Analog Electronic Circuits.
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Lecture-48.
The Differential Pair and the Gilbert Cell as Wideband Amplifier.

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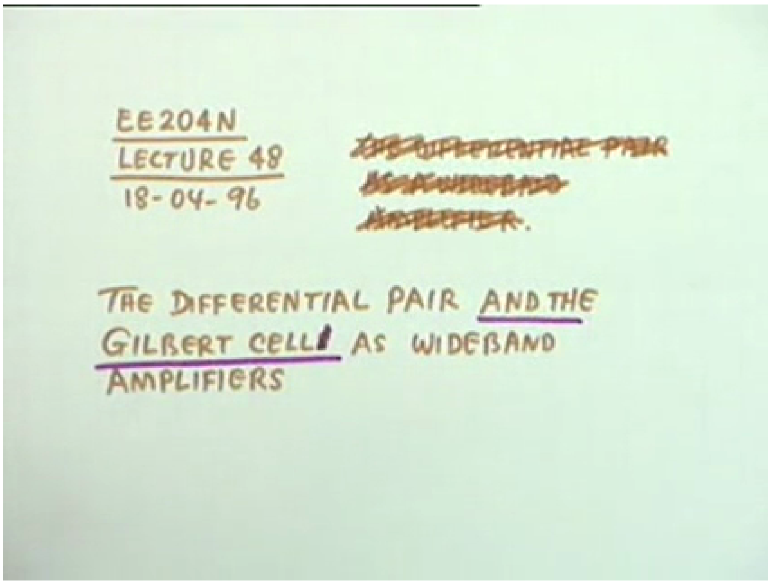

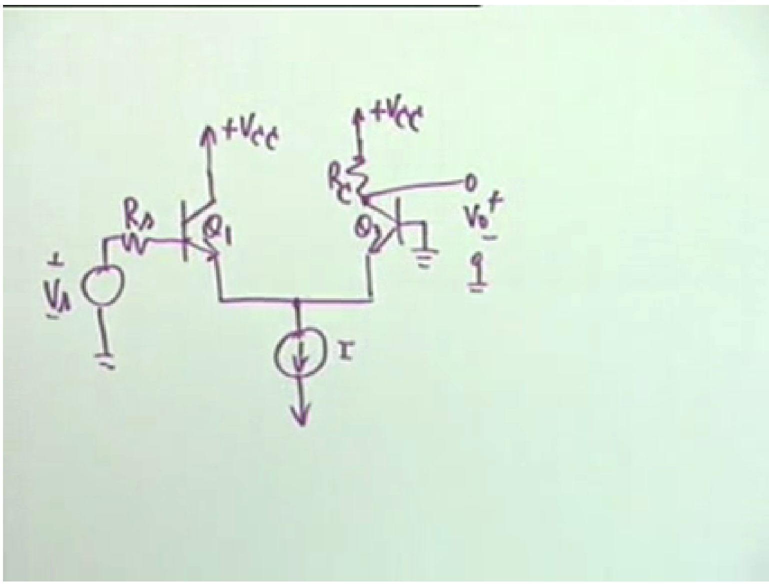

This is 48th lecture and our topic today is the differential pair which we started last time. And we add the Gilbert cell, Gilbert cell, a particular circuit which is very popular in integrated circuit, Gilbert cell. Both often can be used as wideband amplifiers and this is what we are going to look at today. The differential pair as I had drawn the last time please, the 1st transistor is without a load, it is a common collector stage coupled to the 2nd transistor through the emitter and this goes to a constant current source, a current source, the input is applied to Q1, VS, RS and the output is taken from Q2, the load is R sub C and this is the output V0, the Q2 base is grounded, this is the circuit.

We made an analysis, we made the qualitative explanation of why this is wideband. Basically it is wideband because both C mus, their teeth have been taken off, 1 C mu reflects as twice C mu and the other one reflects has only C mu, it does not reflect at all. It goes from collector, it goes from collector to ground because the base is grounded, so there is no Miller effect. There is Miller multiplication Q1 but that is a, it is a very small amount, only twice C mu 1, this is a basic reason. We drew the equivalent circuit and there was a very decent, very elegant result namely that we had found out V Pi 1 was equal to minus V Pi to which made the equivalent circuit look like this.

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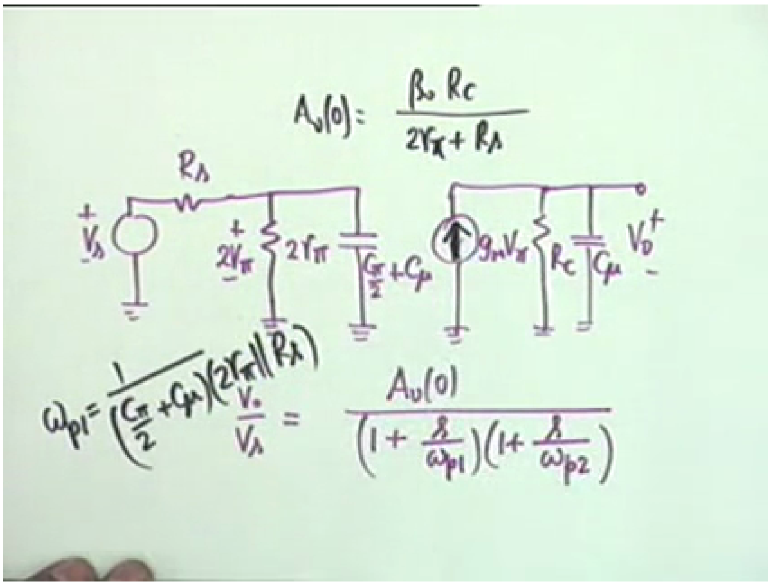

We had gone through this analysis, so I shall not repeat. VS, RS, and then instead of R Pi there was a continuity and it is 2R Pi, the voltage across this is twice V Pi because one V Pi is opposite of the other. The capacitance here, 2C pi in series gives rise to C pi by 2 and then C mu was reflected, then you have the GM V, one has to take care of the fact that this is 2V pi, not V Pi. GM V Pi and you have R sub C and C mu, this is equal to V0. By simply looking at it we could derive the result that V0 by VS is the mid-band gain, let us call it AV 0 divided by…

(())(4:26).

Direction of GM V pi is opposite, yes, thank you so much. In other words V0 and VS shall now be in phase with each other, okay, there is no phase reversal. Wonderful. Okay, by looking at it we could derive the result that this must be of the form 1+ S by omega P1, 1+ S by omega P2 where the values of omega P1 and omega P2 are also obvious. All the 3 quantities are obvious, for example AV 0 we found out that this is beta 0 R sub C divided by 2R Pi plus RS, that is it, 2R Pi plus RS. And that is obvious GM V Pi flows through RC, V Pi is this voltage, twice V Pi is this voltage and therefore we found that out.

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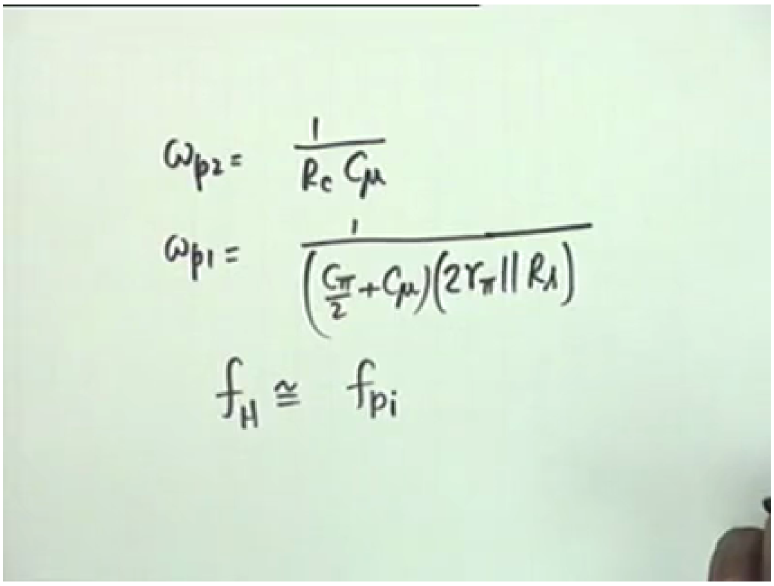

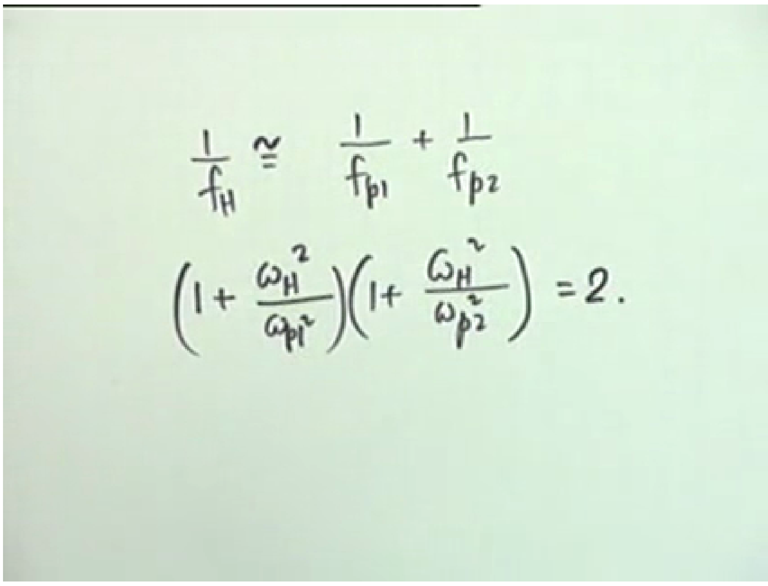

Okay, and omega P1 and omega P2, omega P1 is the reciprocal of this time constant which is C pi by 2+ C mu multiplied by, what, what this capacitor sees is a parallel combination of 2R Pi and RS. So 2R Pi parallel RS and omega P2 is simply the reciprocal of this time constant, that means omega P2 is equal to 1 over RC C mu, agreed. Let me write down omega P1 also, won awards C pi by 2+ C mu twice R Pi parallel RS. Depending on the values of these resistances namely R Pi, RS and R sub C, omega P1 and omega P2 may differ from each other, may also be comparable.

If one of them dominates, that is if one of them is more than 3 times the other, you remember the figure 3 because squares are involved, all right. Then that particular pole shall dominate in determining the high-frequency 3 dB cut-off, that is FH will be approximately equal to FPi, where FPI is the dominating frequency, okay. It is either FP1 or FPC2 depending on whichever is 3 times greater. On the other hand even if, even if this is just 3 times greater is applicable and you have the figures, there is no harm in taking this formula, 1 by FH as 1 by FP1 +1 by FP2, is this correct? Is difficult formula or FH should be FP1 plus SP2.

This is correct for high-frequency. For the low frequencies it will be sum of them, all right. So you could also apply this formula or since it is a quadratic, there is no harm in solving the quadratic equation 1+ omega H square divided by omega P1 square multiplied by 1+ omega H square divided by omega P2 square would be equal to 2, okay. We shall take an example and try to find this out by all the 3 methods and see how good things are.

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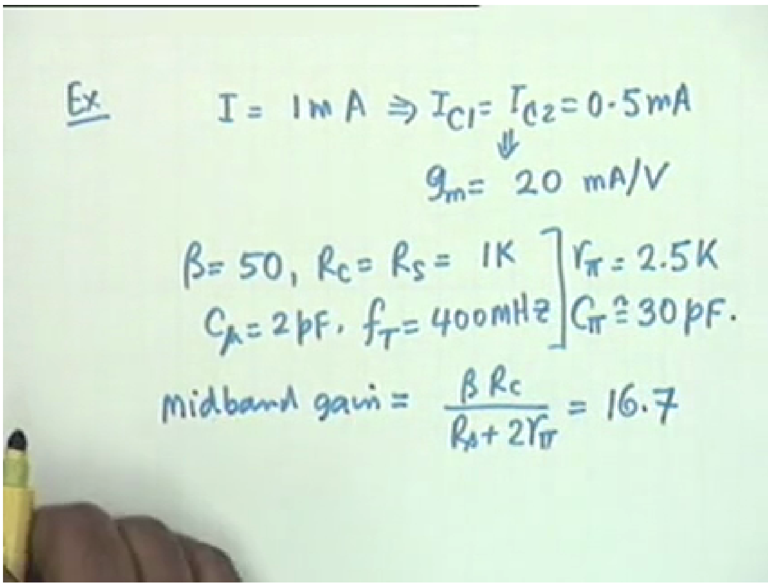

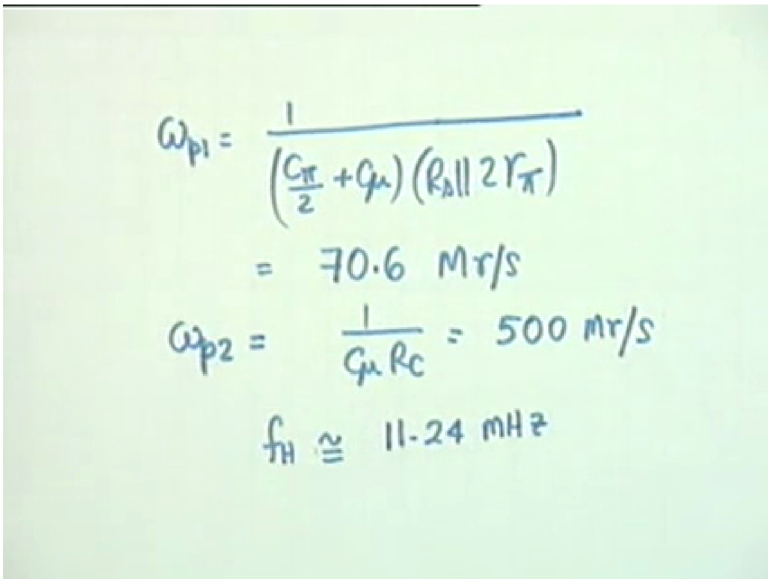

Okay, now let us take an example in which capital RI, the constant current source is 1 milliamperes, which means that I C1 equal to I C 2 equal to half of this because it is a differential pair, 0.5 milliamperes. That therefore gives GM as equal to 0.5 divided by 25, that is 20 milliamperes per volts or millimhos, okay, GM is this. It is also given that beta is 50, R sub C and R sub S, they are equal to 1 K both, C mu is given as 2 picofarads and FT is given as 400 megahertz. From this you can find out R Pi, beta is given and GM is known, so R Pi could be equal to 50 divided by 20 which is 2.5 k and C pi can be found out as GM divided by 2 Pi FT minus C mu and that comes out approximately as 30 picofarads.

Then the mid-band gain, no more negative sign, it is in phase, beta RC divided by RS plus 2R Pi, if you substitute the values, this comes out as 16.7. And then we calculate omega P1 and omega P2. Omega P1 which is 1 by C pi by2 plus C mu, this is also remarkable that instead of C pi we get C pi 2, is in that right. That also contributes to widebanding, C mu, then RS parallel 2R Pi and if you substitute the values, this comes out as 70.6 mega radians per seconds, 70.6 mega radians per seconds.

On the other hand omega P2 is 1 by C mu R sub C and this calculates as 500 mega radians per seconds. So omega P2 is more than 7 times omega P1 and therefore FH would be controlled basically by omega P1 and it should be approximately this divided by 2 pi, so many hertz, and this comes out as 11.24 megahertz. Okay.

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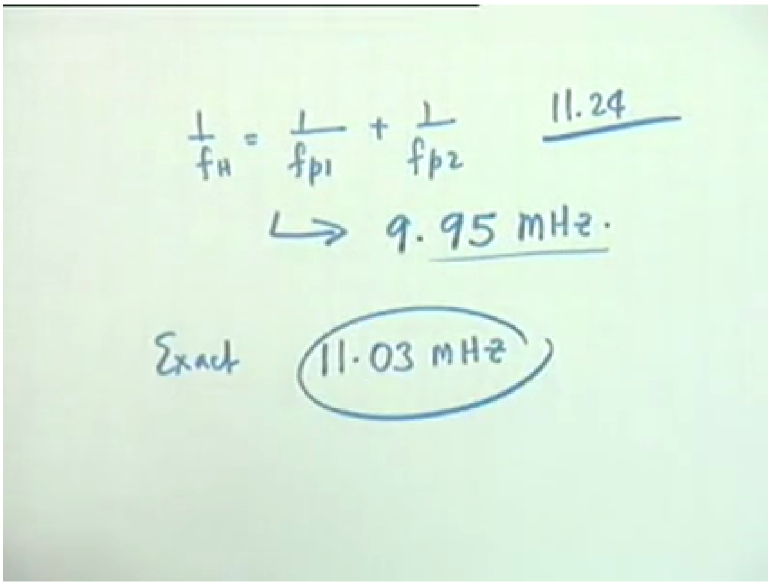

On the other hand if you use the time constant addition formula, in other words if you use the formula 1 by FH equal to1 by FP1 +1 by FP2 and substitute the values, this leads to a figure of 9.95 megahertz. One may think that this is a closer approximation to the actual value, it is not the case. If we ignore omega P2, you got 11.24, we brought in omega P2 in this approximate formula. There is another approximate formula, square, 1 by FH 12. However this one gives 9.95, on the other hand I solved the exact quadratic, solve the exact just to find out which one is closer.

And exact solution gives me 11.03 megahertz. You see it is closer to 11.24, that is why ignoring omega P2. One cannot therefore see which one, formula would give more accurate results, one have to find out. It depends on the situation, one has to find out. There is no rule of thumb is that you use this, then you will get a better… You must notice that in this example the transistors that were used, FT was 400 megahertz friend the bandwidth that we get is only 11.03 megahertz, that is approximately 1 by, 1 by 40, approximately 1 by 40. Suppose, now this happens because we are processing basically voltage and it is always true, it is always true.

Suppose I had processed current, if current was my signal, you know this fact was known to people but it has dawned on people only very recently because in integrated circuits the demand for higher and higher bandwidth led people to think that if instead of voltage we do current processing, if we take current as the signal, then perhaps we can reach very near FT, we can reach very near FT. And this led to a new group of circuits, integrated circuits which are called Transconductance circuits. And instead of operational amplifier we now have in the market, in addition to operational amplifier, more modern chips are OTS, that is operational transconductance amplifier.

And they are basically current processing, okay. We shall at least have an exposure to one circuit and this is what I want to show you, it is the Gilbert cell which basically processes currents instead of voltages. And the idea is the same, that we want to go as close to FT is possible and Widebanding should be possible. One of the qualitative things that, that one thinks in terms of Widebanding by current signal processing is that the parasitics which cause distortion of voltage may not have any effect on the current, okay. It is very qualitative, I am not going into the details of it but it can be shown that parasitics are less effective in current processing rather than voltage processing.

And however in the practical circuits, the engineers obsession because of the historical reasons with voltage processing. If I give you a current chip, current amplifier chip and your need is for voltage processing, microphone apparently generates a voltage, it is a high impedance source, it generates a voltage, even some of the electrodynamics microphones, they generate a voltage. So basically you generate a voltage and perhaps you might like to display it on and oscilloscope. And basically you have to display this voltage, right.

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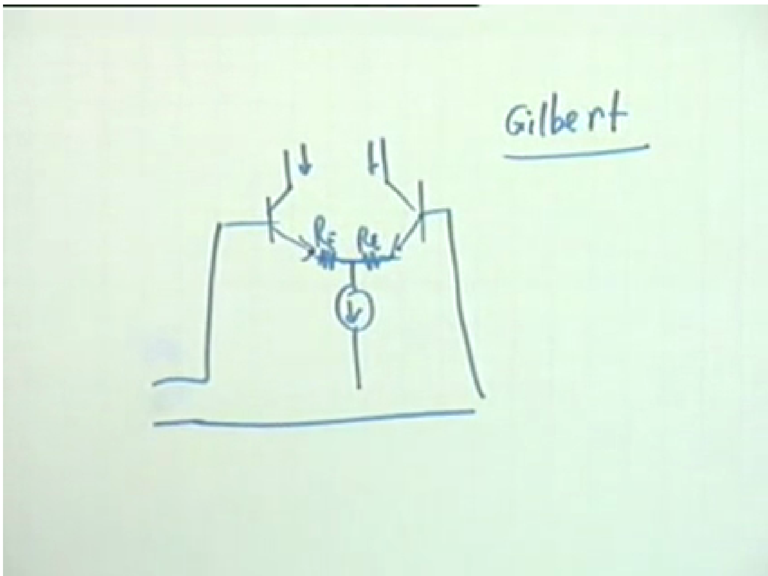

And basically what you do, if I give you a current to current transducer, you have to convert it into a voltage to voltage by a V to I conversion in the beginning and I to V conversion at the end. This is not difficult at all, for example you know that this, this differential pair, with a small resistance here. Suppose we apply a voltage between these 2 points with a small resistance here, suppose we apply a voltage between these 2 points, between these 2 bases, then the voltage is converted to currents, agreed. The voltages converted to currents and the voltage difference between these 2 emitters shall be proportional to the signal current, agreed. The difference between the 2 collecter currents.

You see application of voltage here will cause and unbalance between these 2 currents and therefore it will cause a voltage difference between these 2 points. Basically this is a voltage to current converter, all right, voltage to current converter. And if you want to convert current to a voltage, all that you have to do is to pass them through a resistance. Alright. And therefore it is not difficult. So if you want a voltage amplifier out of a current amplifier, then you require V to I conversion at the beginning and I to V conversion at the end. We will see, we will see how the Gilbert cell, it is after the name of a British scientist called Gilbert, Barry Gilbert, that this is his name, he is still alive.

He gave a very interesting circuit, very strong differential concept which is very much used in number of chips. Gilbert must have become very rich after his patented the circuit and sold out the patent. And the only condition for the Gilbert cell is that it cannot be made in discrete circuits. The reason is that the Gilbert cell for its operation requires very closely matched transistors and that can be achieved only by IC fabrication. Let us look at the circuit, I will draw the circuit only in parts and again I will draw the full circuit and I shall explain away go ahead.

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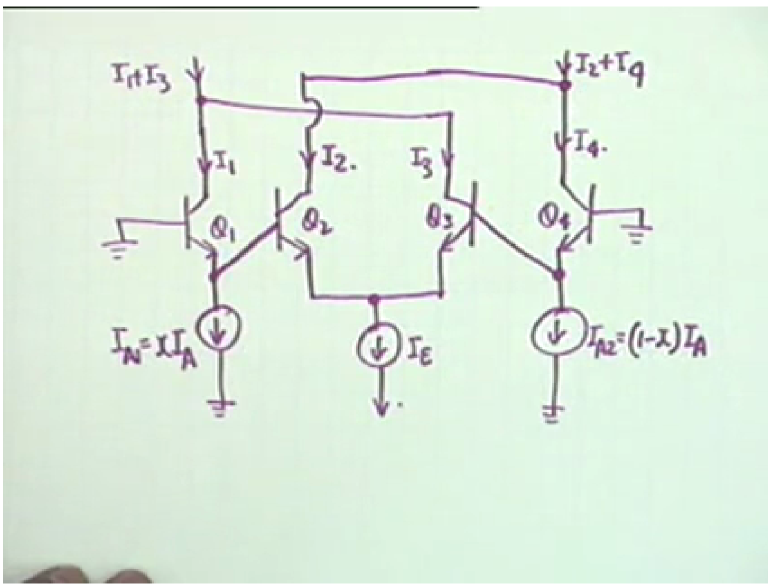

Please try to draw with me. You have a transistor Q1 whose base is grounded, all right. The current here is I1, from this collector, this collector is connected to another collective, we shall call that as Q3, okay. And this current we will call as I 3, so that this current is I1 plus I 3, were it comes, it could come from a supply, it could come from a constant current generator, we do not bother. Okay, the other part of the circuit is you have another transistor Q4, symmetrical circuit, you have another transistor Q4 whose base is again grounded and the current through this is I 4 and this collector is common to another transistor Q1, Q3, Q4, so this should be Q2, all right.

This is connected to this point, this current we shall call as I2, all right, therefore this current would be I2 plus I 4, agreed. It is a criss-cross type of connection, the 2 emitters are connected and this is connected to a constant current generator I sub E. Now the connection between Q1 and Q2, Q1 feeds Q2 and Q4 emitter feeds Q3, okay. No, it is not, it is not a Darlington pair. Well, it is similar to Darlington what it is not quite because the 2 collectors are not brought together. In the Darlington the collectors are connected, Q1 and Q2 collectors are not connected, okay.

If it was, it would have been a Darlington pair. Then of course Gilbert could not have got his patent because Darlington would have objected. Darlington was alive at the time, Darlington died recently, Sydney Darlington. Okay, then these 2, these 2 emitters, I will show this connected to 2 current generators and how these current generators so obtained, I will show in a different circuit. We will call this as IA1 and I will call this as part X of the current I sub A, we will explain what this means later. This is I A2, this is the rest of the current IA, namely 1 minus X I sub A. Some fraction, X is less than 1, okay. Have you drawn this circuit?

Yes.

Okay. I will come back to this. I sub A is a current generator and X I A and 1 minus X I A are created by this difference. Normally the 2 currents will be identical, normally these 2 currents will be identical, that is I A 1 and I A2 would be identical. The difference or the unbalance is created by a voltage source and it is like this. This part of the circuit now I shall draw separately. Is this part of the circuit drawn?

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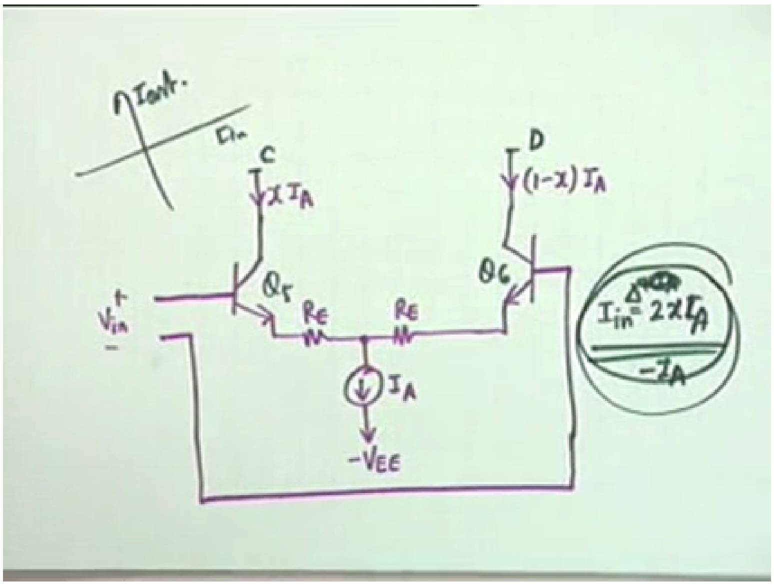

Okay, the 2nd part of the circuit is, I wanted to show you X IA, how it is derived and how 1 minus X I K is derived. This is taken to a differential pair, derived from a differential pair, the emitters are connected through 2 small resistance is R sub E, okay and the middle point is connected to a current source I sub A, this is the current source I sub A which goes to a minus VEE, and negative supply, okay. Now between these 2 bases, between these 2 bases is applied an input voltage V in plus minus. This is how those 2 current sources, X I and 1 minus X I are created.

You can see, you can see that if these 2 are tied together, if the 2 bases are connected together, okay, then this current and this current shall be identical because these are transistors are absolutely identical. We have named, we can name them as Q5 and Q6. The transistors, all transistors are identical, they are made in a chip, it is a chip, all transistors are identical. So what to do is, when you apply a voltage V in here, this causes an unbalance in the 2 collectors currents and this unbalance however, the sum of the 2 must be equal to IA, all right. We assume of course that beta is much greater than 1, that is the base current be ignored.

And therefore the collector currents are approximately equal to the emitter currents and the sum of the 2 must be IA. So when we can is 0, when these 2 are connected together, there is no unbalance, this current is same as this current. And this as you can see is a voltage to current converter, that is an application, in response to application of a single, 2 currents are created which differ. The 2 collectors currents, and in fact the input current here, input current here, it is not the, it is not the actual input terminals that you apply current, no, you apply voltage.

But effectively, this is a V to I converter, voltage to current converter and the input current can be reckoned as a difference between these 2 currents. So, what is the difference, twice X I sub A, agreed. We will find out what the output current is.

(())(25:40).

Vin, we are investigating the DC characteristics, V in could be positive, it could be negative, so it could be sinusoidal, it is a single. But we are, we are investigating the DC characteristics. If there is a change in the voltage between these 2 terminals, what happens? The change, when the change is negligible, it is an incremental characteristics. Pardon me?

(())(26:07).

Where does the circuit fit in, this is where they are connected, this point and this point. If I call this as the point C and this as a point D, then this is C and this is D, okay. I did not want to draw it because it becomes long this year and also becomes complicated. That part of the circuit is basically a voltage to current converter, so I showed it separately.

(())(26:35).

What is, you see these 2 voltages, these 2 currents shall be equal if the 2 bases are under identical conditions, agreed. The base to emitter voltage, based emitter voltage, if they are identical, they are connected symmetrically RE, RE, so they should be identical if these 2 are connected together. As soon as you create, as soon as you apply voltage here, an unbalanced is created, okay. So one of the current increases, the other decreases but the sum of them remains the same.

Is it that the DC currents flowing in the collectors change?

Yes, that is correct.

But in (())(27:13). How is this connection different from DA (())(27:26).

That is precisely what is happening here. We are considering that 1st V in is 0, when V in is a very small quantity. If it is a very small quantity, then obviously the current will be, the 2 collector currents will different. Then the small quantity will make it oscillate between positive and negative.

(())(27:45).

Source, Oh, this I in is not a source in the physical sense, it is an equivalent input current.

Is it the difference between 2 currents?

It is the difference between these 2 currents X IA and 1 minus X IA.

Is it a DC current or (())(28:16).

It is DC, we are investigating the DC characteristics as we always do. If I plot I out versus I in, the DC characteristics, from this I will be able to find out the AC characteristics, okay. As we had done in the case of a differential amplifier. Now there is a point here, the difference is not 2 X IA, plus I A, okay, plus IA.

Minus.

Okay, minus IA, all right. Then we will argue… Pardon me?

(())(29:01).

No, this is not a physical quantity, it is a virtual quantity, instead of putting 2 currents as the input, we have put 2 voltages and they have produced these input currents. This is a, this is the defined quantity IA, all right. Pardon me?

(())(49:21).

Since this is not a physical quantity we do not care where it is or what it is. Alright, we will come back to it, if this bothers you, forget about it, that we will see is the end. All right. Now let us go back to the circuit. Let us go back to this circuit.

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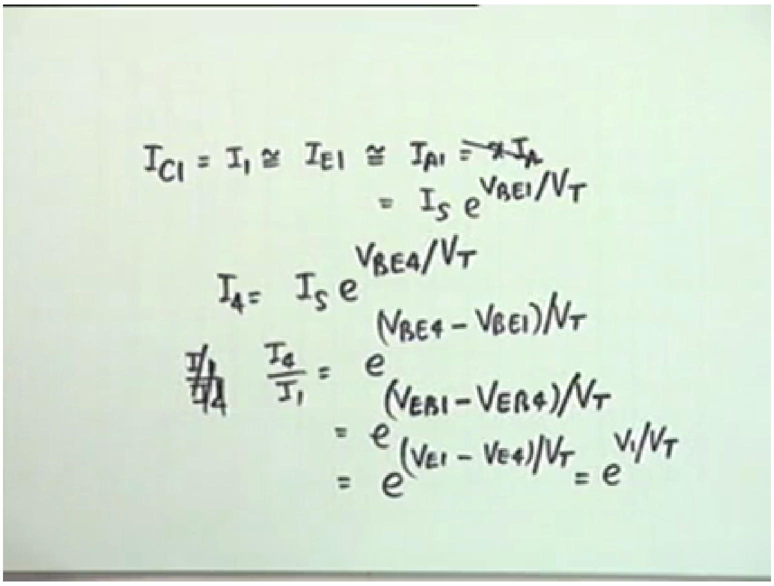

You see the collector current of Q1 Ic1 is equal to, I C1 is equal to I1 is approximately equal to I E E1. And if beta is large, if beta is large this is true and beta is large, the base current to Q2, you see this current supply the base of Q2, which would be negligible and therefore this is approximately equal to I A1 which is equal to X IA. X IA we will bring later but I A1 is equal to I S exponential VBE1 divided by VT, agreed. There is the relationship between the base emitter voltage and the collector current.

(())(30:46).

X is less than 1 because it is the sum of the 2, X IA, 1 minus X IA, that has to be equal to I.

There is one problem here, (())(30:56).

X is a, X cannot increase because the sum of the 2 currents has to be equal to IA, X cannot be greater than 1. It is a differential pair, let me go back to the circuit. It is a differential pair, whatever these 2 current are, they must add to IA and the other current directions and therefore X cannot be greater than 1. Okay, is this point clear, that I C1 can be written in this form.

(())(31:53).

VBE 1, you see this is current, I1, I1 is I C1 and therefore this is related to VBE 1 by this relation. In a similar manner I 4, I 4 which is equal to I sub C 4 can be written as IS e to the power VBE 4 divided by VT. All right. And therefore, therefore I1 by I 4, no, let us take I 4 by I1, is equal to, one has to proceed very carefully, I 4 by I1, if I take the ratio of the 2, it is simply exponential VBE 4 minus VBE 1 divided by VT. All right. I can write this as exponential, if I interchange B and E, the sign would be negative. And therefore I get VEB 1 minus VEB 4 divided by VT.

What is the potential of B1? 0. B4? Also 0 and therefore I can write this as e to the power VE1 minus VE4 divided by VT. Let us call this voltage, the voltage difference between the points E1 and E4 as V1, e to the V1 by VT, okay. Let us identify as this V1 occurs. V1 obviously the difference between this potential and this potential, is not that right. Difference between the potentials, points C and D, is the point clear? And therefore what we have produced is ultimately a difference of potential between Q1 and Q2 which is applied to the differential pair Q2, Q3.

Is in that right, between the 2 bases is applied a voltage V1. Now let us draw the circuit a little more in a little more simplified manner to emphasise this point, because much depends on this point. But you also notice that, all right, we keep this figure.

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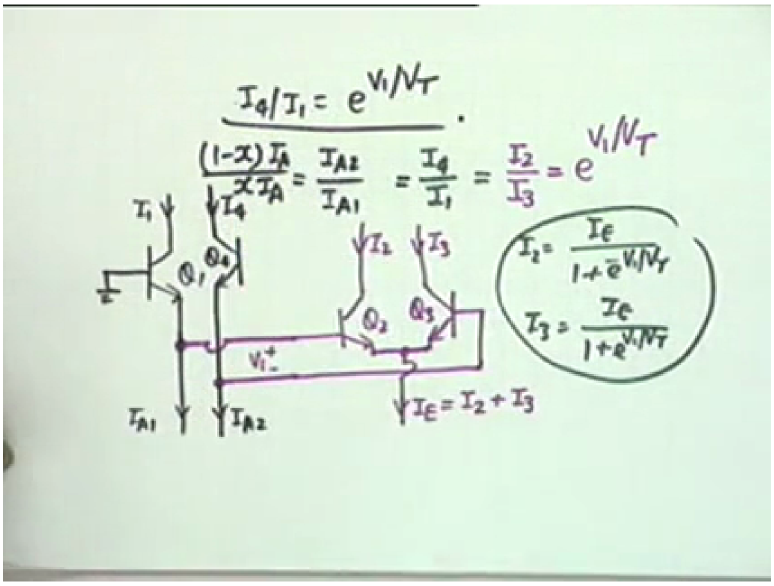

I 4 by I1 equal to e to the power V1 by VT, let us see where this will be one is. I will draw the circuit in a little different manner to emphasise this point. Q1 and the base is grounded, this current is I1 and this current is I A1, I1 and I A1 approximately equal. We also draw Q4 adjacent to it, this current is I 4 and this current is I A2. The difference, now let me use a different colour. The difference between this point which was C in the previous circuit and this is V1. So let us take this line, this is V1, okay, this is the identification of V1, the voltage between the 2 emitters.

And we apply it, apply this difference to the 2 transistors connected in a differential pair, Q2 and Q3, all right, I am just redrawing the same circuit suitable to emphasise the point where V1 is created and how it is applied to another differential pair. This current is I3 and the common emitter, emitters connected together, they are connected to a current generator of value I sub V, I will not show the current generator, I do not want to complicate the circuit. But obviously I sub V would be equal to I2 + I3, all right. Now what is the ratio I2 to I3 is? Can you tell me just by looking at the circuit without doing any calculation the ratio of the 2 collector currents?

E raised to V1 by VT, absolutely correct. And we have then analysis earlier also in the case of a differential amplifier, I2 + I 3 is a constant, I2 + I 3 is this, therefore the 2 quantities can be written as, we have done this earlier, this is a side result, if you wish to remember, I2 is IE divided by 1+ e to the minus V1 by VT and I 3 gives IE divided by 1+ e to the plus V1 by VT, okay. The sum of the 2 is equal to IE. Now we do not require this result, what we require is that I2 by I 3 is e to the V1 by VT and this is equal to I 4 by I1, I 4 by I1 was also equal to E the V1 by VT, all right.

You must give, you must appreciate Gilbert the way he arrived at the result. Now look what happens, if this is so, I 4 by I1 is also equal to IA 2 divided by IA 1, alright, this is also equal to I A2 divided by I A1, right. And what is I A2? 1 minus X IA divided by X IA. Now this has become too complicated, let me the produce the results here.

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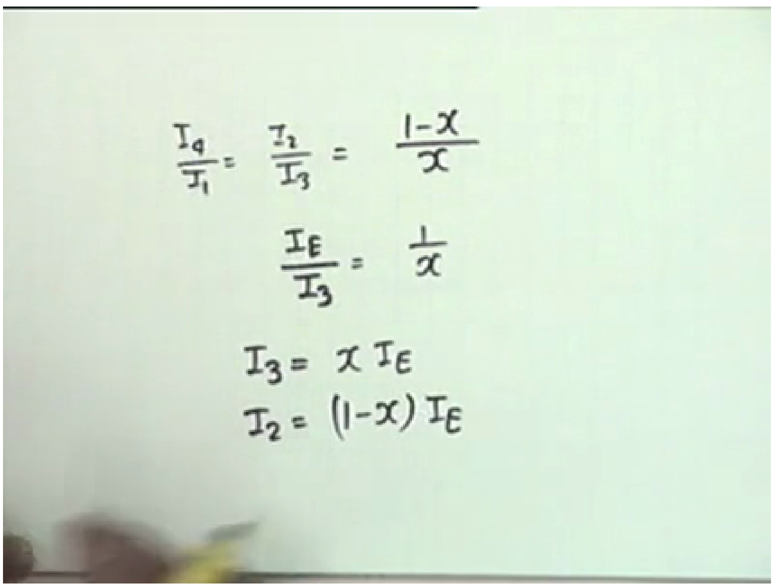

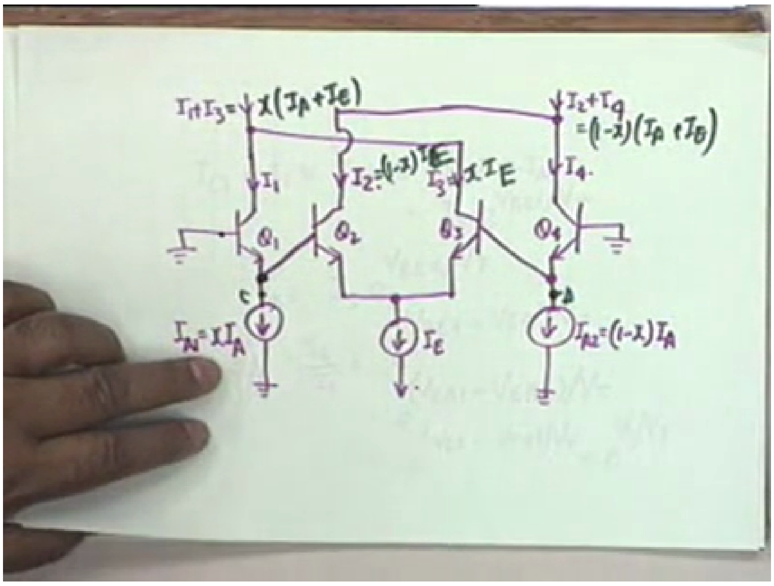

I 4 by I1 is equal to I2 by I 3 equals to 1 minus X divided by X, all right. Let us add 1 to both sides, then I get IE divided by I 3, because I2 plus I 3 gives IE, this would be equal to 1 over X. And therefore I 3 is equal to X IE. And therefore what is I2, obviously 1 minus X times IE, is the point clear? What we have done is that this proportioning of the input current has now reflected in the in the differential amplifier. The proportioning of the input current X I A and 1 minus X IA has now reflected into the proportioning of the collector currents of the differential amplifier, all right.

Let us go back to the circuit and see what has actually happened. Let us go back to the original circuit here. This I2 is 1 minus X IA, all right. And I 3 is, no, IE and I 3 is X IE, okay. So what is I1 plus I 3 then?

1 minus X.

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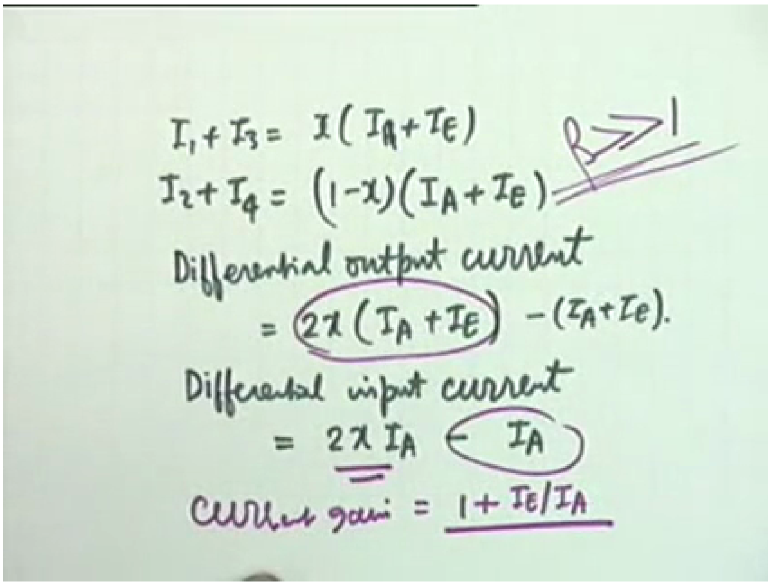

No 1 minus X, this is… X IA plus IE, okay. X times IA plus IE. And what is this current then? 1 minus X IA plus IE. And what is the difference between the 2? What is the difference between the 2? Obviously these 2 are my output currents, I1 plus I 3 and I2 plus I 4 are my output currents. And the difference between the 2 output currents obviously is twice X IA plus IE minus IA plus IE, okay. Let us go, let us write this down. I1 plus I 3 equal to X I B plus, I A plus IE, I2 +4 is equal to 1 minus X IA plus IE and the difference, differential output current, differential output current would be equal to twice X IA plus IE, all right.

Minus IA plus IE and differential input current is equal to twice X, what was it, IA minus IA, okay. Now where is the reflection of the signal? Obviously it is in this term, this is the DC part, this is constant, it does not change, what changes is this part. This is constant, it does not change, this is the changing part, okay. And therefore you can see that a signal dependent current twice X IA get simplified as twice X IA plus IE. And therefore what is the current gain? Current gain is 1 plus IE over IA, all right.

It is exactly similar to the voltage gain of a noninverting operational amplifier, except that the 2 ratios are not those of resistances now, they are of 2 current sources which can be highly stabilised. And this gain can be any quantity greater than 1. In the analysis, we never consider frequency. How could current be affected by frequency? We only consider DC characteristics, right, we never consider current, we never consider frequency. We did not have to consider the effect of capacitors and so on.

Of course it would be dependent on the capacitors but this is the basic principle of current amplification and since current is the signal, the only constraint, the frequency has come in our consideration, can you tell me where? Where? We assumed that beta is much greater than 1. We cannot quite go to FT because FT is the frequency at which beta is equal to1. If we died equal to1, obviously we cannot ignore the base currents as compared to the collector currents. This is the only constraint that has come, alright.

And therefore if beta is, you can go up to 10 let us say, which will be a frequency close to FT, this circuit can amplifier to a frequency much higher than what can be obtained through voltage processing. The question that remains is how to convert this current into a voltage.

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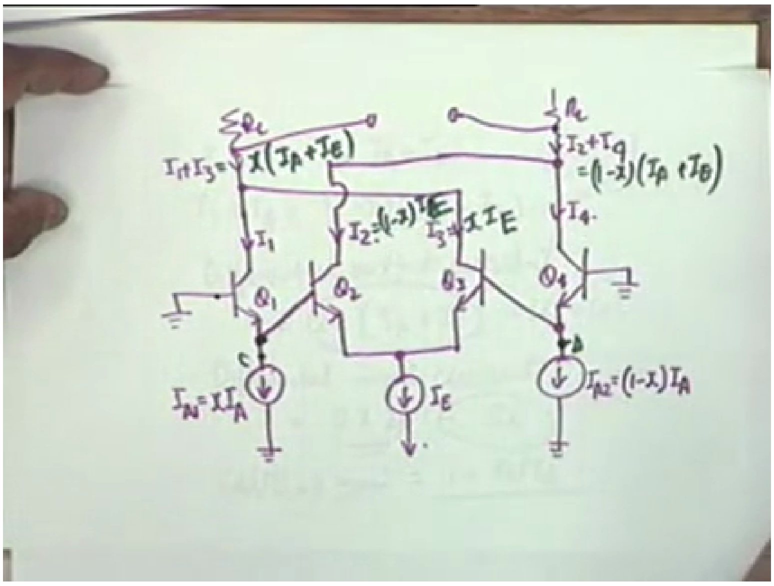

That is very simple, you go back to the surface, our original circuit and all you do is you include a resistance here, a resistance here RL and you take the difference between the 2 voltages, alright. So this is a very simple means of current to voltage convergence. You see the drop here would be VCC minus I1 plus I 3 RL, the drop here, the voltage deal will be minus I2 plus I4 RL and therefore in the difference those DC quantities will cancel out, that IA plus IE, that will cancel out, all right. And therefore you get a voltage which is derived from a current amplifier through a current to voltage conversion. Tomorrow I will 1^{st} draw the total circuit, I will 1^{st} draw the total circuit and then look into the Op-Amp imperfections.