Introduction to Electronic Circuits
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Lecture no 13
Module no 01
Natural Response of 2nd Order Circuits

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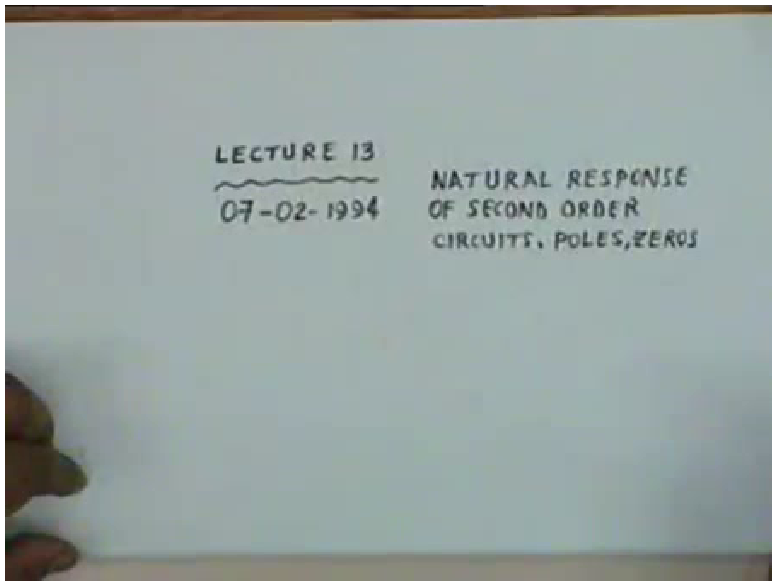

This is the 13^{th} lecture we shall continue the discussion on that problem in the tutorial classes. 13^{th} lecture today is 7^{th} February and we discuss natural response of 2^{nd} order circuit and if time permits then we shall discuss these 2 terms; Poles and Zeros.

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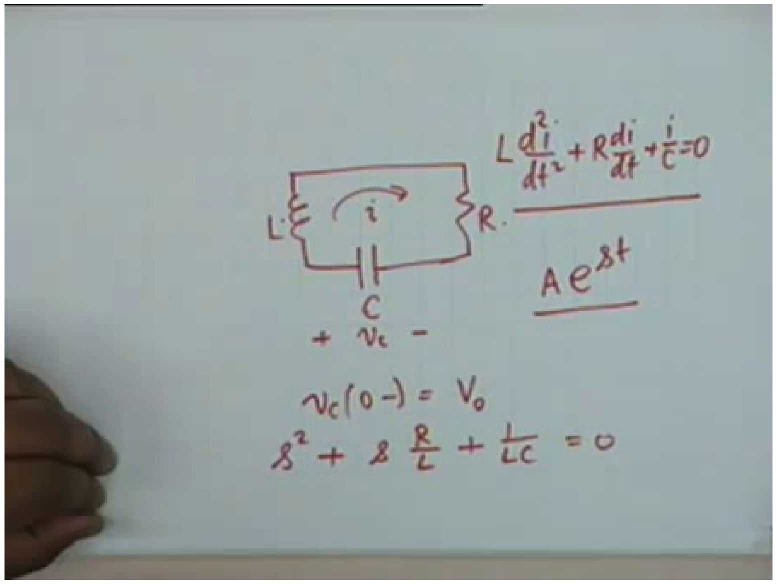

Remember in the 2nd order circuit that we had considered, what we have considered was we had a capacitance which is charged to which is charged to a voltage of capital V let us say V 0 and then at t = 0 is connected across an inductance L and a resistance R, the current that flows in the circuit is i, we are considering this particular LCR circuit. The initial condition is that V c 0 – is V 0 and then naturally as time increases small i at t = 0, that is when the capacitor is switched to this series combination, the current will be 0, current would be 0 and then the current would gradually increase and then decrease, after infinite amount of time it will decrease.

Qualitatively different the phenomena that depending on the relative values of R, C and L the way it does so would be quite different and we have discussed one of the cases that before that let me write the differential equation, differential equation was L d2t dt2 + R di dt + i by C = 0, how readily obtain this equation? We 1st obtain an integro-differential equation that is R i + L di dt + 1 by C integral i dt V 0 – that and then we differentiated this to obtain a homogeneous differential equation that is the right-hand side is equal to 0.

And then I argued that the solution to this equation shall be we can try the solution of the form e to the st some constant multiplied by this and this with this we ended up in a quadratic equation that is small s satisfies a quadratic equation of the form s square + s R by L + 1 over L C = 0.

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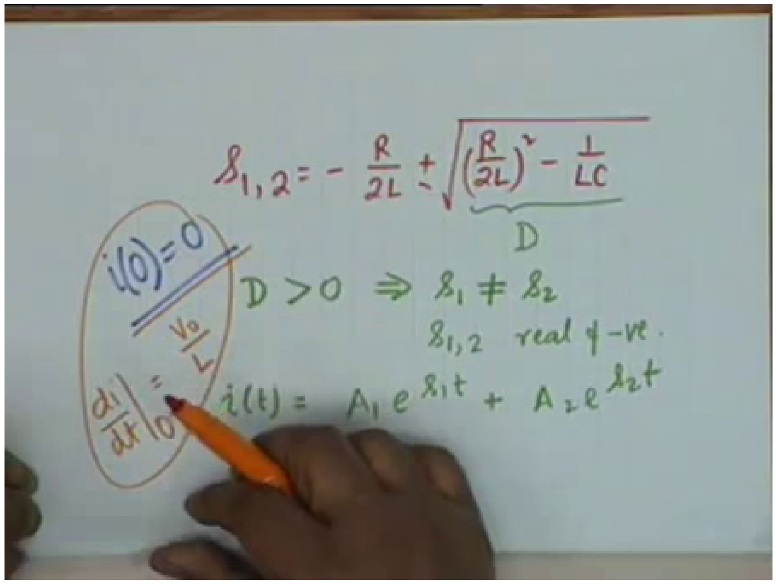

If you substitute this in this equation and clear both sides of A e to the st then this is the equation that you get, which clearly haS 2 solutions and the 2 solutions are s 12 = – R by 2 L + – square root of R by 2 L whole square – 1 over L C and if you if you denote the quantity under the square root sign as capital D then you know capital D stands for discriminant. And we have considered the case when discriminant is greater than 0, this leads to s 1 not equal to s 2 and both s 1 and s 2 are real and negative and therefore the general solution to the current function would be A 1 e to the s 1 t + A 2 e to the s 2 t and we have taken a specific example to show how the current varies with time.

A general solution we had evaluated A1 and A2 from the initial conditions and there are 2 initial conditions to bother about. One of them was that i of 0 was equal to 0 and the other initial condition was that di dt at 0 + = V 0 by L these are the 2 initial condition, this initial condition would apply to every situation whether capital D is greater than 0 whether capital D is equal to 0 or less than 0 it does not matter, if it satisfy this initial condition apply to all conditions all parameter conditions that is values of R, L and C.

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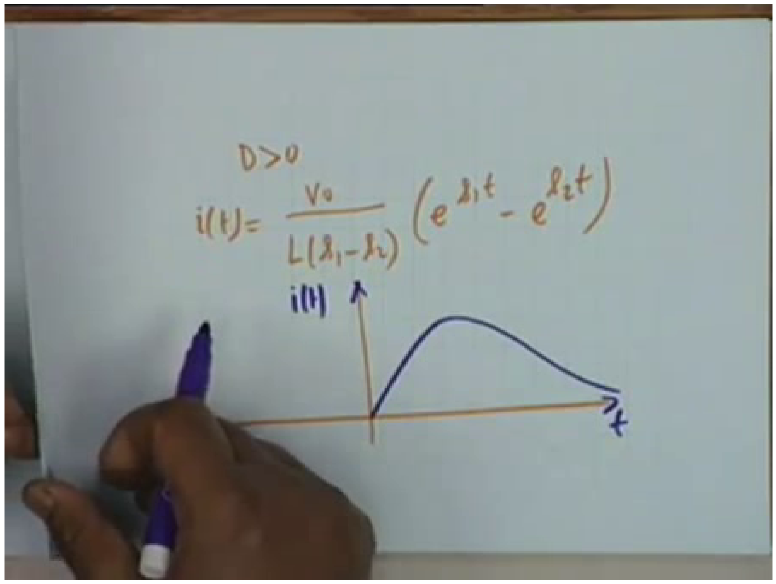

And with this under the condition that D greater than 0 we had shown that the total solution to the current is given by V 0 by L s 1 – s 2 e to the s 1 t – e to the s 2 t. And we have taken a special case and shown that the plot of the current would look like this that it would show a maximum and then it would fall exponentially with time okay, this is one case now let us consider the other case in which D is less than 0 then we shall consider the juncture case that means D = 0 that is the transition from D greater than 0 to D less than 0 or the other way round, we shall consider the transition case at D last instant alright. First let us consider, we had considered capital D greater than 0, let us consider capital D less than 0.

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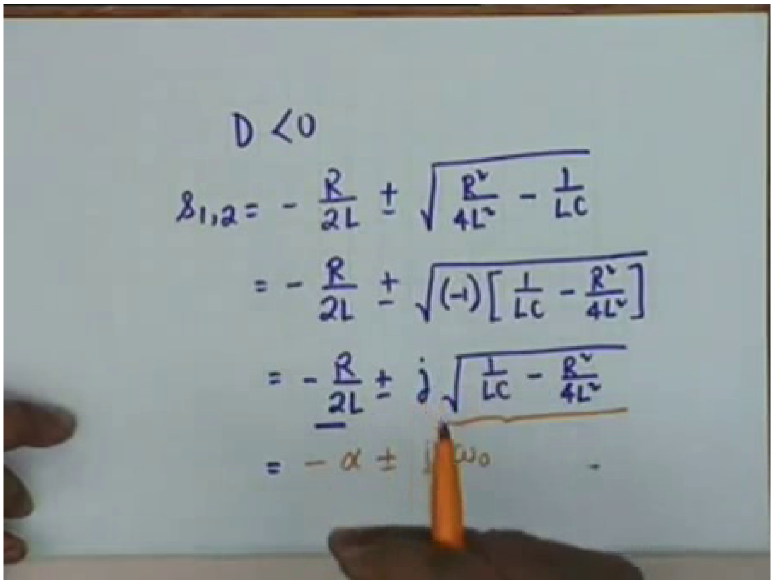

Then s 12, the 2 roots are – R by 2 L + – square root of, D is less than 0 that is R square by 4 L square is less than 1 over L C and therefore this quantity is negative, I can write this as – R by 2 L + – square root of I can take a – 1 common here and then write a positive quantity 1 by L C – R square by 4 L square. And square root of – 1 is j – R by 2 L + – j, in mathematics one writes an i, the small i we use for current in electrical engineering and therefore in electrical engineering we always use a j as square root of – 1. So j square root of 1 over L C – R square by 4 L square this would be the nature of the roots. And you see that unless R is 0, the roots are in general complex quantities, this is a real quantity and this is a complex quantity.

Let us consider let us call the real quantity as – Alpha that is – R by 2 L and + – j let us call this quantity as Omega 0 is a positive quantity, we have taken + – here so the roots are in general complex – Alpha + – j Omega 0. And any complex quantity can be represented in the complex plane where it is a 2-dimensional plane rectangular coordinates, around X axis we represent the real part, around Y axis we represent the imaginary part.

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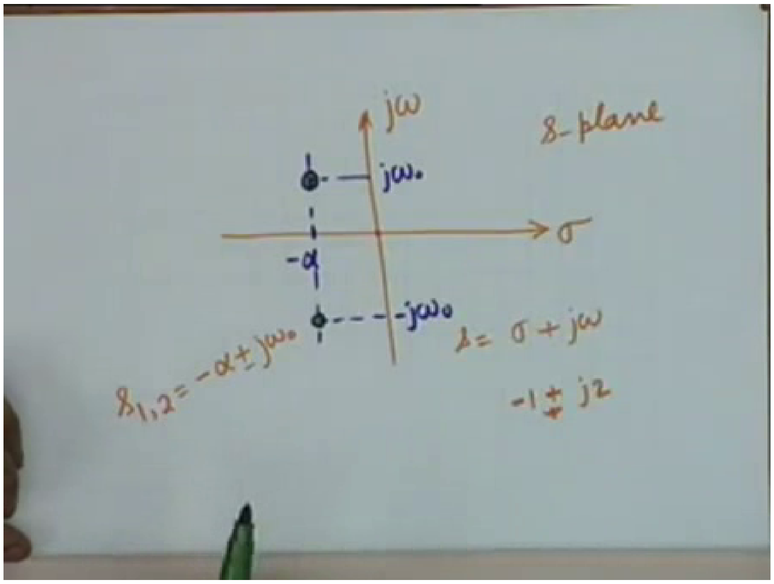

And therefore that plane that complex plane we shall denote as the s plane, as you see these values, these values are values of S. What is S? s was the power of the exponent divided by t, so these are the values of S, s 12 also the roots of the characteristic equation, roots of the characteristic equation and these roots can be represented on the so-called Argand diagram, if you are not acquainted with the name it does not matter but what you do is the real part of S, you consider the s plane, real part of s is represented on the X axis and we call it the Sigma axis and the imaginary part of s is represented on the Y axis the orthogonal axis and you call the Omega axis of the j Omega axis.

This is the usual convention for electrical engineers that you show the imaginary values explicitly Sigma + j mega. For example, this is the 00 point so our s in general is Sigma + j Omega, now if we have let us say – 1 + j 2 then we shall have we shall go – 1 in this direction and you go 2 units perpendicular to it in the upper direction and our point would be somewhere here. But in our case the roots are s 12 = – Alpha + – j Omega 0 so what we do is we will go – Alpha to the left let us say this is with the point – Alpha then we go up and down by a distance of Omega 0 so we have one of the roots here.

And I show this by a circle where this distance is j Omega 0 and you go down by the same distance and put a circle here, this distance is – j Omega 0, so these 2 points these 2 points painted green inside are the 2 roots of the characteristic equation and they can be represented in the complex plane, this plane is called the complex plane because we are representing complex quantities on this plane or the s plane.

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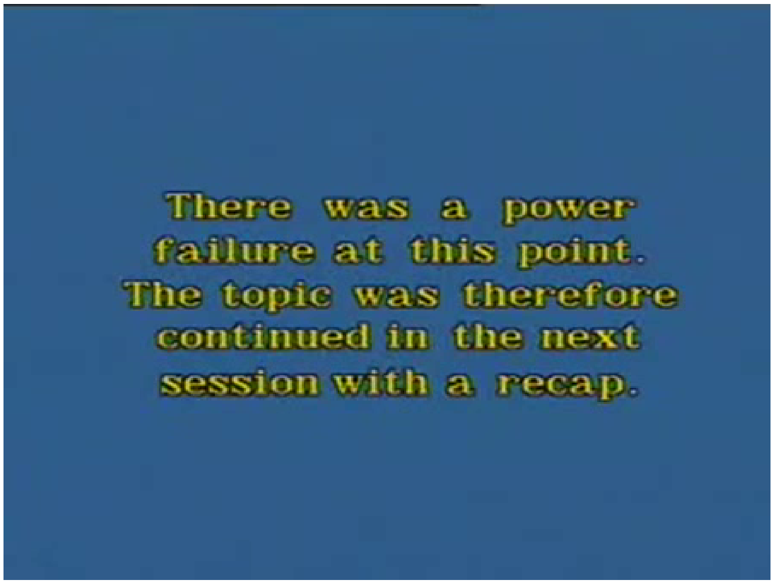

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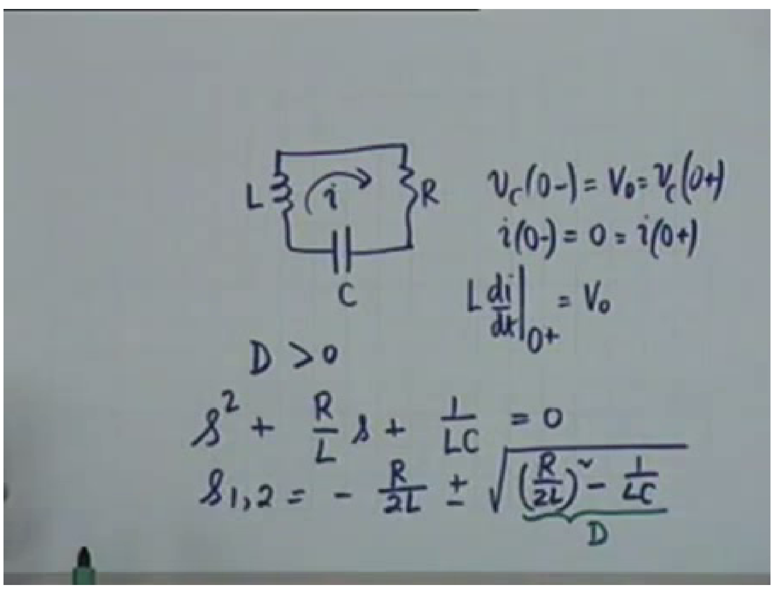

This circuit an RLC series circuit RLC in which V c 0 – = some voltage V0, the current in the circuit if this is the direction of the current i of 0 – = 0, and by continuity of capacitor voltage and inductor current we know that this should be true. The other initial condition is that L di dt at t = 0 + should be equal to V 0, and if this initial conditions we have solved the circuit for the case when the D the discriminant is greater than 0, if you recall the characteristic equation of the circuit characteristic equation was s square + R by L s + 1 over L C = 0 which is s 2 root s 1,2 equal to – R by 2 L + – square root of R by 2 L whole square – 1 over L C and it is this quantity under the square root sign which we have called D.

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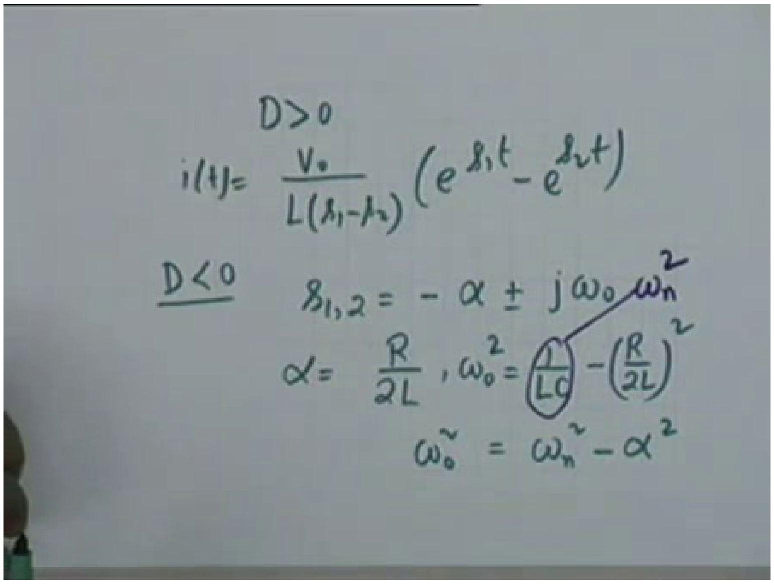

For D greater than 0 we saw that the roots are real, distinct and negative and we saw that the complete solution could be worked out in terms of the initial conditions as if you recall, D greater than 0 the current driven by V 0 by L s 1 – s 2 e to the s 1 t – e to the s 2 t, it got correct? I hope that is correct, yes. And we are trying to see what happens if D is less than 0, we have got that if D is less than 0 then s 12 can be written as – Alpha + – j Omega 0 where Alpha = R by 2 L and Omega 0 square = 1 over L C – R by 2 L whole square. And as you can see, R by 2 L is simply Alpha so I can write Omega 0 square as – Alpha square and this quantity 1 by L C let us use another symbol that is Omega L square.

Let us insert Omega n square as 1 by L C then you will notice that Omega n is simply equal to Omega 0 when Alpha equals 0 that means if there is no resistance in the circuit if there is no resistance in the circuit then Omega 0 is simply equal to Omega n. If there is no resistance in the circuit where are the roots of the characteristic equation? They are clearly imaginary + – j square root of 1 by L C, which means that the roots should be + – j Omega n and therefore Omega n has physical significant.

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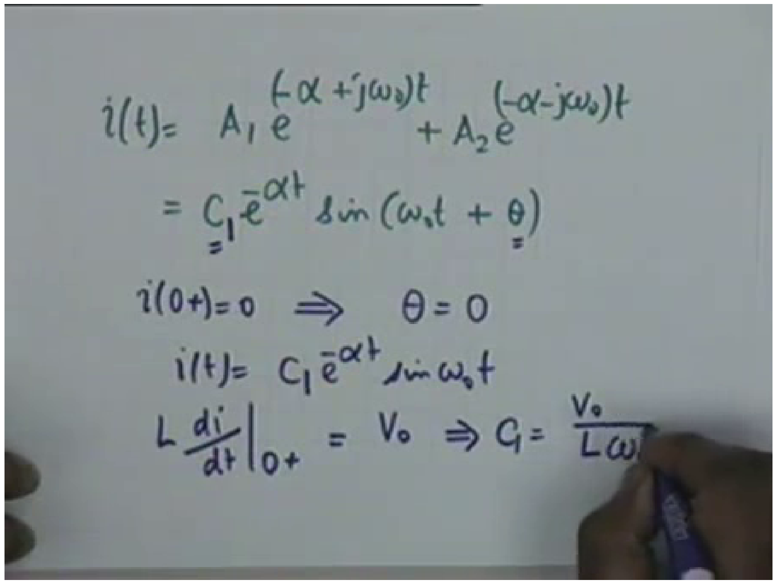

Now under this condition that D is less than 0, we showed last time that the current solution shall be of the form A 1 e to the power – Alpha + j Omega 0 t + A 2 e to the power – Alpha – j Omega 0 t. And we are good but if I take e to the – Alpha t out then the rest of the quantity inside the brackets could be put in terms of cosine and sine and the cosine & sine could then be combined into this particular form Sin of Omega 0 t + Theta, where you see the initial constant, I did these steps last time okay. Now you see the initial constant A 1 and A 2 have now been replaced by modified constants C and Theta, which have to be evaluated from the initial condition.

Let us look at the initial conditions, i of 0 + = 0, this means if I put t = 0 here then you see C sine Theta should be equal to 0, which means that Theta should be equal to 0 alright. And therefore my solution is i of t = C e to the – Alpha t sin of Omega 0 t. Once again we know that L di dt the other equation is that L di dt at t equal to 0 + = V 0, if I substitute i of t = this here and put t equal to 0 then what do you think I will get, I will simply get L times I should use some other constant here, I should not use a C why? C I have used for capacitance so let us use C 1 alright, let us use C 1. Then if I do this if I substitute here the differentiation and put t = 0, can I skip the algebra and write simply that C 1 will be equal to V 0 divided by L that is what it shall be alright, which means is not this okay? If you want you can do the algebra and gladly do it, there is no problem.

“Professor–student conversation starts”

Student: (())(18:06)

Professor: Okay, I am glad you you got that, this should be V 0 divided by L Omega 0 that is correct because a differentiation of this shall give Omega 0 alright, so C Omega 0 cosine Omega 0 t multiplied by e to the – Alpha t, put t = 0 this Omega 0 shall be there thank you very much okay.

"Professor–student conversation ends"

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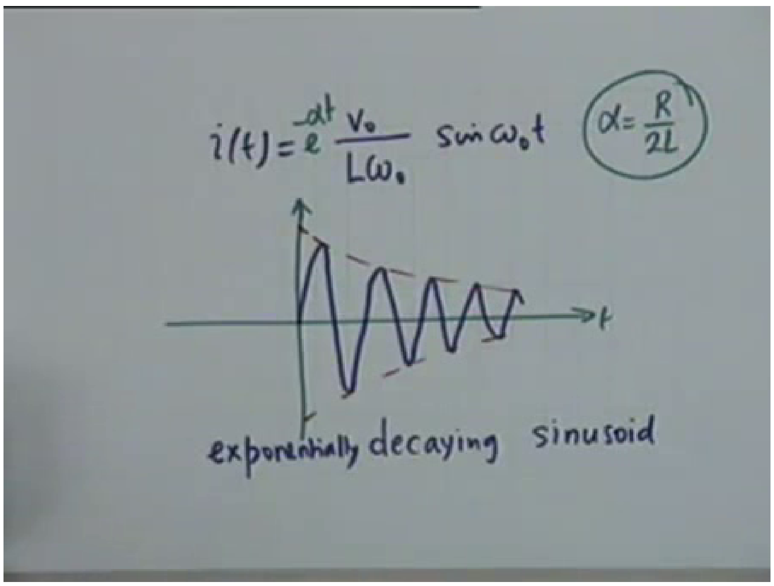

And therefore my current solution would be total solution is V 0 by L Omega 0 Sin of Omega 0 t times e to the – Alpha t that factor shall be there alright. Now suppose sin of Omega 0 t was not there, if this was not there then its dependence on time is exponentially decaying because Alpha is capital if you recall Alpha is capital R divided by 2 L it is a positive quantity and therefore e to the – Alpha decays with time. The presence of sin Omega 0 t means that is a product of an exponentially decaying curve and a sin curve therefore what it means is if I plot i of t versus t, the envelope of the curve shall exponentially decay where else inside there shall be a sinusoid.

If Alpha was 0 if this was not there then it would be a non-decaying sinusoid. On the other hand if Alpha t is there as time increases naturally the amplitude of the sine wave gradually decreases it decreases exponentially and therefore the figure that I shall get it shall start from here, the figure that I shall get will be like this. It starts from here and then gradually the sine wave as time proceeds the maxima and the minima shall gradually decrease; it is not exactly a periodic wave because after one period it does not repeat, the amplitude goes down so this is a decaying sinusoid. How does it decay, the room for decay is exponential so exponentially decaying sinusoid.

It is as if the sinusoid is being damped with time, it is as if the sinusoidal is damped with time and therefore this situation is called Damped oscillation okay. There is an oscillation there is an oscillation but oscillation amplitude generally decays so it is a damped oscillation. Now if Alpha was equal to 0 then you see the oscillations would have been un-damped that is the amplitude would have remained constant whatever the time is. And therefore you now get a physical significance of the quantity Omega 0; Omega 0 is obviously the damped frequency of oscillation or frequency of oscillation under the damped condition.

And you see Omega n then is the frequency of oscillation under undamped condition and Omega n is called therefore the natural frequency of oscillation that is if the damping was equal to 0 then the oscillations for undamped natural frequency of oscillation, the word natural comes because it is not a forced oscillation, it is oscillation because of the initial energy when the system when the system has been left alone and therefore it shows its natural behaviour, so Omega n has the interpretation of undamped natural frequency of oscillation and Omega 0 is the damped natural frequency of oscillation damped natural frequency of oscillation and Alpha is damping coefficient Alpha is the damping coefficient.

What is negative Alpha? 1 over 2 that is correct. And you know that Omega 0 square is Omega n square – Alpha square alright, so all the 3 quantities Omega 0, Omega n and Alpha they have a physical significance and this is the significance. The envelope decays according to this is V 0 by L Omega 0 e to the power – Alpha t, we can take an example at this point this is the general solution, we can take an example.

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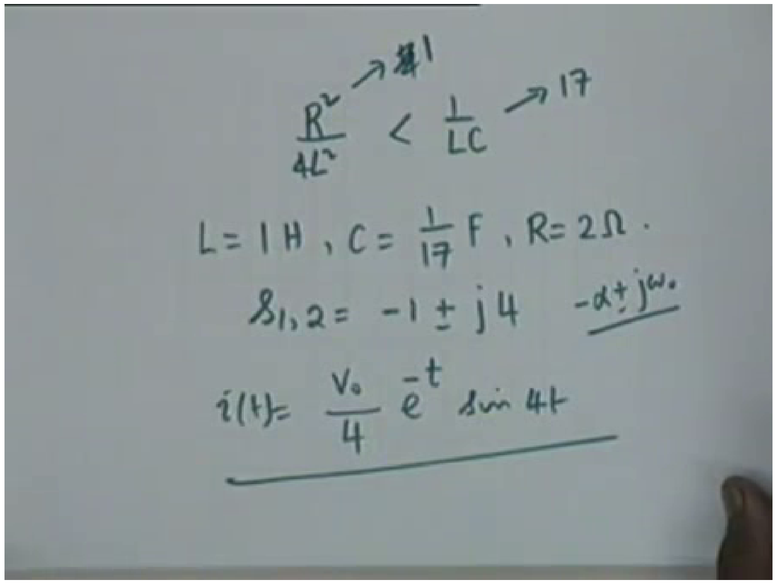

The condition is that R square by 4 L square should be less than 1 by L C, this is the condition under which oscillation takes place and the oscillations shall be damped oscillation. Now, that there are various combinations of R, L and C which satisfy this but one of them let us say C = 1 by 17 farad, we take peculiar value so as to get nice numbers and capital R let us say 2 ohms, then the roots are s 12 = extensive – R by 2 L so – 1 + – now R square by 4 L square is given by 4 and 1 by L C is 17.

Students: That is 1.

R square by 4 L square is one that is right, so 1 definitely is less than 17 and you therefore see that the roots are complex and 17 – 1 is 16, square root of 16 is 4 so – 1 + – j 4. And since we have found out the general solution already, we can write down the current solution as what will be the constant? V 0 V 0 divided by L Omega 0 L is 1 Henry, what is Omega 0? 4 and therefore V 0 by 4 e to the power – what is Alpha? So it is – t, Alpha is 1, the roots are – Alpha + – j Omega 0 alright, so Alpha is equal to 1 then Sin of + 4 t this is the solution alright. If you know the general solution for any combination you can always find this out.

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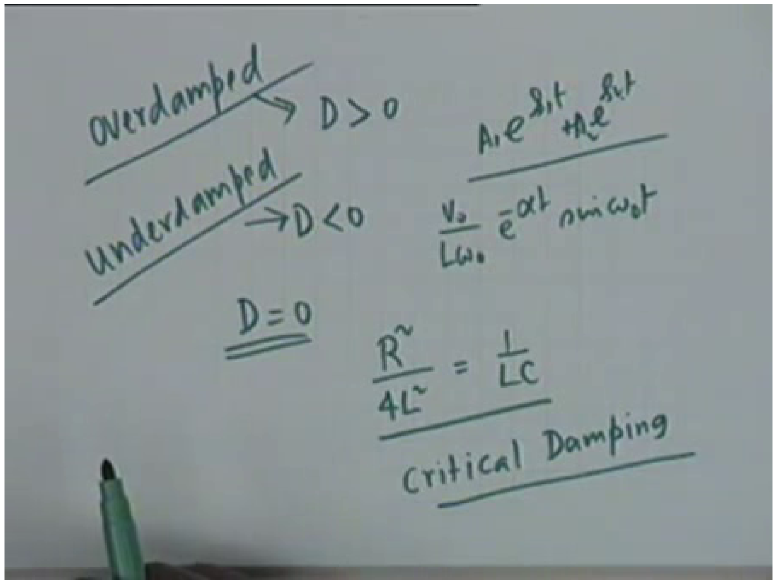

Now we have considered 2 cases 2 cases; that is D greater than 0, in this case there are no oscillations, the solution consists of the sum of 2 exponentials e to the – s 1 t and e to the – s 2 t where s 1 and s 2 are both real and negative alright, e to the s 1 t – e to the s 2 t A 1 + A 2, this is the solution where s 1 and s 2 are both negative, there are no oscillations to the current starts from 0, attains a maximum and then goes to 0. On the other hand, if D is less than 0 then you have oscillations, we have we have V 0 by L Omega 0 e to the – Alpha t sin of Omega 0 t alright.

The borderline between these 2 or the transition case which is which happens at D = 0 occurs obviously when R square by 4 L square is equal to 1 over L C alright, this is a very special condition and the condition on the resistance R to satisfy this is called R critical, it is a critical value of resistance for which this shall be true alright. If this equality becomes an inequality there shall be either oscillations or no oscillations and therefore this is a critical case and this case is called the case of Critical damping critical damping, that is if the damping is slightly less than there shall be what? There shall be oscillations; if the damping is slightly less there shall be oscillations.

Alpha is damping coefficient, if the damping is slightly more there should be no oscillations and therefore this case D greater than 0 is called an over damped case, over damped means no oscillations in the natural response and naturally D less than 0 should be called an underdamped case alright, and D = 0 is the so-called critical damping case critical damping, it determines the boundary between oscillation and no oscillations.