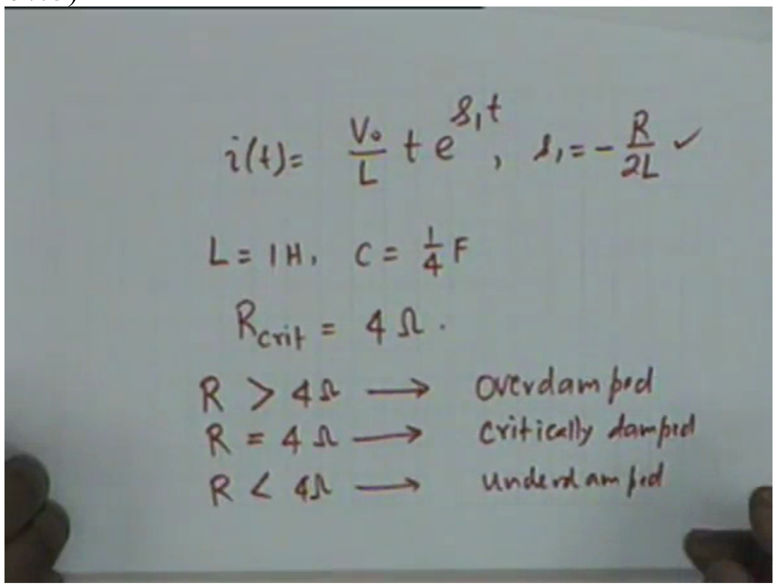

If I differentiate this then I get L A 2 t s 1 e to the s 1 t + e to the s 1 t that should be equal to V 0 at t = 0. And I have differentiated this this expression and I have to put t = 0, if I do that then what do I get as A 2? A 2 is V 0 by L whenever Omega 0 comes alright. A 2 is V 0 by L which means that my total solution is given by V 0 by L t e to the s 1 t where s 1 = – R by 2 L this is the solution to the equation when the 2 roots are coincident. For example, if L is equal to 1 henry and C equal to one quarter Farad, what should be the R critical? Twice square root of L by C that is 4 Ohms. In other words if R is greater than 4 than this case will be which one, over, under? Underdamped.

If R is equal to 4 Ohms of course, this corresponds to critically damped case and if R is less than 4 then it is the underdamped case alright. Now let us look at this situation, let us look at the situation in the s plane alright and I will repeat I will repeat what I had been saying in terms of the complex plane.

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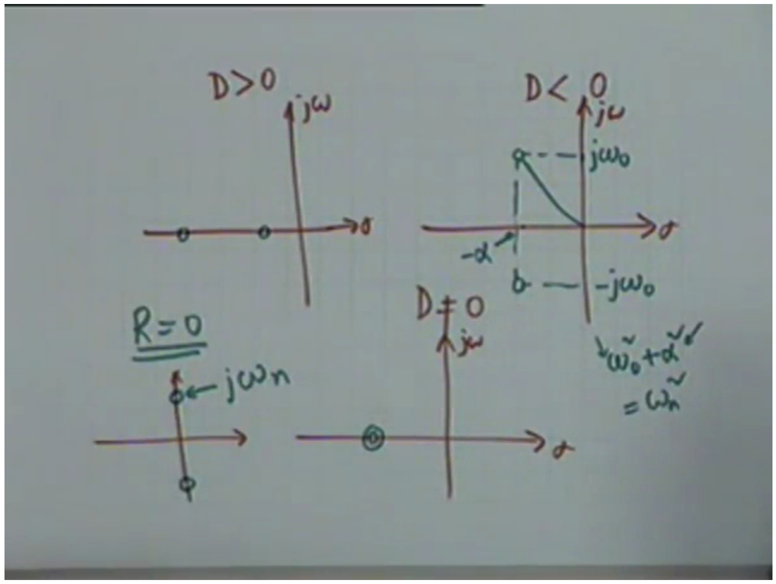

You see if I have, I have 3 situations; one is capital D greater than 0, capital D less than 0 and capital D equal to 0 alright. The roots of the characteristic equations, let me show the axis Sigma, j Omega, j Omega Sigma, Sigma j Omega there are 3 cases and you see in the 1st case D less than 0 there are 2 negative real roots; – s 1 and – s 2. In the 2nd case that is the underdamped case, we have 2 complex conjugate roots, where this distance this point is – Alpha and this point is j Omega 0, this point is – j Omega 0 alright – Alpha + – j Omega 0. On the other hand, if I have the critically damped case where the 2 roots coincides on the negative real axis alright is it okay?

Let us consider a 4th case in which capital R = 0, can you tell me where the roots will be? On the imaginary axis okay. Can you tell me the value? 1 by L C, in terms of Omega n it is j Omega n is that okay right? Now I want you to look at this figure carefully, if this figure is too small on the monitor I can write again the last part, can you see distinctly from the last bench? Okay. Now look at this case D less than 0, this is Omega 0 and this distance is Alpha, so what is the distance of the root from the origin? Omega 0 square…

Student: Omega n square.

+ Alpha square, is a not this simply equal to Omega n square? Okay. Now Omega n depends on L and C only, is not it right, Omega n does not depends on R is that clear? By definition, Omega n is the un damped natural frequency oscillation and underdamped means R equal to 0, it is this

Omega n we are talking of. On the other hand, both Alpha and Omega 0 naturally depend on R is that clear? Because Alpha depends on R, Alpha = R by 2 L therefore, Omega 0 depends on R, Omega 0 square after all is Omega n square – Alpha square, is that clear? I am trying to introduce the conception, please be with me.

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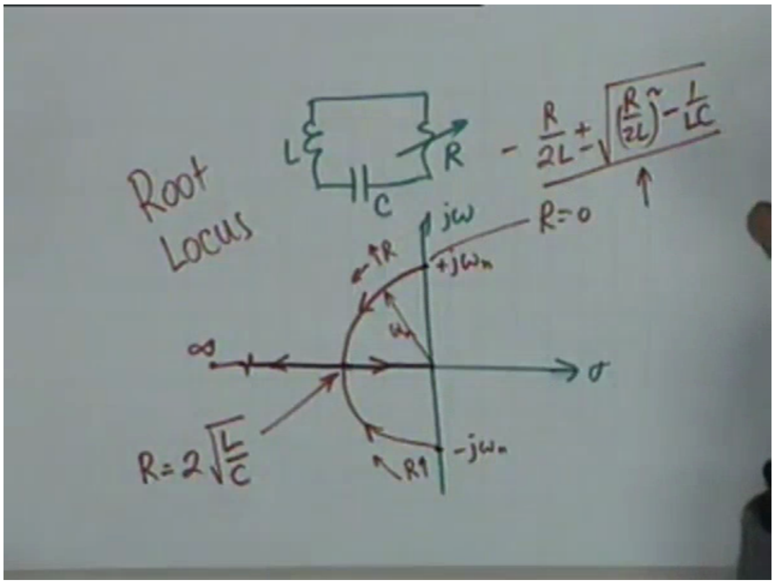

What I have said is Omega n is independent of R. Alpha as well as Omega 0 depends on R and therefore if I take a circuit if I take a 2nd order circuit, C, L and R and if I adjust R if I adjust R, how do the roots change? How do the roots change? Alright, this is what I want to find out. Suppose I used to draw the roots of the characteristic equations on the s plane and when R = 0 when R is equal to 0, start from the 0 value it cannot go below 0 alright, when R = 0 where are the roots? Roots are on the imaginary axis at + – j Omega n is that okay. When r is slightly increase what happens to the roots, they move to the left half plane, this is the right half plane this is the left half plane.

They move to the left half plane such that the distance of the root distance of either root from the origin remains a constant and that constant is equal to Omega n and therefore the movement of the roots shall naturally be on a semicircle where this distance is Omega n alright is that clear? As R is increased, this point corresponds to R equal to 0 and as R is increased, the roots the 2 roots, this root moves in this direction and this root this root moves in this direction such that they are always complex conjugate that they most remains on the circle alright.

Now, and the roots approach each other and meet on the negative real axis, what is the situation, the 2 roots are coincided and therefore R must have reached its critical value so this point shall correspond to R = twice square root of L by C where the roots become coincident. Roots are moving by complex, they are real, negative and coincident alright. What happens after this? They move in the real axis, well what happens is you recall that the roots are – R by 2 L + – square root of R by 2 L whole square – 1 over L C. This point corresponds to capital D = 0 alright, that means the roots are at – R by 2 L.

Now when resistance increases further what happens is that this – R by 2 L is either added to the quantity less than it or subtracted there is a negative quantity which is less than is added to it. What does it mean, it means that the roots one of the roots goes towards the origin and the other root goes towards infinity alright. If capital R is increase beyond critical value, the 2 roots the case is that of over damped case and the 2 roots move along the negative real axis. The root 1 is again negative real, 1 increases in magnitude the other decreases in magnitude, the one that decreases naturally goes towards that the origin and the one that increases goes towards infinity alright, this can also be same from the analytical expression, try to follow this very carefully.

When capital R tends to infinity when R tends to infinity, one of them one of the roots will be 1 by L C can be naturally neglected then and therefore one of the roots will be – R by 2 L – R by 2 L and the other root will be – R by 2 L + R by 2 L, so one of the root goes to the origin, the other goes to infinity is that clear? Okay. So both from the analytical expression from explanation and from the figure you can see that the locus of the 2 roots is given by this semicircle and then 2 straight lines, one going to the origin, the other going to infinity. And this picture this picture which gives the motion of the roots or the locus of the roots as one of the parameters capital R changes in the circuit, this picture is called a root locus for obvious reasons.

It describes the locus of the roots as one of the parameters that is capital R changes. Now if know what is so sacred about capital R, why do not you keep E constant and vary L? We can again obtain another sketch which is exactly this, it will be another sketch but can draw it and I leave that to you as an exercise. Once again you can obtain a critical damping condition and underdamped condition and an over damped condition. You can also draw a root locus for variation of capital C, capital C may also vary and this will be a 3rd kind of sketch. Now is this point clear the concept of root locus? The concept of over damping, under damping and critical damping alright?

Usually measuring instruments, which can be usually model as a 2nd order system, what do you feel we should have as far as damping is concerned?

“Professor–student conversation starts”

Students: Critically damped.

Professor: Critically damped, well why not underdamped? Let us understand.

Student: If we have underdamped system then it keeps oscillating about the actual position and we do not get the exact value in a small-time.

Professor: That is right, it takes quite a bit of time it will go on oscillating and then finally it settles to some value and when it is critically damped then it reaches the final value alright, if it is over damped then it takes very long time to reach a steady state and therefore these constructs are important.

“Professor–student conversation ends”

The concept of root locus as we shall see is extremely important arises from control system arises in in many other physical systems. The other concept that is of extreme importance is that of impedance and this is time to introduce this concept, impedance. I shall 1st give a formal defination, yeah.

Student: Before impedance could you tell us what exactly is the per bother finding the (())(20:25) 2nd order circuit.

Professor: It shows the locus of the roots as one of the parameters is varying.

Student: Sir that is okay, that is obvious but I mean what is its importance?

What is it importance? Right. As you shall see later if you are becoming engineer, in control system in any process control for example, which can be modelled as 2nd order, 3rd order or 4th order system, suppose the roots one of the roots due to a disturbance in parameter, goes to the right-hand plane if the root locus includes the right half plane what will happen all the imaginary axis then what will happen? Suppose the parameter is such that the the roots are on the imaginary axis then the instrument or the process that we are experimenting with will never settle down, it will go on oscillating is not it right?

So the root locus shows the locus of the roots and your design should be such that the roots are in the left half plane. For example, it is that always true that in all physical systems we require a critically damped case, no. Sometimes a bit of oscillation is necessary why? If you want a quick response well after a couple of oscillations it stays it settles little bit a damping a little bit of under damping is quite in order. So the root locus shows where to locate your roots and if you know where to locate your roots then the system design is obvious alright so this is the purpose of the root locus.

Root locus is an analytical tool for designing a system, I have drawn the root locus in the 2nd order system there could be 3rd order, 4th order usually natural systems practical systems are of are 2nd or higher order system usually 2nd or higher-order systems and the root locus is essential for determining the design criteria where should you locate the roots in order to satisfy what you want in order to satisfy your requirements, is that okay? Any any other questions regarding root locus? Alright.

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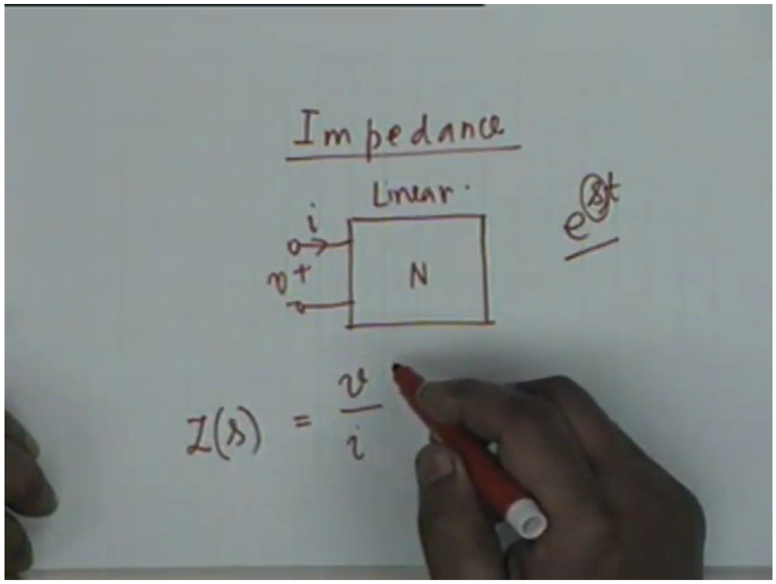

Next we introduce the concept of impedance. The introduction of the definition that I am going to give you now would be a bit heuristic a bit inaccurate a bit unsophisticated but it will serve our purpose in the present moment. The the definition of impedance is like this, consider a 2 terminal network, a network which has only 2 terminals alright and you can see a linear network alright a linear network. When you know that all that you can do is to it to connect a voltage generator and measure the current, the excitation would be a voltage generator, the response should be current or you could connect a current generator here and measure the voltage alright.

Now if your excitation either a current generator or voltage generator is of the form e to the s t and this network is a linear network contains resistance, capacitance and inductance then you know the differential coefficient e to the s t is also of the form e to the s t s times e to the ST. The integral of e to the s t is also of the form e to the s t so if you apply a voltage generator here which is of the form e to the s t the current shall also be of the form e to the s t, is not it right?

Similarly, if you apply a current generator which is of the form e to the s t the voltage developed across this shall also be of the form e to the s t and therefore, the ratio of V by i the ratio of V by i shall be a constant but it will depend on the value of s and therefore, this is denoted by Z of s and this is called the impedance. I must caution you… Let me 1^{st} define it, the impedance of any 2 terminals any linear 2 terminal network is the ratio of voltage to current divided the excitation either the voltage or the current excitation and in brackets either the voltage or the current is of the form of e to the power s t is that clear? It is made true for any arbitrary excitation, excitation must be exponential only then the ratio of V to i is called an impedance. Let us take an example, is the definition clear?

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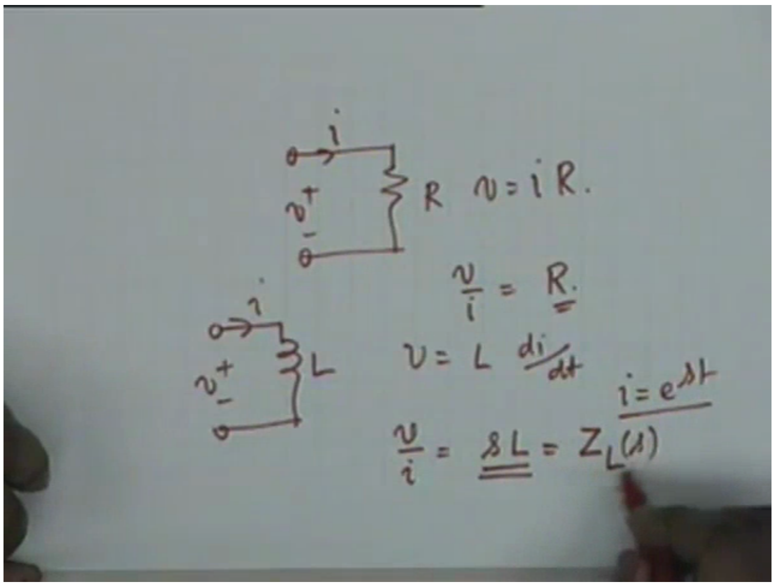

The exponential excitation is an integral part of this definition, if that part is taken out then it does not makes sense alright. For example, if I take a resistance let us say a 2 terminal network a pure resistance R, then you know V = i R and if V is of the form e to the s t, i is e to the s t divided by R. If i is of the form e to the s t then V is e to the s t multiplied by R and V by i is equal to R, this is a constant independent of s alright this is a constant independent of s. On the other hand, if I have an inductor L, V and i then we know V = L di dt and therefore if i is of the form e to the s t then V by i shall be equal to s times L, so the impedance of an inductor of value L is simply equal to s times L alright.

What is the dimension of s? 1 by time, which means s is frequency and s could be real or complex alright. So in general small s is a complex frequency, real frequency real quantity is a special case of a complex quantity, is not it right? So if you live life as complicated, it is the most general situation, if you take imaginary part then life becomes simple alright. So s becomes complex frequency and the impedance of inductor Z L of s = s times L.

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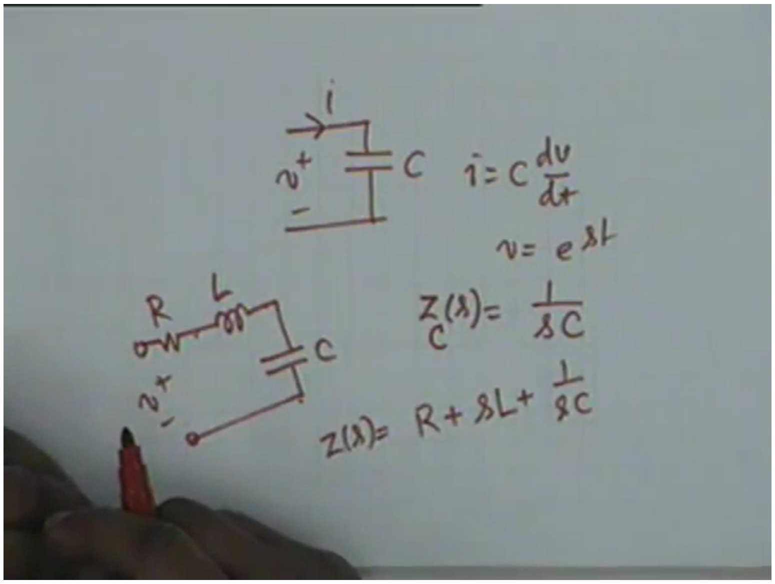

If I take a capacitor C, you know the current i equal to C dv dt and if v is of the form e to the s t then Z of s is given by 1 over s times C, this is the impedance of capacitor Z C s alright then when the impedance is different for inductor and capacitor you can treat inductors and capacitors exactly like resistances alright. The impedance of resistor is R, the impedance of inductor is s times L, the impedance of capacitor is 1 by s C. And if you have C a series combination of R, L and C then your V shall be equal to drop across R + drop across L + drop across C and if you take the voltage or the current of the form e to the s t, you can show that Z of s here of R, L and C in series is simply given by R + s L + 1 over s C that is these quantities cannot be added alright.

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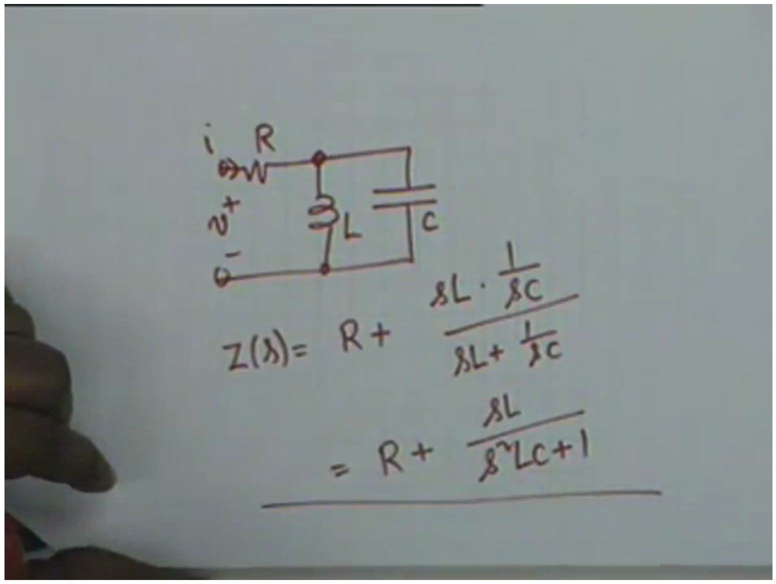

Similarly suppose I have a circuit like this, R, L and C, suppose I have another combination this is not a series combination this is a serious connection of R with parallel connection of L and C. If this is V and this is i, if one of them is exponential then the impedance of the circuit can be written, the manipulation will be exactly like that of resistances, here an impedance s L is in parallel with impedance of 1 by s C and therefore it is R + s L times 1 by s C, R 1 R 2 divided by R 1 + R 2, now instead of r you say Z the impedance, so Z 1 Z 2 divided by Z 1 + Z 2 that is s L + 1 over s C, which is equal to R + s L divided by s square L C + 1, now you ask me what is the use? Why should we do that?

Well if we do this if we define an impedance, one of the advantages is that all kinds of Circuits can now be treated in exactly the same manner that you have treated resistive circuits. For example, Thevenin's theorem with this definition of impedance now Thevenin's theorem is applicable to RLC circuit also Thevenin's Norton and we can also do circuit analysis without writing a differential equation. Once the impedance concept is introduced, d dt shall be replaced by s and integral shall be replaced by 1 over s, e to the s t dt integral is e to the s t over s, d dt of e to the s t is s times e to the s t alright, let us take a specific example before we close this class yes.

"Professor–student conversation starts"

Student: Sir for considering impedance has the applied voltage or current whatever it is, has to be of form e to the s t.

Professor: Exponentially.

Student: But Sir if we consider charging of capacitor, we do not have voltage of e to the s t, but still the voltage across the capacitor can be given as e to the s t so can we say can we define impedance for it?

Professor: That is the generation of e to the s t, generation will be exponential, that question is completely different, and this question is that of combining networks.

"Professor–student conversation ends"

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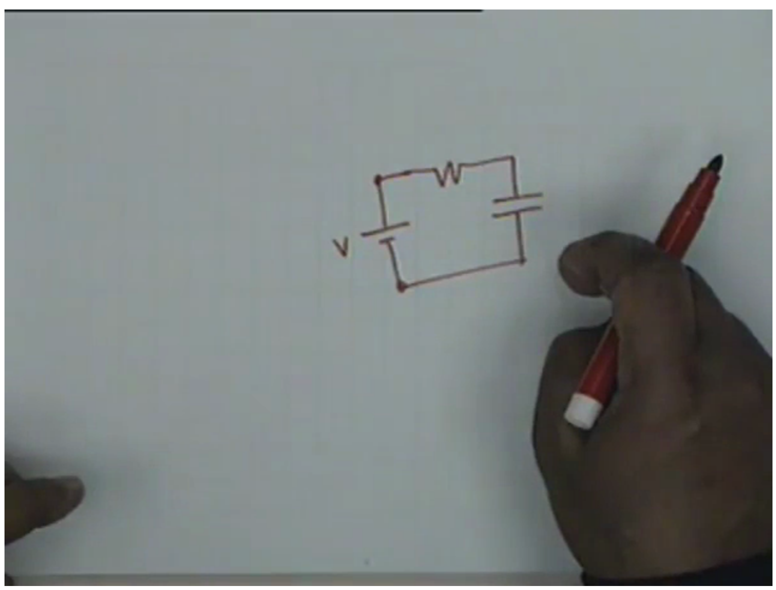

If you simply want to charge a capacitor, here e to the s t does not come into effect because we have voltage source is a constant, it is not of the form e to the s t, it is e to the s t is basically 0 and therefore you cannot introduce impedance concept here alright, impedance concept will come only when the source is an exponential, let us take an example. Let us take the same circuit that we have been talking of.

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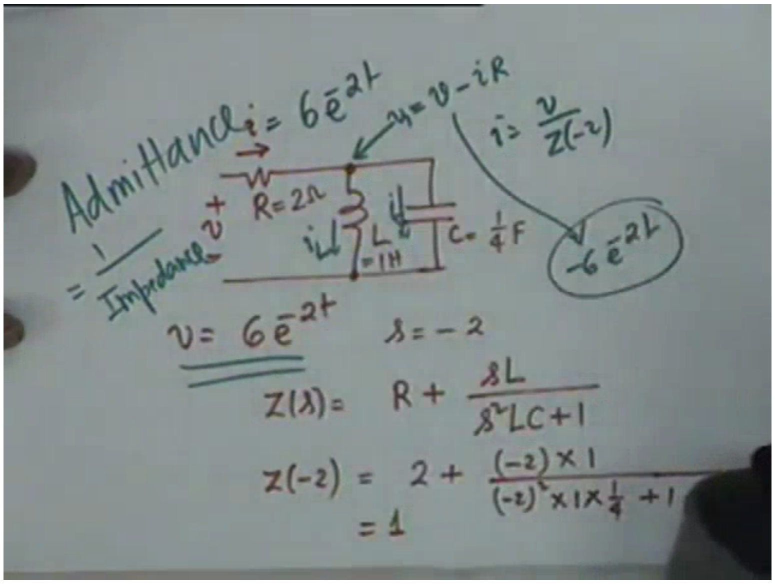

Let us say R, L and C now please pay your attention to this, we are doing something new alright. Suppose R = 2 Ohms, L = 1 Henry and C = one quarter Farad alright and this V this V is given as 6 e to the – 2 t, is not it at the further exponential, it is an exponential with s = – 2. So if I need to find this current let us say i for example, then what I do is I 1st find out Z of s, Z of s as you have seen already is r + s L divided by s square L C + 1. Now substitute the values, Z of -2 alright would be equal to 2 + s is – 2 multiplied by 1 divided by s square is – 2 square times 1 times one quarter + 1, which means Z of – 2 is equal to how much? Alright, Z of – 2 = 1. So what do you think this current shall be then?

V by i is Z of s so it will also be 6 e to the power – 2 t, is that clear? Is it minus? No it is +, I shall be equal to V divided by Z of – 2 and therefore it is 6 e to the – 2 t. What do you think this voltage shall be V 1? It would be V – I R, so what would this be? So this should be – 6 e to the – 2 t and it is voltage. If I know this voltage, may I know this current i L? I L shall be equal to this voltage divided by s L where s shall be equal to – 2. May I know this current i C? Yes, it would be C dv dt that is the differentiation of this so I know all currents and voltages in this circuit. You say the excitation is exponential, I would not have to write the 2nd order differential equation normally, yeah you wanted to ask a question?

Student: No.

Normally in a circuit like this I have to write a differential equation and then solve it, there is A 1 e to the s 1 t +… I know that would be anything like that because I recognize that the excitation is exponential. And if the excitation is exponential, I can work purely in terms of impedance is alright. I will just 5 minutes… I have just introduce another term to you, it is Admittance. I told you, impedance is the ratio of voltage to current, the reciprocal of an impedance is admittance, admittance is 1 over impedance, sometimes it is easier to work in terms of admittance rather than impedance and it is a base that we conclude the class today.