Digital Communication

Professor Surendra Prasad
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture – 13

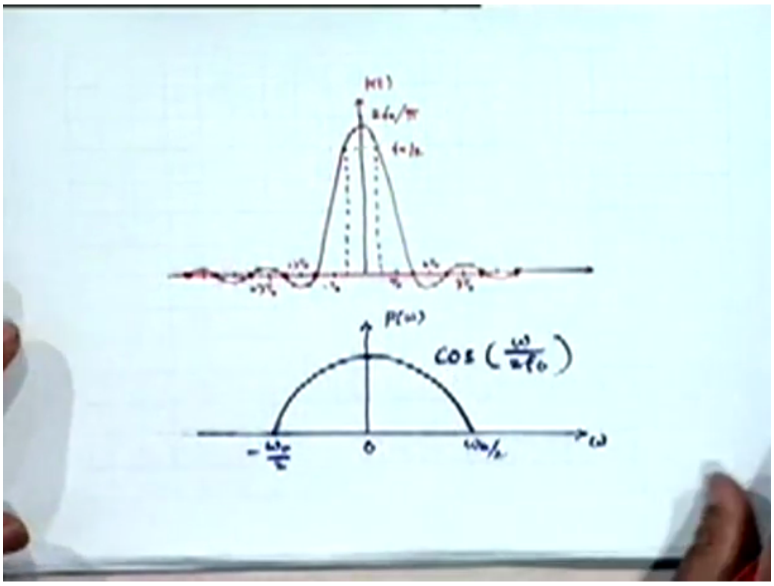
Precoding for Duobinary & Modified Duobinary Systems

Okay you may recollect we were talking about partial response signalling basically we had introduced the concept of duo binary pulse shaping by saying that we would like to achieve the twin objectives of eliminating inter-symbol interference while using minimum nyquist bandwidth and also using a practical pulse shape and by practical pulse shape we mean a pulse shape which is tolerant to timing errors, perturbation errors right.

Because we found that we could do, we could eliminate inter-symbol interference by using minimum nyquist bandwidth, by using the so called sinc pulse but that unfortunately is not a practical pulse shape because it is not tolerant to timing errors, right, because a small timing error can cause a considerable reintroduction of inter-symbol interference which is undesirable and what we said was okay.

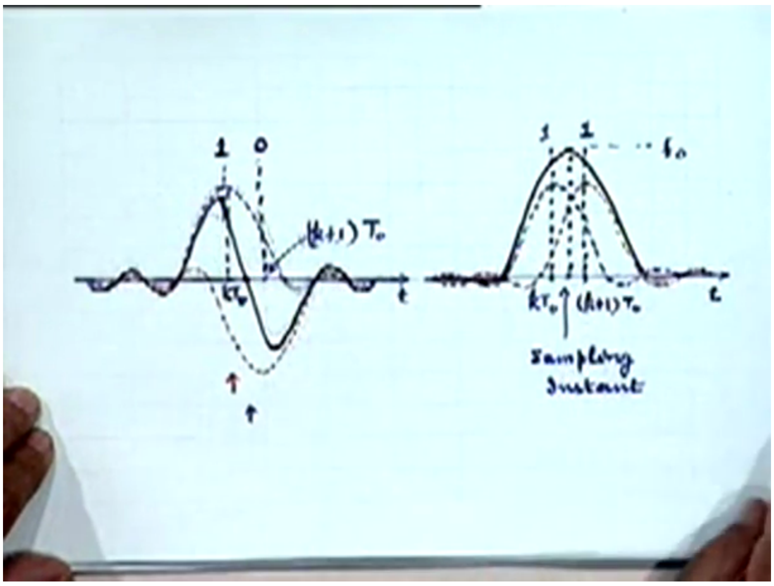
What we can do is introduce some amount of inter-symbol interference in a controlled manner ourselves that is what the duo binary pulse shaping does okay, if you recollect we now and that is what the nyquist second criterion tells us to do although it does not say so in so many words but basically what we are doing is introducing a controlled amount of inter symbol interference at the transmitter.

(Refer Slide Time: 02:47)



You may recollect this was a pulse shape that we recommended in the duo binary case, in which the spectrum is confined to the nyquist bandwidth where it is no longer a rectangular spectrum or the nyquist shape kind therefore it will exhibit inter-symbol interference right and what we do have here is the pulse which is now sampled not at T equal to 0 but at T equal to T0 by 2 right.

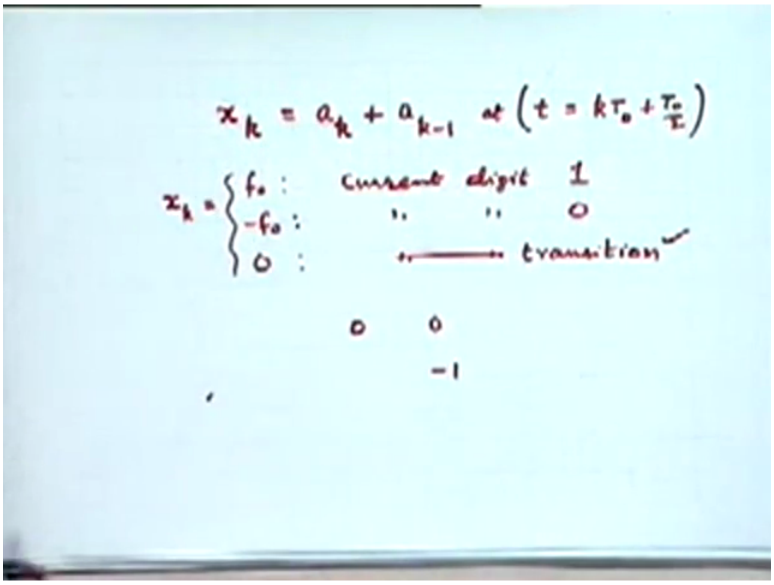
(Refer Time Slide: 03:56)



And if we transmit a sequence of such pulses every T0 seconds and you sample them every T0 seconds displaced by T0 2 from the centre what we will get is interference from immediately preceding pulse right because of preceding pulse and the present pulse values will be identical, for example this was, this are pictures which I am repeating we had shown, I had shown you these last time also, suppose you transmit a sequence of two 1's then this will be the previous pulse, this will be the next pulse, when I am sampling at k plus 1 T0, I get a contribution.

I am sampling here, I am getting contribution not only from this pulse but also from the previous pulse and the two together add up to give me this peak over here right which I am calling somewhere value f not so basically this was a very first problem I gave you in the exam that show that the net to final pulse that you will get the sample value other sampling instance would be the sum of the present pulse, present sample and the present data and the previous data right this is just a one line answer to the question, if you had understood this then the first question was really very trivial.

(Refer Slide Time: 05:25)



Similarly if you transmit a 1 followed by a 0 this is the situation at the sampling instant you will get zero amplitude alright, so this is preview of what we did last time. So xk at the k th sampling instant for the duo binary pulse shape gets contribution from the present value of the symbol a sub k and the immediate past value of this symbol ak minus 1 this is the sample value that you will observe at the timing instant t equal to k T0 plus T0 by 2 right.

So that is the reason that I thought the first question probably should take a couple of minutes only because it was kind of trivial if you have understood it but perhaps you did not. It is alright, alright now based on this we could work out a decoding procedure which we also discussed briefly last time, I did not introduced the decoding from this point of view but it is now obvious how you can carry out a decoding.

For example when I think this is what I told you last time that if we receive xk to be equal to either f0 or okay we can get three possible values right because ak and ak minus 1 each contain two possible values 0 and 1 right, xk can take three possible values, if f0 is plus 1, if you get f0 that means your current digit is 1, if you get minus f0 your current digit is 0 and if you get a 0 your current digit is no we (())(07:29) it is opposite of the previous one or we should say current digit is a transition.

The current digit is decided by the fact that has to be of opposite polarity to the previously transmitted digits right, so this is the way we can carry out a decoding and as you might have appreciated this can lead to error propagation right, particularly when you are making this kind of decision and the received value is 0 and the decision is that whatever was the previous decision compliment of that is the present decision right.

This will lead to possible error propagation in the sense that if the previous decision was wrong then the present decision also will be wrong and this will continue for some time right which is a not a very nice situation is it okay?

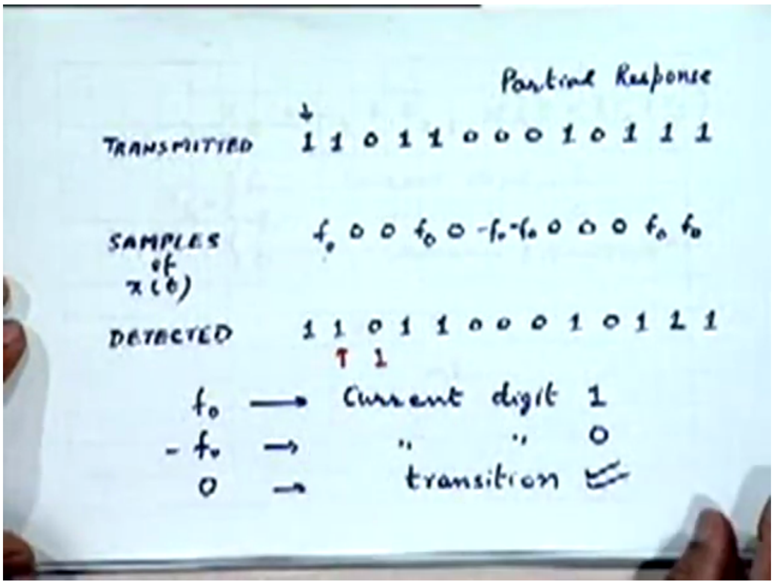
Student: (())(08:23) if we have a transition and suppose if we have a next one is 1 suppose we transfer 0 to 1 next one is also 1 so if we get from minus f0 we are going to be 0 and let us say we have 0 0 1 and like point of this.

Professor: No it does not matter what sequence you have, the point is that the presence of a 0 can only tell you this much that whatever was the pervious decision you must compliment that decision of right, suppose because of noise that decision was wrong for whatever reason occasionally in any digital communication, all this discussion we are doing right now is under the assumption that there is no noise.

Eventually everything is to work in the presence of noise and no matter how carefully you design your system there are bound to be errors now and then, we like them to be as small in number as possible right, but suppose such an error has taken place right now we would not like these errors to be correlated so that introduction of one error causes a sequence of errors after that right, which is precisely what is going to happen in this kind of situation.

For example suppose ak minus 1 you have decided wrongly it is 1 while it was actually 0, a 0 was transmitted suppose by a flock of noise only that mistake right, now let us say that ak, the new value is it was, so it was 0 and let us say that new value is that you transmitted is also 0 right, because you could be transmitting anything right then xk will turn out to be, I should take a (())(10:14) then it becomes 0. If ak minus 1 was 0 that means it was being transmitted as minus 1 right so ak (())(10:27)

(Refer Slide Time: 10:45)



Just let me take a data, I think I have got one here why do not I this (())(10:44) look at this okay this is what you are transmitting, this is your samples that you are receiving getting and this is what you are going to decode them as right, now because you are getting a value 0 here right you are saying that this 1, this decoded digit has to be 0 because of previous one was 1 but suppose the previous one was wrongly detected as 0 for whatever reason.

Suppose at this point I had made a mistake and instead of correctly detecting this as 1, I had detected as 0 right, due to whatever reason, at this point now I will make a wrong decision then it is a 1. (())(11:36) Error detection but not correction right, you will be able to say that there is an error sooner or later but we cannot correct it right.

Student: Sir that was the same case in the line coding scheme also when we had said that there is single error detection.

Professor: That is true but here in addition to all that we have a possibility of error propagation that is one error causing another error.

Student: But chances are that we are going to, I mean obviously

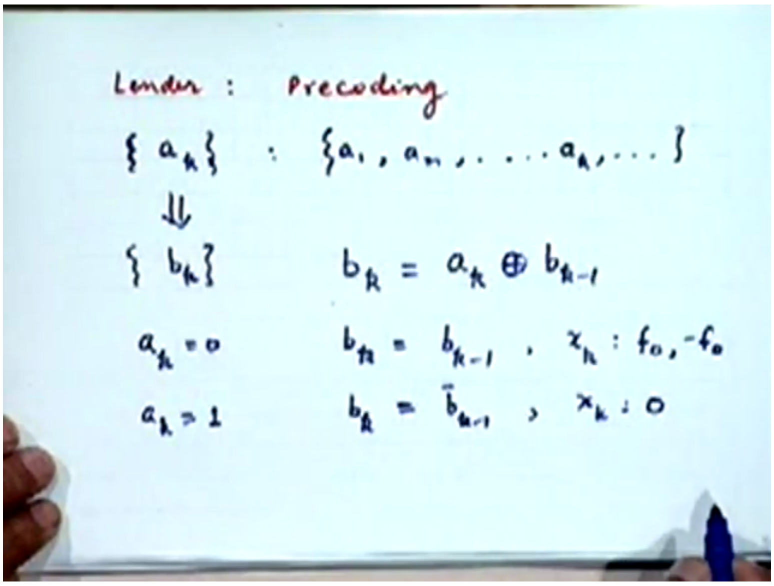
Professor: Not persistently, you may not detect it also.

Student: Sir because if you

Professor: See that error detection capability is for a single error, if multiple errors occur you may not be able to detect because the rule may be still roughly followed, is it (())(12:18) situations odd situations where you may be able to detect there maybe odd situations where you may not be, in fact there will be more often the case when there is error propagation, you may not be able to detect always with surety that an error has occurred because multiple errors have occurred now right.

Some of them may be mutually complimentary in the sense that the net tool will be still followed in most right, so anyway we are not going to detail discussion of the error mechanism but this point is fairly easy to appreciate that error propagation will tend to occur right okay so what can we do about it, Lender suggested, Lender is a name that repeatedly again and again in the context of partial response signalling, he was a pioneer of this particular technique in the 1960s.

(Refer Slide Time: 13:11)



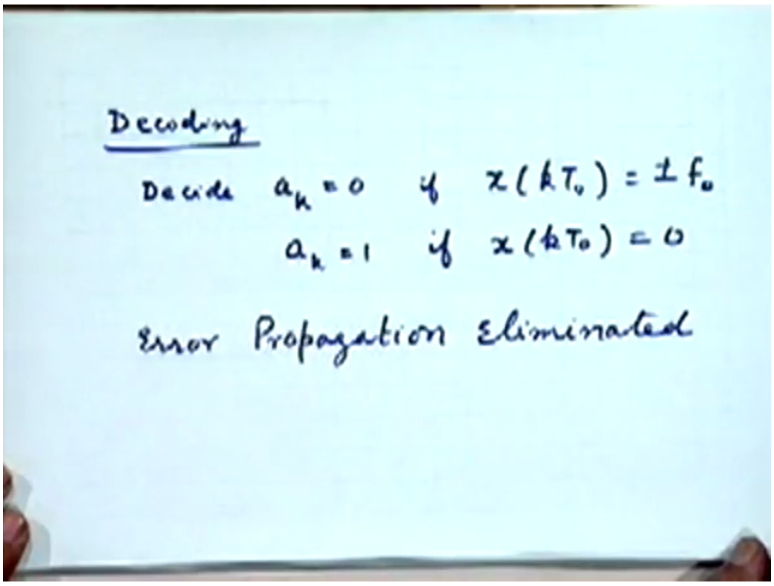
Lender suggested the use of pre coding okay for this purpose that is before we start transmitting your data at the binary level itself at the logical level itself you do certain, a very simple operations in fact one very simple operation which consists of the following thing suppose your data sequence is ak alright, that is some string a1, a2, ak and so on alright you do by pre coding a transformation of this data sequence and this other moment I am taking this data sequence to be in the logical form, 0 1 form, not level form but simply logical form represented, each bit represented by its value either as 0 or as 1.

This is transformed to another binary sequence at the logical level bk such that the k th transform bit bk is equal to ak the present input data bit exclusive or with the previously transmitted digit bk minus 1 right, so b sub k is what you transmit now which is the exclusive or of and the true value of the present bit a sub k and the previously transmitted bit b sub k minus 1 alright.

Well you will have to assume some value for b k minus 1 which you can arbitrarily assume to be either 0 or 1, it does not matter you have to start somewhere if you do this let us see what happens, let us remember that okay let us just try to, suppose ak is 0 what will happen to bk? bk will be bk minus 1 right that is whatever you transmitted previous time instant we are transmitting the same bit now, what will be the sample value of the waveform that you get now, x k?

It will be two things adding together right, remember, right it will be either both plus 1 or both minus 1 right, so it will be f not or minus f not, the value of xk can be now would be either f sub 0 or minus f sub 0, is that okay? Clear to both of you? Clear to everybody here? Similarly if ak is 1, bk is the compliment of bk minus 1 very good and obviously xk value will turn out to be 0.

(Refer Slide Time: 16:52)



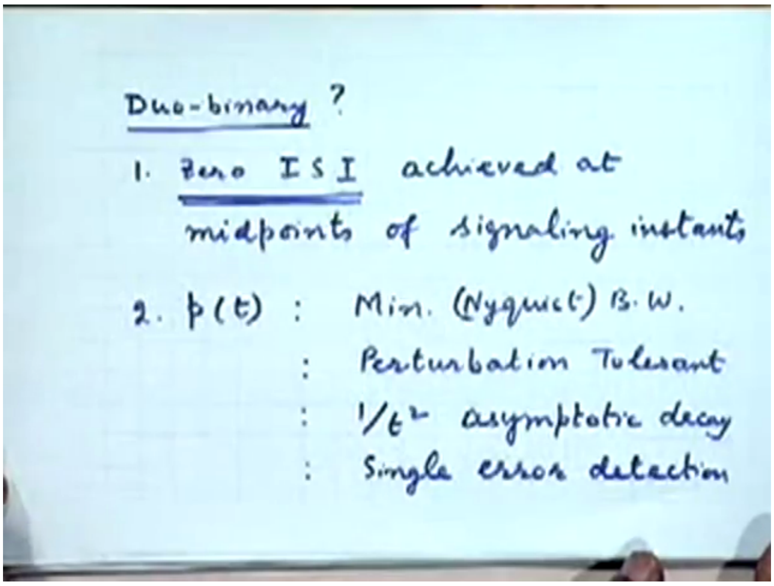
So the decoding is very simple now, you decide, I think let me write it again, the decoding process will now consist of the following, decide that ak equals 0 if x k T0, I will just write x k T0 but although strictly I should write x k T0 plus T0 by 2 right, is equal to plus minus f sub 0 and ak is equal to 1, excuse me, if x k T0 is 0, so the k th digit is now decided by looking at the k th sample value alone.

There is no decision that a transition is to take place right, we are able to explicitly determine the value of ak by just looking at the value of xk so there is no question of error propagation now, is it clear? So error propagation eliminated by such pre coding okay so very neat way of taking care of error propagation, in fact this kind of technique is also used in some other different modulation schemes particularly phase shifting and things like that which we will see later it is called differential encoding in that context right but we will see about that, talk about that separately when the time comes, any questions?

Student: (())(18:37) which has same value that 0 and T not instead of minus T not by 2 and T not by 2.

Professor: There are many other kinds of pulse shape which you can have and I am going to motivate some of the them now and talk about it I will not take an exhausting discussion of this because I think there is only a limited amount of time we should spend on this topic and then go on for the next topic but I will give you enough food for thought and you can probably design a lot of pulse shapes of your own right.

(Refer Slide Time: 19:22)

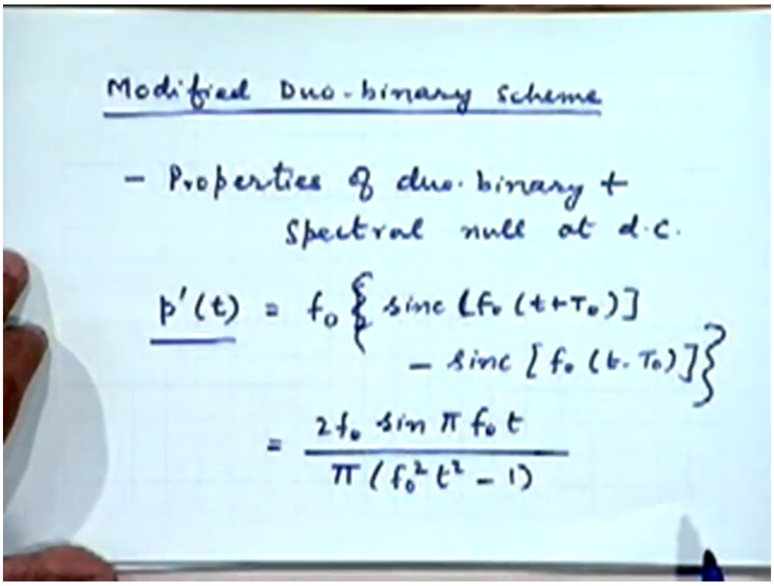


I will just talk about them in a few minutes okay before doing that there is some rise, what we have achieved by duo binary pulse shaping, what have we achieved? First we have obtained zero inter-symbol interference right provided we sample at midpoints of what I have called signalling instants right, we have done this of course this zero ISI has to be taken a bit carefully because it is not strictly 0, it is effectively eventually 0 because we are able to decide correctly in spite of the fact that we have deliberately introduced some inter-symbol interference right.

Secondly our pulse shape is such that it uses minimum nyquist bandwidth right, minimum nyquist come in the brackets, minimum possible bandwidth which is a nyquist bandwidth and which is perturbation tolerant right, why is it perturbation tolerant? Because the spectrum decays smoothly rather than sharply at edge of the nyquist bandwidth, in fact if you remember the expression for pg asymptotically it decays as 1 by t square not 1 by t cube, 1 by t cube was for raised cosine family with r equal to 1 right which occupies twice the nyquist bandwidth.

So that is our rate of decay, asymptotic rate of decay and finally as before we still have some single error detection capability right, single error detection is possible, without pre coding we have the problem of error propagation which we can easily avoid by using pre coding of the kind that we just discussed right, so this summarizes our discussion on duo binary, it seems to have all the nice features that we like it to have except for one which you may remember and that is it does not have a spectral null at DC which is the only other feature we would like it to have which it does not have.

(Refer Slide Time: 22:38)



Now this is precisely the problem that Lender addressed himself to again and came up with what he called a modified duo binary system which has practically all the nice features of the duo binary scheme that we have just discussed and in addition it introduces a spectral null at omega equal to 0 right, let us see how we go about doing this, let me first take the mathematics of it and then we will look at the picture.

So basically the motivation is that we would like to have the properties of the duo binary scheme repeated plus a spectral null at DC right, which is desirable, now let me motivate it like this, if you were to look at the duo binary pulse shapes structure a bit carefully you would notice that it is really a composite function, this pulse shape p t has a composite pulse shape consisting of two nyquist pulses, two sinc pulses separated by T0 seconds.

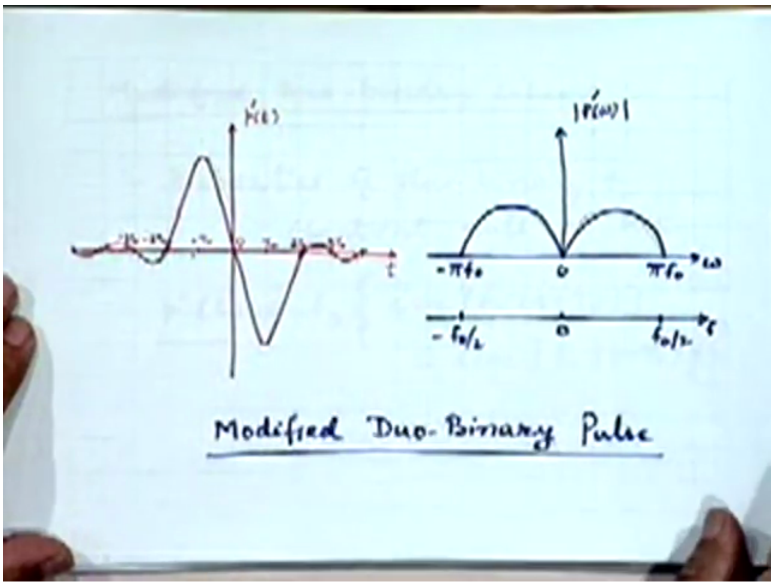
Remember that, one of amplitude f0 or not f0 it is amplitude is some 2 f0 by pi or something right whatever the amplitude it does not matter, both of the same amplitude but displaced from each other by T0 seconds, only the sampling instants are different, chosen to be different. This in fact forms the basis of designing other pulse shapes in general right, we could select a linear combination of such time shifted nyquist pulses or sinc pulses to construct other kinds of partial response signals right.

And that is the basic idea of also that modify duo binary system as well as many other partial response systems which people suggested later right, so let us therefore construct a pulse shape p prime t which has the desirable property of having a spectral null and you also know what we should do to have spectral null, we have done that discussion before, the area under the pulse should be 0 right.

For example in the Manchester codec scheme right, split waves coding scheme, line coding right so similarly the motivation here is not to use rectangular pulses but to use sinc pulses so as to limit yourself in bandwidth to the nyquist band but at the same time has the advantage of zero area right, so I think now it is very easy to appreciate why Lender define a pulse shape p prime t in a manner that I am just going to be write here, maybe I should write it, okay.

Takes two sinc pulses, one located at minus T0 other located at plus T0 and subtracts them out of course this is a pulse shape defined for which is centred at t equal to 0 for defining the pulse shape at t equal to 0 centre the T equal to 0 what is taking is the difference between a sinc pulse located at minus T0 and another at plus T0 okay, if you were to, maybe we can do that later and I can first show you what the pulse shape will look like right.

(Refer Slide Time: 27:16)



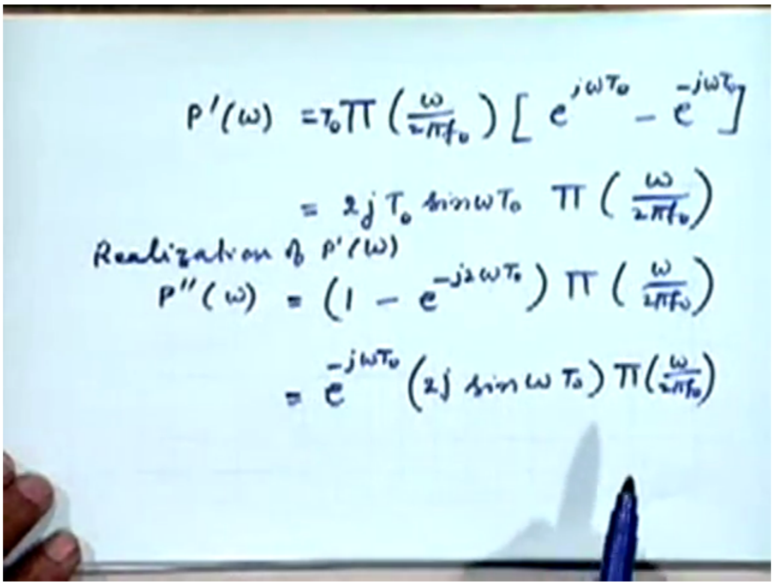
Obviously at T equal to 0 it will go through a 0 just like the Manchester coding goes to a transition in the middle of the pulse right only thing is instead of a sharp transition like that you have a gradual transition like that right because you are subtracting two sinc pulses the difference is like that right and we would like it to be like this so that the spectrum is limited, bandwidth is limited in a, when we discuss Manchester coding we were not worried about the total limitation, absolute limitation of bandwidth to within the nyquist band right.

You are generally saying that bandwidth should be small but we have not restricted the fact that the bandwidth should be strictly within this interval, correct, so this is the pulse shape p prime t which I have just mathematically defined for you, if you were to plot it this is what it will look like it has a peak here at minus T0 and peak at plus T0 and a 0 value at 0 okay if you were to simplify a bit mathematically, I will show that picture again in a few minutes.

This will turn out to be very similar to what we have for the duo binary case that is we have a instead of a cosine function here I have a sine function here naturally because you are going to a 0 at T equal to 0 and the rest is all very similar right amplitude is 2 f not by pi and everything else is same right and the asymptotic rate of decay is again t square, all those nice properties seems to be there obviously the bandwidth will be limited to nyquist band, is it obvious or not?

Because each of these components are limited to nyquist band f not by 2 right both of them have rectangular spectrum with bandwidth f not by 2 but together the spectrum will be something different when you combine it but whatever it is it cannot spell outside the nyquist band right because outside the nyquist band both of them are 0, that is why we have chosen sinc pulses to form other signals, other kinds of pulse shapes, because each other sinc pulse itself is limited to the nyquist band.

(Refer Slide Time: 30:25)



That is the motivation, that is our general technique that we can use to construct in fact a whole host of partial response signals what you have discussed here are only the duo binary and the modified duo binary but there can be many other second construct right, so this is the pulse shape if you have to evaluate the spectrum the Fourier transform of this obviously it will be the rectangular spectrum multiplied by e to the power j omega T0 minus e to the power minus j omega T0 right because the first pulse is located at minus T not right and the other is at plus T not right so we get this which simplifies to 2 j T sin omega T not into the rectangular function okay, questions, if you have any problems speak out.

Sure why not, this pulse, sinc pulse has a rectangular function as a Fourier transform right which is this, this is the Fourier transform, what is the this rectangular function as width of, f not by 2 right corresponding to f not by 2 you have the sinc pulse this sinc pulse is being defined for that bandwidth only, similarly see this sinc pulse and this is your resultant spectrum in fact is not it obvious from here after I have written it down.

If it was not obvious otherwise it should be obvious from here because you are getting some sinusoidal function here as a function of omega multiplied by this rectangular function right so the whatever spectrum you have it is your ultimately limited by this rectangular function which is multiplying this function right but when you linearly combine two spectra which are itself, which each of them are separately limited to the nyquist band that is the zero outset on nyquist band, no matter how you linearly combine them resulting spectrum will also be confined because Fourier transform is a linear transform right.

If it is 0 outside the band both of the (())(32:49) terms are 0 individually the sum will also be 0 right, yes please, oh it should be T not thank you. Any other question, and that is what it looks like if you are to plot, you have a sine omega t not function confined to a nyquist band right and you can see the nice spectral null that we wanted at f equal to 0 okay can we have the discussion pointing this way it will be nice so that everybody else also can participate in it.

Okay I am sorry, maybe I have made some mistake in some step okay you are right as of here from here to here, I think the T not comes here itself right that is the mistake I have made right T not comes here itself, okay now let us see what kind of things are happening here in time domain, what the moment we using a linear combination of sinc pulses of this kind we are certainly got a new pulse shape which is band limited to the nyquist band.

But how is it behaving? What kind of inter-symbol interference is being introduced? Certainly something is being introduced right, suppose I decide to sample this pulse at T equal to T not for the current decision then what I really get is a contribution from pulse which is removed from it by two symbol intervals, two bit intervals right so let me show this pictorially or derive it pictorially for you first.

Let us write this p omega, let us try to realize this function p prime omega, one practical realization would be something like this, let me call it p double prime omega so I am talking of realization of p prime omega okay, you can call it p double prime omega which is slightly different from p prime omega in the following way, everything is same except that this function that I vision here is really e to the power minus j omega T0 times what I should have got here.

I have ignored this function, this constant T0 in this, it requires certain amount of delay for realizing because you do not have to prove it strictly speaking it is not really realizable as such because you have an infinite pulse response and things like that but what I have got here is not that particular problem pointed out here but a simple way realizing right.

Basically I have taken this expression and taken e to the power minus j omega T not outside right and sorry e to the power j omega T not outside you get this expression, this and this are related by that.

Student: You would not multiply e to the j omega, when we alleged we do not take this such factor e to the j omega T not what you have taken over here.

Professor: Just a second, I made some mistake somewhere.

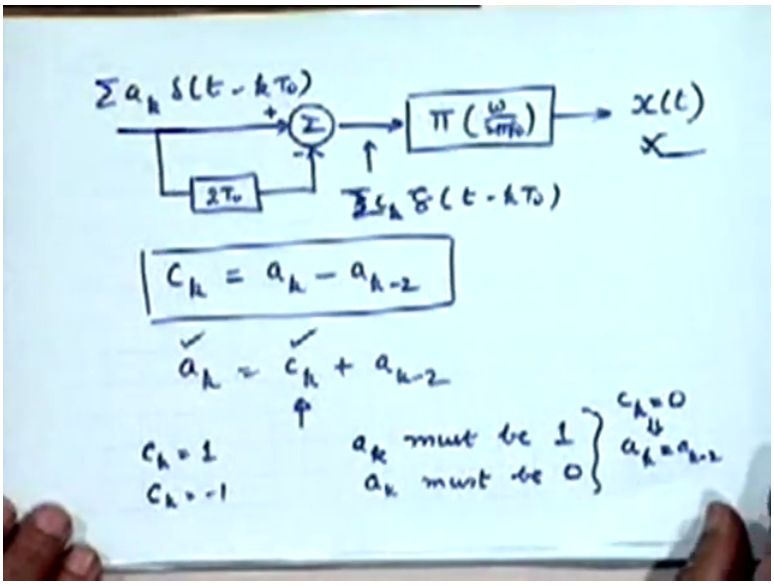
Student: This is a different thing e double prime omega, this is a different thing here, it is only delayed.

Professor: Yeah but I would have liked this to come from this, e to the power minus j omega T not, how is it alright?

Student: Because you take e to the minus j omega T not common so you get e to the j omega T not minus e to the (())(38:03)

Professor: Oh yes you have to divide by 2j, okay I am sorry I also got confused but yes you take this out e to the power j omega T not and then multiply and divide by 2j and that resulting expression will turn out to be e to the power minus j omega T not right, so anyway what is that the reason why I wanted to come to that expression is if I were to draw, picture this by a block diagram I could do that very easily now.

(Refer Slide Time: 38:59)



It was only for that convenience that I did this, so what you do is look at this expression again p prime omega right you want to realize p double prime omega as a filter as a pulse shaping filter such that the transfer function of the filter is this right such that if I feed an impulse to it the output pulse shape will have this spectrum because impulse has constant spectrum alright so if I feed my data impulses here as a sequence alright then this transfer function you can I think see it easily is realized by this.

Oh yes, if this is plus this signal have, should be minus because there is a subtraction sign 1 minus e to the power minus j omega 2 omega T0 into this right, so it was to bring this expression, the reason for bringing this expression to this form was to realize it by a block diagram of this kind in a very straight forward.

No this is still the overall pulse function that you are talking about but it helps me to interpret what I am doing basically from that point of view I have drawn this diagram okay this mind you, this xt here is the waveform that you are receiving just before your sampling the waveform at taking your decision at the receiver and this is what you are transmitting so in between you have everything else the transmitting pulse shaping filter, the channel and the receiving filter.

As before that discussion, that picture remains the same we are not changing the picture at the moment so let the input data sequence be ak which is being represented by a series of impulses over here so what is it that you are going to get here let me call this obviously here also I am going to get some impulse sequence right except that it is going to be some of the impulse sequences, one coming directly and the other coming through this delay.

Let me call this ck, sigma ck delta t minus k T not obviously ck will be related to ak by ak minus ak minus 2 this is the k equation here just like we had discussed ak plus ak plus 1 earlier there ak minus 1 earlier we have this is the corresponding equation, successive symbols are not interfering, symbols which are interfering are removed by 2 intervals that was clear also by construction right and ak is you can write it as ck plus ak minus 2.

So let us see based on this what can we say how we will carry out that decoding right or whether or not it is possible from ck to come to a, first of all it is clear I think before you even discussed that point what we should appreciate is that these ck's which are transmitted after sampling will come back towards as cks right because what is the pulse shape that is being used for transmitting them? The sinc pulse, of course finally what we are really transmitting is not the sinc pulse because finally you are transmitting the net effect of all this.

But if we now think of the fact that we are transmitting not ak's but ck's then it will look to us as if ck's are being transmitted by sinc pulses right and therefore the sinc pulses satisfies the nyquist first criterion, they will be received without inter-symbol interference right, is it clear, so ck's will be able to observe at the receiver when we sample this every t not seconds except for this delay of 2 T not that we have got away here.

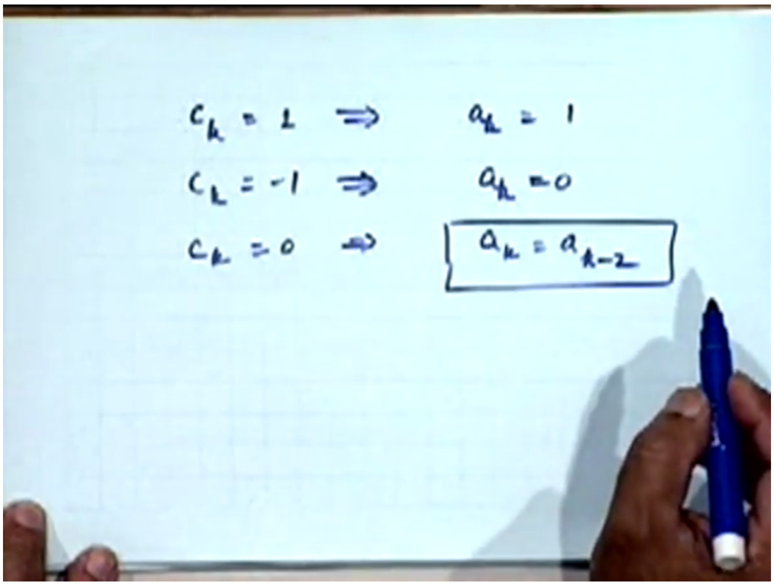
We will see samples which are precisely ck's so our observed quantities at the receiver are going to be the symbol ck's and what we will like to see is whether from ck's we can correctly decide what ak's were which is our actual problem, the decoding problem and that comes from this equation suppose we say we get ck equal to 1 now what can we say about ak, ak must be?

Student: Sir we are subtracting something we can

Professor: Is it obvious or not? Everybody says so but not giving me correct complete picture alright let me proceed further we will come back to it if necessary, if ck is minus 1, ak must be 0 we are talking about ak as a binary entity here 0 1 kind of thing and the third possibility is that ck is okay I think we should have, I should have said this earlier, what is the number of levels of ck?

Suppose ak takes 0 and 1 values once again we have three level system right just like the duo binary system so third possibilities is ck is 0 when ak will be same as ak minus 2 so that means ak is ak minus 2 so based on these equations (())(45:45) rewrite it so that there is no problem.

(Refer Slide Time: 45:49)



Our decoding strategy then becomes ck is equal to 1 implies ak is 1 it is equal to minus 1 implies ak is 0, ck equal to 0 implies that ak is equal to ak minus 2 this are the three possible values that you will receive, you will sample and these are the corresponding decisions you have to take right and therefore a unique relationship exists between the sample values of ck and the decoded values of ak except for the fact that once again there is a possibility of error propagation, because if you receive ck equal to 0, if you have made a wrong decision at k minus 2 you will perhaps again make a wrong decision perhaps, not necessarily, okay.

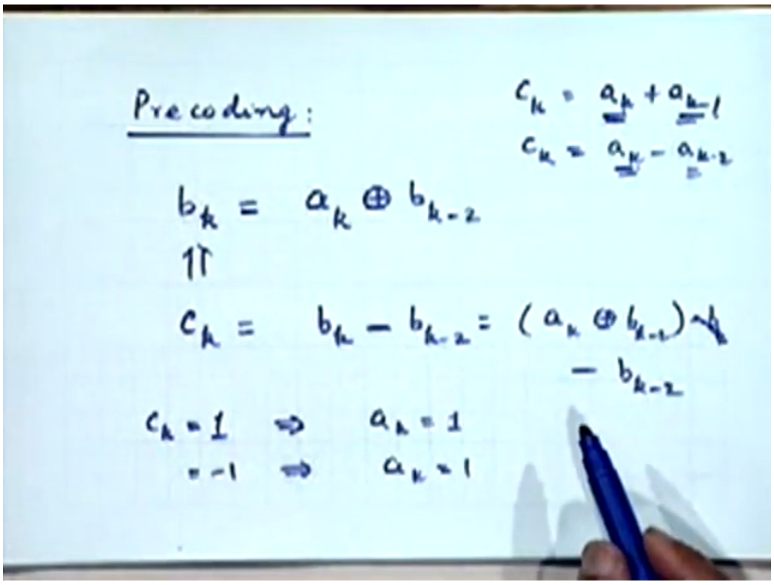
So once again one can do pre coding and things like that in fact we will come to that in a few minutes at the moment let us proceed further, we will come to pre coding in a minute but before I come to pre coding what I like to discuss with you is what have we achieved by doing all this, we achieved the spectral null that we wanted but have you continue to achieve perturbation tolerance, do you continue to have that property perturbation tolerance, we will have to look at the spectrum right.

The spectrum I have already shown you, it has that half sinusoid shape within the nyquist interval it does not look, it does not seem like it has a sharp cut-off right so therefore it must be perturbation tolerance in fact from the pulse shape expression it again becomes obvious that it is decaying at the rate of t square, 1 by t square right as t tends to infinity, so we now have a waveform which has the desirable features of introducing a controlled amount of inter-symbol interference such that the data sequence can be decoded uniquely plus it is perturbation tolerance, plus it is strictly limited to nyquist bandwidth right.

So these two were remarkable achievements at that time in 1960s when Lender proposed the use of these for data communication for effectively doubling the capacity of existing telephone trunks for data transmission right existing cable trunks or digital trunks for a data transmission overnight by the use of duo binary scheme one could double the transmission capability, is not it?

Because before that, before this concept of partial response signalling came to practical use the only perturbation tolerance waveforms that were typically being used were the raised cosine family with r equal to 1 right and that had twice the nyquist bandwidth so by just using a small modification of the waveform by just doing a little bit of manipulation of the transmitter we could now have a situation for the same bandwidth we could effectively double the transmission rate.

(Refer Slide Time: 49:37)



We could have twice as many channels on the same cable system which was a very remarkable primary achievement at that time in 1960s. Let us come now to the perturbation tolerance sorry the pre coding aspects of this, you suggest to me what kind of pre coding should be used here I think you can tell me.

Let me make a few enquiries okay Nikhil what is your opinion? Let us say we want to create a sequence bk from the sequence ak right how should we go about it, it should be ak exclusive or with not ak minus 2, bk minus 2 right that is correct okay and that is a good guess I am sure they have only guessed it because of the similarity of this equation with the previous one, see the only, what is the difference between duo binary case and this case?

In the duo binary case we had ck equal to ak plus ak minus 1 on the channel this is what is happening on the channel and in this case we have ak minus ak minus 2 so because of this kind of index value that are coming to picture here and the index values coming to picture here it seems reasonable to try a pre coding scheme of this type actually there is a general theory of pre coding from which you can take care of more general situations than this but at the moment it is sufficient to appreciate that this kind of pre coding perhaps will serve our purpose.

Let us see whether on or it will, so we will use this sequence bk as a input to a modified duo binary system what we will be transmitting there is not ak's but bk's but from the observed ck's now you will be able to precisely decode the value of ak without referring back to previous transmitted pulse, values which is what we will like, let us see how, your ck is now going to be equal to not ak minus ak minus 2 but b sub k minus b sub k minus 2 right.

Which you can write as a sub k plus b sub k minus 2 minus b sub k minus 2 alright, excuse me, now when ak's and bk's, if they are, we are giving that ak's and bk's are binary symbols 0 and 1 values but this subtraction is not modulo, this is the real subtraction right that is the difference so let us see what kind of decisions we can make now, suppose ck is 1 what is the meaning, it implies ak is equal to 1 right because the ck is 1 that means bk must be 1 and bk minus 2 must be 0 right.

Similarly if it is minus 1, no again it is 1, ak is still 1 right because you see how can ak be 1, ck be minus 1 when this is 0 and this is minus, this is 1 so ck is minus 1 now how can. I am sorry there is a mistake, no this is fine, I know this is fine but the argument if ak is 1 and bk minus 2 is 0 we can have this as either plus 1 or minus 1 that is a question point right.

Student: Both should be 1, bk minus 2 and ak should be equal to 1.

Professor: I always get confused at such points, just a second I think it is very simple, bk is 0

Student: When ak and bk minus 2 is 1, both are 1, sir we should do this reverse way, sir first said is ak is equal to b by 1 then what happens?

Professor: Yeah I think, okay we will start from there next time.