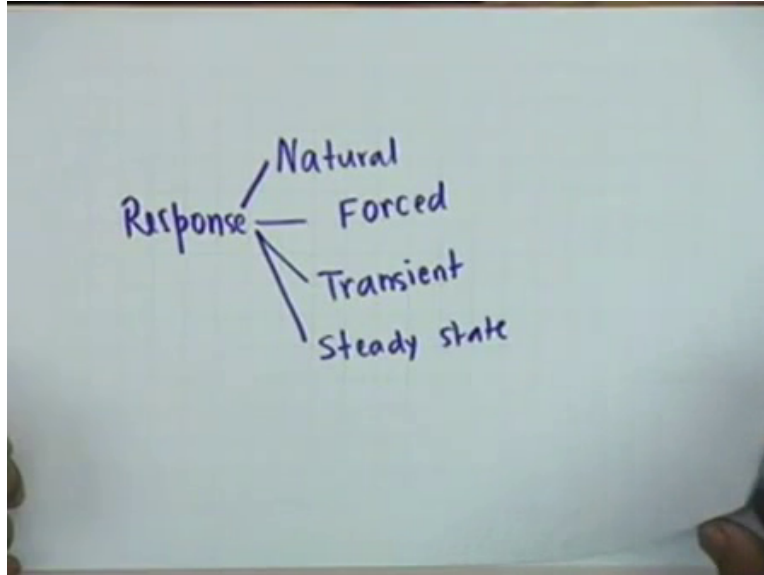


Introduction to Electronic Circuits
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Lecture no 12
Module no 01
Natural Response

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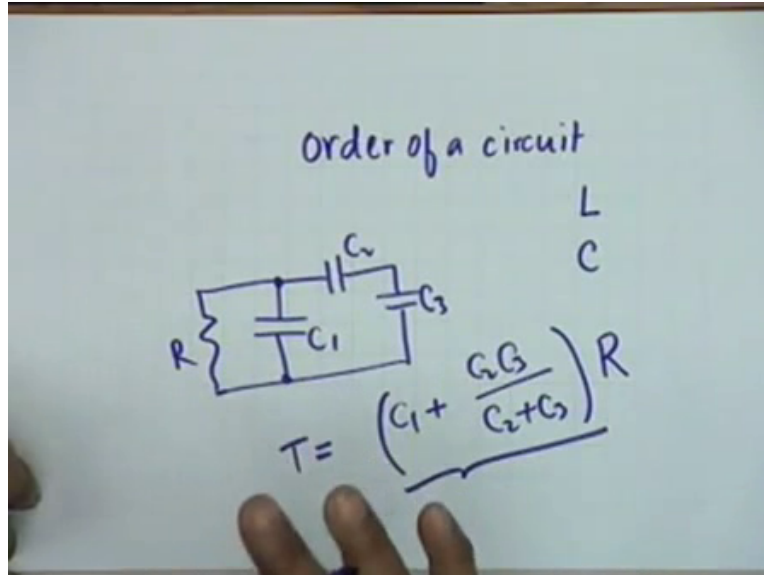


12th lecture on Natural response of Electrical circuits, we have already defined the various kinds of responses that an electric circuit electrical circuit can have and I have said that natural response is the one which arises due to internal stored energies and the circuit being left to itself to settle, there are no external forces or external excitations. If there are external excitations then the response that we get is called Forced response and in general the circuit may have excitations voltage sources or current forces or both and also internally stored energies like electrostatic energy in capacitors and electromagnetic energy in inductors and therefore in general the response of an electrical circuit shall be a combination of natural response and forced response.

And then to other terms which we introduced were the transient response that is the response that occurs at the beginning of expectation and after long time has passed the response that you get from an electrical circuit is called the steady-state response, so the 4 kinds of responses natural, forced, transient and steady-state. Sometimes natural response and transient response are

wrongly identified with each other they are not necessarily the same. Similarly forced response and the steady-state response are also not necessarily the same.

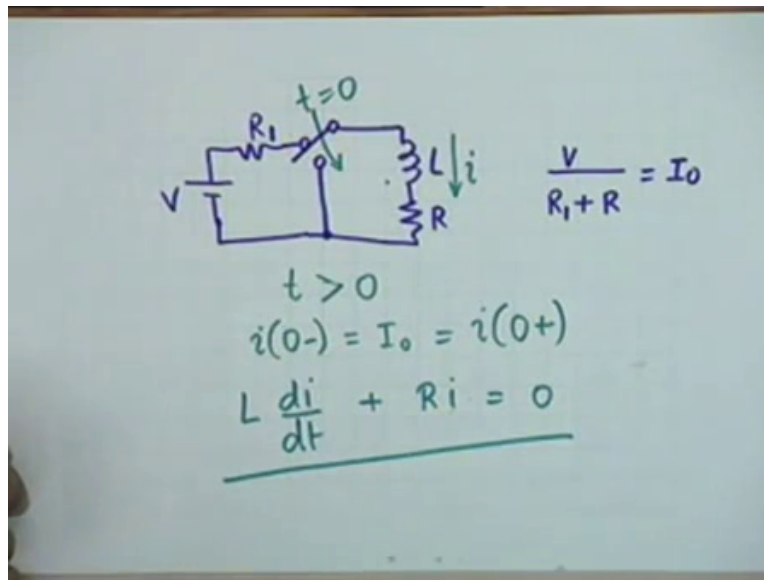
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The previous day I had also introduced to you terms like order of a circuit and I said that a 1st order circuit is one which has effectively 1 energy storage element that is either one inductor or one capacitor alright, there may be more than one capacitors or there may be more than 1 inductors trivially connect to each other which behaves as single energy storage element and of the examples that I get was something like this. If you have let us say 3 capacitors like this and there is a resistance here then you know these 3 capacitors behave like a single capacitor it will be $C_1 + C_2 C_3$ divided by $C_2 + C_3$ and therefore this is the effective capacitance connected across resistance R and this behaves as a 1st order circuit with an effective time constant of this, total effective capacitance multiplied by the effective resistance across it.

Similarly, there may be more than 1 resistor here but effectively they behave like 1 resistor. Similarly, we can have a resistance-inductance circuit which will also be a 1st order circuit or a multiplicity of inductors connected in a trivial manner so that they could be reduced to one effective inductance then that is also a 1st order circuit and as an example I have taken this circuit.

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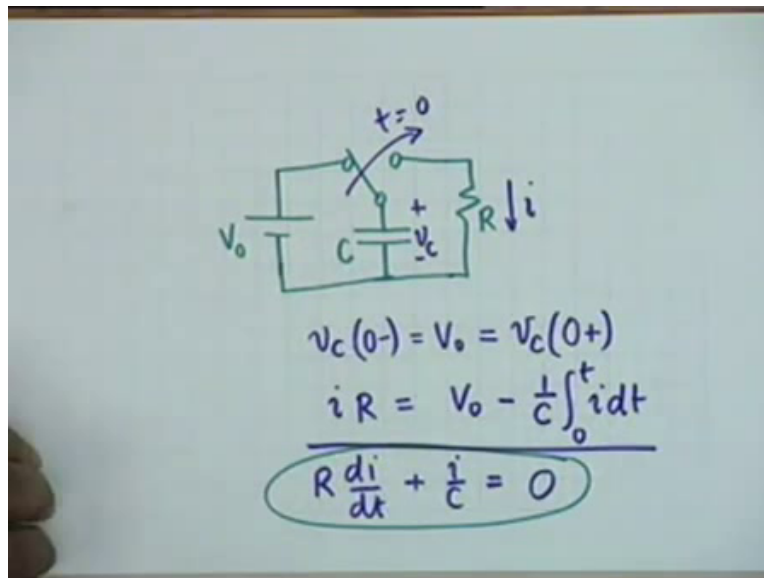
Let us say we have a battery V and a resistance R , which is connected to let us say an inductor L in series with a resistance R alright. A switch is left in this position for a very long time so that the current that passes through this circuit will be given by V by $R_1 + R$ the inductance is not effective in dropping the voltage when the current is steady. So in the steady-state the current in the circuit shall be given by V by $R_1 + R$ and we call this as I_0 . At $t = 0$, this switch has a connection here at t equal to 0 this switch is moved to the downward direction alright, this is how we indicate this.

At t equal to 0 this switch is moved to the downward position so that the initial current in the inductor, the inductor was carrying a current of I_0 , now the inductor and the resistor in series they are connected across a short-circuit and therefore the circuit will have a natural response, it is an example of a 1st order circuit having a natural response that is we are interested in the behaviour of the circuit at t greater than 0 . The initial condition is that if I consider this current as i then i of 0^- – that is just before the switch is closed is equal to I_0 , i of 0^- – this before the switch is thrown in the other position, i of 0^- – this is the indication of the significance of ‘-’ sign here this is I_0 and you know the energy of an inductor cannot change instantaneously and therefore this will be i of 0^+ also alright.

And you can see you can see that as time passes, the inductor shall lose its energy to the resistor, the resistor will dissipate that energy and gradually this current should diminish with time and at

infinity at time $t = \infty$, the current should diminish to 0. The differential equation that one can write is simply the application of KVL that the drop across the inductance + the drop across the resistance should be equal to 0 because there is a short-circuit and therefore the drop across the inductance is $L \frac{di}{dt}$, the drop across resistance is $R i$ and this should be equal to 0, this is the 1st order differential equation and this is what defines the order of a circuit. If a circuit's natural response can be described by 1st order differential equation then it is a 1st order circuit.

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This is not the only one, we could have a capacitor resistor circuit for example, let us say we have a battery V_0 connected like this connected across a capacitor C and then there is a resistor waiting to receive energy from the capacitor C , this is the capacitor C and then we know that if the switch if the switch is left in this position for a long time in this position for a long time then the capacitor voltage V_c if I call this V_c with this polarity, the capacitor voltage V_c at 0^- shall be equal to be V_0 , it will assume the voltage of the battery.

And if the switch is thrown to the other position at $t = 0$ then the capacitor which has stored energy will now try to send the current through R let this current be i alright. It will try to find current through R and in the process the capacitor will discharge or it will lose its energy, gradually this energy will be lost in the resistance as heat dissipation and gradually as time increases the current will decrease, the voltage across the capacitor will decrease and it will take infinite amount of time for the capacitor to completely relax that is completely lose the energy

that it has contained. And once again since the capacitor cannot change its energy instantaneously, the voltage across the capacitor at 0^+ shall also be equal to V_0 .

And if you see the differential equation if you see the equation describing this, once again once again it is an application of KVL or common set alright. It says that if the current i R the drop across the resistance $i R$ should be equal to the drop across the capacitor that is V_c , now what is V_c ? V_c had started at V_0 at $t = 0$ and as time proceeds, this current i it is important to understand this, this current i has to flow in a closed-circuit and therefore this current i charges the capacitor in the opposite direction which is equivalent to say that the capacitor is using charge. What is this charge, it is equal to $1/C$ times the charge integral 0 to t $i dt$ this is the equation, is this equation clear?

The charge the voltage across the capacitor at any instant of time would be the initial voltage – the voltage that it would have acquired in the opposite direction because of the current i flowing in a closed loop. Now, this is not a differential equation, it is an integral equation, in general in general we may get an integro-differential equation. In the previous case in the LR case it was an differential equation pure differential equation, in the RC case it is a pure integral equation nevertheless one can make it into a differential equation by 1 more differentiation. $R di/dt + i/C$ if you differentiate this it is simply i/C that would be equal to $dV_0/dt = 0$. So this is also this is also as you can see of 1st order differential equation and therefore the circuit is also a 1st order circuit.

Let us see what kind of solution you get, you are acquainted with the solution to such differential equations but in the context of electrical circuit we used to bring in some more physical concepts and I want you to pay attention to this.

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$$R \frac{di}{dt} + \frac{i}{C} = 0 \quad \text{HDE 1st order.}$$
$$R \frac{di}{dt} = -\frac{i}{C}$$
$$\frac{di}{dt} = -\frac{1}{RC} i$$
$$i(t) = A e^{-\frac{t}{RC}}$$

Let us see the 2nd circuit 1st, where our differential equation was $R \frac{di}{dt} + I \text{ by } c = 0$. Now the right-hand side of this equation is 0, which means that this is a homogeneous differential equation, homogeneous differential there is no force in turn. Well there cannot be a force in turn because we are investigating natural response and it is quite logical that our right-hand side is 0 that this is a homogeneous differential equation of the 1st order, 1st order homogeneous differential equation. Now in order to solve this equation we know mechanically how we do it, you put an integrating factor and things like that or you take the this quantity to the right-hand side and integrate, change of variables and so on you know all this.

We shall do it in a slightly different manner which is more physical in character, which gives more light into the physical happening in the circuit. We argue that any solution to this the differential equation has to satisfy this; $\frac{di}{dt} = -i \text{ by } C$ any solution has to satisfy this, which means that $\frac{di}{dt}$ shall be equal to $-1 \text{ by } RC$ times i are alright. So what you usually do is bring i here $\frac{di}{dt}$ by $i - 1 \text{ by } RC \text{ dt}$ then you integrate both sides and so on, but no we are not going to do that. What we are going to argue is now comes the physical picture into the situation.

What we argue is that my solution must be of a form such that $\frac{di}{dt}$ cancels $i \text{ by } C$ or $\frac{di}{dt}$ and $-i \text{ by } RC$ must be identical and a function which amply qualifies eminently qualifies for this kind of a job is the exponential function. You know that the exponential function the differential coefficient is again an exponential function and therefore we argue that it must be of the form e

to the power from constant s small s times t . Time has to be there time is an independent variable, it is an exponential function of time it could be a constant multiplied by time and then we argue that in general the solution must be of the form, some arbitrary constant multiplied by e to the st , so is this physical reasoning clear that the solution must be of this form $A e$ to the st , A is arbitrary constant which will be determined by the initial condition and the boundary condition of a problem.

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$$R \frac{di}{dt} + \frac{i}{C} = 0 \quad // \quad i = A e^{st}$$

$$R(A s e^{st}) + \frac{A}{C} e^{st} = 0$$

$$R s + \frac{1}{C} = 0$$

$$s = -\frac{1}{RC}$$

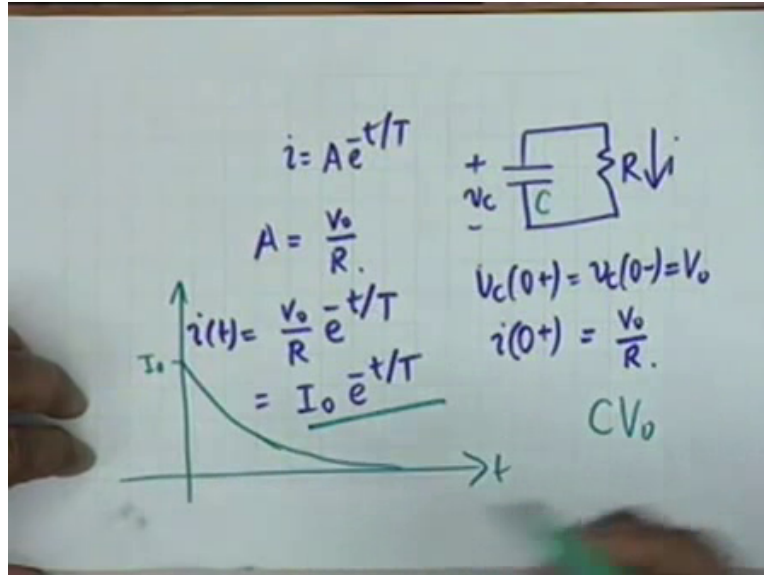
$$i(t) = A e^{-t/(RC)} = A e^{-t/T}$$

Exponential function we took because di/dt dv/dt of this must be of the same form as i same form as the function and therefore it must be an exponential function. And if I substitute this in the original equation then what I get is, our original equation is $R di/dt + i/C = 0$ and I substitute $i = A e$ to the st this is called a trial solution, we try a solution of this form then I get $R A s e$ to the $st + A/C e$ to the $st = 0$, where $A e$ to the st cannot be equal to 0 because this is the form of the solution, if this is 0 then of course the current is identically equal to 0.

Current identically equal to 0 obviously satisfies the equation that cannot be a solution because we know if the capacitor has an initial energy it shall send a nonzero current and therefore i cannot be 0 therefore $A e$ to the st cannot be 0, the only other alternative is that R times $s + 1/C$ should be equal to 0 which means this s should be equal to $-1/RC$ alright s should be equal to $-1/RC$ in other words, the solution to the equation shall be of the form $A e$ to the

power $-t$ divided by RC and you notice that RC the product RC must have the dimensions of time and therefore we denote this as a time constant and we say $A e^{-t/T}$.

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The only problem that remains is to find out capital A alright, that is found out from the initial condition, you know that if I draw the circuit again, this is a C in parallel with R , this is i and you know that $V_c(0+)$ is the same as $V_c(0-)$, we recall that this is equal to V_0 and therefore what is $i(0+)$, it must be V_0 by R , our solution is $A e^{-t/T}$ so $i(0+)$ is obtained by $t = 0$ which means that A must be equal to V_0 by R and therefore the total solution is V_0 by $R e^{-t/T}$ alright. Now let us call this as $I_0 e^{-t/T}$ okay, now if you plot this current versus time, then it starts at I_0 it starts at I_0 and falls exponentially and you know how to define the time constant that it is the time after which the current has fallen by how much? $1/e$ of its original value and so on and so forth.

The point is, the current is decaying exponentially, it is only on infinite time $t = \infty$ that the current shall diminish to 0. What is the charge that flows? After all the capacitor started with a certain amount of charge which was equal to C times V_0 alright, so how much charge does the capacitor lose in the process alright that obviously would be obtained by integrating over from 0 to infinity.

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$$q = I_0 \int_0^{\infty} e^{-t/T} dt$$
$$= I_0 T = \frac{V_0}{R} \cdot RC$$
$$= \underline{\underline{V_0 C}}$$

Now so the charge that has passed shall be equal to $I_0 \int_0^{\infty} e^{-t/T} dt$ and it is very easy to see that this is simply equal to I_0 times capital T okay you integrate this and put the limits I_0 times capital T . I_0 is V_0 by R and capital T is RC and you can see that this is precisely the total charge that has that the capacitor loses in the process of sending the current through the resistance is precisely the charge that it started with, so it loses its whole charge, is that clear? This is the physical reasoning that I wanted to bring into the solution of the differential equation, you should not do it blindly must have the physical picture in mind.

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$i \downarrow$

L

R

$i(0^-) = i(0^+) = I_0$

$L \frac{di}{dt} + Ri = 0$

$Ae^{-t/T}$

$i(t) = I_0 e^{-t/T}$

$T = \frac{L}{R}$

In exactly similar manner we can take care of the inductance resistance circuit L-R which is short-circuited with current i here $i(0^-) = i(0^+) =$ let us say I_0 and the differential equation satisfied by the circuit is $L \frac{di}{dt} + Ri = 0$. If you proceed exactly similarly that is you put $i = A e^{-st}$, incidentally what should be the dimension of s ?

Student: 1 by t .

1 by t second to the -1 and 1 by t is frequency and therefore, small s is called a frequency. We will come to this a little later, the physical interpretation is small s and you will see how nicely it tunes with physical concepts of frequency. But if you try a solution of this form and obtain and substitute this initial condition that is $i(0) = I_0$, you shall be able to show that $i(t)$ is exactly of the same form that is $I_0 e^{-t/\tau}$ by capital τ where capital τ is now $= R$ upon L , instead of R/C it is R upon L alright and the same kind of argument about the total charge that passes in the circuit, if you integrate this it will be I_0 times capital τ . This also incidentally brings in another definition of capital τ , is not it right? What is it what is the definition?

Another definition of capital τ , one definition of time constant is the time required for the current to drop by $1/e$ value at $t = 0$. Another definition could be in terms of charge that it is effectively the initial current, if initial current had passed for a time equal to capital τ then the charge that is lost would be equal to the actual charge lost by the capacitor or actual charge flowing in this circuit for infinite amount of time, is that clear? Now let us go back here.

“Professor–student conversation starts”

Student: What is the time constant?

Professor: Time constant you see can be written as $\tau = V_0 C$ divided by I_0 if I go back to the original problem.

Student: In that inductor thing it should be L by R .

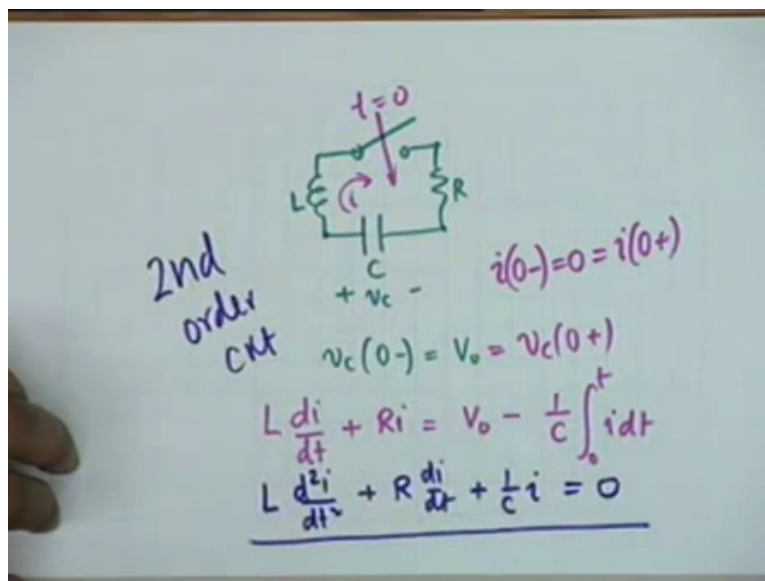
Student: Yes sir, L by R not R by L .

Professor: Why did you take so long? Thank you, this is the correct story alright.

“Professor–student conversation ends”

Now an interpretation of the time constant, it is more evident in the capacitor resistor circuit. The capacitor started with the total charge of $V_0 C$ and you see if it had if it had sent a current at the rate if it had sent a constant current I_0 then it requires only time equal to capital T, so you can define capital T as that amount of time is the time required for the capacitor to lose its charge if it maintains a constant current at the initial value, this is another interpretation or definition of the time constant. Now in a about the 1st order circuit, now let us look at 2nd order circuit.

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Let us consider a capacitor C, which is charged to a voltage V_0 alright. And then and then let us say at let us say at t equal to 0 this capacitor is 1st charged to a voltage V_0 maybe by connecting a battery across this battery of voltage V_0 , it has in this circuit in the circuit an inductance and a resistance and a switch. If the switch is open then the capacitor sits pretty it does not have to lose charge, but suppose we the switch is closed at $t = 0$, the switch is thrown into the lower position at $t = 0$ then obviously the capacitor cannot sit pretty it shall have to lose charge, a current shall flow like this, capacitor is charged with this plate positive so this charge will effectively behave like a battery, you remember the Thevenin’s equivalent circuit which will tend to find current in this direction.

And as time passes as time passes the current this current i has to pass through the capacitor in the opposite direction which means that the capacitor gets charged in the opposite direction

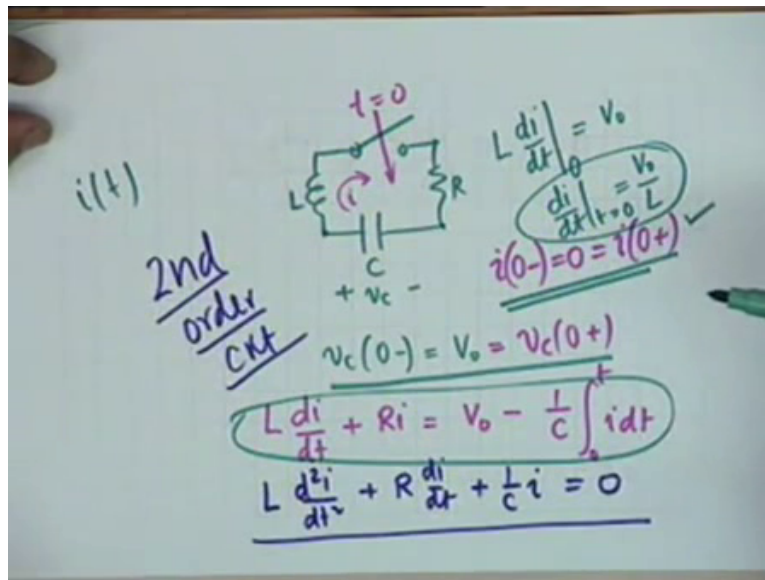
which is equivalent to saying that the capacitor loses charge and as time proceeds it loses more and more charge and therefore the current gradually diminishes the charge diminishes and after infinite amount of time the circuit will again be relaxed that means it shall have no initial energy. Let us look at the equation, what kind of equation does this circuit okay.

Obviously, once again it is a simple application of KVL and you notice that the inductor voltage $L \frac{di}{dt} + R i$, this should be equal to $V_{sub c}$, but $V_{sub c}$ starts with V_0 then this charges that is the current charges in the opposite direction $\frac{1}{C} \int_0^t i dt$ this is the equation that is obeyed by the circuit and you notice that this is an integro-differential equation with the initial conditions that, what is the initial condition? i at $0^- = 0$ and since this is the current through the inductance which cannot change instantaneously, i of 0^+ shall also be equal to 0 one initial condition and the other is that $V_{c 0^-} = V_0 = V_{c 0^+}$ alright is the other initial condition.

Now the integro-differential equation as I have told you is always converted it can be converted into a differential equation by one more differentiation. We have to get rid of the integral sign so what I get is $L \frac{d^2i}{dt^2} + R \frac{di}{dt}$ and in the process V_{dt} of V_0 becomes equal to 0 and therefore we get simply i by C which is taken to the left-hand side becomes $\frac{1}{C} i = 0$. If you notice, this equation is also a homogeneous differential equation of the 2nd order and therefore this circuit is called a 2nd order circuit and we are indeed talking of natural response, the initial energy in this circuit comes from the capacitor voltage.

It could also, instead of the capacitor voltage it could also have come from an initial electromagnetic energy in the inductor. For example, inductor could have current of i_0 and there are $t = 0^-$ $t = 0^+$ it is connected across the capacitor and the resistor exactly the same way and we would have the same kind of differential equation a 2nd order differential equation thus defining a 2nd order circuit.

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Let us write this equation again, $L \frac{d^2i}{dt^2} + R \frac{di}{dt} + i \text{ by } C = 0$. I want to make a point regarding initial conditions, can we go back to the previous part? Before I pass to the to the solution of the equation, this see what I am going to do is to solve this solve this differential equation for i alright so i of t . And i of t I shall obtain in terms of unknown constant which I shall try to evaluate from the initial conditions. One of the initial conditions is that i of $0 = 0$, i of $0 -$ and $0 +$ are the same. The other initial condition I said, it is not on i might is on the capacitor voltage, can you convert it to a condition on i ? This is very illuminating.

By looking at this equation alright, let us consider $t = 0 +$, at $t = 0 +$ what is the value of i ? It is 0, $\frac{di}{dt}$ is not 0 not necessarily, i can be 0 but it can be rising so then what about this integral? 0 and therefore do not you see that my initial condition is $L \frac{di}{dt}$ at $t = 0$ should be equal to V_0 and therefore $\frac{di}{dt}$ at $t = 0$ should be equal to V_0 by L , this is the other initial condition. So there are 2 initial conditions, it is a 2nd order circuit, it stands to reason that you have to solve the 2nd order differential equation you require 2 initial conditions alright and the 2 initial conditions are that the initial current is 0 and that the initial rate of change of current is simply initial voltage across the capacitor divided by inductor, the resistance does not come into play anywhere in this.

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$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = 0$$
$$i = A e^{\lambda t} \checkmark$$
$$\lambda^2 L + \lambda R + \frac{1}{C} = 0$$
$$\lambda^2 + \lambda \frac{R}{L} + \frac{1}{LC} = 0$$
$$\lambda_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Why not? Because i of 0 is 0 so there is no drop in the resistance, are these 2 initial conditions clear? Can we go ahead to the solution of the circuit? Once again once again we shall proceed from a physical reasoning rather than integrating directly physics or you see an integrating factor as we do in mathematics we do not want to leave side of the physical situation and so we say alright, we notice that i , $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ must be of the same functional form otherwise their sum cannot be 0, if one is a Sine, the other is a Cosine and 3rd is an exponential obviously it cannot it cannot cancel. And therefore we argue once again that i must be of the form of $A e^{\lambda t}$.

Now if I substitute this here and clear $A e^{\lambda t}$, $A e^{\lambda t}$ cannot be 0 clear $A e^{\lambda t}$ then what we will get is $s^2 L + s R + \frac{1}{C} = 0$, do you see this? And will do a couple of algebraic steps. I quickly substitute this, take 2 differentiations, I shall get s times R here and s^2 times L , $A e^{\lambda t}$ I cancel because it cannot be equal to 0 and this equation yes...

“Professor–student conversation starts”

Student: Sir, at $t = 0$ – like $0 + i(0)$ (33:31) $A(0)$

Professor: $A(0)$ at...

Student: (0)(33:36)

Professor: That is $t = 0$.

Student: Yes sir you have to apply that boundary condition

Professor: Okay, however we shall apply that boundary condition later alright. This is the 1st term this is not the 1st order circuit so this cannot be the complete solution, we are trying the form of the solution, you say is this does this qualify as the form of the solution. Let me clarify this, in the 1st order circuit we are trying a complete solution, we know the solution has to be of the form $A e^{st}$. Here obviously since it is a 2nd order circuit the solution must be more complicated than $A e^{st}$ because at $t = 0$ as you rightly pointed out since $i(0) = 0$, A has to be identically equal to 0 so this cannot qualify as the total solution.

But does this qualify as a form of the solution this is what we wish to find out and the physical situation immediately bring in the total solution. As you see here differential equation $s^2 R + s L + \frac{1}{C} = 0$ this is a quadratic equation, so there are 2 values of s which satisfies the equation there are 2 values of S , original it was only, in the 1st order circuit it was only one value of S , here there are 2 values and these 2 values are s_1, s_2 you know what it means, subscript 1, 2 that we write both of them simultaneously and you can see that this is $-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$, there are 2 possible values of s and therefore there are 2 possible solutions of the original differential equation and these solutions are...

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$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = 0 \checkmark$$
$$\lambda_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$
$$i(t) = A_1 e^{\lambda_1 t} + \underline{\underline{A_2 e^{\lambda_2 t}}}$$

linearly independent

$$i(0) = A_1 + A_2 = 0$$

My equation is $L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = 0$, and the 2 solutions are $-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$. And therefore the solutions are $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$, both of them shall satisfy the original differential equation and multiplying by any arbitrary constant will not affect the solution and therefore we say these constants are let us say A_1 and A_2 , both of them individually satisfy the original differential equation so the sum of them should also satisfy the original differential equation and this qualifies as the general solution to the original differential equation. We shall not apply the initial conditions till we reach this point till we get the general solution to the equation.

$e^{\lambda_1 t}$ was a trial solution, we see that indeed it is of the correct form but it does not satisfy the initial conditions therefore, why is it so? Because it is a 2nd order circuit and therefore there must be 2 linearly independent solutions, is this phrase known to you? What does linear independence mean? By multiplying this by constant I cannot get the other solution, this $e^{\lambda_2 t}$ cannot be obtained from $e^{\lambda_1 t}$ by simply multiplying by a linear constant alright. These are 2 independent solutions and now we can apply initial conditions. The 1st initial condition is that $i(0) = 0$ that is $A_1 + A_2$ should be equal to 0, which tells me that A_1 should be equal to $-A_2$ alright.

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$$i(t) = A_1 (e^{\lambda_1 t} - e^{\lambda_2 t})$$
$$\left. \frac{di}{dt} \right|_0 = \frac{V_0}{L} = A_1 (\lambda_1 - \lambda_2)$$
$$A_1 = \frac{V_0}{L(\lambda_1 - \lambda_2)}$$
$$i(t) = \frac{V_0}{L(\lambda_1 - \lambda_2)} (e^{\lambda_1 t} - e^{\lambda_2 t})$$

In other words, the general solution therefore shall be of the form $A_1 e^{\lambda_1 t} - e^{\lambda_2 t}$, A_2 is simply A_1 , and the other initial conditions is that di/dt at 0 should be equal to V_0 by L , which means that A_1 now make a ddt and put $t = 0$, ddt of $e^{\lambda_1 t}$ is $\lambda_1 e^{\lambda_1 t}$ and if you put $t = 0$ then it is simply λ_1 - similarly λ_2 is that clear? I have omitted the differentiation step and therefore $A_1 = V_0$ by $L(\lambda_1 - \lambda_2)$ is that okay? And therefore my total solution i of $t = V_0$ by $L(\lambda_1 - \lambda_2)$ times $e^{\lambda_1 t} - e^{\lambda_2 t}$ this is the total solution total solution to the equation.

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$$\lambda_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$
$$D = \left(\frac{R}{2L}\right)^2 - \frac{1}{LC}$$

$D > 0$	real, distinct, -ve.
$D = 0$	" , coincident, "
$D < 0$	Complex, distinct, conjugate
$R = 0$	pure imaginary, " . "

$\sqrt{-5} = j\sqrt{5}$
 $j = \sqrt{-1}$

Now so far I have not said anything about the nature of these 2 quantities s_1 and s_2 , if you recall s_1 and s_2 are given by $-\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$. The nature of these 2 solutions obviously this is constant obviously shall depend on the discriminant that is D which is $\frac{R^2}{4L^2} - \frac{1}{LC}$. If D is different than 0 then obviously the 2 roots shall be real, distinct, anything else that we can say? Negative, is not that right? $-\frac{R}{2L} +$ a quantity which is less than $\frac{R}{2L}$ obviously it shall be negative quantity. When the sign is negative, $-\frac{R}{2L} -$ obviously the whole thing is negative, so all are distinct, this is a real distinct and negative.

If $D = 0$, obviously that is possible by combination of R , L and C , you could make them equal to 0, they are real, coincident we do not say equal they coincide they call us upon each other and they are still negative. Okay, if D is less than 0 these roots are complex, why complex because there is a real part and D is less than 0 so this will be J times okay square root of -5 is J times square root of 5, where $J = \sqrt{-1}$ alright, so you will have $-\text{real quantity} + -J$ times another real quantity, so it will be complex, distinct we can say positive or negative conjugate here.

Suppose $R = 0$ then the roots are not only complex but purely imaginary purely imaginary because this is 0, this is 0, square root of $-\frac{1}{LC}$ so purely imaginary, to emphasise you say purely okay, purely imaginary, distinct and conjugate. We shall talk about this pardon me...

“Professor–student conversation starts”

Student: (())(42:18)

Professor: They are of course conjugate.

Student: Sir when D is 0 and there are different solutions for the differential equation, we will have full dependent solution and when D is 0 s_1 will be equal to s_2 .

Professor: In the case of repeated roots in the case of repeated roots as you know in the differential equation we have to have $A_1 + A_2 t$ multiply by e to the multiply by the exponential but we are not bothered about that, we will go by this and see what exactly comes from the physical picture alright, we will look at it from more a physical point of view because it is

introduction to electronic circuits that you are learning, not solution to differential equation. We have to solve the equation to be able to find a solution.

“Professor–student conversation ends”

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$D > 0$ s_1, s_2 real, distinct & -ve.
 $i(t) = \frac{v_0}{L(s_1 - s_2)} (e^{s_1 t} - e^{s_2 t})$
 $[L] = [C][R][R] \left(\frac{R}{2L}\right)^2 > \frac{1}{LC} \Rightarrow R > \frac{2}{\sqrt{LC}}$
 $R > 2\sqrt{\frac{L}{C}}$

Let us consider this case D greater than 0 that is roots are s_1, s_2 are real, distinct and negative alright. And you already know that the current solution is equal to v_0 by $L(s_1 - s_2) e^{s_1 t} - e^{s_2 t}$. D greater than 0 means R, L and C are such that this quantity R by $2L$ whole square let us look at this, this is also interesting, D greater than 0 means what? Let R by $2L$ whole square should be greater than 1 by LC , which means that R square should be greater than $4L$ by C is that right, that is R should be greater than 2 square root of L by C .

The 2 things I want you to notice that it depends on the total resistance of the circuit okay, if the total resistance in the circuit is large compared to is greater than twice square root of L by C , well the roots will be real and distinct. If you if you take a resistance, Rheostat for example in the circuit and gradually reduce it then you can see all the phenomena that is you can see real effective real distinct negative roots, effect of real coincident negative roots, effect of complex roots and effect of purely imaginary roots. If you simply go and adjust this resistance, when it is 0 you will see completely imaginary roots and so on alright.

The other thing that I want you to notice is the square root of L by C obviously has the dimension of resistance, is not it right? So if you wish to express the dimensions of L in terms of the dimensions of C and R square then obviously it would be this right, L would be equal to C R square. Now let us take a numerical example for this case, let us suppose...

“Professor–student conversation starts”

Student: (0)(45:41)

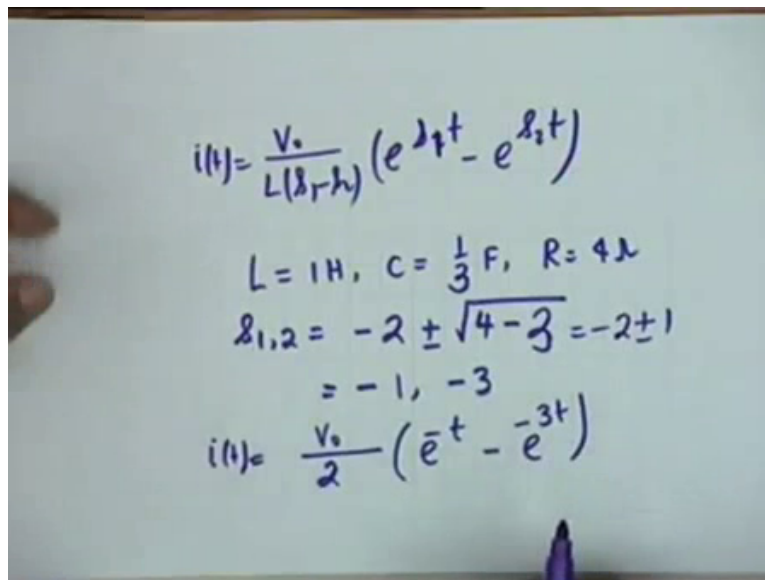
Professor: About what?

Student: How do circuit behave?

Professor: How does the circuit behaves, this is what I am going to show you, this is what I am going to show you with numerical example.

“Professor–student conversation ends”

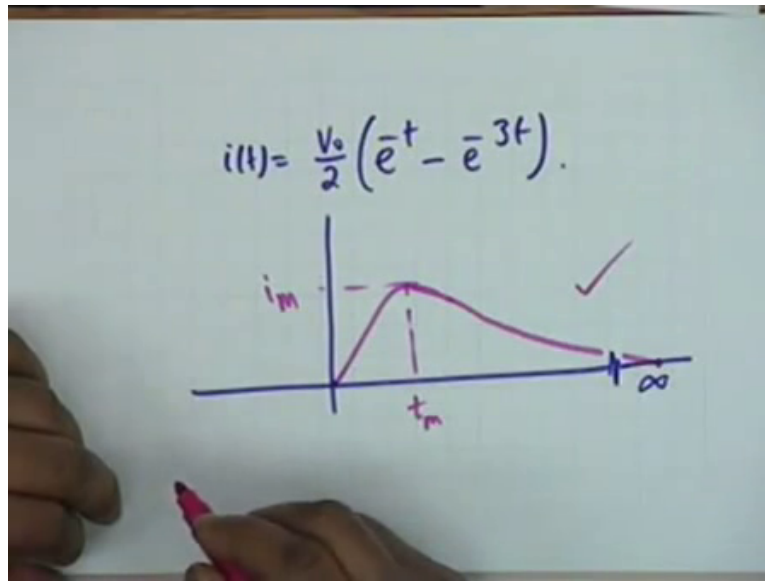
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$$i(t) = \frac{V_0}{L(\lambda_1 - \lambda_2)} (e^{\lambda_1 t} - e^{\lambda_2 t})$$
$$L = 1\text{H}, C = \frac{1}{3}\text{F}, R = 4\Omega$$
$$\lambda_{1,2} = -2 \pm \sqrt{4 - 3} = -2 \pm 1$$
$$= -1, -3$$
$$i(t) = \frac{V_0}{2} (e^{-t} - e^{-3t})$$

Well, we have an example in which let us say $L = 1$ Henry, $C = \text{half Farad}$ and R equal to 4 Ohms alright. If this is the combination of elements then you see s^2 is $-R$ by $2L$ so it would be -2 is that okay, 4 divided by 2 into $1 - 2 + -$ square root $4 - 1$ over LC , 1 over LC I have made a mistake. Let us take would you pardon me let us take C as one third Farad, I want to keep

life simple alright, let us take C as one third Farad then this will be $4 - 3$, which means that it is $-2 + -1$, the roots are real, distinct and negative, so the roots are -1 and -3 . And therefore, i of $t = V_0$ by L is 1 , $s_1 - s_2$ is obviously 2 is that right? $-1 + 3 = 2$ and then e to the $-t$ $s_1 - 1$ is -1 and s_2 is -3 so e to the $3t$ okay.

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Let me write it down again, i of $t = V_0$ by $2 e$ to the $-t - e$ to the $-3t$, how do you think the current will behave with time? How shall it vary with time? Obviously, at 0^- the current was 0 and at 0^+ the current will be 0 , put $t = 0$ it is 0 . At infinity when t goes to infinity this term goes to 0 , this term goes to 0 , $0 - 0$ is 0 so at infinity it must again be 0 , let me show infinity on a finite plane okay by a break. Let us say this is the point at infinity so infinity is also 0 , in between the current is positive it does not change direction it cannot be 0 and therefore it must have a maximum it must have at least one maximum, in a higher order circuit it can have more maximum and minimum alright so it must have at least one maximum.

So it must go to a maximum and then goes to 0 , we show a break because you are showing infinity on a finite plane alright. So the current attains maximum value somewhere and when came by differentiating this, find out at what value of time at what value of time the current attains maximum and then what happens. One can also find out the total charge that is to this circuit, what do you think will be the total charge? It will again be C times V_0 that is one third V

0. If you integrate this from 0 to infinity we shall get the same expression. On Monday we will consider the case of a complex root that is D less than 0 and we will have fun more fun there.