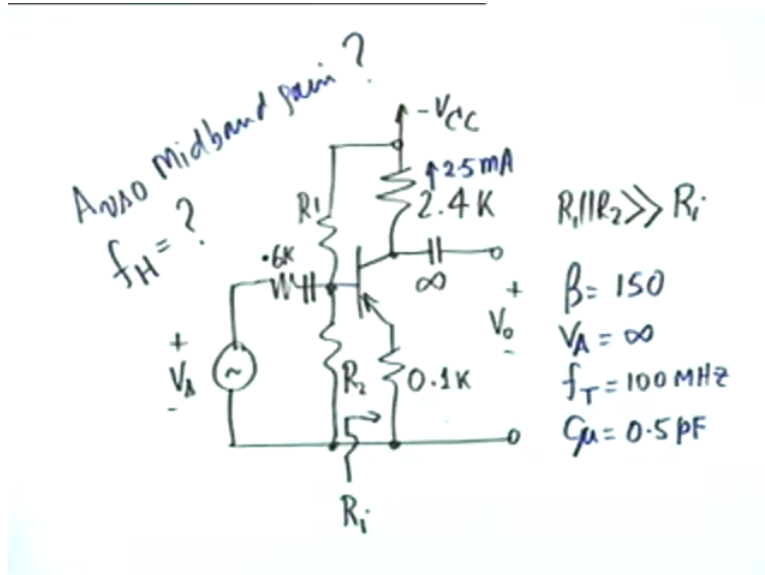


**Analog Electronic Circuits**  
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**Department of Electrical Engineering**  
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**Lecture no 19**  
**Module no 01**

**Problem Session 5 on Frequency Response of Small Signal Amplifiers**

And we have a problem session 5 and Frequency response of small signal amplifier, I shall work out only one problem today just one problem but it is a loaded problem, the problem has a problem okay we will see what this problem is take down with me.

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There is a  $-V_{cc} - V_{cc}$  what does it mean what kind of transistor? NPN alright smart, a resistance of 2.4 K, we have a transistor as guess it is a PNP, there is a small resistance here 0.1 K as you shall see creates the problem and this is unbypassed this is unbypassed. The output is taken from here through a capacitor which goes to infinity so the output is  $V_o$  and the biasing is done in the usual manner but this resistance  $R_1$  parallel  $R_2$  is much larger than  $R_i$ , where  $R_i$  is the input impedance looking into the transistor, which means that  $R_B$  can be ignored okay, these values are also not given but this is given 0.6K and the voltage source is her  $V_s$ , the data that are given are that this current is 2.5 milliampere the collector current is 2.5 milliampere DC of course 2.5 milliampere and Beta is given as 150,  $V_{sub A}$  is given as infinity which means that r

0 is infinity effect of  $r_0$  can be ignored, even with this simplification you will see what the problem is.

FT is given as 100 megahertz, you remember what FT is, FT is the transition frequency that is the frequency at which the common emitter short-circuit current application factor reduces to unity okay and  $C_{\mu}$  has been measured independently as 0.5 puf this is all the data that is given. What is required is  $A_{vs0}$  that is the mid band gain to be found out and the high frequency 3 dB cut-off point  $f_H$ , these are only 2 quantities which are to be found out, the simplification looks simple but even then there is a problem as we shall see.  $V_A$  is the early voltage, you recall  $r_0 = V_A$  divided by  $R_{sub C}$ , is  $V_A$  is given as infinity that means  $r_0$  goes to infinity okay.

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Handwritten calculations:

$$g_m = \frac{2.5 \text{ mA}}{25 \text{ mV}} = 0.1 \text{ S}$$

$$r_{\pi} = \frac{150}{0.1 \text{ S}} = 1.5 \text{ K}$$

$$C_{\mu} = 0.5 \text{ pF}$$

$$\omega_T = 2\pi \times 10^8 = \frac{g_m}{C_{\pi} + C_{\mu}}$$

$$C_{\pi} = 159 \text{ pF}$$

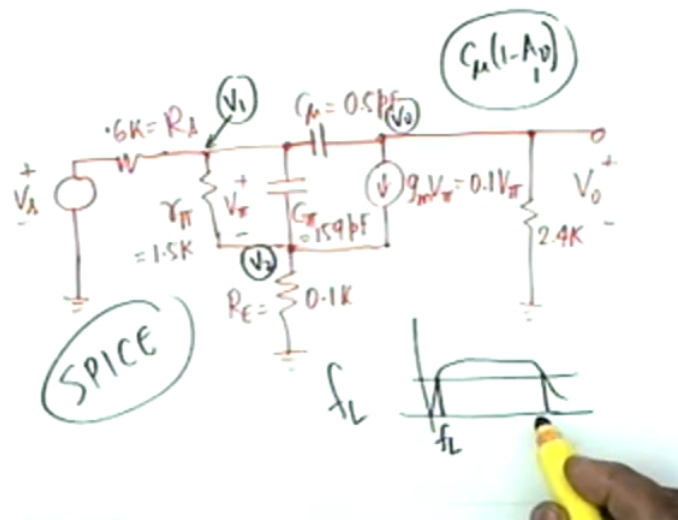
Annotations in the image: An arrow points from  $0.1 \text{ S}$  to  $g_m$  in the  $\omega_T$  equation. Another arrow points from  $0.5 \text{ pF}$  to  $C_{\mu}$  in the  $\omega_T$  equation.

Now in drawing the equivalent circuit we require the value of  $R_{\pi}$ , we require the value of  $g_m$  and therefore the 1<sup>st</sup> thing we do to solve the problem is to find out  $g_m$ ,  $g_m$  is 2.5 milliamperes that is  $I_{sub C}$  divided by 26 millivolts let us take it as 25 to keep life simple okay 25 millivolts and therefore this is 0.1 mho okay. Therefore  $R_{\pi}$  which is  $\beta$  by  $g_m$  is 150 divided by 0.1 mho which = 1.5 K and  $C_{\mu}$  of course has been given as 0.5 puf, we also require the value of  $C_{\pi}$  that is one of the ideas I have in mind when picking up this problem, how to calculate  $C_{\pi}$ ? Now if FT is  $C_1$  you know  $\omega_T$  which =  $2\pi$  multiplied by FT, which is 10 to the power 8 hertz this =  $g_m$  divided by  $C_{\pi} + C_{\mu}$ , now  $g_m$  is known 0.1 mho and  $C_{\mu}$  is known 0.5 puf

and therefore this relationship gives you the value of  $C_{\pi}$  alright the value of  $C_{\pi}$ , and  $C_{\pi}$  in my calculation comes out as 159 pF it is fairly large value.

And we are pretty sure that it is  $C_{\pi}$  which will dominate the high frequency 3 dB cut-off, no we cannot be sure why not because  $C_{\mu}$  will cause Miller effect and  $C_{\mu} (1 + g_m R_L)$  may be much larger than  $C_{\pi}$  so large  $C_{\pi}$  does not necessarily mean that  $C_{\pi}$  will dominate,  $C_{\pi}$  at the most will contribute okay, now let us draw the equivalent circuit.

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By looking at the original circuit, the equivalent circuit becomes  $V_s$ , 0.6 K then  $R_B$  is negligible effect of  $R_B$  can be ignored and therefore we shall have  $R_{\pi}$  that it is a PNP transistor does not change the equivalent circuit, PNP transistor only means that the biasing has to be done properly okay. The voltage here is  $V_{\pi}$  then we have the  $C_{\pi}$  this is  $R_s$ ,  $R_{\pi}$  is 1.5 K,  $C_{\pi}$  is 159 pF then we have the  $g_m V_{\pi}$  which = 0.1  $V_{\pi}$  alright and  $r_o$  is ignored, is there a load resistance, no everything is same as far as incremental equivalent circuit is concerned it depends on the polarity of  $V_s$  okay, with this polarity this is the direction, the direction you see directions are not changed this is also one of the points that I wanted to mention.

This is only the DC biasing, as far as DC is concerned this equivalent circuit is valid for both PNP as well as NPN okay there is no load but there is a load of 2.4 K this is  $R_C$  and this voltage is  $V_O$  and the complication is this 0.1 K, in addition there is this inevitable  $C_{\mu}$  which

= 0.5 puf and this resistance is  $R_{sub E}$ . Now  $C_0$  is not given so we assume it to be 0 okay, the complication is this we cannot apply miller simplification here that is the problem, we cannot apply Miller simplification miller simplification if  $R_E$  was = 0 or it was bypassed then it was perfectly alright this would have been the input voltage this is the output voltage so you can find the gain and  $C_{Mu} 1 + g_m R_L$ .

Here the gain shall be controlled by  $R_E$ , not only that what capacitance is reflected shall also be controlled by  $R_E$  and it is not only capacitance, there would be a resistance of also across it both of which shall be frequency dependent and therefore in this situation we cannot apply miller simplification so what are the alternatives then? What are the alternatives then? One of the alternatives is that you take this load  $V_1$  and this load  $V_2$  and this is  $V_0$ , this is  $V_S$  so you have to write 2 node equations or 3 node equations okay. The 0 is this voltage, we have to write 3 node equations and solve 3 by 3 matrix.

Student: Sir, Can you once again please repeat why can we not apply miller simplification?

Miller simplification was if you recall  $C_{Mu} 1 - A_v$  okay this was the miller voltage, what is  $A_v$ ?  $A_v$  is this output divided by this input. Now the input here is from here to ground not across  $r_{Pi}$ , the output is not across  $g_m V_{Pi}$  it is this and therefore the Miller simplification which was derived on the assumption that  $R_E$  was bypassed is not true we cannot make that simplification you must recognise this fact. If this was shorted perfectly alright, we could have done that but not otherwise, unfortunately this is the point that I wanted to make. Fortunately there is an alternative technique that is available whenever Miller cannot be used that technique is more general, it can be used always although a fairly rough approximation although a fairly rough approximation.

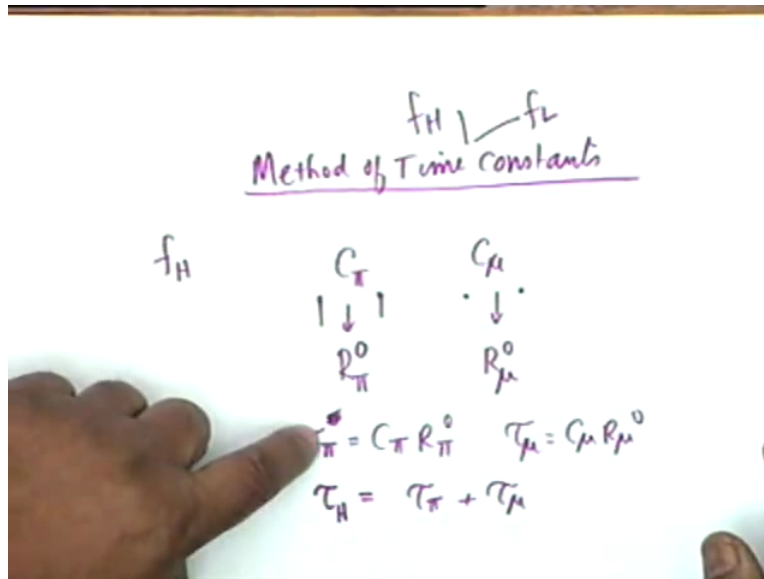
And this simplification I will give the logic later alright 1<sup>st</sup> let me say the principle. The principle is as I said in electronic circuits and also in other fields logic can be supplied if the method works some logic can be worked out, logic may not be necessary to workout, if the method works it works that is it okay suppose there is not more than 10 percent deviation we do not care then we do not care. Now the method that is available let me tell you that this is a very simple circuit 1 transistor, a couple of resistors and 2 capacitors have to be taken.

Even here we do not want to write the node equation okay because in design on paper and pencil, node equation writing and inversion of matrix is complicated, you can use one of the subroutines one of the algorithms that is available a package like SPICE this is the most commonly used algorithm for this purpose for network analysis but even there conditions the numerical calculations may be ill conditioned and you may not get the result that you want, it has to be applied with precaution and it may not be available, for a simple circuit like this we do not want use SPICE. Now as you go ahead you will receive when 2 transistors are involved if it is a cascade or cascode or some other combination as we shall see later the problem becomes tougher then you have to write 6 to 7 node equations.

If it is an integrated circuit card containing some 1000 transistors okay then of course it is an ocean there is nothing that you can do except to go to the computer and believe in that box blindly alright. The problem is very tough in integrated circuit design where indeed there is very large number of transistors, even an Opamp contains as many as 19 to 20 transistors connected in a very complicated manner, you know for biasing you require current mirrors, every transistor will have to have a current mirror unless there is a current repeater then there will be level shifting, there will be power amplification, there will be voltage amplification, there will be buffer, all kinds of things okay.

And if you draw the equivalent circuit and try to analyse it by Miller or by some other means, you will go absolutely crazy and therefore but  $f_H$  and  $f_L$  that is the gain if I plot it, it would be something like this, there would be a mid-band at which the gain will be nearly constant, we want to know what this frequency is and what this frequency is  $f_H$  that determines the character of the amplifier, that determines whether the amplifier is suitable or not. For example, if it is an audio amplifier then you want to know if  $f_L$  is of the order of 16 hertz or no,  $f_H$  is of the order of 16 kilohertz or no okay this is the range that you want to amplify and you must know what is  $f_L$  and what is  $f_H$  is. The problem is the reverse way, you are asked to design an amplifier for the specified  $f_L$  and  $f_H$ .

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Now given an amplifier particularly in multiple transistor circuits as in indicated circuits, this method that I am going to talk about is the only method and many textbooks do not even mention this okay, Burns and Bonds do not mention this so please do follow me carefully. The method is called the method of time constant and we shall use this to solve this circuit. Method of time constant; it can be used to determine  $f_L$  as well as  $f_H$ , both  $f_L$  and  $f_H$  can be determined by the method of time constant and the method is this. If your problem is to determine  $f_H$  then identify the capacitors in the circuit identifier capacitors, in this particular case the capacitors are  $C_{\pi}$  and  $C_{\mu}$  okay.

Now assume  $C_{\mu}$  to be open and find out the Thevenin's resistance that is seen by  $C_{\pi}$ , assume  $C_{\mu}$  assume all other capacitors to be open if there are more than 2 assume all capacitors except one to be open and find out across  $C_{\pi}$  what is the Thevenin resistance okay, let this resistance be called  $R_{\pi}^0$ ,  $R_{\pi}^0$  is not  $R_B$  parallel small  $R_{\pi}^0$ , here  $R_{\pi}^0$  is the Thevenin resistance seen by  $C_{\pi}$  and to distinguish between that  $R_{\pi}^0$  and this  $R_{\pi}$  and also to indicate that all other capacitors are open, we use a superscript this is not a power, it is not  $R_{\pi}$  to the power to 0, it is  $R_{\pi}^0$  super O okay O for open circuit, not this capacitor but other capacitors. Similarly, you find out you open  $C_{\pi}$  and find out what is  $R_{\mu}^0$  that is the Thevenin resistance or the resistance seen by  $C_{\mu}$ . I am mentioning Thevenin because independent sources are eliminated, in this calculation independent sources are eliminated okay.

Then after you find these 2 resistors, this will not be very tough this will not be very tough because if it contain some controlled sources and resistors that is all, nothing else no other capacitors and therefore it is a purely resistive circuit and resistive circuit solution is not bad problem at all. So what you do is you find out these 2 time constant  $\tau_{\pi}$  which =  $V$  can simply say  $\tau_{\pi} C_{\pi} R_{\pi}$  and  $\tau_{\mu}$  time constant which is the product of  $C_{\mu}$  and  $R_{\mu}$ , add them to find out  $\tau_H$  which =  $\tau_{\pi} + \tau_{\mu}$  and call this  $\tau_H$ , Pardon me...

Student: We put a code  $\tau_{\pi}$ .

No, I do not have to it is understood, it is understood that time constant is calculated under open circuit condition for all other capacitors. Once you calculate the time constants for the various capacitors there may be more than 2 for example in the ordinary common emitter circuit in short-circuited emitter resistors there are 3 capacitors;  $C_{\pi}$ ,  $C_{\mu}$  and  $C_0$ , fortunately  $C_0$  is not given here we will bring that complication later okay you must follow this carefully because this is the only method available for IC design okay. After calculate the time constants add them up at them up and call the total time constant as  $\tau_H$  this capital H subscript stands for high frequency okay.

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$$\omega_H \cong \frac{1}{\tau_H}$$

$$f_H \cong \frac{1}{2\pi \tau_H}$$

$$C_1, C_2, \dots, C_n$$

$$\tau_H = \sum_{i=1}^n C_i R_i^0$$

$$f_H \cong \frac{1}{2\pi \tau_H}$$

“Professor–student conversation starts”

Professor: And then Omega H, the high frequency 3 dB point in radian per 2<sup>nd</sup> is approximately = 1 over Tau H, it could be...

Student: Inverted 2 Pi...

Professor: Not 2 Pi, 2 Pi will come if we find out f H, f H would be 1 over 2 Pi Tau H okay.

Student: Sir, Can you give justification for it?

Professor: I will come to the logic later I will come to the logic later, 1<sup>st</sup> I want to make the calculation then I will supply the logic now please listen to me.

“Professor–student conversation ends”

If instead of 2 capacitors if it is a complicated circuit with C 1, C 2, C n, n number of capacitors okay maybe 3 transistor circuit we have 9 capacitors then okay. Then what we do is we find out Tau H which = C i, R i o, where i goes from 1 to n alright and then f H = approximately an approximation 2 Pi Tau H okay. Now quickly a look at the logic since the question has been raised a look at the logic.

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The image shows a whiteboard with handwritten mathematical derivations. At the top left, the word "Logic" is written and underlined. Below it, a circled letter 'A' is followed by a fraction: the numerator is "Const" and the denominator is  $(1 + j\frac{\omega}{\omega_1})(1 + j\frac{\omega}{\omega_2}) \dots$ . This is followed by an equals sign and another fraction: the numerator is "Const" and the denominator is  $1 + j\omega(\frac{1}{\omega_1} + \frac{1}{\omega_2} + \dots)$ . A handwritten note "~~order term~~" with an arrow points to the denominator of this second fraction. Below this is an approximation symbol  $\approx$  followed by a final fraction: the numerator is "Const" and the denominator is  $1 + j\omega/\omega_H$ . A hand holding a yellow highlighter is visible at the bottom right of the whiteboard.

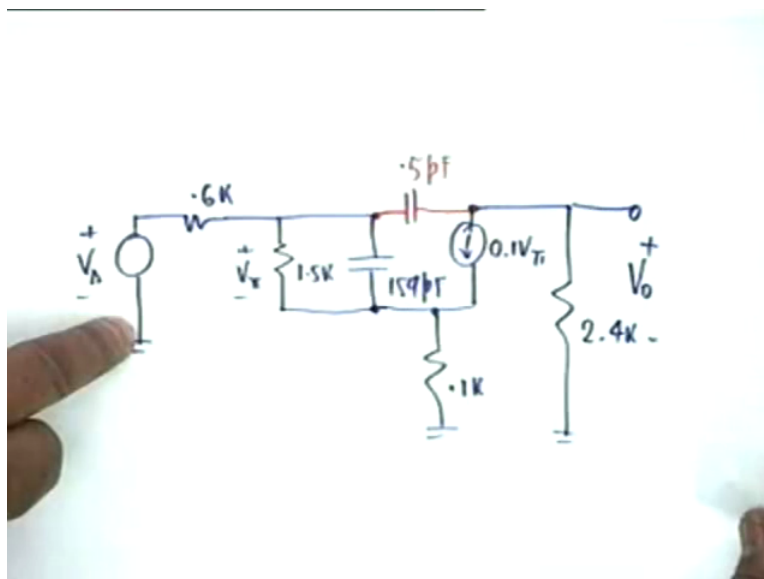
Now if we are calculating high frequency response, each capacitor contributes to a pole each capacitor contributes to a pole and therefore due to n capacitors the high frequency response



would be of the form some constant divided by  $1 + j\Omega$  by  $\Omega$  times  $1 + j\Omega$  by  $\Omega^2$  and so on alright, which can be written as constant divided by this is the logic, logic is also rough the approximation is rough the logic is also not very strict logic okay, so I can write this as  $1 + j\Omega$  over  $\Omega$   $1 + 1$  over  $\Omega^2$  and so on + higher order terms higher order in  $\Omega$  that is we will have  $\Omega^2$   $j\Omega^3$  and so on and so forth alright. And if we ignore if we ignore all these higher-order terms which we usually can do for frequency around  $\Omega_H$  then obviously this can be written approximately as constant divided by  $1 + j\Omega$  by  $\Omega_H$ .

And obviously  $\Omega_H$  obviously  $1$  over  $\Omega_H$  is approximately =  $1$  by  $\Omega$   $1 + 1$  by  $\Omega^2$  and so on.  $1$  by  $\Omega_H$  by definition is  $\tau_H$  and therefore  $\tau_H = \tau_1 + \tau_2$  and so on that is the logic. The logic is that you retain only the 1<sup>st</sup> order terms in the denominator and neglect the rest. And you remember that when  $\tau_1$  was calculated, the other capacitors were left open and therefore these are all open circuit time constants so I can use the subscript 0 okay these are all open circuit time constants that is all other capacitors are left open. We shall apply it to the circuit that we have already calculated by Miller and also an exact expression and we shall see what kind of an approximation it affords but before that let us complete this example.

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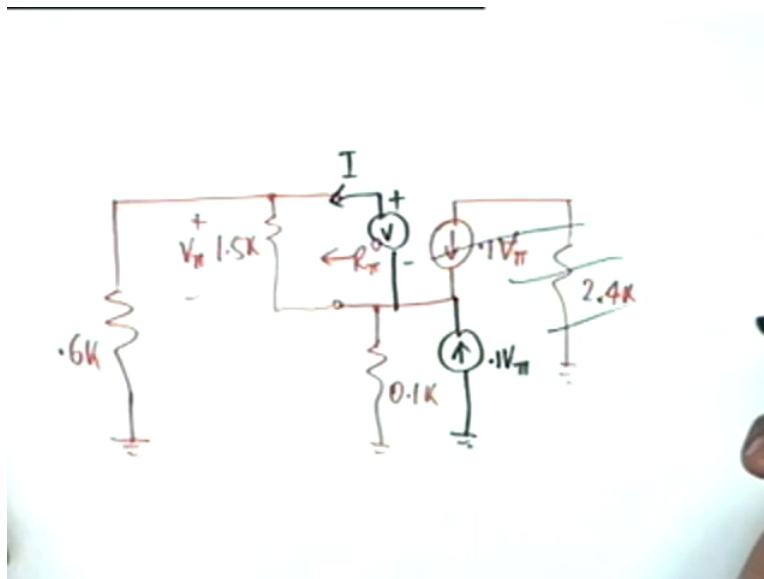


The equivalent circuit if I draw it once again was 0.6 K yes... Oh I will come to that I will come to that... let me finish this then I will come to assume okay. We have 1.5 K,  $V_{Pi}$ , then 159 puf, 0.1 K then 0.1  $V_{Pi}$  that is  $g_m V_{Pi}$  and 2.4 K, this is  $V_0$ . Now in order to calculate...

Student: C Mu is where?

Oh, how could I forget that, I will put it in red because this is the source of all problems, C Mu is 0.5 puf. Now if I want to calculate  $R_{Pi}$ , what I want to do is the resistance seen by 159 puf and therefore what I will do is and we will leave 0.5 picofarad open so let us see what the circuit becomes, also it is the Thevenin's equivalent so therefore  $V_s$  shall be shorted.

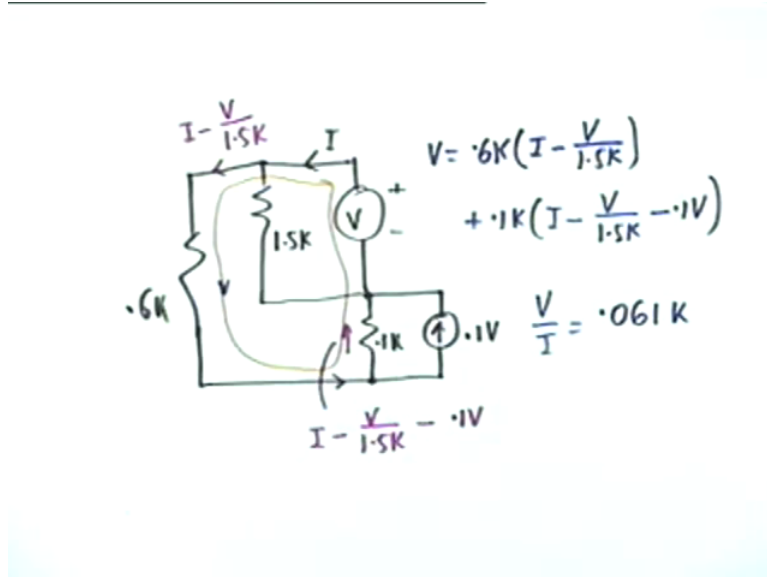
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So the equivalent circuit determine  $R_{Pi}$  becomes this  $R_{Pi}$  is across  $C_{Pi}$  we have a 1.5 K and this is  $V_{Pi}$  then we have from here a 0.6 K to ground agreed because  $V_s$  is shorted Thevenin resistance and we have from here by 0.1 K, then we have from here 0.1  $V_{Pi}$  and we have 2.4 K alright. Since 2.4 K in in series with a current source it has no effects and therefore I can replace a whole thing whole thing by a current source 0.1  $V_{Pi}$  okay, I can ignore the whole thing the 2.4 K drops out of consideration alright. So what we have to do now is to connect voltage source, I want to calculate this resistance, this resistance cannot be calculated by combination of resistances because there is a controlled source there is a dependent source so what I want to do is I connect a V here and I find the current I and  $R_{Pi}$  shall be  $V$  by I okay let

me draw this circuit it has become a little messy, let me draw this circuit on a clean slate, this is not a slate, this is a plate

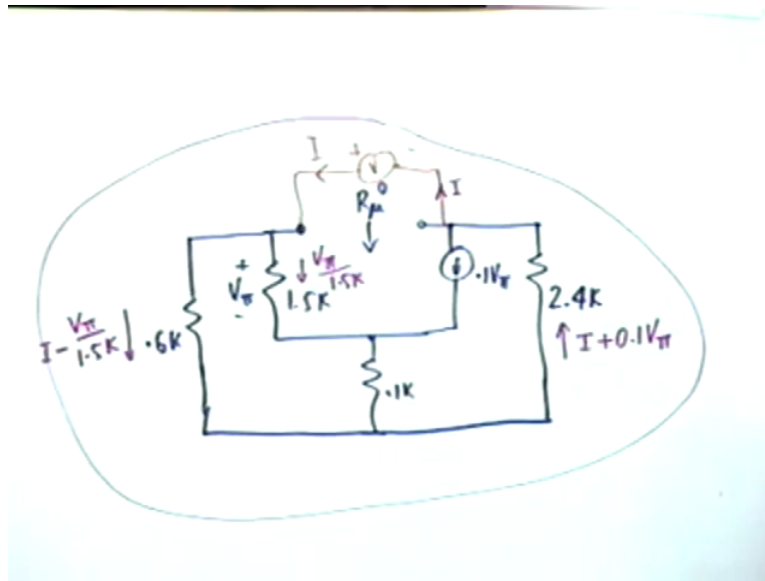
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1.5K, this current is I and then we have 0.6 K, 0.1 K, to emphasise that these are connected I am not drawing the ground, I am connecting them together and then we have a current source 0.1, what is V Pi? V Pi is also V so why keep another variable just put V alright so I have to calculate I now these are ways of one has to improvise without writing any loop equation node equation or simultaneous solution one can do it by inspection, what is this current?  $I - V$  by 1.5 K alright and pardon me... Is that okay then what is this current?  $I - V$  by 1.5 K - 0.1 V, this current comes here breaks up into 2 parts, one is 0.1 V and the other is this alright.

So if I write a loop equation around this, let me use a different colour I want a lighter colour okay around this if I write the loop equation my equation becomes  $V =$  drop in 0.6 K, 0.6 K multiplied by  $I - V$  by 1.5 K alright + the drop in 1 K which would be + 0.1 K multiplied by  $I - V$  by 1.5 K - 0.1 V alright, is this point clear? This loop equation this equation contains only V and I and therefore we can find out the ratio of V by I and my calculation shows that  $V$  by  $I = 0.061 K$  it is only 61 ohms that is what C Pi see alright okay. Then we have to calculate R Mu 0, well that is not all that one has to do is to be careful in drawing the equivalent circuit.

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If I draw  $C_{\mu 0}$  well  $R_{\mu 0}$ ,  $R_{\mu 0}$  is across  $C_{\mu}$  and  $C_{\pi}$  is open and therefore what we have is a  $1.5\text{ K } V_{\pi}$  remains, we have a  $0.1\text{ K}$  and I will not draw the ground I will show them connected a little later then we have a current generator  $0.1\text{ V}_{\pi}$ , now  $2.4\text{ K}$  can no longer be ignored,  $2.4\text{ K}$  because what you have to do is to connect a voltage source here  $+ -$  and calculate the current  $I$ , is my circuit complete? No, there is a  $0.6\text{ K}$  and these come together. What I will do now is to write what we have to find out is  $V$  by  $I$  and the intermediate variable is  $V_{\pi}$  so let us 1<sup>st</sup> identify the currents. What is this current? Current  $I$  comes here and the current that goes here is  $V_{\pi}$  divided by  $1.5\text{ K}$ ,  $V_{\pi}$  is no longer  $= V$  okay so this current must be  $I - V_{\pi}$  divided by  $1.5\text{ K}$  alright then what is this current?

Student:  $I + 0.1\text{ V}_{\pi}$

Wonderful, how did you calculate this because this current must be  $I$  and this current is  $0.1\text{ V}_{\pi}$  so this current must be  $I + 0.1\text{ V}_{\pi}$  outright therefore, we can write a loop equation like this a big loop the outside loop okay, it will be this drop  $+ \text{ this drop would be } = V$ . And the other equation that we can write is this loop okay, what about this loop? Take any 2 independent loops, you cannot write this loop because there is a current generator here you must be careful about this okay, we do not know what the drop across this is so I cannot write KVL here okay so the only other loop that is available to me is this for which I know this current okay I know this voltage

this voltage across 0.6 K this would be the voltage across this which is V Pi + the voltage across this now therefore I have to find the current, what is this current?

Okay 0.1 + 1 by 1.5 K times V Pi so we know this current no... You must be careful about the scale, what is the unit of this 0.1? mho, and therefore when you write 1.5 do not write 1 by 1.5 it that will be absolutely wrong, it is 1 by 1500 mhos okay that is why I am retaining the dimension, ultimately when you solve you do that. So you write 2 loop equations from which you eliminate V Pi this algebra I leave to you, my calculation gives R Mu 0, if there is any equation now ask me you may not get another problem of this type I mean I may not solve another problem of this type.

Student: ( ) (35:31)

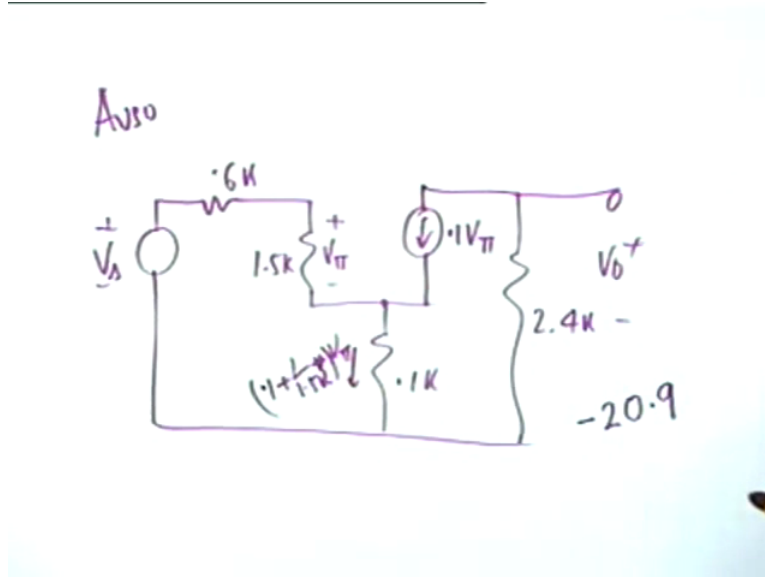
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$$\begin{aligned}\tau_H &= \tau_{\pi}^0 + \tau_{\mu}^0 \\ &= (0.061 \times 159 + 15.5 \times 0.5) \text{ ns} \\ f_H &= \frac{1}{2\pi\tau} = 9.11 \text{ MHz}\end{aligned}$$

Oh because the 1<sup>st</sup> loop contains V Pi, V and I do not want V Pi, I want only the ratio V by I so I need a 2<sup>nd</sup> loop to eliminate V Pi alright and my value comes R Mu o as 15.5K therefore the total Tau, Tau H which = Tau Pi o + Tau Mu o that = 0.061K multiplied by 159 puf, what would be the unit of this? 10 to the - 9 which is nanoseconds so this would be in time it is nano second + 15.5K multiplied by 0.5 so many nanoseconds ns and therefore f H which = 1 over 2 Pi Tau = 9.11 megahertz it is a good amplifier it goes right up to 9.11 megahertz. And as far as the other part of the problem was to find out A vs o the mid band gain well, that should be simple mid

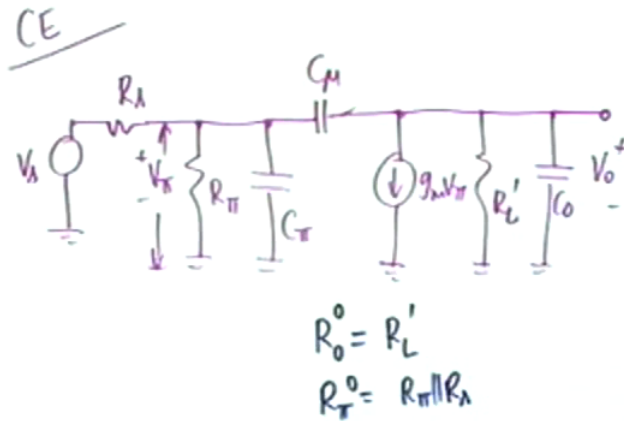
band gain you ignore all the capacitors, draw the equivalent circuit let me do it, I will tell you what the value is.

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My equivalent circuit for  $A_{v_{s0}}$  is  $V_s$ ,  $0.6\text{K}$ ,  $1.5\text{K}$ , this is  $V_{\pi}$ ,  $0.1\text{K}$ , we have already done it I suppose we must have done it is this unbypassed resistor unbypassed okay approximately we want the exact gain oh that is a good clue, what should what is the gain that you expect?  $-2.4$  divided  $0.2$  which is  $20$  okay now see what happens here, this is  $2.4\text{K}$  and this is  $V_o$  can you solve it by inspection? Yes there is nothing much you see this is  $\beta + 1$  by  $R_{\pi}$  this is  $0.1 + 1$  over  $1.5\text{K}$   $V_{\pi}$  this current and then you can write the loop equation, find out  $V_{\pi}$  and the output voltage  $V_o$  is  $-0.1 V_{\pi}$  multiplied by  $2.4\text{K}$  so you can find out, this is almost by inspection the value comes as  $-20.9$  the approximate value was  $24$  it is not too bad it is not too bad, so this calculation I leave it to you.

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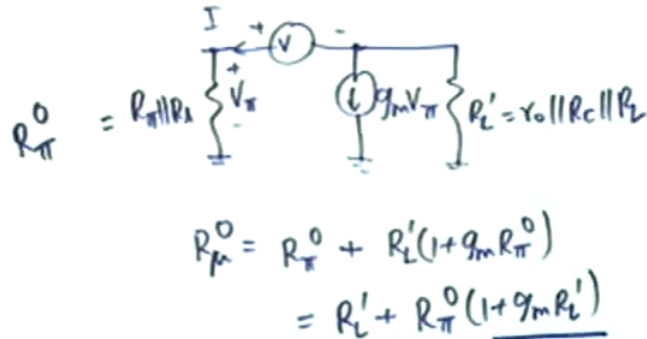
Now the other part of the calculation well let me illustrate this principle so that it sticks to your mind by considering the common emitter amplifier with bypassed R E, let us see what happens there if we apply the same principle. With bypassed R E the ordinary C E amplifier at high frequencies you have V s, R s, then you have R Pi which is the parallel combination of R B and r Pi, this is V Pi and then you have C Pi, C Mu, no Miller I am trying to calculate by the method of open circuit time constant now I shall put an adjective open circuit because in the low frequency case it will be the short-circuit time constant. Now let us see C Mu you have g m V Pi, now there is no problem in incorporating r 0 and therefore we say R L prime which is the parallel combination of small r 0, R sub C and R sub L and then of course you have the output capacitance C0, this is V0 okay.

Now to calculate, a couple of things we can calculate directly you can see directly what these are for example, what is R o o that is what is the resistance seen by C o? Simply R L prime agreed because there is a current generator and this voltage is 0 A vs is 0 okay however, the other 2 are not obvious. To calculate R Pi no... We also know what C Pi sees, what does C Pi sees?

Student: R pi parallel R s.

Professor: That is right R Pi parallel R s that is right it is only C Mu that gives us the problem let us see what this problem is.

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From C Mu you calculate you find out V and find the current I, this is the terminal of C Mu, V Pi is open so you have R Pi what is this resistance? R Pi parallel R s, and on the other side this is V Pi on the other side you have g m V Pi in parallel with R L prime so that is not too bad, you can see that V Pi is simply I times what is this resistance? R Pi o so V Pi is I times R Pi o and this is g m V Pi, this is R L prime so it is not difficult to find out what the current is you have to write one equation that is all okay, let me leave this calculation to you, you can show that R Mu o = R Pi o + R L prime 1 + g m R Pi o, I hope I am right R Pi + R L prime or I can write it the other way round, I can also write this as R L prime + R Pi o 1 + g m R L prime, this is simple circuit analysis so I will skip this.

After you find out the 3 resistances then your Tau H pardon me... Well, let us go directly to Omega H. Omega H becomes R Pi parallel R s times if you add them up see I wanted to write in this form why? Because I have this g m R L prime here so I will multiply this by C Mu okay and you can see that C Mu 1 + g m R L prime that C M the Miller capacitance come okay that is why I wanted to write this in this form alright.

Student: Sir what is R L prime?

Professor: R L prime is the parallel combination of small r 0, R C and R L.

Student: In the question we do not have any R L.



Professor: We are not doing any equation we have gone to the ordinary common emitter amplifier with bypassed R E, I wanted to show that the method of time constant open circuit time constant works here also okay.

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$$\omega_H = \frac{1}{(R_{\pi} \parallel R_s) C_T + (C_0 + C_{\mu}) R_L'}$$

$$\frac{V_o}{V_s} = \frac{(-g_m R_L') \frac{R_{\pi}}{R_{\pi} + R_s} \left(1 - \frac{j\omega}{g_m / C_{\mu}}\right)}{1 + j\omega [(R_{\pi} \parallel R_s) C_T + (C_0 + C_{\mu}) R_L'] - \omega^2 R_L' \dots}$$

So if you add them up and take the reciprocal this is the result that you get;  $C_T + C_0 + C_{\mu} R_L'$ . If you recall the exact analysis that we had done for the same circuit, our expression was from exact analysis node analysis our expression was  $-g_m R_L'$  let me recall  $R_{\pi}$  divided by  $R_{\pi} + R_s$  times  $1 - j\omega$  divided by  $g_m$  over  $C_{\mu}$  divided by, you recall this we had put  $\omega = \omega_3$  and we said  $\omega C$  is very large therefore we shall ignore this term so this calculates this qualifies is the A vs 0 and in the denominator we had  $1 + j\omega$  multiplied by  $R_{\pi} \parallel R_s C_T +$  exactly this expression  $C_0 + C_{\mu} R_L'$  then  $-\omega^2$  and  $\omega^2$  term  $R_L'$  et cetera et cetera. And we have said that usually this term can be ignored and therefore  $\omega_H$  is the reciprocal of this quantity.

Do not you see that the method of open circuit time constant gives exactly the same result, is not Miller, what does Miller give? What does Miller approximation give? Miller gives only this part, Miller gives  $R_{\pi} \parallel R_s$  multiplied by  $C_T$  alright so the 3 approaches are quite similar and it brings in confidence by the method of open circuit time constant perhaps is a bit better than Miller approximation. However, if Miller approximation can be applied we do not go to method of time constant because that is a bit time consuming method of open circuit time constant is a

bit time consuming have to calculate  $R_{Pi 0}$ ,  $R_{Mu 0}$ ,  $R_{L0}$  okay then add them up and then take the reciprocal, if Miller is available nothing else is required okay and you see the results are not far from the exact values.

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$$V \approx \frac{\text{Const} \left(1 + j \frac{\omega}{\omega_4}\right)}{\left(1 - j \frac{\omega}{\omega_1}\right) \left(1 - j \frac{\omega}{\omega_2}\right) \left(1 + j \frac{\omega}{\omega_3}\right)}$$

$$\frac{\omega_3}{j\omega} \times \frac{1 + j\omega/\omega_4}{1 + j\omega/\omega_3} = \frac{\cancel{\omega_3} + \frac{\omega_3}{\omega_4}}{1 - j \frac{\omega_3}{\omega}}$$

$$\frac{\omega_2}{j\omega} \times \approx \frac{\text{Const}}{1 - j \omega_3/\omega}$$

Now about the low frequency response okay, in the low frequency response let me 1<sup>st</sup> give the logic alright. In the low frequency response, the response will be of the form constant divided by  $1 - j \Omega_1$  by  $\Omega$  so at DC = the value = 0, then we have  $1 - j \Omega_2$  by  $\Omega$  and in addition we had a factor of... Due to C E we had  $1 + j \Omega$  by  $\Omega_3$  and  $1 + j \Omega$  by  $\Omega_4$ . First let us look at this term let us look at this term due to C E, now you know which one is greater  $\Omega_3$  or  $\Omega_4$ ?  $\Omega_3$  is much greater than  $\Omega_4$  well above 50 times so I can write this  $1 + j \Omega$  by  $\Omega_3$  divided by  $1 + j$  these are some tricks of the traits usually not mentioned in books okay, there is only a C Mu who will tell you do remember.

I can write this as let us multiply both numerator and denominator by  $\Omega_3$  by  $j \Omega$  let us do that, if I do that than in the denominator what do I get? I multiply by  $\Omega_3$  by  $j \Omega$  so I get  $1 - j \Omega_3$  by  $\Omega$  alright and if I multiply the numerator by  $\Omega_3$  by  $j \Omega$  then I get  $\Omega_3$  by  $j \Omega +$  what do I get?  $\Omega_3$  by  $\Omega_4$  agreed is the point clear?

Student: No.

Professor: No... What is the problem? I multiplied this, I multiplied this both numerator and denominator okay.

Now notice the trick, notice that around  $\Omega_3$  around  $\Omega_3$  this term magnitude is approximately unity whereas this is approximately 50 and therefore I can write this as approximately a constant, it is this frequency which are of concern a constant divided by  $1 - j\Omega_3/\omega$ . Why is the largest frequency of concern? Because we are calculate thing the low frequency response, the largest frequency determines the low frequency response. So even if C E gives a kind of a low pass combination with high pass and as I had mentioned the characteristics is like this, we can approximate it by a high pass characteristics like this goes to 0, this is what we have done here agreed.

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$$\begin{aligned}
 & \frac{C\omega t}{(1-j\frac{\omega_1}{\omega})(1-j\frac{\omega_2}{\omega})(1-j\frac{\omega_3}{\omega})} \\
 = & \frac{C\omega t}{1 - \underbrace{j(\omega_1 + \omega_2 + \omega_3)}_{\text{higher order terms}} + \dots} \\
 \approx & \frac{C\omega t}{1 - j\omega_L/\omega}
 \end{aligned}$$

So my low frequency transfer functions would be of this form  $1 - j\Omega_2/\omega$ ,  $1 - j\Omega_3/\omega$  approximately and I can write this as constant divided by  $1 - j(\Omega_1 + \Omega_2 + \Omega_3 + \text{higher order terms})/\omega$ , which diminish as  $\Omega$  increases alright and if we want to calculate f L which will be dominated by this term obviously if we ignore this I can write this as constant divided by  $1 - j\Omega_L/\omega$ , and  $\Omega_L$  therefore is simply the sum of the 3 poles.

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$$\omega_L = \sum_{i=1}^3 \omega_i$$
$$\frac{1}{\tau_L} = \sum_{i=1}^3 \frac{1}{\tau_i^\infty}$$
$$f_L = \frac{1}{2\pi\tau_L}$$



That is  $\omega_L = \sum_{i=1}^3 \omega_i$  in terms of time constant  $\frac{1}{\tau_L} = \sum_{i=1}^3 \frac{1}{\tau_i^\infty}$ . In the previous case in the high frequency case the time constant were added, now the reciprocals of the time constants are added okay the reciprocals of time constant and  $f_L$  is simply  $= \frac{1}{2\pi\tau_L}$  alright. Now under what conditions will these time constants be calculated, obviously short-circuit and therefore these time constants are written with a superscript of infinity, short-circuit means the capacitors are infinite or capacitors are shorted, in the previous case capacitors were open, this is also the only method that is available for a very complicated circuit engineering method engineer's approximation. These resistances as I said can be calculated almost by inspection and therefore this gives a quick calculation and this method is known as the method of short-circuit time constant.

We shall apply both these methods when we come to complicated circuit that is Cascade, Cascode and dialling tone connection and CB-CC, CB-CE all-time the connections when you consider wide banding techniques and that is the point to stop today.