Digital Communication
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Lecture No 31
Performance of M'ary Digital Modulations

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We will now take up the last topic in digital modulations that is of interest to us, namely get a feel for the performance of M-ary digital modulations. We have fully taken care of the binary modulations.

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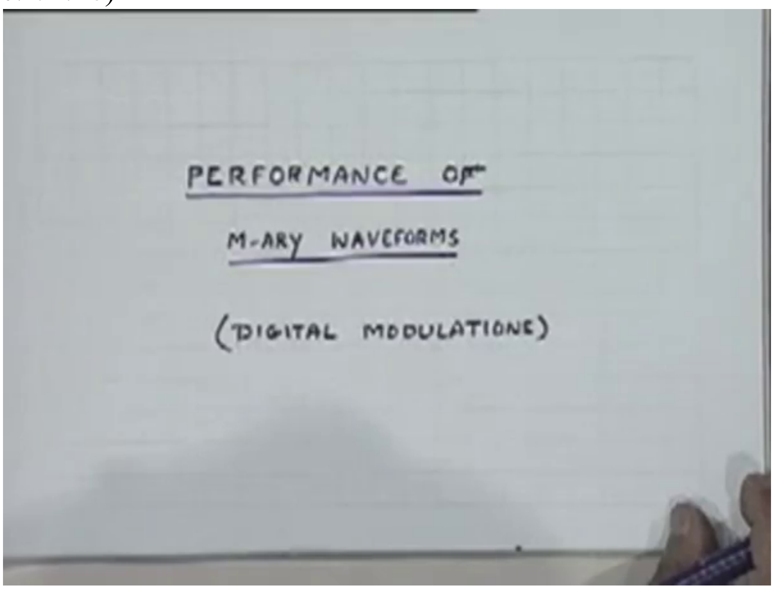

The only thing left is M-ary digital modulations. We have seen the receiver structures, primary digital modulations both for orthogonal as well as

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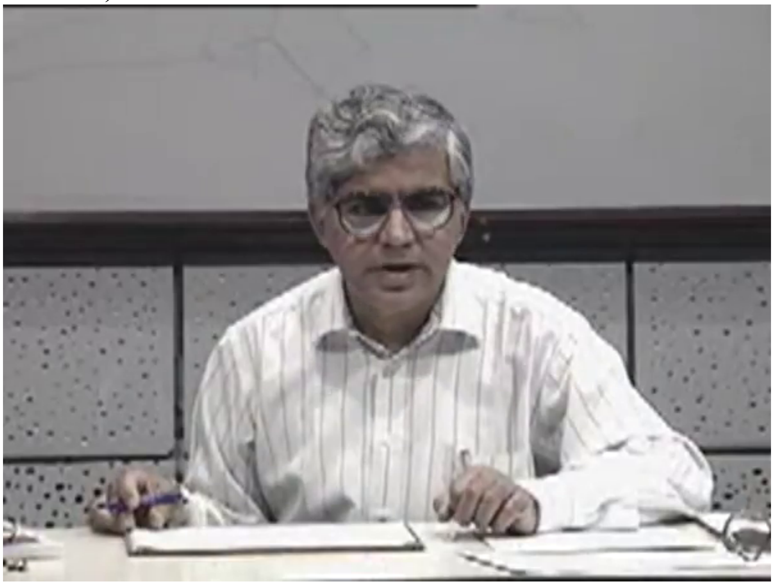

other M-ary waveforms, M-ary modulations based on two dimensional signal constellations, right? Today we would like to take up the performance.

Now because of shortage of time, I will not be taking up detailed performance analysis of every, all aspects of M-ary waveform performance, M-ary digital modulation performance. However I will go through the broad approach and also by now you are quite reasonably familiar with the techniques that need to be used and I will be therefore leaving a number of things for self-study in this. So as we go along I will tell you precisely what you have to read yourself. I will be talking about the major result over here, and it will be very easy for you to do that because the basic ideas are similar to what you have been doing so far, Ok.

So we will take up the performance of M-ary waveforms. And to start with I will take M-ary orthogonal waveforms.

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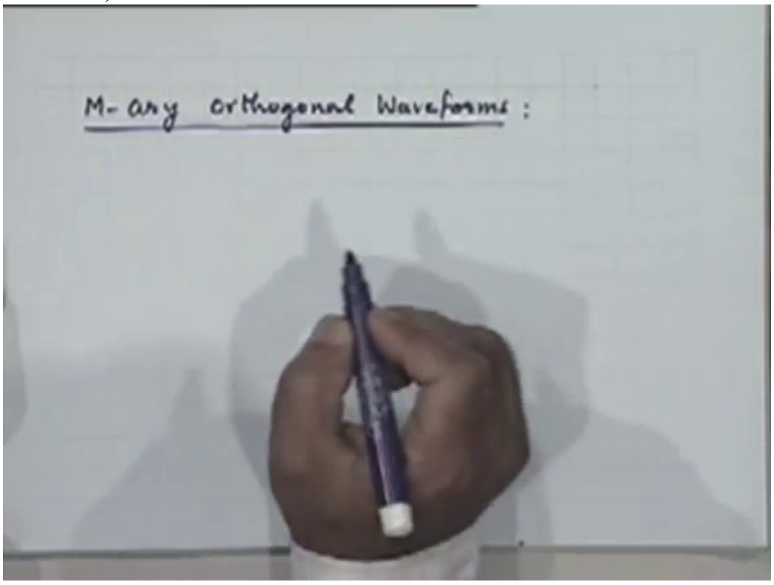

And in this class of modulations we will consider the performance of the coherent T modulators or the coherent receivers.

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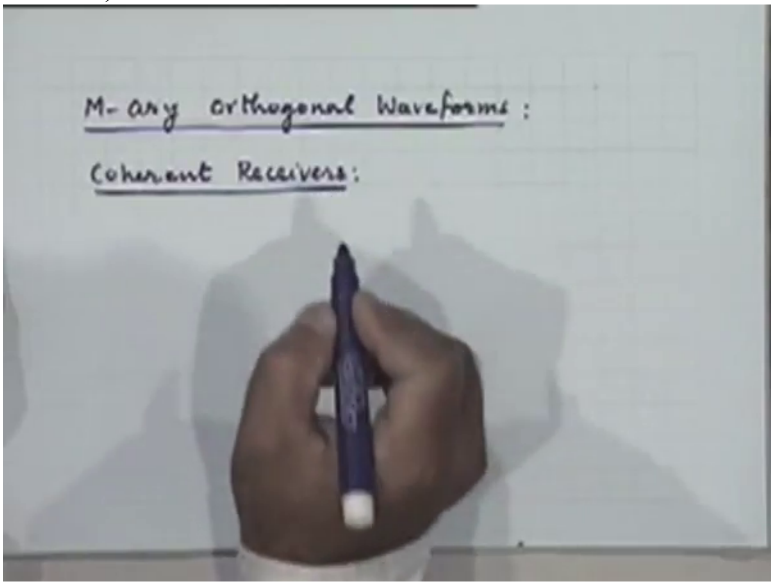

The analysis for the, the corresponding analysis for the non-coherent receivers will be very similar and it will be very easy for you to work it out yourself, read it out yourself.

Alright, now let us quickly recapitulate the decision statistic that we have to use for making our decisions for the case of coherent receivers in using orthogonal waveforms, that is we are looking at, what is the structure of the coherent receiver? I do not have the picture here but it is very easy to remember that picture. We have

(Professor – student conversation starts)

Student: A bank of

Professor: You have a bank of matched filters, right. And you have sampled the outputs of these bank of matched filters at the time instant t equal to l t for the lth symbol and depending on which matched filter produces the largest value of this sample you will decide for that particular signal have been transmitted,

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Ok, right.

That is the structure we have in mind and the decision statistic, I did go through this maths at that time. But it is very easy to even intuitively remember what it was. The decision statistics is to look at the matched filter output u sub m l T, right, index m denoting the mth matched filter, l T denoting the time index corresponding which you are looking at the signal 0:04:23.4. This will be equal to E sub p into delta m sub l m plus n m sub l, right

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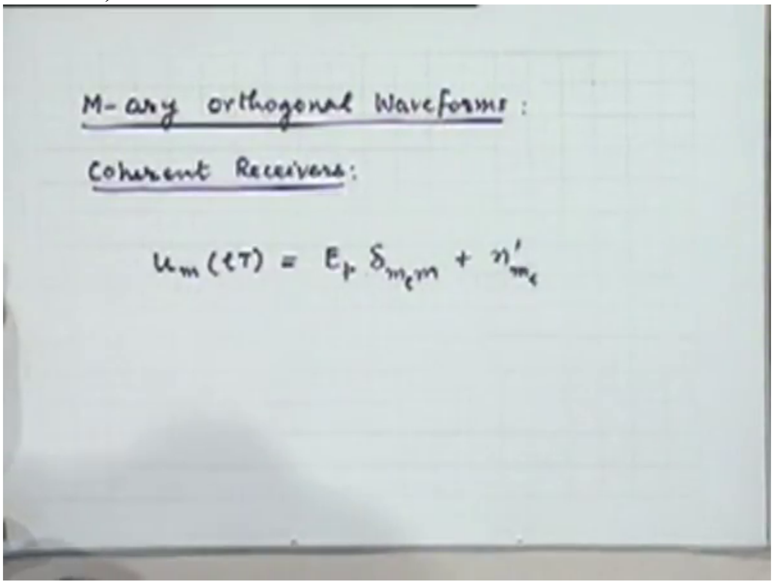

where m sub l indicates the symbol actually transmitted in the lth T interval, right.

m sub l will also take the values from zero to

Student: n 0:05:03.9

Professor: Capital N minus 1, right. So will l. The index m denotes which matched filter output we are looking at, and m sub l denotes the actual symbol that was transmitted in that interval. So obviously this, this we will expect this contribution from the pulse to come only in the m sub lth filter output, right?

(Professor – student conversation ends)

That is why this 0:05:30.6 delta function is appearing in this expression. And if we assume that the input noise is white Gaussian, then this matched filter output noise as sample of this time instance will be a Gaussian random variable, with variance N zero by 2, right? And also the noise outputs of all the m matched filters would be mutually uncorrelated. We had seen that property earlier. Same white noise going through orthogonal matched filters, the corresponding sample value with the noise variables which you will get will be all uncorrelated, independent.

So we have already seen that for white Gaussian input noise n t, these random variables n sub m l prime are, and since they are Gaussian they are also independent, independent Gaussian random variables.

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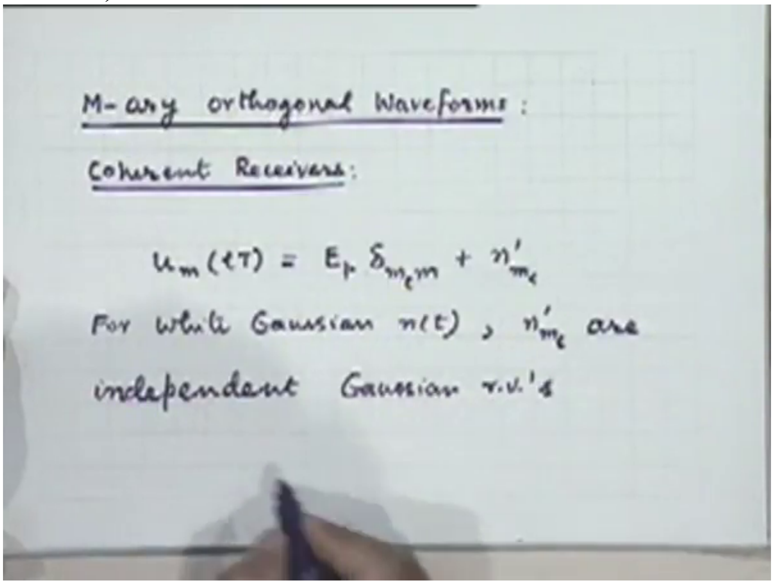

The expected value of n sub m l prime squared will be equal to N sub zero by 2.

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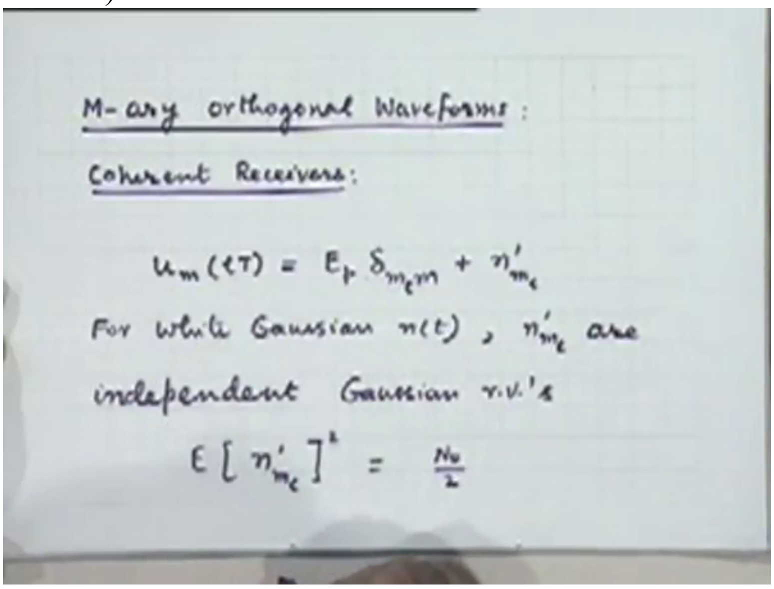

Alright, this is the decision statistic based on which we can easily do our error analysis at least in approximate way. I will only consider approximate way here.

There are both kinds of analysis possible. One approximate method and the other more exact method. But the approximate method itself is reasonably good under certain situations and also it gives some interesting insights. So I will consider the approximate method first.

What is the error event before we come to the method?

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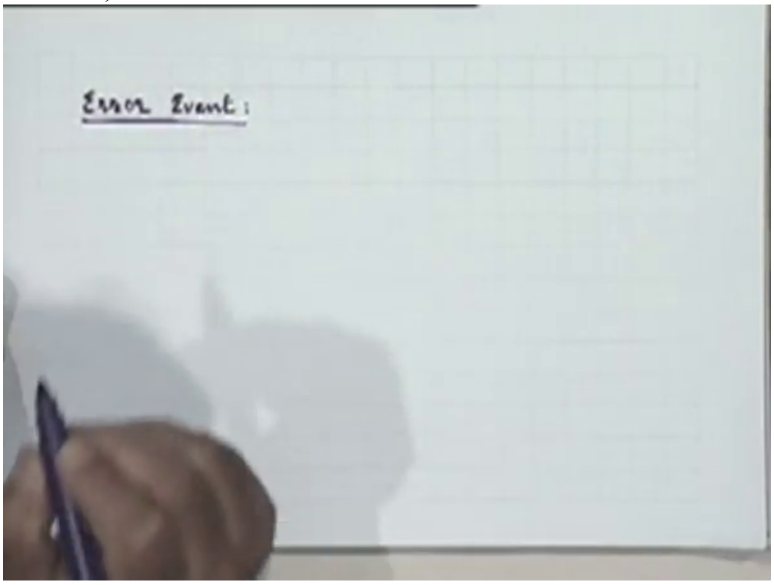

The error event in this case is described as follows.

Given that, let us say a particular symbol m is a true index. I am slightly changing the notation now because earlier m was denoting the matched filter output

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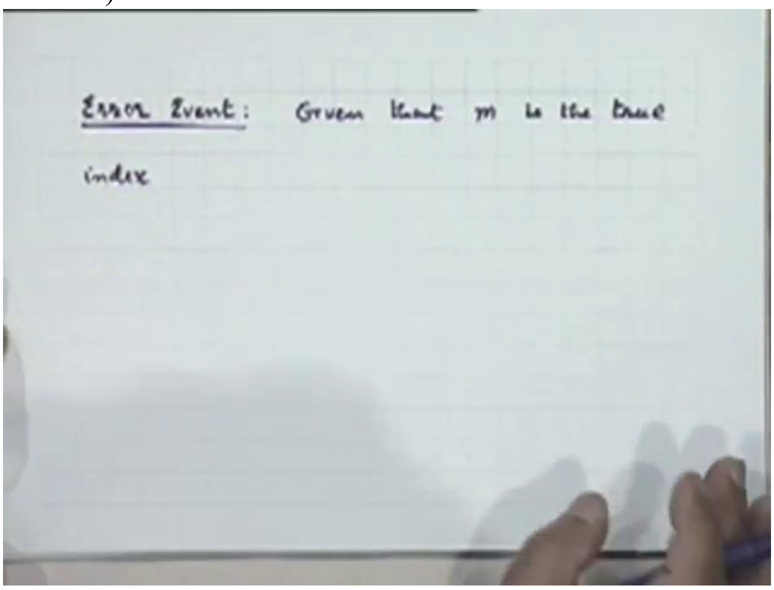

corresponding to the mth filter. But I am now simplifying the notation saying given that m is the true index, the transmitted index, right. The error event is that some other matched filter output exceeds the matched filter output corresponding to this index, right? So some other matched filter output, actually the real part of that exceeds the output of the mth matched filter, right?

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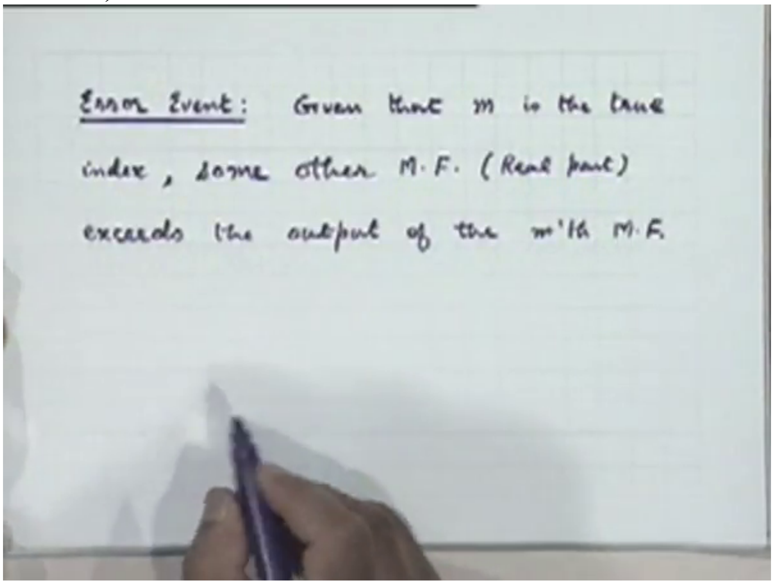

This is a reasonable statement of the error event based on which we can try to compute the error probability. You all agree with this?

True index is m but the largest output is not coming from the corresponding matched filter but some other filter, right? The approximate method uses what is called a union bound method for calculation. Approximate valuation of p sub e

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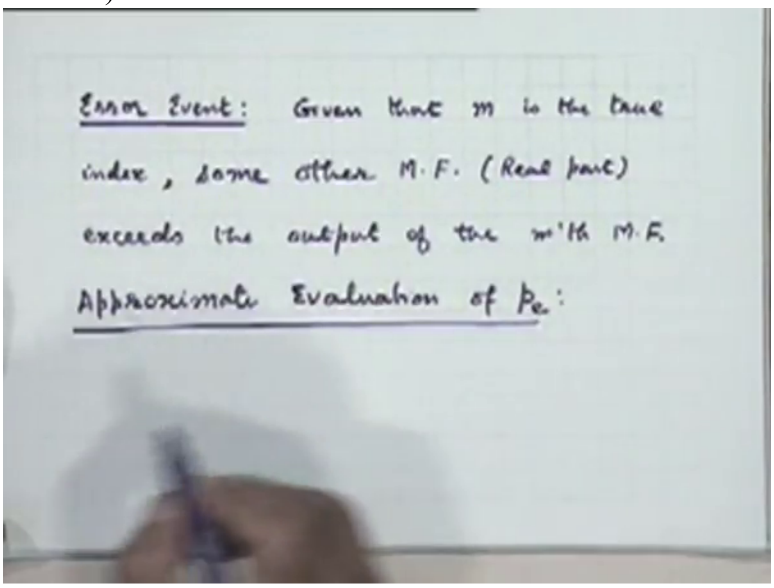

is based on what is called a union bound argument.

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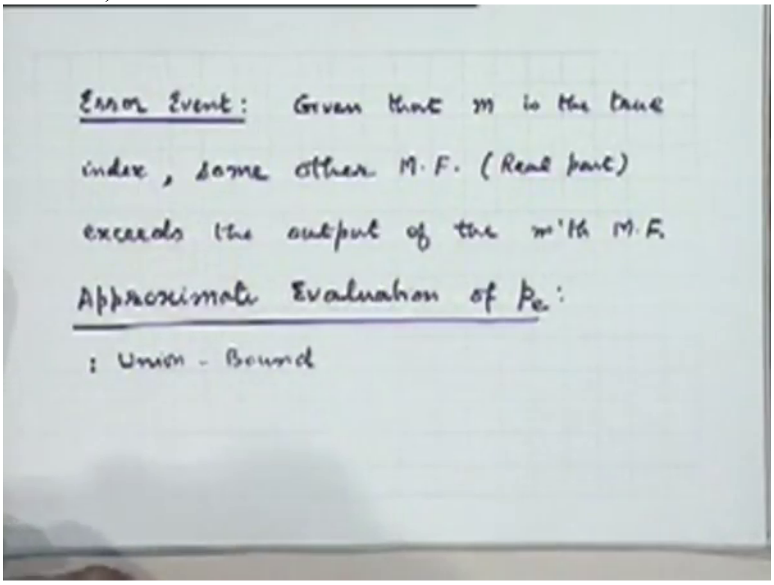

The union bound argument is basically this. If m is your true index then there is certain probability that, that, let us say we will go through a count of all the other possible indices leaving m, right?

So we will go through all the possible ways by which error can happen. That is, m is the, let us say m is the non zero value. Or let us say m is the zero index, just for the simplicity of our discussion. Let us say m was zero, right, the very first index. Then the error event is that something other than the zeroth matched filter is producing largest output. Union bound says we will calculate individual probabilities that the first, the matched filter corresponding to index 1 produces output larger than zero.

Similarly we 0:10:26.2 compute the probability that output, index, matched filter corresponding to index 2 produces larger output and so on and so forth. And we just add up all these probabilities.

(Professor – student conversation starts)

Student: That will be pessimistic

Professor: That is obviously pessimistic answer, answer upper bound, right. That is what a union bound does. It is an approximate method. And there is an error in it; we will try to appreciate that. But have you understood the argument? The argument is that we calculate the probability of every one of the other indices producing the output larger than the matched filter output and the sum of all these probabilities is taken as the overall error probability, right?

This is not a true probability error, error probability calculation. This is approximate calculation. That is, you are taking the union of all possible error events as the actual; error event is being broken up into a number of events which if disjoint would have given rise to a correct result. But they are not really disjoint.

Student: Sir, what is that...?

Student: Sir we are not taking the union. We are taking the addition. Because if we would have taken the union, it would have been exact 0:11:40.1.

Student: Union and this thing, disjoint...

Student: But Sir, why are not 0:11:44.8 disjoint?

Student: 0:11:51.4

Professor: Let us put it this way. It is possible that more than 2 matched filters produce an output which is greater than the mth filter output.

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Student: You simply 0:12:07.3

Professor: Whereas, so therefore what is going to happen is that is going to be counted twice in this situation, right? You appreciate that? Because this is also producing and that is also producing and individually we are taking the probability, you know, we are only looking at one of them. We are not looking at disjoint event at all.

Student: Therefore it is optimistic

Student: No

Professor: That is a pessimistic thing, because that is being counted twice.

Student: Yes

Professor: The probability of two, each of these is being calculated individually irrespective of the other, right? It is being counted twice. And so on.

(Professor – student conversation ends)

Of course, theoretically can happen at more than two also will do it but of course the probability of that is very, very small. So we don't have to worry about it. So that is the basic idea of the union bound. That is, you define let us say the conditional probability of this event, I am going to denote by p sub m prime given m as the probability that output sample of the m primeth, it should be at least read as m primeth matched filter is greater than the corresponding output of the mth matched filter, Ok given that

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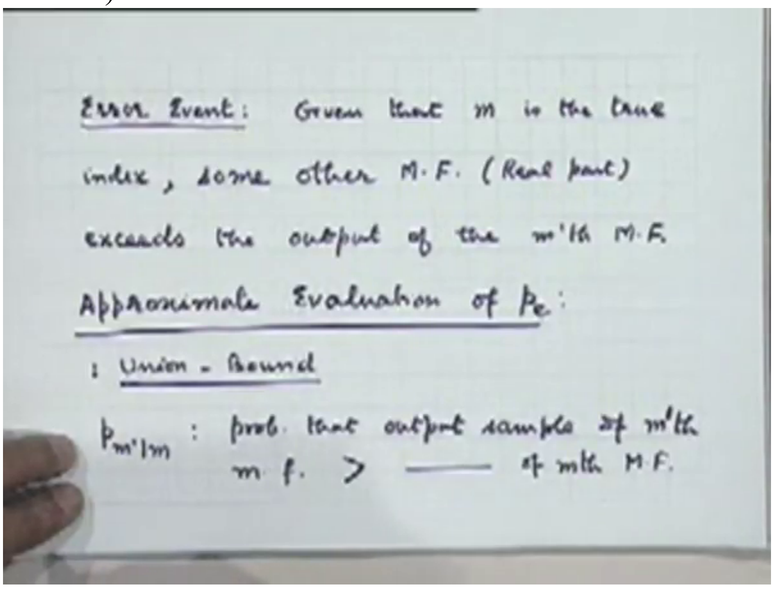

m was transmitted.

This indicates the probability that some other matched filter m prime is producing the larger output than the mth filter, alright? Then what the union bound says is that the overall error probability will be simply the sum of all this for different values of m prime.

(Professor – student conversation starts)

Student: Not equal to m.

Professor: Not equal to m.

So union bound tells us that each p sub e is actually less than or equal to, because it is a pessimistic, this sum of all the conditional probabilities for m prime not equal to m.

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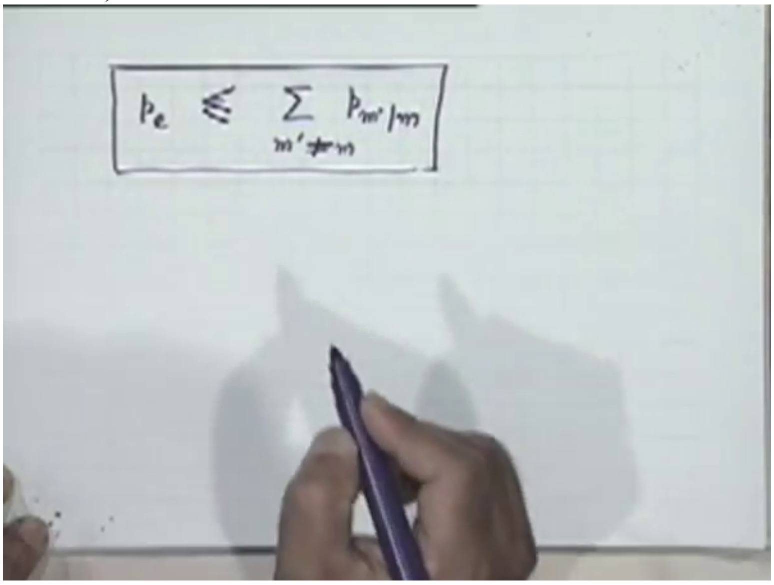

That is union bound.

Student: Sir, this can be greater than 1 also.

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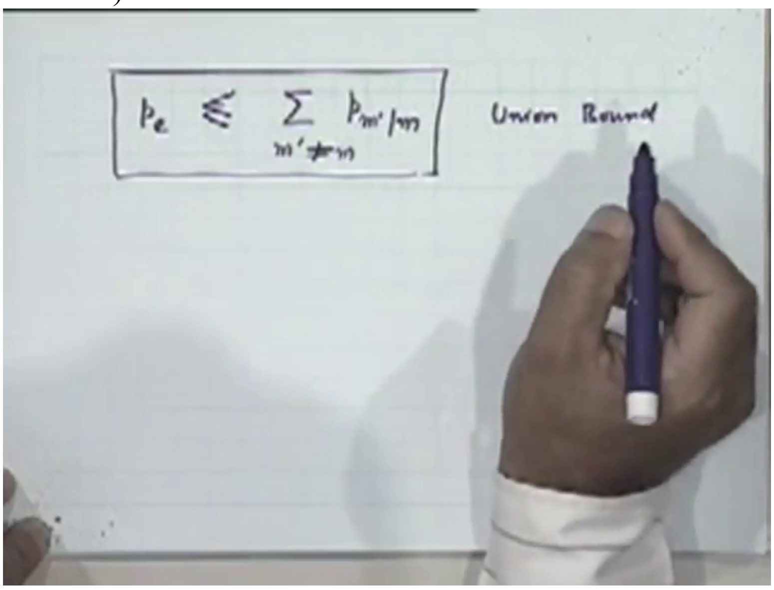

Professor: How can it be greater?

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Student: If we sum in their probabilities, we have no restriction that it has to be less than 1.

Professor: Oh, it can be greater than 1?

Student: Yes sir.

Professor: Yeah, of course. A bound can be greater than 1 which is alright. Because after all, p sub e is going to be, is the probability. And probability is going to be less than, anything less than 1 can also be bounded by something greater than 1. In that case that bound is going to be useless, right? True, I agree with that but let us see how useless or useful this is.

(Professor – student conversation ends)

Ok, so is the union bound argument clear to everybody? Alright. So this probability we already know, p m prime by m, given m. This is the same probability that we considered for the binary orthogonal case. I mean instead of taking, essentially now taking a pair at a time, isn't it? So it essentially becomes the same question as if you are considering binary orthogonal scheme as far as one particular value of m prime is concerned, right?

Therefore p m prime given m is nothing but, well whatever result we got for the binary coherent orthogonal scheme which was I think, right?

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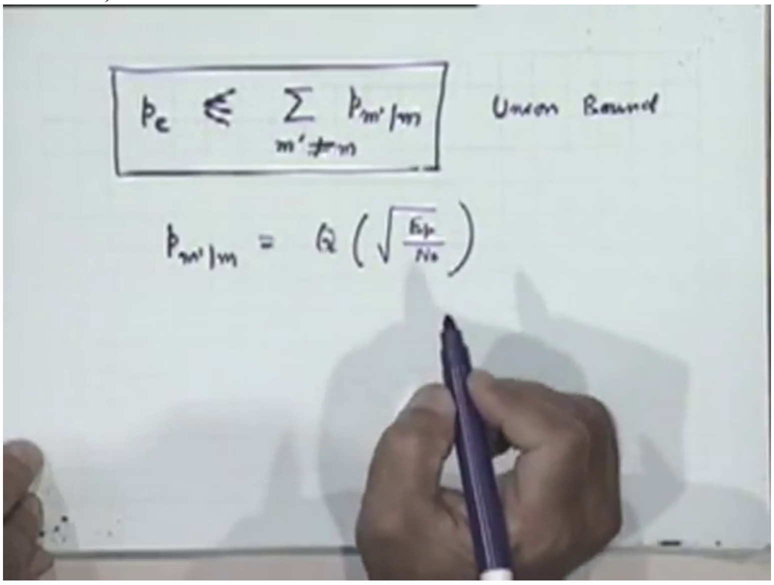

Remember it was only slightly different from the corresponding result for coherent b s k, right? The 3 d B difference was there. So if you remember the form of one of them, you can write the result for the other. So this was the result we did for binary p s k, sorry binary f s k coherent demodulation.

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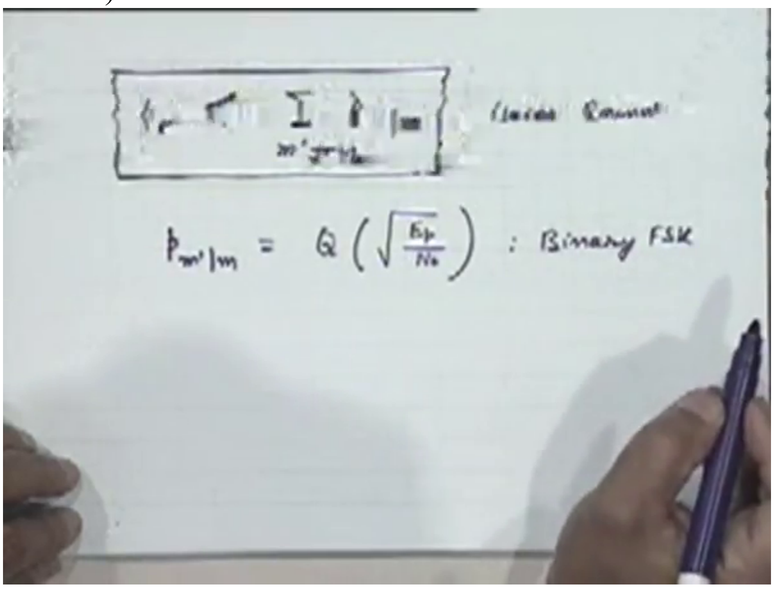

Therefore for M-ary orthogonal waveforms, the union bound tells us, now this is going to be same for every value of m prime and you are adding for how many terms?

(Professor – student conversation starts)

Student: m minus 1

Professor: M minus 1, so it is going to be equal to m minus 1, less than or equal to M minus 1 times, you like to express this in terms of bit, average bit energy. Now we know that E p is, or E p by log 2 M is your

Student: E b

Professor: E sub b, right.

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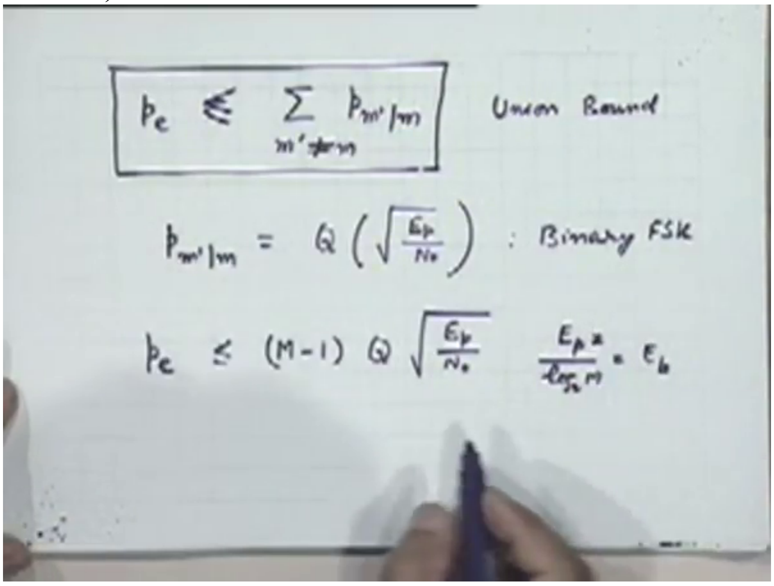

So E sub p can be substituted by that and this result becomes M minus 1, this should be in brackets log 2 m E sub b by N zero,

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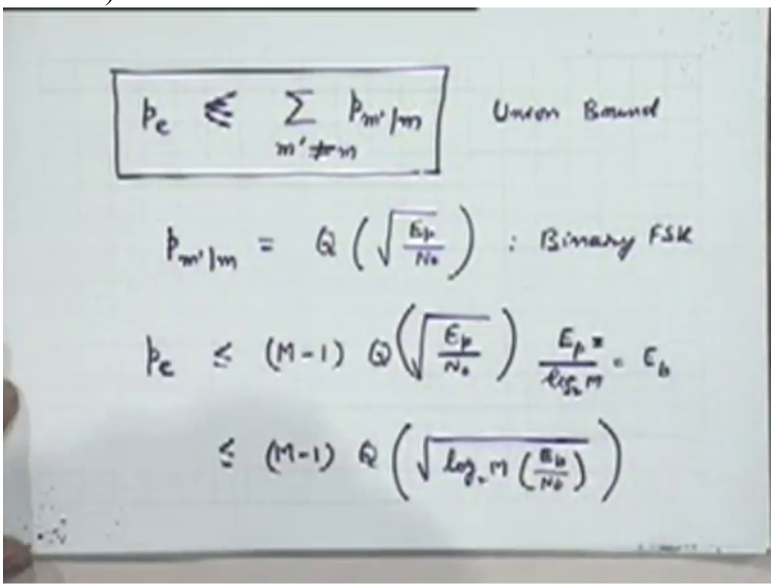

Ok.

Student: You have written E p is equal to log 0:17:33.0

Professor: Yes, E p upon log 2 m, because it is energy for k bits, k equal to log of M to the base 2. That is equal to the average bit energy, right? So that is the result we get for the performance of M-ary orthogonal coherent demodulations

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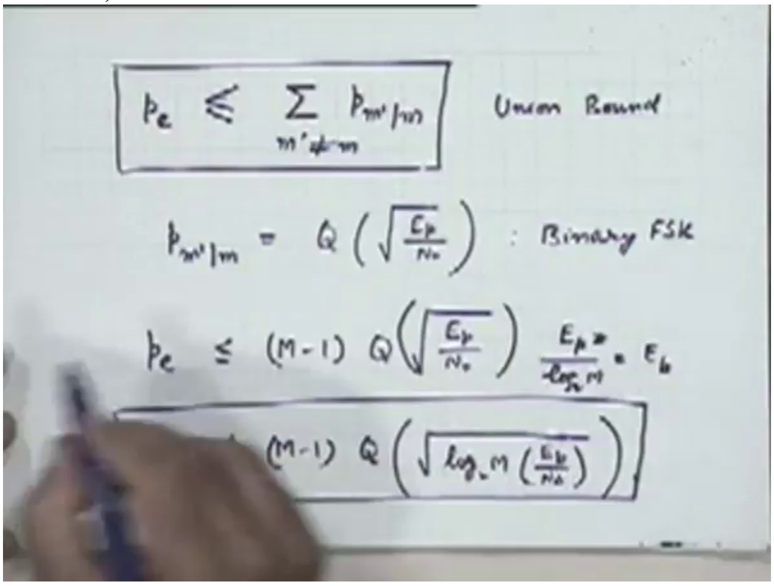

making use of the union bound.

(Professor – student conversation ends)

Now this becomes an exact expression asymptotically, that is as the signal to noise ratio E b by N zero is increased; it actually becomes a very close or in fact equal relation for E b by N zero tending to infinity. Therefore the union bound is not too bad, right? Why is it so?

Because after all remember, what are the kind of events we did not consider? The event that simultaneously more than 2 matched filters other than m producing the output larger than m. That probability will become smaller and smaller as signal to noise ratio becomes larger and larger, right? Therefore that possibility of counting that twice or more than once, that becomes remoter and remoter, right?

Therefore this becomes a true asymptotic expression for the error probability 0:19:01.1, right? Of course for the smaller values of signal to noise ratio 0:19:06.1, this, there is a considerable amount of approximation; or for that matter for smaller values of m.

(Professor – student conversation starts)

Student: 0:19:16.0 with m?

Professor: It is not directly, but that is fine.

Student: How 0:19:25.4 p m prime given m is equal to q of root of E b by N 1?

Professor: Ok, this is a, this result, what I am saying is, this is precisely the same as, it has to be the same as what we obtain for the binary f s k case.

Student: Now the distance between two

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signals did not 0:19:40.6 matter?

Professor: Yes. Every signal has the same energy E sub p, right, orthogonal waveforms. We are only considering a pair of orthogonal waveforms now which is what the binary modulation scheme is, right? So there is no difference when you, basically this becomes a pair-wise calculation. m was transmitted but m prime was taken to be larger, gave the larger output. This is the only event we are considering. And if we just look at this event as precisely as if you are looking at only binary f s k or binary orthogonal signaling, Ok, any other doubts or questions?

Student: Sir at high S N R, this is approaching to infinity 0:20:20.5

Professor: That is right; as S N R tends to infinity, asymptotically. When I say high, actually I am talking about an asymptotic result here, alright.

So what do you see from here? You see that, what happens? Let us see what happens to error probability as m is increased? From this bound what do we learn?

Student: As in what?

Professor: It decreases or increases?

Student: Increases

Professor: As m increases?

Student: m increase

Student: It is coming inside also.

Student: Increase

Student: Sir, it will become more and more exact

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Student: Sir it will decrease

Professor: Ok. We will come to this point.

Student: Because m is inside also and

Student: Outside also

Professor: So one has to see which one is more important.

Student: Actually 0:21:00.8 log to m will be dominant, decrease....

Professor: Ok the behavior will be different depending on

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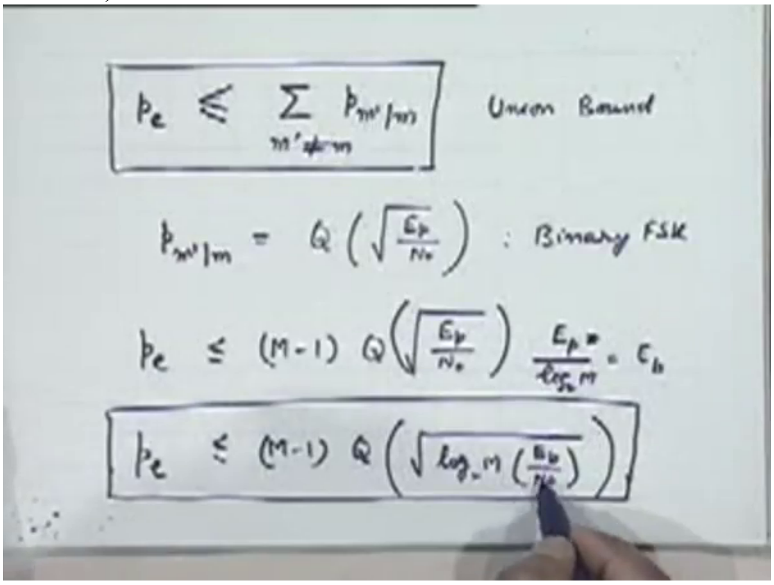

what is the value of E b p by N zero is.

Student: E b by N zero

Professor: Ok? We will see that. We will come back to this question. Keep that in mind.

In fact I will take up that question right away. Here is a plot of, but before I give the plot, let me talk about the, a little bit about the exact expression which I will not prove here. I will leave that as an exercise for self-read.

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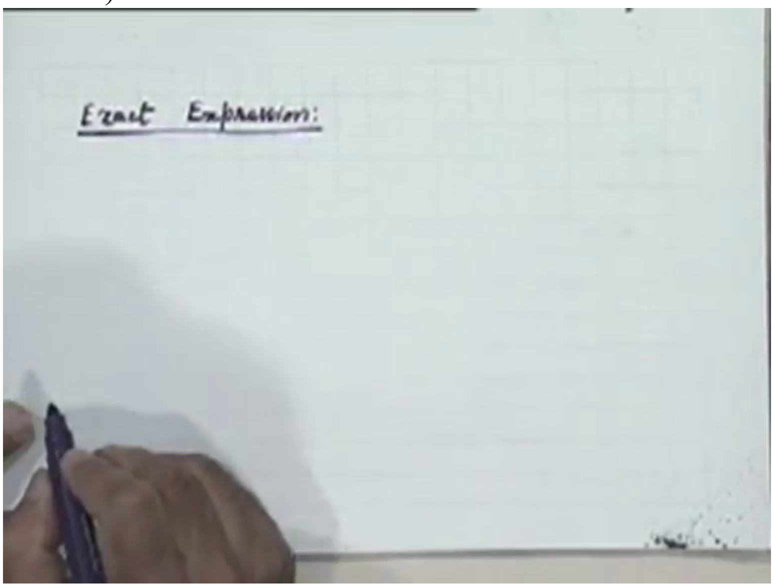

For M-ary orthogonal systems, the precise expression for the error probability is something very, it is an extension of what we did for the binary case but the final result is this. It looks like a complicated looking integral, Ok

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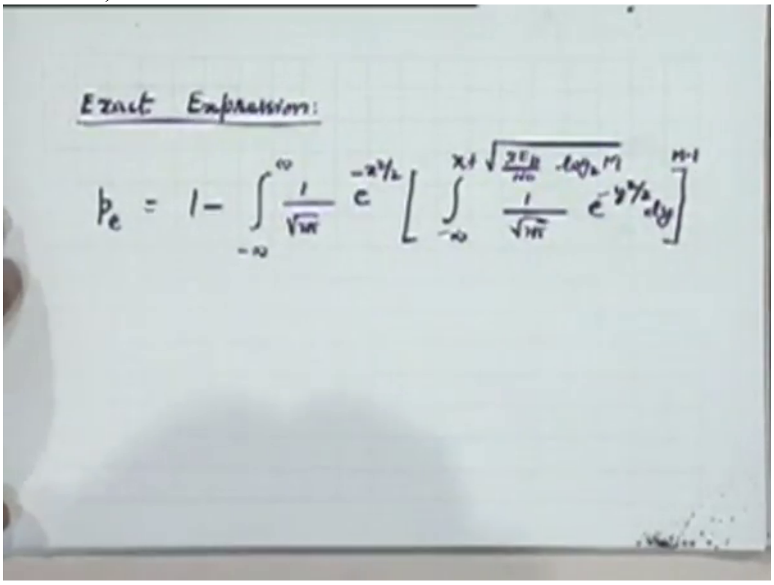

this is a more precise expression. Not readable?

Student: It is not readable over here.

Professor: Ok let me read it out for you. This or maybe I can rewrite it. May not be able to fit it into the space that I have available with me. Is it more readable now?

Student: Yes

Professor: This is where I get into trouble now.

Student: So this n naught is not in the

Professor: N sub zero

Student: Sir, N zero, N sub zero is not in the...

Professor: No it is in the square root sign; everything is under the square root, Ok. I think it is still missing up.

Student: How does it come, Sir?

Professor: This to the power M minus 1.

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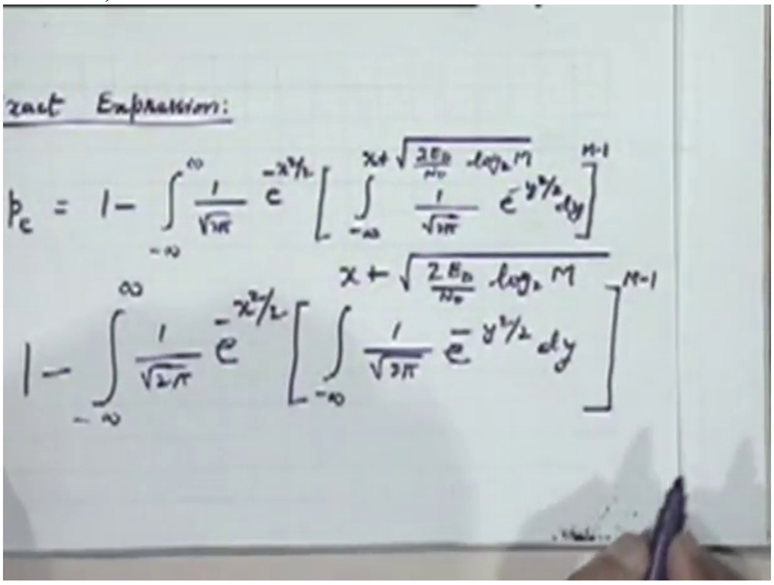

This square bracketed, expression to the power M minus 1 0:23:36.0 This is only the,

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, is it more readable now?

Student: Sir, one d x is 0:23:45.2

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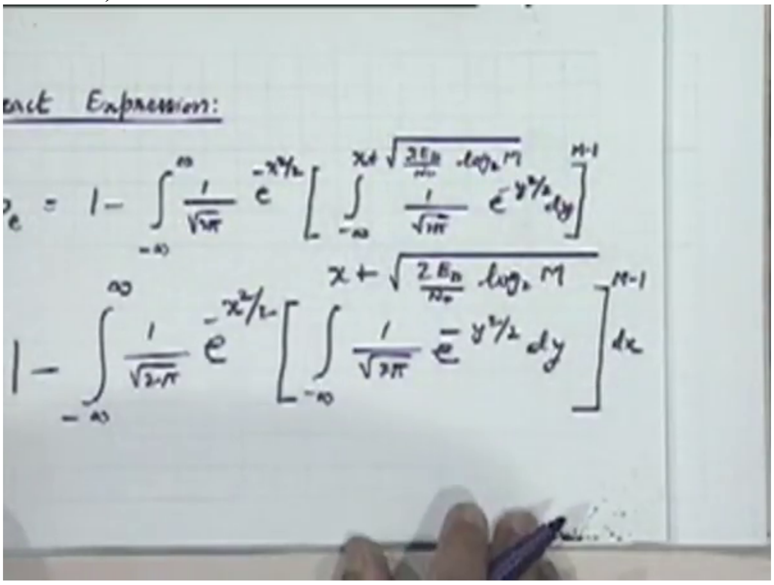

Professor: Yes

(

0:23:49.2

Professor: And they have not even told me.

)

Professor: Alright?

Student: Is that 2 root 0:24:13.5 E d by N zero? Here does it 0:24:16.4 by N zero?

Professor: Well the final expression, not possible to get it in a closed form, it is again in the form of complicated Q function. So you cannot make out anything from this. You cannot directly compare this non-closed form result with the closed form result, Ok.

Student: Approach to....

Professor: Approach is very similar to what we did for the binary case. It is an extension to that for the M-ary case. So please read that. Ok.

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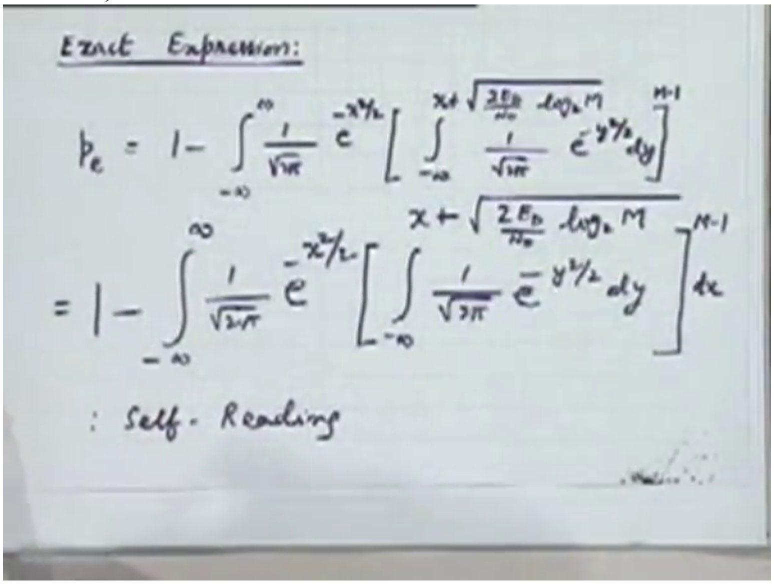

This I am leaving out for self-reading. It is something that you can easily understand therefore I will

Student: In the photo shot you have not given any

Professor: I will give you that. I will give you the notes.

Now another result, alright now let me finish with this. This is a plot

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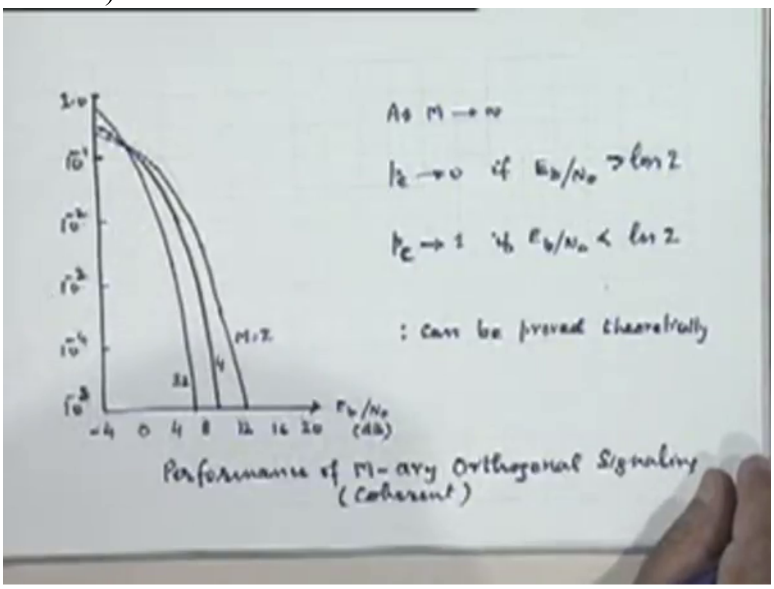

of this error probability, Ok. I plotted here for E B against N zero for different values of N. So as you can see, as you increase the value of M, the curve tends to become lower and lower provided you are above some

Student: E B by N naught is greater than

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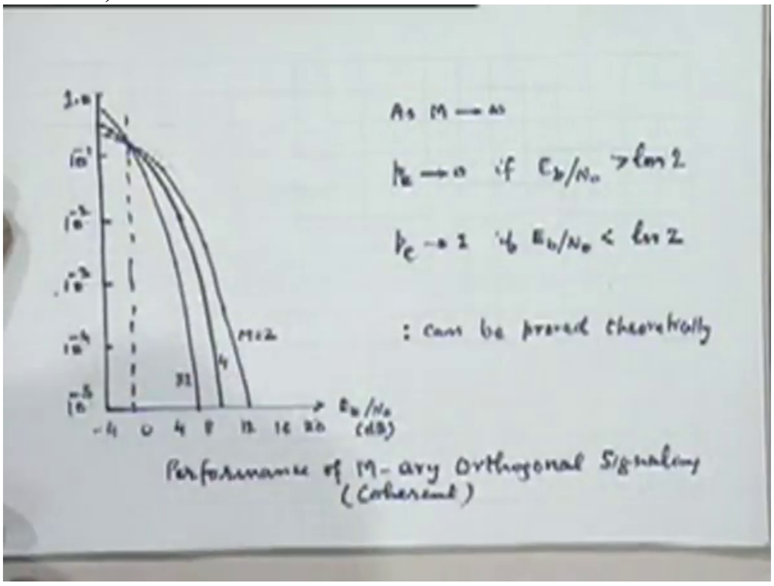

Professor: E B by N zero is greater than some minimum value, Ok which is about minus 1 point 6 d B or something. Now this value is minus 1 point 6 d B. All these curves intersect at this point,

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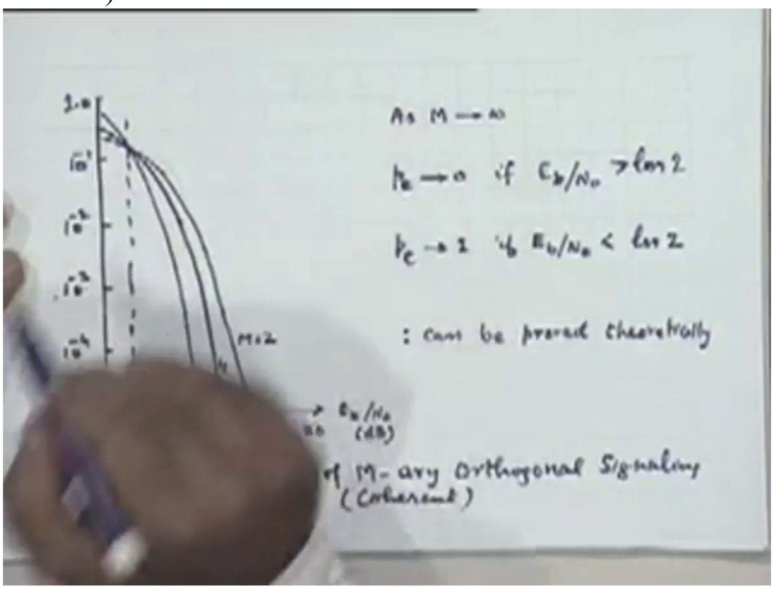

Ok.

(Professor – student conversation ends)

So the answer to that question which I asked, sorry, the answer to that question that I asked you some time ago as to what happens to the error probability as M tends to infinity, the answer is the error probability tends to zero provided the E B by N zero is on the right of this point, that is l, this point is actually l n 2, right, that is how it is minus 1 point 6 and it tends to 1 as, if your E b by N zero is less than this value.

So there is a threshold value of the signal to noise ratio. If you are above the threshold value of the signal to noise ratio, increasing M always improves the error probability for a given E B by N zero. Ok and this you can prove theoretically and is also proved in the book, I would like you to read it up yourself.

(Professor – student conversation starts)

Student: Sir, which book?

Professor: Same, same. Ok.

And that is a very interesting result, remarkable result. I hope you appreciate that. Because unlike what you might be thinking for M-ary modulations in general, particularly when we talk in the context of, let us say 2 D M-ary modulation schemes later as we will see, the error probability does not increase with increasing value of M, which you might expect to happen, right? But for orthogonal modulation schemes that does not happen. Because they are orthogonal, right?

Because every time you add a new value of, add a wave form, you are going into a orthogonal direction in the signal space. There is a price to be paid for it, right? Can you guess what that price might be?

Student: Mathematical complexity is more.

Professor: More in terms of bandwidth. It will be basically in terms of bandwidth. They have got to

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expand the bandwidth. Because you are adding newer and newer dimensions to your signal space that means your signals would be such that it occupies higher and higher frequencies typically. Otherwise it is very, this is of course not obvious from this discussion. I have not got gone into bandwidth calculations at all. But it is something that one can appreciate to some extent, because you are adding dimensionally to the signal space by adding more and more orthogonal wave forms, then something has to be paid as a price and that price is in terms of bandwidth.

Student: Sir in this definition can we correlate with 0:28:31.6

Professor: In bandwidth

Student: Sir, are you saying the 0:28:33.6 bandwidth will work? If they are having...

Professor: I have not given an exact argument because I have not gone into bandwidth calculation at all, but roughly given a bandwidth there are only a certain few number of waveforms that you can design which will be mutually orthogonal. You want to add some more waveforms then you necessary go for higher bandwidth, Ok. So essentially, look at it only intuitively from that point of view and therefore the M-ary orthogonal modulation schemes are asymptotically very good as M tends to large values but for a price which you may not be able to pay in real life practice. So that is something to keep that in mind.

Now there is another related result here regarding simplex signals which you have discussed before. Do you remember what was the motivation for introducing simple signals?

Student: That is energy...that is more...

Student: Energy is more....

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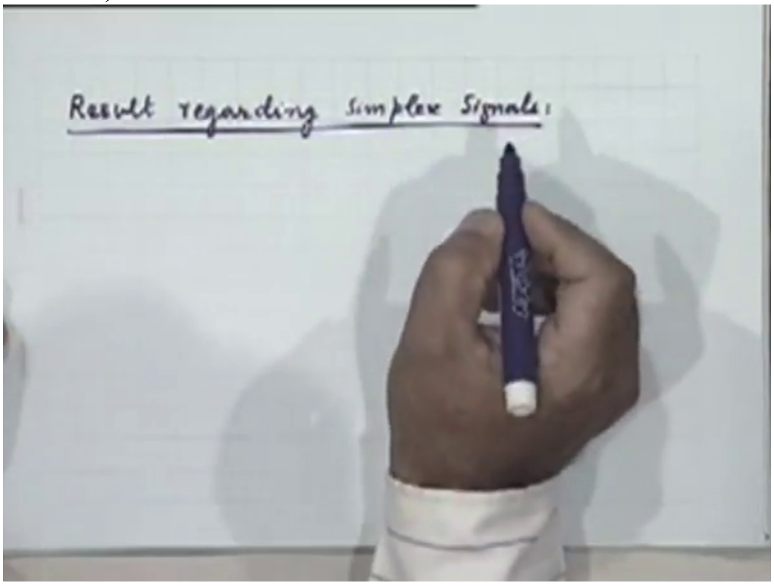

Student: Average energy is...

Professor: The motivation was, and this is a result now you can prove. I would like you to read it yourself. That it will give you a M-ary simplex sets of wave forms with the optimum receiver, coherent receiver will yield the same error probability, same error rate as in orthogonal set, with an average energy which is less, Ok.

(Professor – student conversation ends)

And the relationship is, the average pulse 0:30:03.3 energy will be 1 minus 1 by M into E sub p, rather E sub p.

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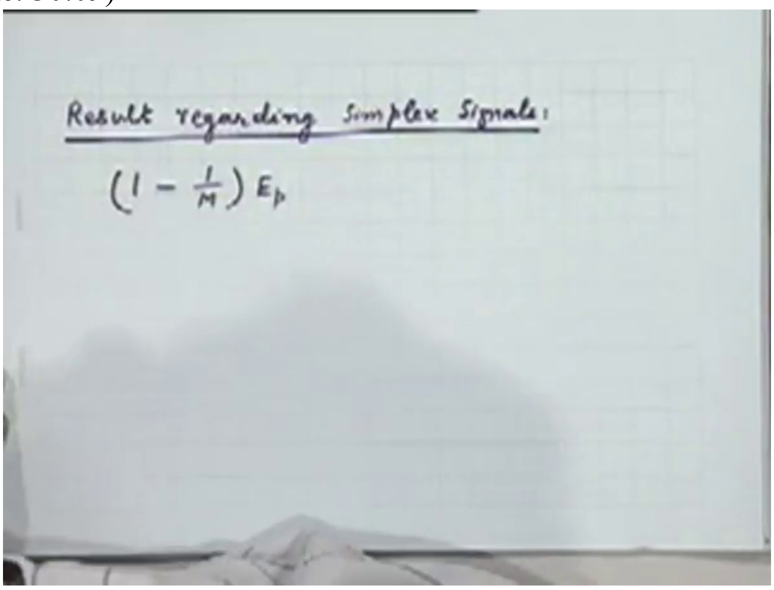

So if E sub p, the energy of the pulse required is E sub p for the orthogonal case, it will be this value for the simplex case. Of course as M tends to infinity, as M becomes larger and larger this difference becomes smaller and smaller.

Ok, so asymptotically with M, both have similar performances. For a finite M simplex set holds an advantage over the orthogonal set, Ok. So this result again, I would like you to read from the book for yourself. It is a fairly simple proof and you can easily appreciate it. Finally before I leave orthogonal signals, let me discuss one point regarding the error rate calculation. The error rate calculation that we have done so far is with respect to symbol error rate, right? It will be an interesting question to ask how is the symbol error rate in this case related to bit error rate?

(Professor – student conversation starts)

Student: The bit error rate will be 1 by M, 1 by rho

Student: k times

(Refer Slide Time: 31:26)

Professor: No, don't jump to conclusions. Just think about it.

Student: Bit error rate will be 1 by log 2 because if we use Gray code then there will be one bit field error.

Professor: There is no particular significance of Gray code in M-ary orthogonal signaling. It is very important for two dimensional M-ary signaling, right? But for orthogonal signaling there is, there is, everything is orthogonal to everything else, right? There is no nearest neighbor as such. Every orthogonal waveform is as close or as distant from every other waveform as any other 0:31:59.2. There is no preferential distance relationships. Distance is precisely zero in terms of orthogonality, right?

So there is no significance of Gray coding and Gray coding this thing for the orthogonal schemes. So don't jump to conclusions therefore. So what we can say about bit error rate, bit error versus symbol error probabilities?

Student: k minus 1

Professor: Leave the world of speculation and try to

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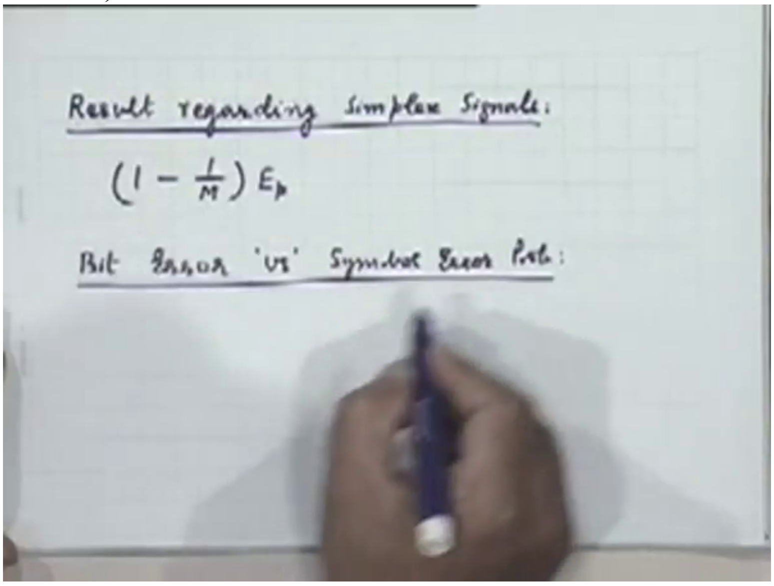

see what result we can get.

Student: 1 by 2...

(Professor – student conversation ends)

Now, let us go through some logical arguments. The first point is when a symbol error occurs; any one of the other M minus 1 symbols could be obtained, right? The erroneous symbol

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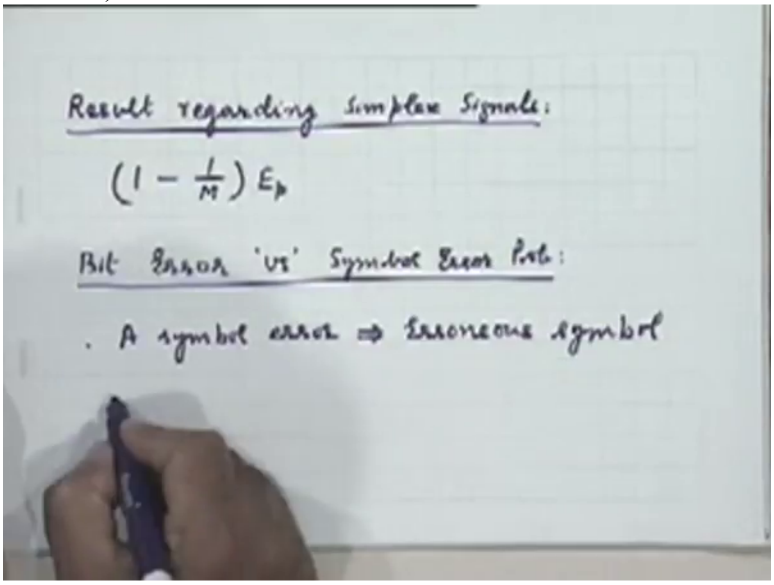

could be any of the other m minus 1 symbols

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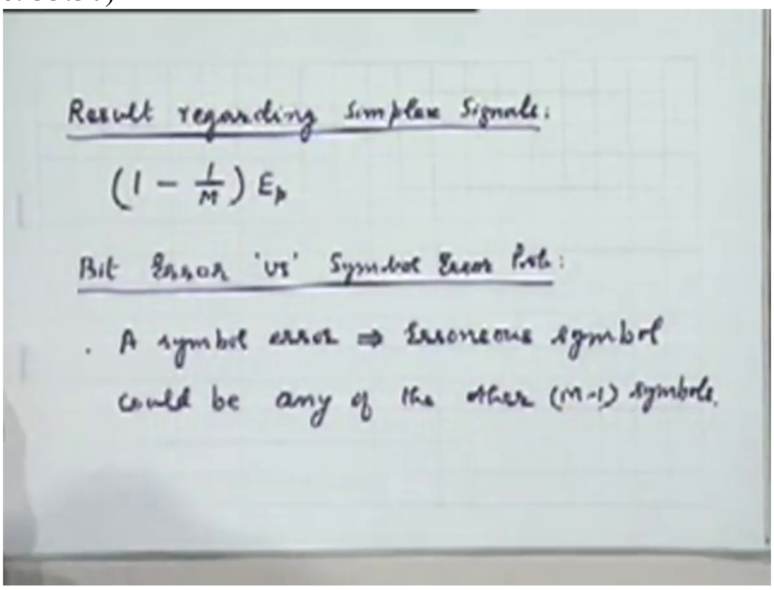

other than the true one. That is the first thing to appreciate, right?

So a symbol error implies erroneous symbol could be any of the other M minus 1 symbols, right? Also this probability of going from, causing the symbol error, and the probability of the particular symbol error is going to be same, no matter which symbol I consider, right. Whether I consider M as the true index, M equal to zero as the true index, or M equal to 1 as the true index, or so on, this symbol error probability is going to be same because all the waveforms are symmetrical, symmetrically placed with respect to each other in the signal space.

(Professor – student conversation starts)

Student: Then 0:34:08.1 assume that the distribution also the in the same coordinate?

Professor: Assuming also they are a priori, probability also same, all signals have same, equal, a priori probabilities, right? Therefore the second point is that this probability is independent of the particular symbol transmitted, right? That is, it does not matter what is the true value of m?

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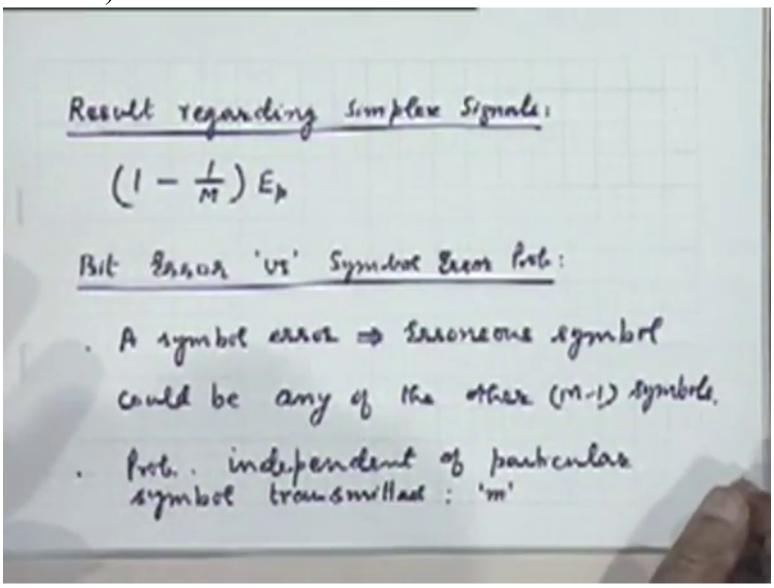

Therefore I can take a convenient value of M for which I can do the calculation more easily and then the result would hold for any other value of M. Because the symmetry of the problem, it does not matter whether I do this way or any other way, right?

A convenient value of m is to consider is m equal to zero.

Student: All zeroes

Professor: All zeroes, right. Or the index m is equal to zero which will correspond to a k-bit word of all zeroes, right? So choose m equal to zero and the binary k tuple implied by this is a bit sequence of all zeroes, let us say.

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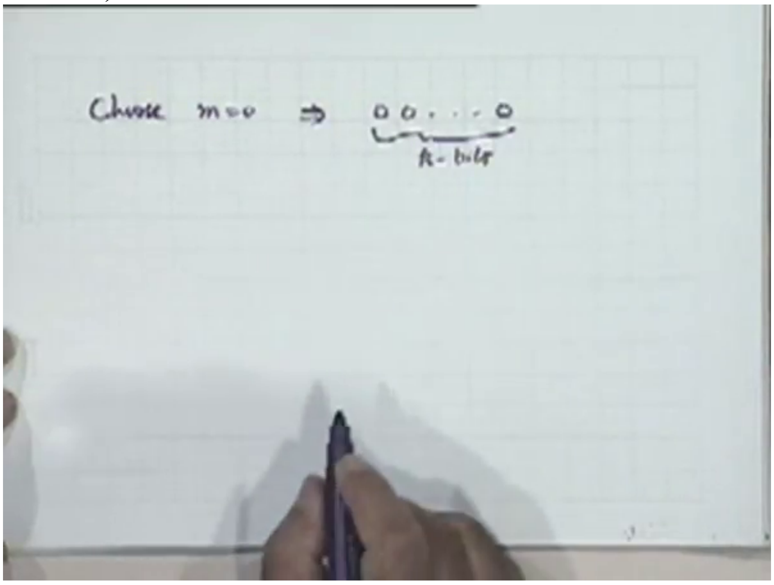

Then the erroneous words, the possible erroneous words are all the other possible words, right? Because is the true value, all the error values will be m equal to 1 to m minus 1. And that will correspond to all other bit sequences other than the all-zero bit sequence, right?

(Professor – student conversation ends)

Therefore I can now count how many different error patterns exist, right? For example, if the error pattern equals to m equal to 1, then this is zero zero zero zero... let us say 1, and only one error occurs, right? If it is something else, two errors may occur. Depending on what error pattern, what value of m has been actually selected, right?

Therefore the average number of bits that can go in error will depend on, what is a decoded word, what I can do is I can count the number of ones, and all the other words, right and divide by

(Professor – student conversation starts)

Student: m minus 1

Professor: m minus 1 that is the average value, average number of bits which can go in error if there is no specific preference for one over the other, on an average. That is the basic criteria.

(Professor – student conversation ends)

So fix any symbol which we have done, chose m equal to zero, and let us say, the expected number of errors per symbol, I think I should say bit errors, expected number of bit errors per symbol

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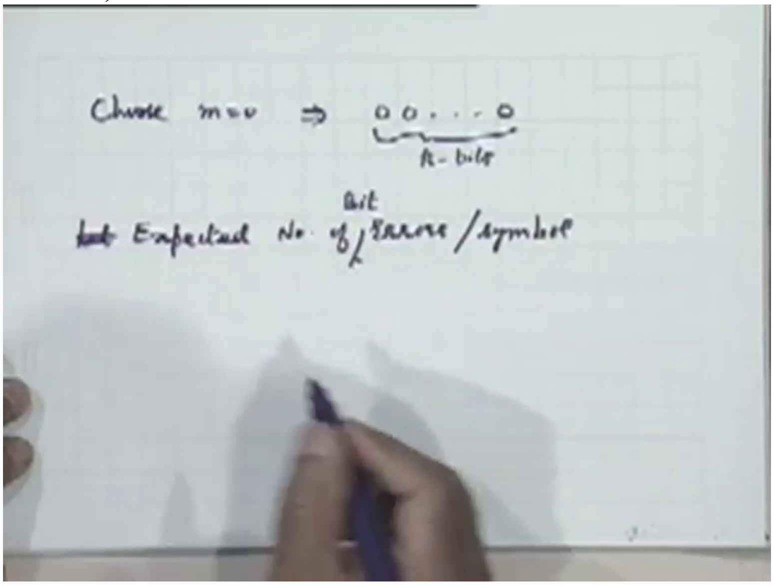

is nothing but the expected or average value, average number of 1s in all non-zero symbols, right?

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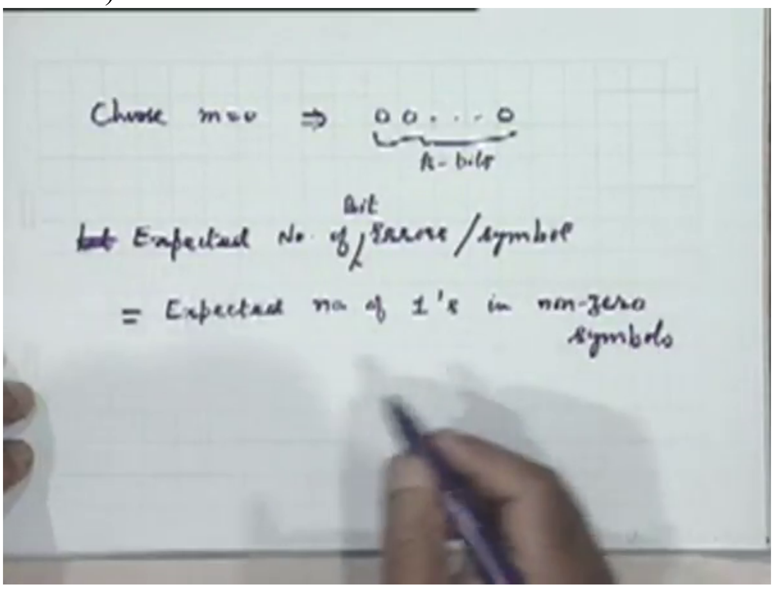

So I have to just count the total number of bits, total number of 1s in all the non-zero symbols and divide by M minus 1,

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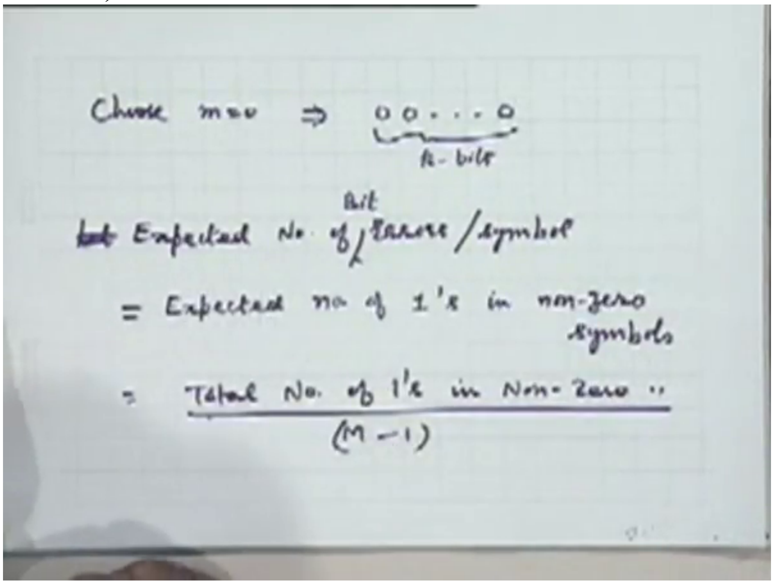

right? And it is very easy to check, this is the argument that you can, I mean, you just have to sit down and do it and verify.

One can count the total number of ones in all the non-zero symbols as equal to half k into two to the power k.

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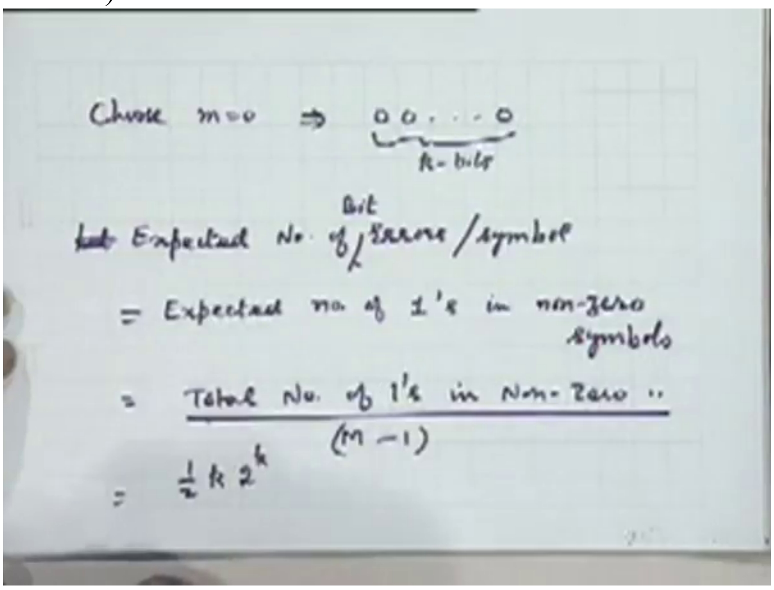

This is like obvious, right? Because the maximum value is k, the minimum value is zero, average value is k by 2, minimum value is 1, average value is k by 2 and 2 to the power k minus 1, sorry, 2 to the power k such words

(Professor – student conversation starts)

Student: 0:38:43.8

Professor: So there is something which can verify more, with more precisely, divided it by M minus 1

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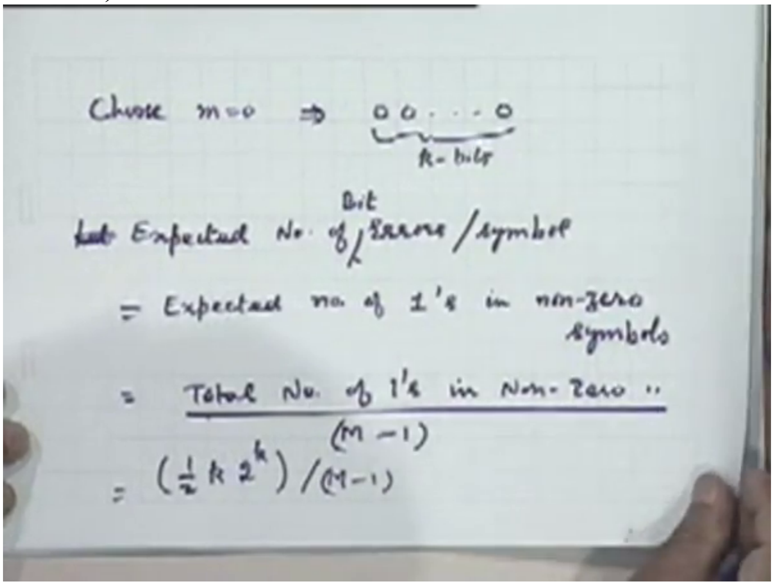

and substituting for M one can write this as equal to k times 2 to the power k minus 1,

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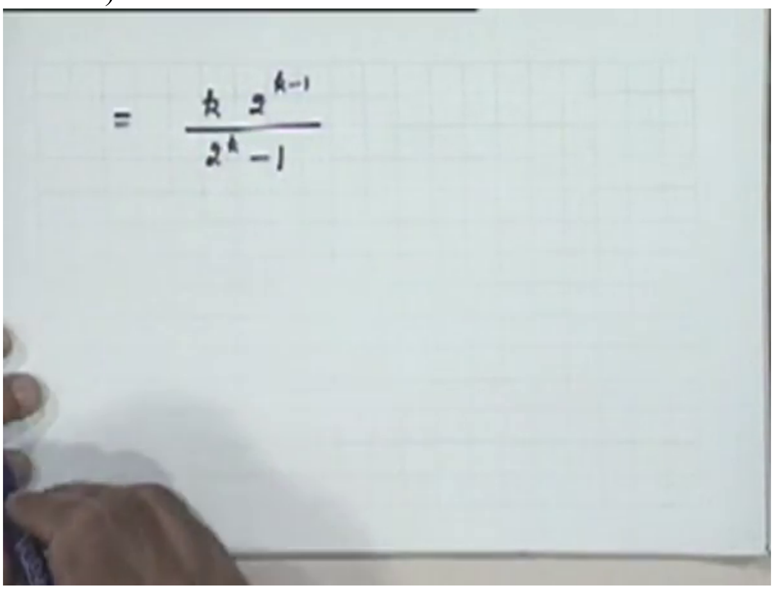

I am writing this half of 2 to the power k as 2 to the power k minus 1, Ok, divided by 2 to the power k minus 1, right? This is therefore, what is the significance of this?

(Professor – student conversation ends)

This is a figure which tells me what is the average number of bits that will go wrong when the symbol is wrongly decided, right? That is, out of k bits that are present in the word, so many bits will be, on an average wrong, clear.

Therefore what is the bit error probability? Divide by k, right, clear, this the number of bits that will go wrong for every k bits, every group of k bits. For per bit, the error probability, bit error probability which l will denote by P sub e comma b will be 2 to the power k minus 1 upon 2 to the power k minus 1 into P sub b which is the symbol error probability which is what we have calculated earlier,

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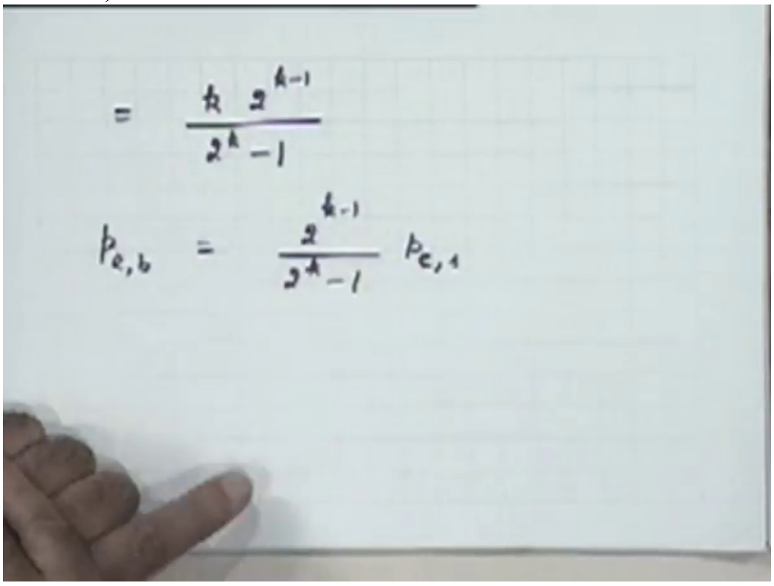

Ok. So that is the res/result, that is the connection between bit error probability and

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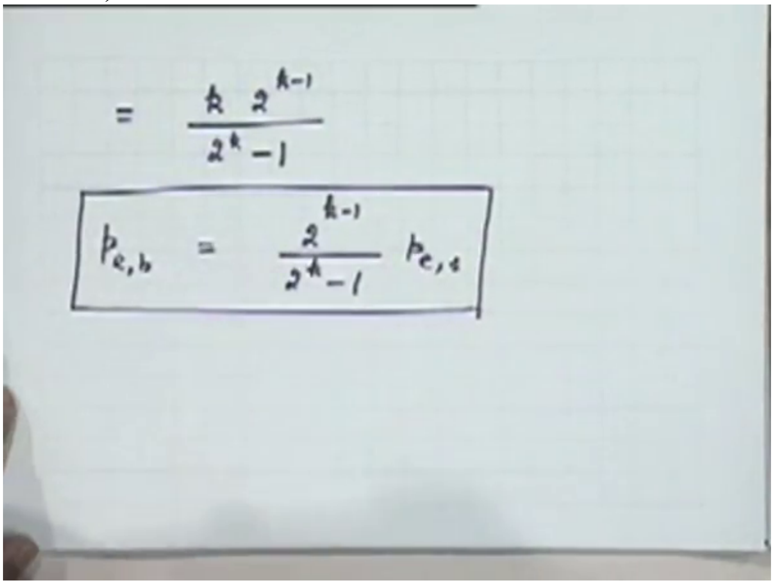

symbol error probability.

(Professor – student conversation starts)

Student: Sir, how do you 0:40:25.7

Student: No, cannot divide

Professor: I have divided by k. Why? Because this is the average number of bits that will go wrong for a group of k bits, isn't it? For per bit, the average number of bits which will go wrong is divided by k.

Student: It tends to half.

Professor: It tends to half provided your k is very large, right. And if M becomes very, very large then bit error probability is precisely half of symbol error probability, Ok, quite true.

Ok all the rest of results concerning M-ary orthogonal signaling namely the exact result, the corresponding results for the simplex family and the non-coherent receiver results. They are very similar in nature. I would like you to study on your own, Ok. The detailed behavior is also similar to what we have discussed earlier. So read everything else regarding M-ary orthogonal signaling. For example, non-coherent receiver error calculations, Ok,

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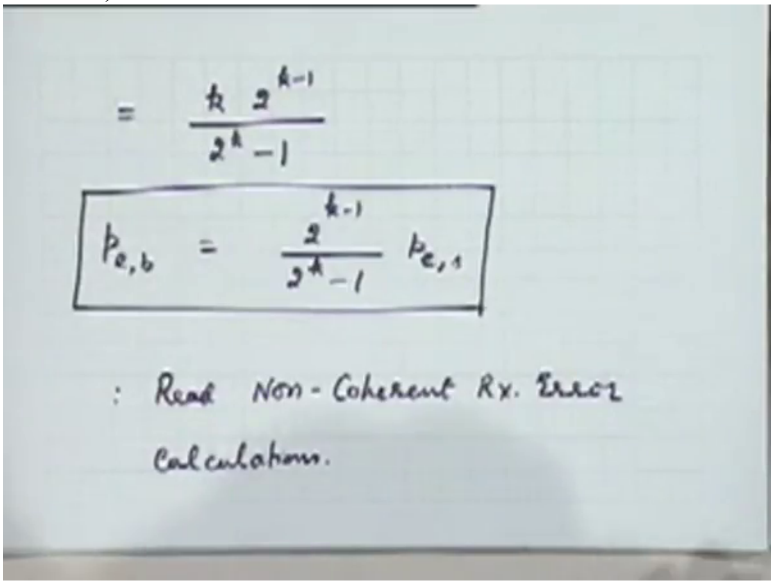

because I would like to finish with this topic today.

Student: 0:41:56.0 You would be giving the photo copies?

Professor: Yes.

(Professor – student conversation ends)

Ok finally let me come to error rates for M-ary signal constellation. When I say this, I usually imply automatically that I am talking of two dimensional signal constellations,

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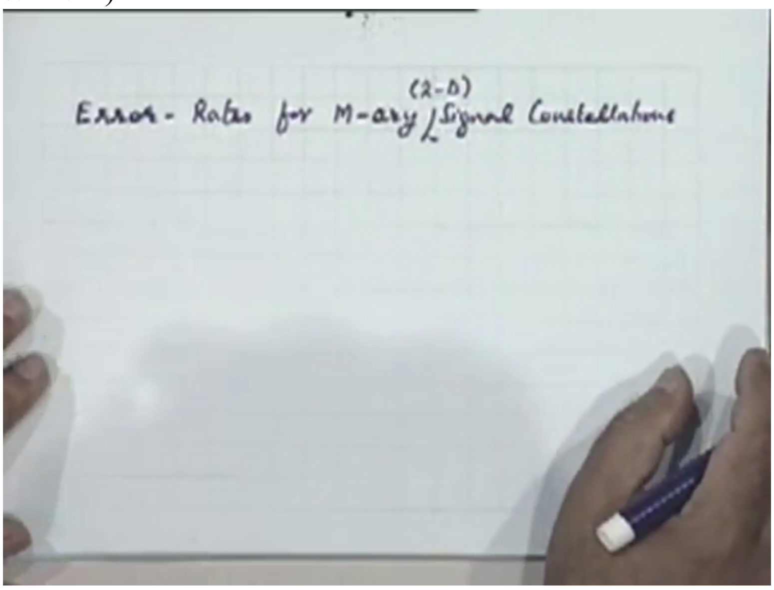

right, which are very popular. The reason why they are popular is because one does not expand in bandwidth as per the increases in value of m. The bandwidth is more or less fixed, because you are using the same pulse shape, no matter what is the value of m. And therefore the bandwidth is under control. Whereas in orthogonal signaling scheme one has to necessarily design a group of wave form which are orthogonal.

So more than one pulse shape is involved. In fact m pulse shapes are involved. And the combined average bandwidth of all these m pulse shapes will be quite large, right? In fact that is a price which may be at times very difficult to pay. That is the reason why you find very rarely M-ary orthogonal schemes for very large values of m under practical use, Ok. Although theoretically they are of great significance, Ok.

So let us return to this two dimensional signal constellations where we use, if you remember basically

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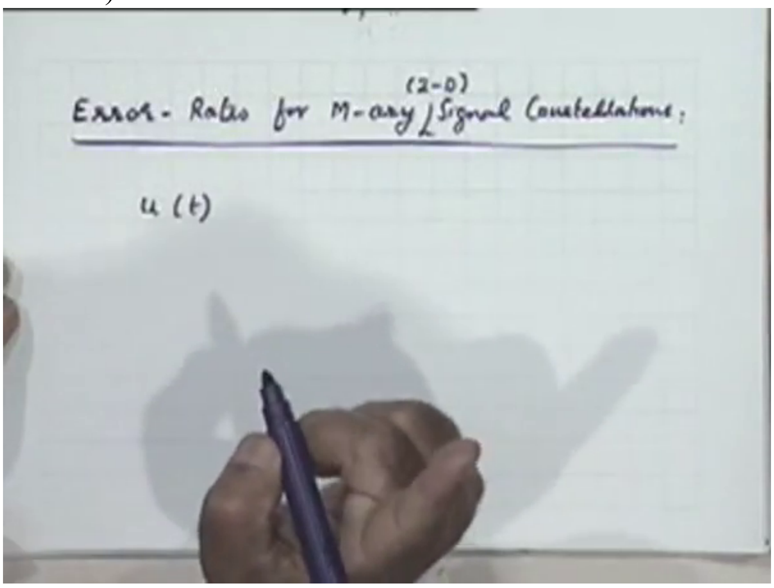

a single matched filter, right and then the i q outputs of this complex matched filter are decoded to find out which symbol was actually transmitted, depending on which decision region it lies in, right? Because the two-dimensional signal space is divided into a number of decision regions, each decoded point will lie in one of these decision regions and our decoding strategy, demodulation strategy is to choose the symbol in which, corresponding to the decision region in which the point actually lies, the complex output actually lies, right?

Let me recapitulate the maths for you. The matched filter output before sampling is some waveform like this.

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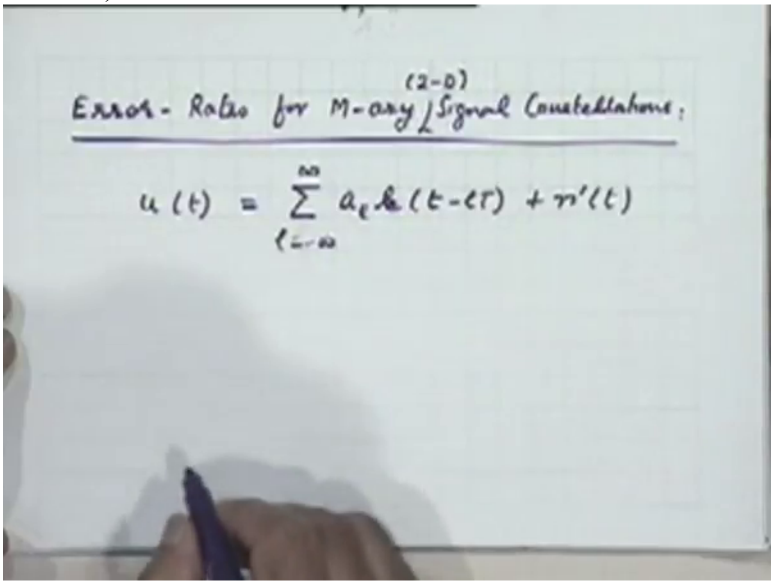

We are using a single matched filter, single matched filter either at pass pulse 0:44:32.0 or base pulse, Ok. Then if it is Nyquist pulse; that is an assumption that we have always been making, then u l T is going to be a sub l plus m prime, let us say n prime l.

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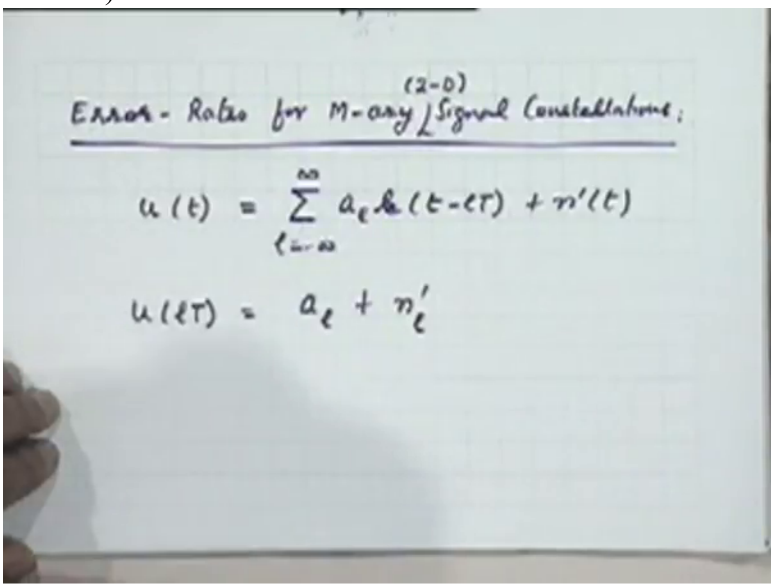

Because the only contribution that will come is from the Nyquist pulse in the corresponding interval t, from l T minus 1 to l T, alright?

So a l is your,

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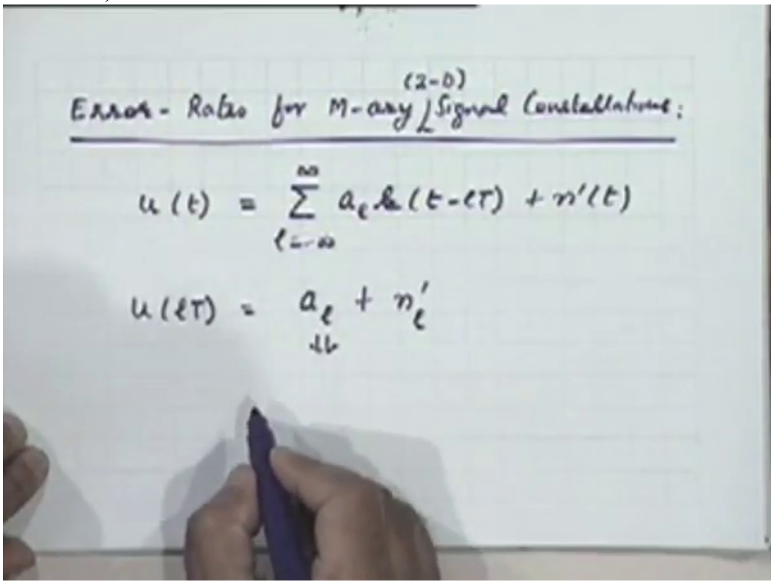

a point in the signal constellation that you actually transmitted and it is coming along with complex Gaussian noise

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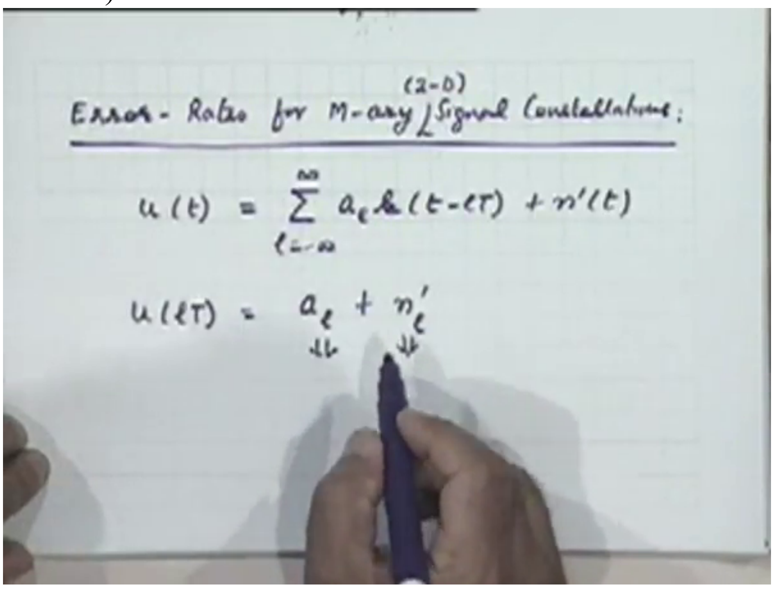

or in other words this is actually complex variable. It has a real part and imaginary part, both are uncorrelated, therefore it has a Gaussian distribution with zero correlation coefficient, Ok. So u l is the sum of, it is a complex variable. It is a sum of a l and n l prime. This is a point in the constellation, signal constellation that was transmitted in the lth symbol interval,

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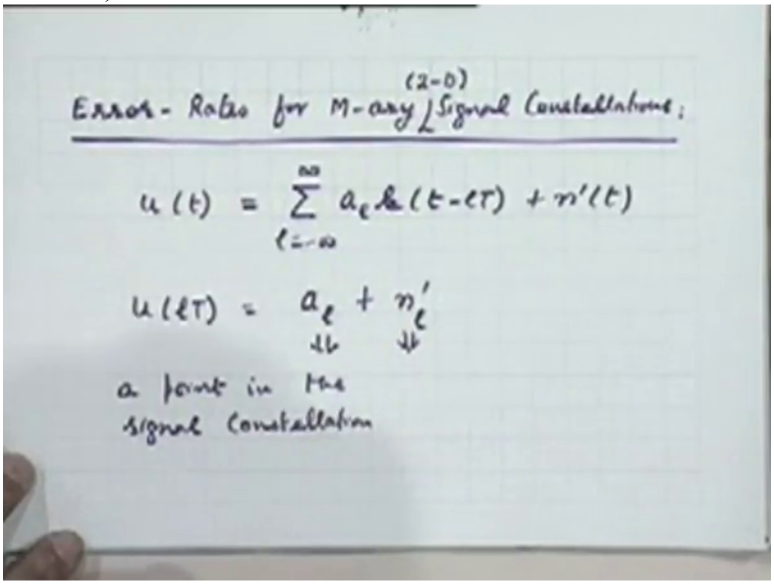

right?

And this is a complex valued Gaussian random variable

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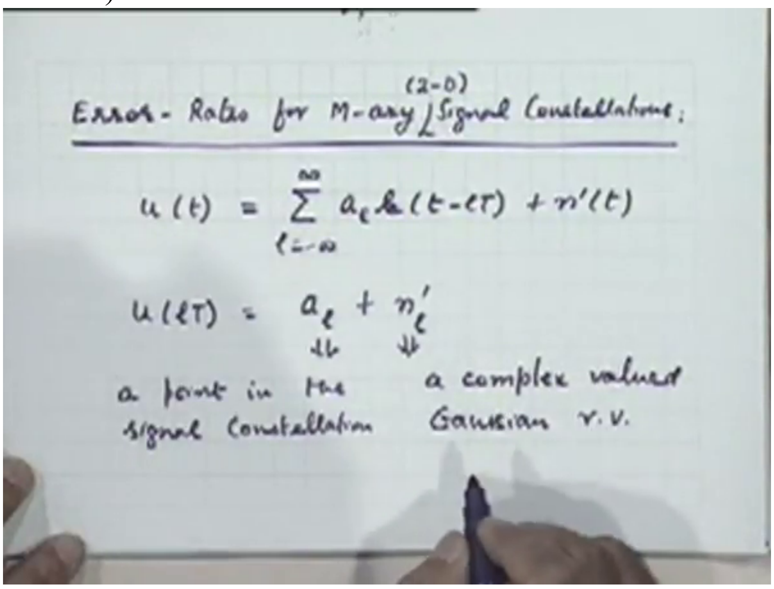

and each component of this complex variable has a variance of sigma square or N zero by 2.

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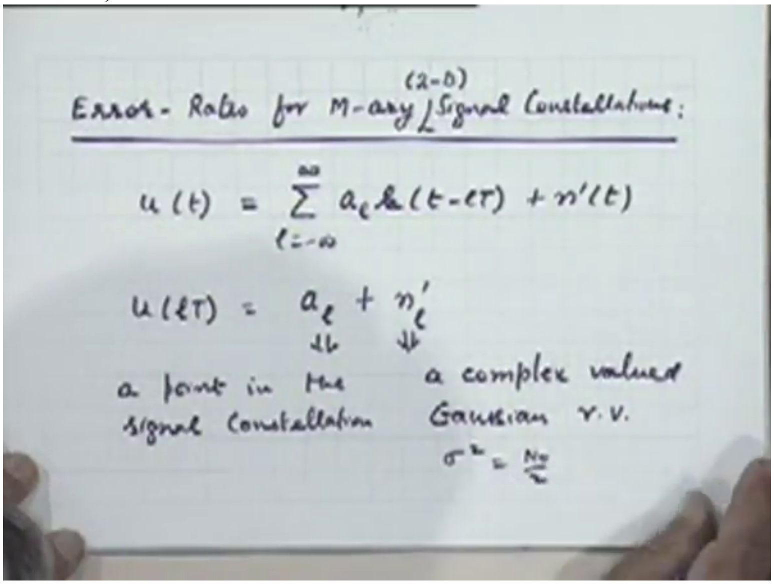

Therefore what can you say about the u sub l? Let me write this as u sub l.

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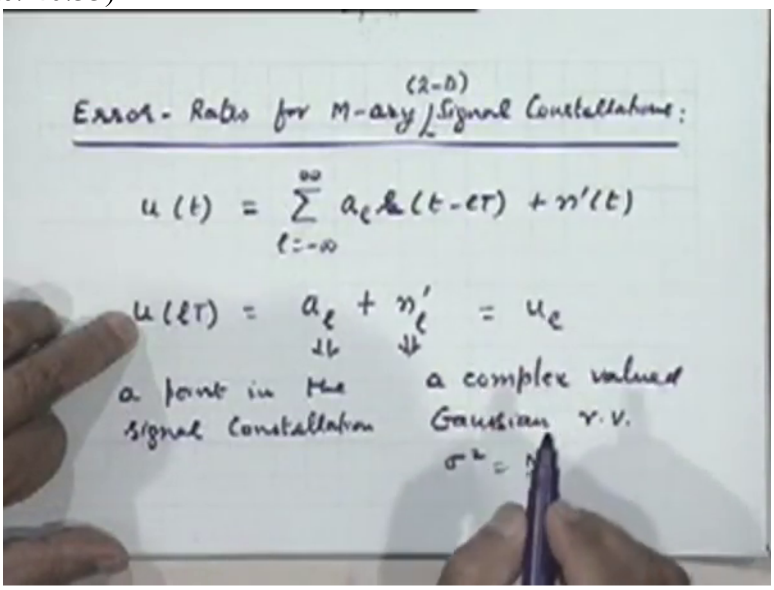

It is a complex Gaussian random variable whose mean is u sub l right? So therefore suppose if we have m possible symbols there are m possible density functions defined for each of the possible transmitted symbols, right?

Let me illustrate this for the case of; let us say a 8 phase p s k system. That is an example of a two dimensional signal constellation of this kind? So we will have a situation like this, for 8-ary p s k.

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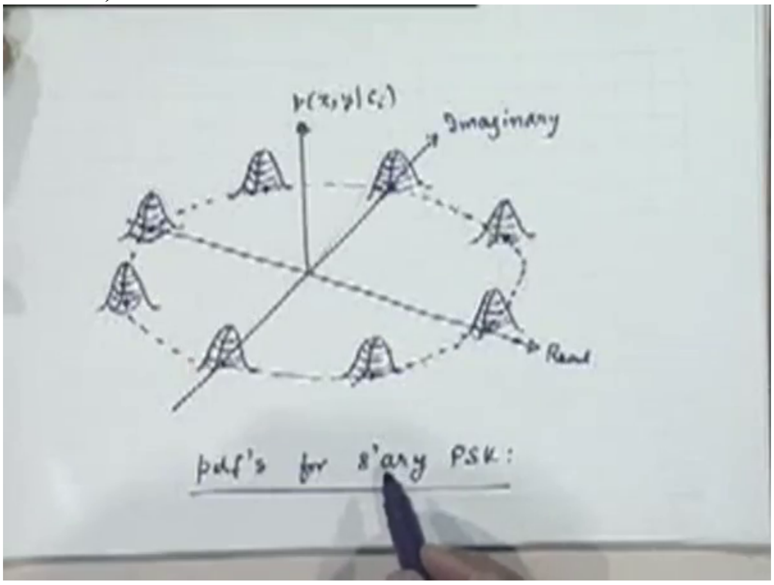

These are the 8 points in the signal constellation diagram lying on a circle, right and around each of these mean values we will have a two dimensional Gaussian s d function coming up which describes the density function of u sub l,

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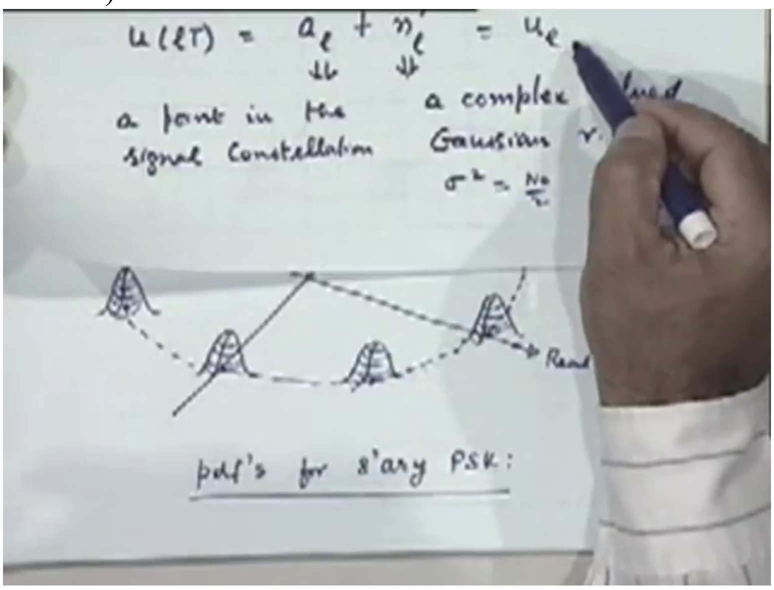

right? Essentially this density function is imposed by the presence of noise n sub l. And what we have to understand is now how we will do the error calculation for this kind of situation.

Of course I will do this exercise simply for 8-ary p s k. I will just tell you the result for general two dimensional modulation schemes. I don't think we have time to complete even this today so I will start from this and quickly finish this next time and then go on to, the next topic that we will be taking up is a brief introduction to information theory and problem.