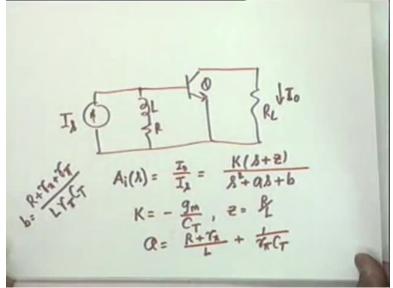
Analog Electronic Circuits Professor S.C. Dutta Roy Department of Electrical Engineering Indian Institute of Technology Delhi Module No 01 Lecture 40:Widebanding by Using an Inductance

We continue our discussion on widebanding by using an inductance.

(Refer Slide Time: 1:06)



The circuit that we were considering is that of a shunt peaking. We have current source I sub S, L and R. This goes to the base of a transistor Q, R sub L and this is I0, the current is I0. And by drawing the equivalent circuit, we have shown that the current transfer function AIS which is equal to I0 by IS is of the form KS + Z divided by S squared + AS + B where K is equal to - GM by...

Student: CT.

Professor: CT - GM by CT. Z equal to R by L, A is equal to I do not remember. R + RX divided by L + 1 over R pi CT. And B is equal to R + RX + R pi divided by L R pi CT okay. This is the transfer function and by taking the magnitude squared we had reduced it to the following form. (Refer Slide Time: 2:42)

$$|A_{i}(j\omega)|^{2} = \frac{K^{2}z^{2}}{b^{2}} \frac{1+\frac{\omega^{2}/z^{2}}{1+\omega^{2}\frac{\omega^{2}/z^{2}}}}{1+\omega^{2}\frac{\omega^{2}/z^{2}}{b^{2}} + \frac{\omega^{4}}{b^{4}}}$$

$$B_{z}^{2}g_{z}^{1}M = |A_{i}(jo)|^{2}$$

$$= |A_{i}(jo)|^{2}$$

$$H = \frac{A_{i}(j\omega)}{A_{i}(j\omega)} = \frac{1+\omega^{2}/z^{2}}{1+\omega^{2}\frac{\omega^{2}-2L}{b^{2}}} + \frac{\omega^{4}}{b^{2}}$$

$$A_{i}(jo) = -\frac{\beta_{0}R}{R+Y_{R}+Y_{R}}$$

AI J Omega magnitude squared we had reduced it to the form K squared Z squared divided by B squared 1 + did we do this? Omega squared by Z squared and then1 + omega squared A squared - 2B divided by B squared + omega 4 divided by B to the B squared again. Okay. And you notice that thisquantity obviously represents the gain magnitude squared at omega equal to 0. Is not that right? Omega equal to 0, this quantity becomes one. So I can write this asAI J 0 squared multiplied by the same quantity. And ifI take the normalised value,AI J Omega divided byAI J 0 whole squared, then this becomes 1 + omega squared by Z squared 1 + omega squared A squared - 2B divided by B squared + omega 4th divided by B squared, whenAI J 0, the value, if you substitute the values, it becomes - beta 0 R divided by R + RX + R pi. You have to substitute for K, Z and B. And this is the value. We shall require this later, AI J 0. This is the gain, mid-band gain. Orit starts from low frequencies because there is no coupling capacitor in the circuit. Yes?

Student: Sir, put beta 0.

Professor: Oh beta 0, beta is the same as beta GM R pi. I should not use beta 0. Beta is GM R pi okay? Now the question that we have left unanswered is what should be the relative values of A and B. What should be this coefficientin order that is magnitude squared function is maximally flat at Omega equal to 0? So what we want to do is,let us call this as M squared M omega squared magnitude,normalised magnitude squared.

(Refer Slide Time: 5:23)

We wish to have : $\frac{dM}{d\omega} = 0 \qquad \frac{d^2N}{d\omega^3} = 0, \dots$ $M^{1}(\omega) = \frac{\omega^{2}}{1+\omega^{2}} + \frac{\omega^{4}}{b^{2}} + \frac{\omega^{4}}{b^{2}}$

What we want to do is if I want what maximally flat, I want DM D omega equal to 0. We wish to have and we wish the next differential coefficient also equal to 0. If possiblethe 3rd, 4th and so on okay. As many as possible. Now how many should be possible? How many should be possible? You see.

Student: 2.

Professor: 2 or 3?

Student: 2.

Student: 3.

Professor: Only 2 are possible. The 3^{rd} automatically cancels as you will see. The 3^{rd} automatically cancels. All right. Let us see. I can write, the way to visualise this is it is a rational function, M squared omega is a rational function. You convert this into an infinite series. That means what you do is, you divide 1 + omega squared by Z squared by 1 + omega squared A squared - 2 B divided by B squared + omega 4 by B squared. Divide the numerator by the denominator. Make a long division. And naturally, the quotient would be an infinite series. Is not that right? Quotient would be an infinite series. Well, another way of looking at this is write this as omega squared by Z squared multiplied by 1 + omega squared A squared - 2 B divided by B

squared + omega 4 by B squared to the power - 1. And this is an infinite series. Multiplying by this obviously shall be an infinite series. What would be the constant term in the infinite series? 1. And the coefficient of omega squared would be 1 by Z squared - this right? Then there would be a coefficient of omega to the 4th and so on and so forth.

(Refer Slide Time: 7:28)

$$M^{*}(\omega) = 1 + \omega^{*}\left(\frac{1}{2^{*}} - \frac{a^{*}-2b}{b^{*}}\right) + 60^{\circ}(1)$$

$$+ \cdots$$

$$dM_{d\omega} = 0 \text{ at } \omega = 0$$

$$d^{*}H_{d\omega} = 0 \text{ at } \omega = 0$$

$$\frac{1}{2^{*}} = \frac{a^{-}-2b}{b^{*}}$$

Now let us look at this. M squared omega1 + omega squared 1 by Z squared - A squared - 2 B divided by B squared + et cetera. Obviously there would be an omega 4th term and you can find out the coefficient and so on. Okay. Obviously DM D omega is 0 at omega equal to 0 because

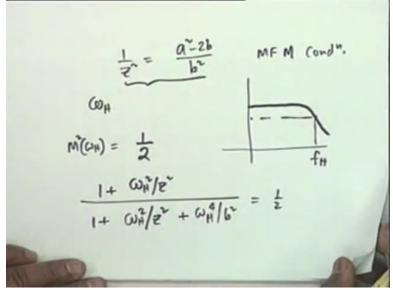
this becomes 2 omega, 4 omega Q and put omega equal to 0, it becomes 0. Obviously no effort is needed. But suppose we want to make D2M D omega 2 also equal to 0 at omega equal to 0, then it requires after 2nd differentiation, this will be left with 2 multiplied by this constant.

So this constant has to be equal to 0. So this is the condition, 1 by Z squared equals to A squared - 2 B divided by B squared. This is the condition that is needed for the 2^{nd} order flatness. As far as the next differentiation is concerned, the 4 omega cubed 12 omega squared, then 24 omega, obviously that is automatically 0 at Omega equal to 0. The degree of the rational function is 4. Is not it right? The rational function that you started from, the highest power is 4 and in general, this is true that if the degree of the rational function is 4 then the maximum possible number of flatness is the degree - 1. 3^{rd} order of flatness, okay.

And it is also true now note this very carefully. It is also true that if I can write a normalised function like this with a constant equal to 1 okay, then the condition for maximal flatness is that the corresponding coefficients in the numerator and denominator should be equal as far as possible. Okay? For example here, we have found out that the condition for maximal flatness is 1 by Z squared equal to A squared - 2B by B squared. The next coefficient is 0. Obviously, 0 cannot be equal to 1 by B squared because then B requires to be infinity.

So the maximal flatness condition is that the corresponding coefficient if you want it 1, as much as possible, the next one is omega squared, I can make them equal. But I cannot go further. So this is the condition for maximal flatness. If this was an 8th order functions, then maximal flatness would have demanded that up to omega to the 6, the coefficients are equal. Okay? We will see, we will come to thisanother example of condition for maximal flatness a little later.

(Refer Slide Time: 10:46)



But this condition1 by Z squared equal to A squared - 2 B divided by B squared is the condition for maximal flatness, maximal flatness in the magnitude. So it is called MFM condition MFM condition and under this condition, the omega H, now what does this mean? It means that if you permit no peak in the frequency response, no peaking in the frequency response, then this gives the maximum possible value of F sub H. Agreed? It gives the maximum possible value of F sub H.

If this condition is not obeyed, then it will go either peaking or it willcome down in a less flat fashion. It will come down more rapidly. So the maximum possible omega H is obtained under this condition and this can be obtained by putting M squared omega H equal to the value of omega H can be obtained.

Student: (())(11:51)

Professor: Not root 2. There is a square here. So it should be equal to half. But you see, which means that omega H squared by Z squared divided by 1 + should I writeA squared - 2 B divided by B squared? No, that is not needed. I write omega A squared by Z squared. That simplifies calculations.+omega H2 the power 4th divided by B squared. This should be equal to half and obviously, this will give you a 4th order equation to be solved but fortunately, this is a quadratic in?

Student: Omega H squared.

Professor: Omega H squared. And therefore you can solve for omega and H and the final result is the following.

(Refer Slide Time: 12:39)

$$A_{i}(0) = \frac{-\beta_{a}R}{R+Y_{a}+Y_{\pi}}$$

$$G_{H} = \begin{cases} \frac{1}{2} \left(\frac{b}{2}\right)^{2} + \sqrt{\frac{1}{4}\left(\frac{b}{2}\right)^{4} + b^{2}} \\ \frac{b}{2} = \frac{Y_{\pi}+Y_{\pi}+R}{RY_{\pi}C_{\pi}} \\ b = \frac{Y_{\pi}+Y_{\pi}+R}{LY_{\pi}C_{\pi}} \end{cases}$$

Simplified result is omega at sea coast to half B by Z whole squared + actually you will have 2 quadratic equation has 2 solutions. One of them will come out negative, the other will be positive. Omega H squared cannot be negative, so the choice is unique. This is the + sign we have to use and then we have one quarter B by Z whole to the 4^{th} + B squared, this total quantity to the power half. Half is, the reason for half is obvious because you solve for omega H squared okay? Andone should remember that B by Z is equal to RX +R pi +R divided by R pi CT. Andone should also remember the value of B which is R pi + RX + R divided by LR pi CT.

Okay? So you can find out omega N. The other equation that is required for designing is AI 0 which is equal toas I already mentioned, beta 0 R I am sorry, I keep on writing beta 0. Beta R divided by R + RX + R pi. These are the 2 equations that are needed for design of this amplifier for a given omega H and for a given mid-bandgain. We will work out an example to illustrate that. Any question on this? These are all design equations. And now we can proceed.

(Refer Slide Time: 14:29)

Example $C_{\mu} = 5 \beta F$ $\beta = 50$ $\gamma_{\pi} = 250 \Omega$ $\gamma_{\pi} = 250 \Omega$ $T_{A} = 100 \Omega$ $C_{\pi} = 75 PF$ $R_{L} = 500 \Omega$ |Ai(0)| = 25 $d find f_{H}$

Supposeit is given that C mew equal to 5 puff, beta equals to 50, a rather low beta, GM is equal to 0.2 mho which means that R pi is equal to 250 ohm. Is not that right? Beta and GM are given. Sometimes the quiscent collector current will be given. If that is given, then you find out GM as IC by 25. IC in milliamperes divided by 25 millivolts. Okay. Then RX is given. RX, there is no way to derive it. It has to be given, measured. C pi many a times C pi will not be given but FT will be given. And if that is the case, then you know omega T is equal to 1 over R pi C pi + C mew.

This is the transition frequency or unity gain frequency. C mew is given, Omega T is given, R pi is derived from the value of GM and beta and therefore C pi can be found out. All right. C pi is given here and RL is 500 ohms, this is given. The mid-band gain needed, AI 0 is required to be 25. Design the circuit. Design the circuit and find what FH does it obtain. What is the high-frequency cut-off?

Student: Sir (())(16:17)

Professor: It maybe, it may not be also. We will work out another problem in which FH and midband gain are given or FH is given, you are required to find out the mid-band gain., As far as the gain is concerned, the absolute value of is not very critical, is not very important okay. What is important is up to what frequency can you go. (Refer Slide Time: 16:43)

$$25 = \frac{B_{0}R}{R+Y_{0}+T_{0}}$$

$$R = 350 \Omega.$$

$$L \leq \alpha^{2} - 2b = \left(\frac{b}{z}\right)^{2}$$

Now this constraint that 25 should be the mid-band gain this makes it 25 equal tobeta R divided by R + RX + R5. RX is 100 ohm, R pi is 250, beta is 50 and therefore you can find out R. Capital R is the series resistance of the inductance, capital R and capital R comes out as 350 ohms. To find the inductance L yes?

Student: Sir, this capital R is not a discrete element. It is a part of the inductor.

Professor: It is a part of the inductor, that is correct.

Student: While designing the inductor, we had assumed that this is the maximum resistance it can offer.

Professor: Itshould not get more than this. Now in practice in practice suppose your capital R suppose the oil can be made with lesser value of inductance all right? Then you can either redesign the circuit that means, you sayyou are asking for a gain of 25, I will give you more. You are asking for an FH of so many mega radians per second, I will give you more. The people would be happyprovided that is not in the frequency range which you do not want okay.

Now ifit is a strict specification, that is you have to use R equal to 350 and the actual R that you get in series with inductance is less, then you add a small resistance there okay. So capital R 50-50. Now the next question is find L. L will be given by the maximal flatness condition, that is A

squared - 2 B should be equal to B by whole squared. L would be obtained from this relationship. And if I write down the expression for A and B and Z in this, I get an equation like this.

 $\frac{1}{L} + \frac{1}{\gamma_{\pi}C_{T}}^{2} - 2 \frac{R + \gamma_{1} + \gamma_{\pi}}{L \gamma_{\pi}C_{T}}$ $= \left(\frac{\gamma_{\pi} + R + \gamma_{A}}{R \gamma_{\pi}C_{T}}\right)^{2}$ L= 27.23 MA

(Refer Slide Time: 18:44)

R + RX divided by L, this is A. 1 over R pi CT A squared - 2B. B is R + RX + R pi divided by LR pi CT. This should be equal to B by Z squared which is R pi + R + RX divided by RR pi CT whole squared. This is the equation. And if you look at the equation, obviously yes?

Student: What is small r?

Student: Capital R. It should be capital R.

Professor: Where did I write small r? Oh , I am sorry, this should be capital. Thank you for pointing this out. I know capital R, I know it is 350. I know capital R, I know RX,I know CT. CT is C pi + C mew 1 + GM RL okay. I know everything here except L and that L occurs in a quadratic equation. Is not that right? There is a squared okay. So it becomes 1 by L squared and this is 1 by L, so you can solve for 1 by L from this as a quadratic equation. There will be 2 solutions. Most probably, one of them will not be acceptable and the other will be acceptable.

If both are acceptable, there are situations when both are acceptable, then you can use either of them but when you put in the circuit, one of them will satisfy, the other will not. Then it becomes slightly difficult. But in most of the situations, it would be obvious which value is acceptable.

Now if you substitute all the values and solve for this, this requires crunching out numbers. In all practical problems, you have to crunch out numbers okay. There will not be an (())(20:34) to round off the values such that the gain becomes 40 or 23 or something.

So you will have to do this and I can tell you, it took me quite some time to calculate it out. 27.23 Micro Henry, this is the order of inductance that is needed and 27.23 Micro Henry is not very difficult to make. Also from the equation, now our design is complete. Okay? Our omega H was not specified. It was required to be found out.

(Refer Slide Time: 21:08)

$$\begin{aligned} \omega_{H} &= \begin{cases} \frac{1}{2} \left(\frac{b}{2}\right)^{2} + \sqrt{\frac{1}{4} \left(\frac{b}{2}\right)^{4} + b^{2}} \end{cases} \\ f_{H} &= 2.76 \text{ mHz} \\ \text{GBW} &\cong 69 \text{ mHz} \\ f_{T} &= \frac{9m}{2\pi (G_{T} + G_{A})} &= \frac{398 \text{ mHz}}{398 \text{ mHz}} \end{aligned}$$

So you substitute the values of the quantities B by Z which we have already found out + square root of one quarter B by Z whole squared + not whole squared, whole to the 4^{th} + B squared whole to the power half and this comes out as, FH comes out as 2.76 megahertz. Ourgain was 25 mid-band gain. So the GBW, gain bandwidth product that we have achieved is 25 multiplied by this which comes as 69 what would be the unit?

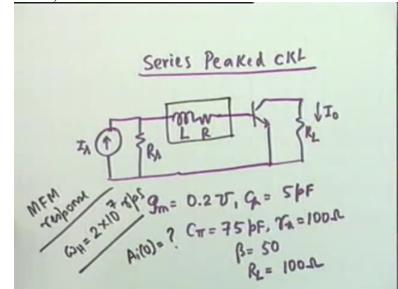
Student: Megahertz.

Professor: Megahertz, 69 megahertz. Compare this with FT. FT is GM divided by 2 pi. Do not forget the factor 2 pi. C pi + C mew all right? This calculates out to 398 megahertz. Okay. So approximately we have gone up to 20 percent of FT. Our GBW has gone up to 20 percent of FT. Is that clear?

Student: Sir, how did you get GBW?

Professor: How did I get GBW. I multiply this by 25, the mid-band gain multiplied by the bandwidth. That is the gain bandwidth product, 69 and it is still within very safe limits. In theory one should be able to go right up to 400 megahertz. If your gain is 1, we should be in theory we should be able to go up to 398 megahertz. But things conspire to bring it down. Even with use of an inductance, we can go only upto 69 megahertz. If we did not have an inductance, then it would have comedrastically down okay. It is the inductance which hasextended the bandwidth to 69 megahertz.

(Refer Slide Time: 23:36)



Nowwe consider a problem. The problem concerns a series peaked circuit. I shall indicate the basic steps and you work out the rest. In a series peaked circuit, the inductance and its losses, the coil is connected in series with the base rather than in shunt. Sowe have a current source RS.it can be a current source or voltage source okay. I have included the source resistance. And then you have an inductance L and its resistance R. This is the coil. This is the coil and this goes to the base and then you have an RL, the current is IO.

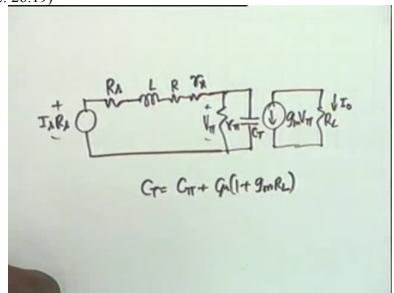
The problem is GM is given as 0.2 mho as in the previous circuit. C mew is 5 puff. It is the same same transistor which is being used for series peaking. 5 puff, C pi is 75 puff, RX is 100 ohm,

beta is 50 and RL is 100 ohm. In the previous case, it was 500. Is not it? What was RL in the previous case? Was RL effective?

Student: No.

Professor: No, it does not depend on what RL is because our transfer function is current transfer function. Current is GM V okay. Now what you are required to find out isdesign the circuit for maximally flat magnitude response. Here we do not care about the mid-band gain but we care about omega H. Omega H is required to be 2 times 10 to the power 7 radians per seconds. These are the specifications and then it says find out the mid-band gain. Mid-band gain is nota requirement but in general the design problem in practical problem, both mid-band gain and omega H arespecified and then you will have to work under those constraints okay.

(Refer Slide Time: 26:19)



Now if you draw the equivalent circuit in a routine manner, what you get is IS RS +- and then a series RS. Then you get L, R and RX. We take account of this because it is possible to take account all right? Then we have R pi and then let us say we take Miller effect into account and this is CT. CT is equal to C pi + C mew 1 + GM RL. Someone said, it does not come into effect, the load resistance. It does in CT right? In the total capacitance it does. Then you have this is V pi, then you have GM V pi and RL and this is the current I0.

We have ignored here the loading effect of C mew. We have ignored here. But in a strict sense if RL is for example very large, if RL is very large, if the impedance of RL is comparable to that of C mew, then you cannot ignore C mew. In fact you will see occasions where this time constant will also have to be taken into account. But continue with this analysis, if you analyse this, it is very simple, I0 is - GMV pi, find out V pi from here and then put it in the appropriate form.

(Refer Slide Time: 28:07)

$$\begin{aligned} \frac{T_0}{T_A} &= A_i(A) = \frac{K}{S^2 + aS + b} \\ K &= \frac{-\beta R_A}{LC_T Y_T}, \quad \alpha &= \frac{T}{L} + \frac{1}{C_T Y_T} \\ R &= R_A + R + T_A \end{aligned}$$

The form that comes is the following. I0 by ISwhich is A sub IS, the current gain, that can be put in this form, K divided by S squared + AS + B. Note the difference where let me down theconstants 1^{st} . K is equal to - beta RS divided by LCT R pi, a is equal to small r divided by L. What is small r? Small r is theseries combination of all the 3. Big RS, big R + small rs. Small r is the combination of these 3 resistances. They all come in series, so we can combine them.+ 1 over CT R pi and we will do r + r pi divided by LCT r pi.

Okay. The current gain isan expression K by S squared + AS + B. The next thing we do is we want maximally flat magnitude response and therefore we take the magnitude and put it in the form in which the numerator as well as denominator starts with 1 okay.

(Refer Slide Time: 29:38)

So I get A I J Omega divided by A I J0 magnitude squared as 1 over will let me down what is A I J0, that is equal to - beta RS divided by r + r pi I am omitting these algebraic steps. I can write this as 1 + Omega squared A squared - 2 B divided by B squared + omega 4th divided by B squared. Now tell mewhat is the difference between this transfer function and the previous one?

Student: There was a omega squared in the numerator.

Professor: There was a in the numerator, there was a 0 in the previous case okay. There was a 0 at S equal to - Z.- R by L. How did this 0 come? Because the inductor was shunting. And at S equal to - R by L, the inductor and series combination was behaving as a short. That is how the 0 came. Here, there is no 0 because there is no shunting effect. Okay. That obviously changes the maximal flatness condition. What will be the MFM condition now?

Student:(())(30:56)

Professor: Pardon me?

Student: A squared equal to 2B.

Professor: A squared equal to 2B. As simple as that because the numerator coefficient is 0. I cannot make 1 by B squared equal to 0, so this is themaximal flatness condition. And if I put this in terms of the elements, then my condition becomes L + CT r r pi whole squared equal to twice r

+ r pi LCT R pi. This is one and under this condition, what would be omega H? What would be omega H? Look at this expression. This becomes 0. So the magnitude square becomes 1 by 1 + omega 4 by omega 4 by B squared.

Student: Omega 4 by B squared equal to 1.

Professor: So this should be equal to 1. So omega H should be equal to squirm out of?

Student: B.

Professor: B.That is it. As simple as that, which is equal to square root of r + r pi divided by LCT r pi. These are your design equations and of course the other isAI J 0 is equal to - beta RS divided by r + r pi. Suppose these are your design equations, 1, 2 and 3. Suppose omega H andAI J 0, mid-band gain are specified, then you can find out a relationship between R and L. Is not that right? If Omega H is specified and AI J 0 is specified, this contains small r, this also contains small r and therefore you will get a relationship between small r and L. Okay?

You have to substitute that here to obtain therequired value of L. Here, omega H has been specified. What else?

Student: R pi.

Professor: Pardon me?

Student: (())(33:10)

Professor: So, the other things are specified. So only specification here is omega H. Therefore from here you get a relation between L and R which you substitute here to get the value of L, the numerical values. I shall only give you the final results. The final results are, I have gone through these quadratic equations and all that.

(Refer Slide Time: <u>33:41)</u>

$$R + R_{h} = 26.92 \Omega$$

$$L = 20.94 \mu H$$

$$|Ai(0)| \approx 3.57$$

$$GBH = 7.14 \text{ Mybs}$$

$$GT = 2500 \text{ Mybs}$$

Final results are that R + RS is 26.92 ohms.Capital R is represents the loss in the inductor and RS is the internal impedance of the source okay R + RS. So obviously, RS would be a low value and you are right the current source actually is a voltage source. Low valued resistance shunting a current source means a voltage source. And the inductance required is 20.94 Micro Henry. AndAI 0, the mid-band gain under this condition is approximately 3.57 only which means the gain bandwidth product, GBW is 7.14 mega radians per second, the gain bandwidth product.

Whereas omega T is calculated had 500 mega radians per second. Series peaking therefore is not as efficient as shunt peaking. This simple example demonstrates. We will continue this discussion next time.