

Analog Electronic Circuits
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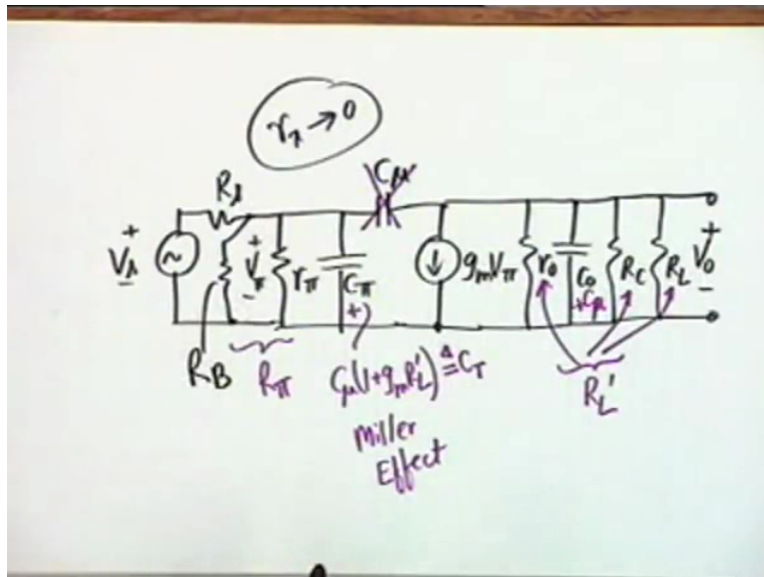
Lecture no 17

Module no 01

High Frequency Response of Small Signal Amplifiers (Contd)

This is the 17th lecture and we are going to continue on High frequency response of small signal amplifiers. If you recall the equivalent circuit of a common emitter amplifier at high frequencies was drawn yesterday like this.

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V_s, R_s , you have you ignored small r_x so you had r_{π}, C_{π} and this voltage is V_{π} , then you have us $C_{\mu}, g_m V_{\pi}$, finally and R_o then C_o and R_c and R_L , this is V_o , R_B we have ignored, not R_B , oh there is an R_B oh there of course this is here R_B , any other omissions?

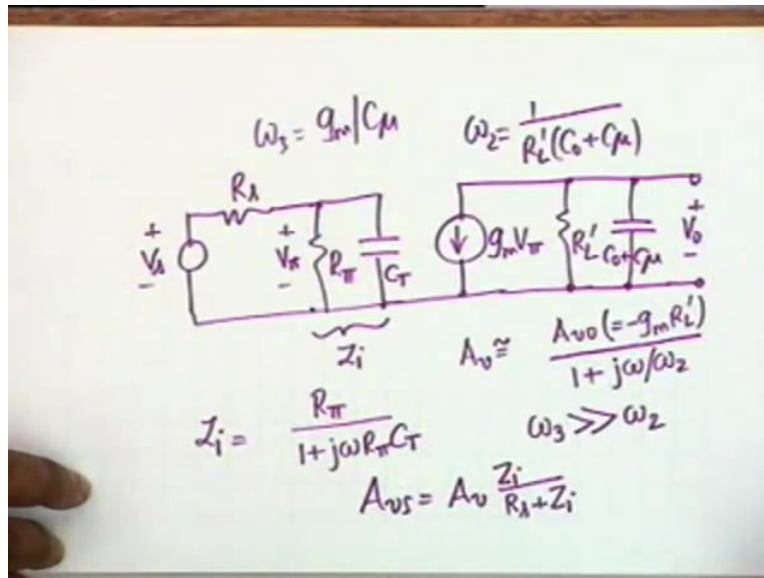
Student: r_x .

Professor: r_x we have ignored, r_x tends to 0 that is what we have resumed.

And we simplify R_B and r_{π} these 2 parallel combination we said R_{π} and the parallel combination of R_o, R_c and R_L these 3 we called it R_L' . In addition we had shown that this cross connection for the bridge capacitor C_{μ} could be taken care of approximately my

including a capacitance here at the input $C_{Pi} + C_{Mu} (1 + g_m R_L')$, C_{Mu} approximately reflects to the input side as this capacitance and this is due to the so-called miller effect and we had called this total capacitance $C_{Pi} + C_{Mu} (1 + g_m R_L')$ as C_T and C_{Mu} district on the output side by adding a capacitor to $C_0 + C_{Mu}$, this is an approximate miller equivalent circuit.

(Refer Slide Time: 4:01)



If I draw it a little more neatly yes okay now if I draw it a little more neatly it shall look like this; V_s, R_s, r_{Pi}, C_T , this is V_{Pi} and then you have no more bridging $g_m V_{Pi}$ in parallel with the resistance R_L' and the capacitance $C_0 + C_{Mu}, V_o$. I would like to remind you that this simplification of $C_0 + C_{Mu}$ on the output side was obtained by ignoring a frequency Ω_3 which was $= g_m$ by C_{Mu} . We saw that at frequencies around Ω_2 which is $1 / (R_L' (C_0 + C_{Mu}))$ at around his frequencies the effect of Ω_3 is negligible and therefore Ω_2 was the cut-off frequency of this output circuit okay. And we had found out that $A_{sub v}$ was simply $= A_{vo}$, what was A_{vo} ?

Student: Energy...

Student: mid band...

Professor: Yes what is value... $- g_m R_L'$.

This is the A_v is the voltage gain between V_0 and V_{Pi} which is same as V_i okay divided by $1 + j\Omega$ by Ω_2 and this was an approximate value under the condition that Ω_3 is much greater than Ω_2 agreed, this is just to refresh your memory how this was obtained. Now as far as the input circuit is concerned, if I call this combination as Z_i , combination of r_{Pi} and C_T then obviously $Z_i = R_{Pi}$ divided by $1 + j\Omega r_{Pi} C_T$ agreed, therefore A_{vs} which will be the order of A_v and V_{Pi} divided by V_s okay would be Z_i divided by $R_s + Z_i$ agreed, this is A_{vs} , A_{vs} is the gain between V_0 and the actual source voltage. Z_i as you can see is also the input impedance of the circuit, this is what the source with its internal resistance sees when it looks into the circuit Z_i that is why Z subscript i , i is for input impedance, no longer R_i it is Z_i okay.

(Refer Slide Time: 7:49)

$$A_{v0} = \frac{-g_m R_L' R_{\pi}}{R_{\pi} + R_s}$$

$$A_v = \frac{A_{v0}}{1 + j\omega/\omega_2} \cdot \frac{R_{\pi}}{R_{\pi} + R_s (1 + j\omega R_{\pi} C_T)}$$

$$= \frac{A_{v0}}{1 + j\omega/\omega_2} \cdot \frac{R_{\pi}/(R_{\pi} + R_s)}{1 + j\omega C_T (R_{\pi} || R_s)}$$

$$= \frac{A_{v0}}{(1 + j\frac{\omega}{\omega_2}) (1 + j\frac{\omega}{\omega_1})}$$

Now if I calculate this, A_{vs} would be $= A_{v0}$ over $1 + j\Omega$ by Ω_2 and then Z_i divided by $R_s + Z_i$ this would obviously be R_{Pi} divided by $r_{Pi} + R_s$ multiplied by $1 + j\Omega R_{Pi} C_T$ agreed, I have simplified I have struck off the denominator of Z_i okay. This I can write as A_{v0} divided by $1 + j\Omega$ by Ω_2 multiplied by let me write this as R_{Pi} divided by $R_{Pi} + R_s$. Then in the denominator R_{Pi} divided by $r_{Pi} + R_s$ so in the denominator what you will get is $1 + j\Omega r_{Pi} R_s$ divided by $r_{Pi} + R_s$, which is the parallel combination of R_{Pi} and R_s , is that okay. R_s times r_{Pi} and I am dividing by $R_s + r_{Pi}$, I have omitted one algebraic step okay.

This I can write as A_{vs0} that is A_{vs} at mid band which means that you put ω tends to 0 so this term, this term disappears all that you are left with is A_{vs0} multiplied by r_{π} by $r_{\pi} + R_s$. So A_{vs0} is the product of let me write it down; $A_{vs0} = -g_m R_L'$ that is A_{vs0} multiplied by R_{π} divided by $r_{\pi} + R_s$ that is it okay. And divided by $1 + j\omega$ by ω_2 multiplied by $1 + j\omega$ by ω_1 , you define a new frequency ω_1 which is the reciprocal of C_T multiplied by r_{π} parallel R_s so ω_1 is defined as 1 over $C_T r_{\pi}$ parallel R_s okay, let me clear this expression write it again.

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The image shows a handwritten derivation of the voltage gain $A_{vs}(j\omega)$ on a whiteboard. The equation is written as:

$$A_{vs}(j\omega) = \frac{A_{vs0} (1 - j\frac{\omega}{\omega_3})}{(1 + j\frac{\omega}{\omega_2}) (1 + j\frac{\omega}{\omega_1})}$$

Below the equation is a pole-zero plot on the s -plane. The horizontal axis is the real axis (σ) and the vertical axis is the imaginary axis ($j\omega$). There are poles marked with 'x' at $-\omega_2$ and $-\omega_1$, and a zero marked with 'o' at ω_3 . The relationship $\omega_3 \gg \omega_2 \gg \omega_1$ is written below the plot. A hand is pointing to the plot with a yellow marker.

A sub v_s a function of $j\omega = A_{vs}$ divided by $1 + j\omega$ by ω_2 multiplied by $1 + j\omega$ by ω_1 . Actually we should have a $1 + j\omega$ by ω_3 also in the numerator but ω_3 we have ignored because ω_3 is much greater than ω_2 . Now we shall show, okay you see that there are 3 such factors; $1 + j\omega$ by some constant okay or $1 - j\omega$, actually the numerator it was $1 - j\omega$ by ω_3 , is not that right in the numerator if you recall $g_m - j\omega C_{\mu}$ so it was $1 - j\omega C_{\mu}$ by g_m , there is a '-' sign there but these 3 factors corresponds to the capacitances because there are 3 capacitances in the circuit normally you expect that there shall be 3 such factors; $1 + j\omega$ by ω_1 , $j\omega$ by ω_2 , either + or minus, it can occur in the numerator it can occur in the denominator alright, 3 capacitors so 3 terms like this.

Do you also recall poles and zeros? Okay, naturally this function has 2 poles okay at $S = j\Omega_1$ is to be replaced by S so $S = -\Omega_2$ and $S = -\Omega_1$ and if I had included the other term, the term that I had ignored $1 - j\Omega_1/\Omega_3$ there was a term like this, you see that the pole-zero diagram is like this. Sigma $j\Omega_1$ pole zero diagram, one is a 0 at $S = \Omega_3$ okay due to this factor, there is a pole at $S = -\Omega_2$ somewhere here a pole is denoted by a cross and the pole at $-\Omega_1$. I have intentionally shown Ω_1 closer to the origin than Ω_2 , we shall indeed show that Ω_2 is usually much greater than Ω_1 .

Student: () (13:36)

Professor: No.

$j\Omega_1$ is to be replaced by S if I find this is a function of small s Laplace variable, A vs of small s would be $1 - S/\Omega_2$ divided by $1 + S/\Omega_1$ so the poles are at $S = -\Omega_2$, $-\Omega_1$ and 0 will be at $S = +\Omega_3$ okay. I have shown Ω_2 as far away from the imaginary axis from the origin than Ω_1 and this is in general true, Ω_3 unfortunately I have shown it here but Ω_3 should have been somewhere here okay Ω_3 is much larger than Ω_2 that means this distance would be much larger than this distance okay. So Ω_3 is much larger compared to Ω_2 and Ω_2 is much larger compared to Ω_1 , these are the 3 relative values we shall take an example to show that these are approximately valid.

But you say we have ignored this, now if Ω_2 is much larger compared to Ω_1 then naturally Ω_1 is the critical frequency which will determine the frequency response of the amplifier, in other words if this can also be ignored at frequencies around Ω_1 then naturally this is again a low pass filter 1st order low pass filter that is effects of $C_0 + C_{\mu}$ okay the effect of C_{μ} , C_{μ} determines Ω_3 they are all swamped by the effect of C_T , C_{π} alone cannot do it, C_{π} is comparable C_{π} is of the order of 10 pF, C_0 could be 5 pF okay 1 is double the other but C_T swamps everything, C_T is $C_{\pi} + C_M$ Miller capacitance and it is the Miller capacitance which swamps everything else. Miller capacitance is C_{μ} although a small quantity, it gets magnified by $1 + g_m R_L$ prime the mid band gain okay.

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Ex

$$\omega_1 = \frac{1}{(R_{\pi} \parallel R_s) C_T}$$
$$R_{\pi} = R_B \parallel r_{\pi} = 110 \text{ K} \parallel 2.6 \text{ K} \\ = 2.54 \text{ K}$$
$$\omega_1 = \frac{1}{(2.54 \text{ K} \parallel 6 \text{ K}) [10 + 2(1 + 57.7)] \text{ pF}}$$
$$= 1.62 \times 10^7 \text{ rad/s}$$
$$\omega_2 = 9.7 \times 10^7 \text{ rad/s}$$

So our approximate 3 dB cut-off frequency high frequency 3 dB cut-off would be Ω_1 , which is $1 / (r_{\pi} \parallel R_s) \times C_T$ okay Ω_1 . Now let us take an example, for the example that we have been considering, r_{π} is $R_B \parallel r_{\pi}$, what was R_B ? 220 K and 220 K so it is 110K parallel r_{π} was 2.6 K and this = 2.54 K. So $\Omega_1 = 1 / (2.54 \text{ K} \parallel R_s)$ is what was R_s , 600 ohms that is what we took R_s so 0.6 K okay multiplied by now C_T , C_T is $C_{\pi} + C_{\mu}$, C_{μ} was 2 puf multiplied by $1 + g_m R_L$ prime which was calculated earlier as 57.7, we did calculate this so many puffs and this works out to 1.62 multiplied by 10 to the 7 radian per second.

Recall that Ω_2 was calculated as 9.7 times 10 to the 7 radian per second, now this value obviously is not much larger than 1.62, it is about 6 times okay, are we justified in ignoring Ω_2 when Ω_2 is only 6 times, there is a justification. I told you that in electronics much greater than means a factor of 10, this is not even 10 this is 6, how come you can justify ignoring this, justification is as follows...

Student: Square.

Professor: Square that is right.

(Refer Slide Time: 19:06)

$$|A_{vs}| = \frac{|A_{v0}|}{\sqrt{\left(1 + \frac{\omega^2}{\omega_2^2}\right)\left(1 + \frac{\omega^2}{\omega_1^2}\right)}}$$
$$\frac{|A_{v0}|}{\sqrt{1 + \omega^2\left(\frac{1}{\omega_2^2} + \frac{1}{\omega_1^2}\right) + \frac{\omega^4}{\omega_1^2\omega_2^2}}}$$

$\omega_2^2 \gg \omega_1^2$

You see what happens is, if I write A_{vs} as A_{v0} the magnitude, magnitude divided by square root of $1 + \Omega^2$ by Ω^2 multiplied by $1 + \Omega^2$ divided by Ω^2 alright. Now this can be written as A_{v0} square root of $1 + \Omega^2$ by $\Omega^2 + 1$ by Ω^2 + Ω^2 divided by Ω^2 + Ω^4 divided by Ω^2 okay. If Ω by Ω^2 is of the order of unity okay if Ω by Ω^2 is of the order of unity then this term can be ignored this term will be much smaller than this term and in this term you see the relative values are of Ω^2 and Ω^2 , not Ω and Ω by themselves. So all that you need is not Ω much greater than Ω^2 but Ω^2 much greater than Ω^2 right, and in this case it is the ratio is 36 which is much greater than 10.

“Professor–student conversation starts”

Student: Sir could you please repeat the justification for ignoring the last term.

Professor: Ignoring the last term okay, suppose Ω is approximately the same as Ω^2 , then Ω^2 by Ω^2 is 1 and Ω^2 by Ω^2 would be 1 over 36 which can be ignored.

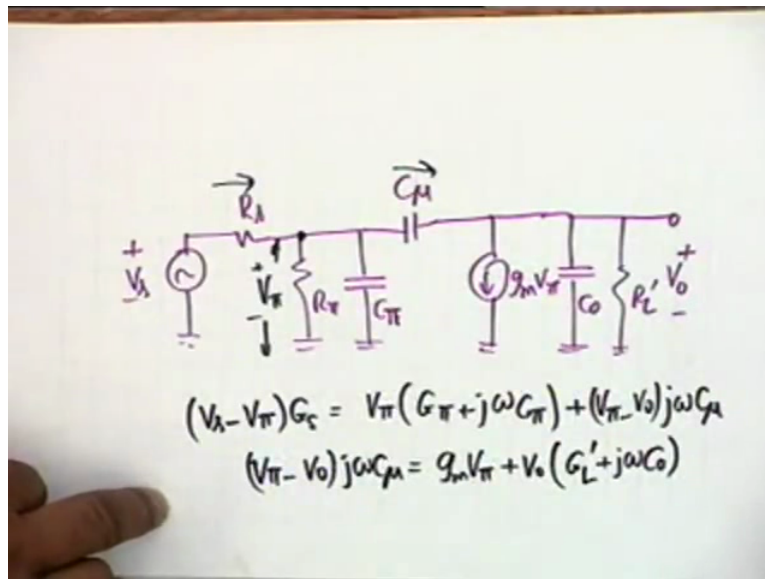
Student: But this is a particular case in which Ω is in the range of Ω^2 .

Professor: No, this is the range that we are interested in let me tell you why.

“Professor–student conversation ends”

What happens is it falls like this and what I want is, this is A_{v0} let us not make a mistake, this is A_{v0} and what we want is the frequency, this frequency at which the magnitude is A_{v0} divided by root 2. Now I at above this frequency at above this frequency the effect of Ω_2 is negligible this is what I want to show, so I am finding for frequencies near the frequency which will be obviously close to Ω_1 alright. Therefore in this particular case as in other cases to come, Ω_1 square as long as it is much less than Ω_2 square, as long as it is 1 decade below Ω_2 square, you understand one decade below means ratio of 10 we are perfectly in shape we are perfectly in business, which means that Ω_1 and Ω_2 need to differ only by a factor of 3.2 agreed, square of which is approximately 10 okay.

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So this simplification is a great simplification and one has to use this in engineering design. Now it is also of interest now to look back and see what approximation does this affect, what kind of error does this introduce? If you recall, our exact equivalent circuit was R_s , let us draw this again, r_{π} , C_T , okay r_{π} , C_T , then we have C_{μ} ... oh I am sorry C_{π} I want the exact analysis I want to find out how much error does it introduce take an idea okay C_{μ} then $g_m V_{\pi}$ then C_o and R_L' , this is our exact equivalent circuit before we took Miller effect into account

alright. Now to analyse this to analyse this what one can do is to write 2 node equations that is all that is required, one is in terms of V_{Pi} this voltage is V_{Pi} and the other node voltage is V_0 so you have to write 2 node equations and solve for V_0 by V_s if you do that and simplify, I am skipping this step...

Student: Sir we just we explained other equation.

Professor: We explained what?

Student: The node equation...

Professor: The node equation you see there are 2 V_s is known, I want to find out V_0 okay, now V_s and V_0 are not directly connected, they are connected through another load V_{Pi} another load whose voltage is V_{Pi} therefore I have to write 2 node equations, let me write this equation. $V_s - V_{Pi}$ multiplied by G_S okay this is the current that comes in, this would be $= V_{Pi} G_{Pi} + j \Omega C_{Pi}$, this is the current that goes into r_{Pi} and C_{Pi} + the current that goes into C_{Mu} it would be $V_{Pi} - V_0$ multiplied by $j \Omega C_{Mu}$ agreed, this is the 1st node equation. The 2nd node equation is $V_{Pi} - V_0$ $j \Omega C_{Mu}$ it is this current that would be $= g_m V_{Pi} + V_0 G_L + j \Omega C_0$ that is it. These are the 2 node equations and all that we have to do now is to eliminate V_{Pi} , so take V_{Pi} from one equation and substitute into the other.

The expression that you get, now what kind of expression do you expect that the degree of numerator and the degree of denominator, what will be degree of numerator? 1 because I had only V_0 , the degree of the denominator would be 2 so I shall have Ω square term also and their expression is this, please do write it down.

(Refer Slide Time: 26:24)

$$\frac{V_o}{V_s} = \frac{\left(-g_m R_L' \frac{R_{\pi}}{R_{\pi} + R_s}\right) \left(1 - \frac{j\omega}{g_m / C_{\mu}}\right)}{\left\{ 1 + \frac{j\omega}{\left[(R_{\pi} \parallel R_s) C_T + (C_0 + C_{\mu}) R_L' \right]^{-1}} \right.}$$

$$\left. - \omega^2 R_L' (R_{\pi} \parallel R_s) (C_{\pi} C_0 + C_0 C_{\mu} + C_{\mu} C_{\pi}) \right\}$$

minus $g_m R_L'$ this is a V_0 as in the approximate expression multiplied by R_{π} divided by $R_{\pi} + R_s$, this takes account of the mid band gain at the input so this whole quantity is A_{vs0} as in the earlier case, then we have $1 -$ as we have found out $j\omega$ divided by g_m by C_{μ} this was ω_3 okay and divided by now here comes the difference, $1 + j\omega$ divided by what I get here is $R_{\pi} \parallel R_s$ times C_T well it does not stop here, in the approximate expression we had simply $j\omega$ by ω_1 and ω_1 , I beg your pardon I made a mistake okay now it is okay alright. I have left a gap intentionally I will tell you why the gap phase, in the approximate expression it is simply this and this inverse, no this quantity was taken as ω_1 ...

No no no ω_2 is different ω_2 comes in 1 by $R_L' C_0 + C_{\mu}$, this inverse was taken ω_1 . In the exact expression there is in addition a term which is $C_0 + C_{\mu} R_L'$ prime that is the term which contributes to ω_2 also occurs here, and it is quite understandable because even in the approximate expression the coefficient of $j\omega$ should have been 1 by $\omega_1 + 1$ by ω_2 approximate expression, in the exact expression the 2 add to each other okay so this we shall call ω_1' , it is not quite ω_1 it is ω_1 modified ω_1 and then you have an ω^2 term and this square term is like this, $-\omega^2$ it is still in the denominator $R_L' R_{\pi} \parallel R_s$ multiplied by $C_{\pi} C_0 +$

$C_0 C_{\mu} + C_{\mu} C_{\pi}$ this is the expression okay, normally it should have been Ω^2 divided by Ω_1^2 multiplied by Ω_2^2 in the approximate expression.

Student: This is the denominator?

Professor: This is all in the denominator okay this is all in the denominator.

And what we have done is we have ignored this we have ignored this which can be ignored in an actual circuit and instead of Ω_1 prime = this we have ignored this term okay and the condition, well before we say before we find out the condition under which this can be ignored let us take the exact expression and substitute the numerical values and see what sort of approximation you get.

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ω	Approx $\{A_{vs}\}$	Exact
10^7	$39.7 / 148^\circ$	$37.9 / 144^\circ$
2×10^7	$29.4 / 129^\circ$	$26.8 / 124^\circ$
10^8	$7.47 / 109^\circ$	$6.48 / 103^\circ$

$\omega_1 = 1.62 \times 10^7$
 $\omega_2 = 9.7 \times 10^7$

$$A_{vs} = -57.7 \times \frac{2.54}{2.54 + 6}$$

$$= -46.7$$

This have been calculated and the values of A_{vs} have been found out at the magnitude not the magnitude the total expression, obviously we have to feed it into a computer and calculation would be too large. At various values of Ω , now would you give me the value of Ω_1 , what did we calculate 1.62 times 10 to the 7 and what is Ω_2 , 9.7 times 10 to the 7 okay, so what we did was calculate at $\Omega = 10$ to the 7 just before Ω_1 at 2 times 10 to the 7 at 3 frequencies we calculate that is immediately after Ω_1 , around Ω_1 and then at a frequency nearly = Ω_2 , let us we calculated 10 to the 8; 3 frequencies we calculated and

we calculated the approximate value that is by using the Miller model and the exact value that is by using this horrible looking expression okay alright.

The values that we get are very illuminating, the approximate value is 39.7 magnitude and the angle instead of 180 degree, at mid band the angle is 180 degree will now become less than 180, it becomes 148 degree, it is a complex expression find out its magnitude and its phase, obviously you have to feed it into a computer okay. And the exact value comes 37.9 angle 144 degree, you can calculate the percentage but you see they are visibly good approximations okay. At 2 times 10 to the 7 do you expect the gain to be more than 39.7 or less than 39.7? Less and the value is 29.4 129 degree and the exact is 26.8 you see the error is increasing why, because you are going closer to Ω_2 , if we had calculated at 10 to the 6 the error would have been much less alright, so 26.8 and the angle is 124 degree. And far away that is nearly at Ω_2 the gain is still less 7.47 the angle is 109 degree, the exact value is 6.48 angle 103 degree.

The angle does not change substantially okay between the exact and approximate you see this is 4 degree, this is 5 degree, this is 6 degree, the angle does not change substantially and 4 degree is what for 144 degree, so the percentage error is much less it is in the magnitude. Now one argue that we will not obviously use the amplifier with a gain of 7.47, when the mid band gain is, what is the mid band gain? A vs 0...

“Professor–student conversation starts”

Student: 57.7

Professor: No... 57 point okay - 57.7 multiplied by R_{Pi} , what was R_{Pi} ? 2.54 divided by 2.54 + 0.6 and this value no I have not calculated. Anyway, this value as you see it is approximately 56 okay 2.54 divided by 3.14 okay it is greater than 50, obviously we will not use the amplifier when the gain becomes 7.47, we will go approximately up to 2 times 10 to the 7 not more than that. So 3 dB cut off being taken as Ω_1 is justified even by exact analysis yes... Pardon me...

Student: It is - 46.7.

Professor: - 46.7 okay so if at mid band we require 46.7 obviously we will not go beyond let us say 30 on this side, on the lower side also there will be a decrease of gain just to be calculated later yes

Student: Sir if the frequency higher then we should have larger error between exact and approximate results.

Professor: Yes.

Student: But in this case we see that $(\omega)(35:10)$ 10 to the 7 as frequency, the error is more as compared to $(\omega)(35:14)$.

Professor: Okay that is a good point.

Student: Percentage error.

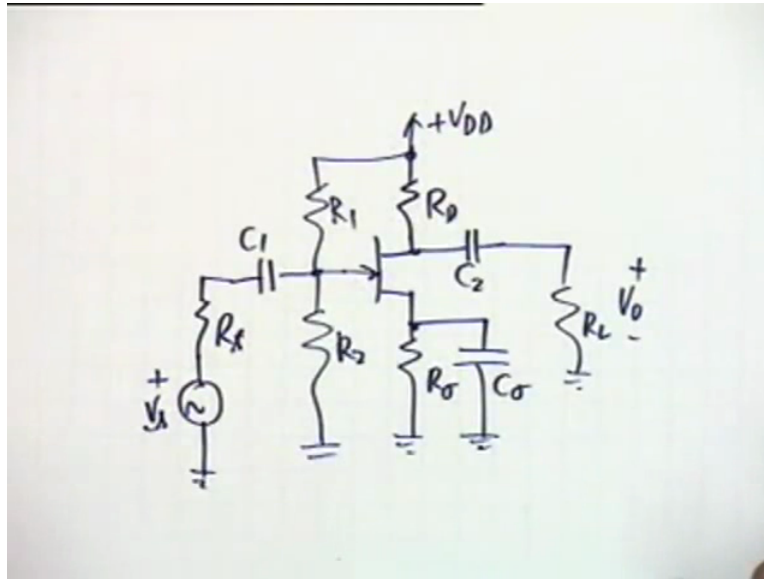
Professor: Percentage error that is what comes not the absolute error okay, you have answered the question this is like a self-interview. You know what is self-interview?

Student: No.

Professor: After the class.

“Professor–student conversation ends”

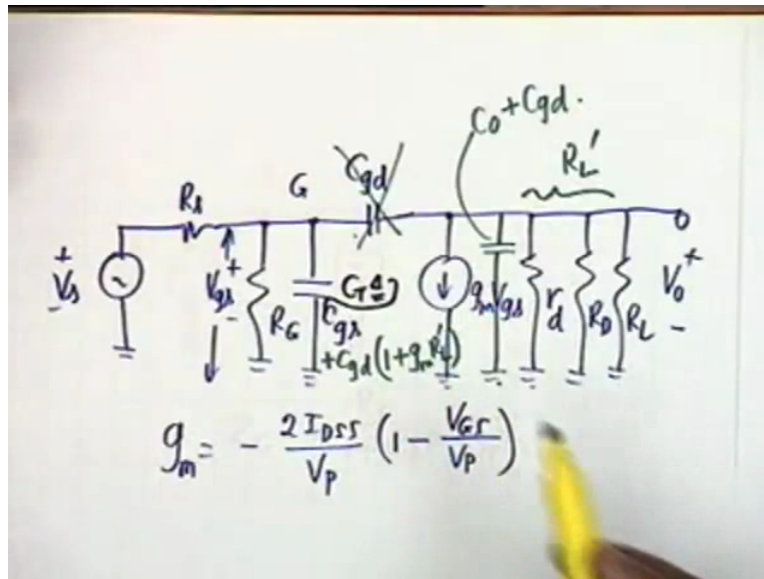
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Okay now as far as an FET amplifier is concerned, the analysis is exactly similar so I shall only recall the steps in the analysis and give the results, the algebra I will not carry out. The circuit let us draw it as low harmonic repetition, the circuit is V_{DD} , R_D , you have a drain source, the sources bypassed, the common source amplifier with C_S , there is a capacitor C_2 which goes to R_L , the voltage is V_o and the biasing is done by 2 resistors R_1 and R_2 , the sole purpose being to establish a voltage at the gate okay, I have shown an N-channel JFET, the sole purpose of R_1 and R_2 is not to feed the current like BJT, this must be understood, BJT requires a base drive base drive not voltage but the current because otherwise $I_{sub C}$ shall be 0, $I_{sub C}$ is approximately β times 0, but here this is not the case.

Here what we want to establish a voltage is a voltage at the gate and a voltage at the source such that V_{gs} is negative or positive? Negative in an N-channel JFET okay and then you have the capacitor C_1 , R_S and V_S okay this is the circuit.

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And the equivalent circuit the equivalent circuit would be V s AC equivalent circuit would be V s, C 1 is taken as short so R s, it goes straight to R sub G which is a parallel combination of R 1 and R 2 and then you have a capacitance of value C gs okay then from the gate to the drain you have a capacitor C gd and the voltage here is V gs, the current source here would be g m times V gs, where g m now has to be calculated as if I recall g m is – twice I DSS divided by V P multiplied by 1 – capital V GS divided by V P wonderful, you have to calculate this okay. Then this goes to ground what we have here? A dynamic r d dynamic drain resistance and we have the parallel combination of R D and R L, this voltage is V 0. As usual we do simplification by combining these into R L prime that is all that is not r Pi here okay.

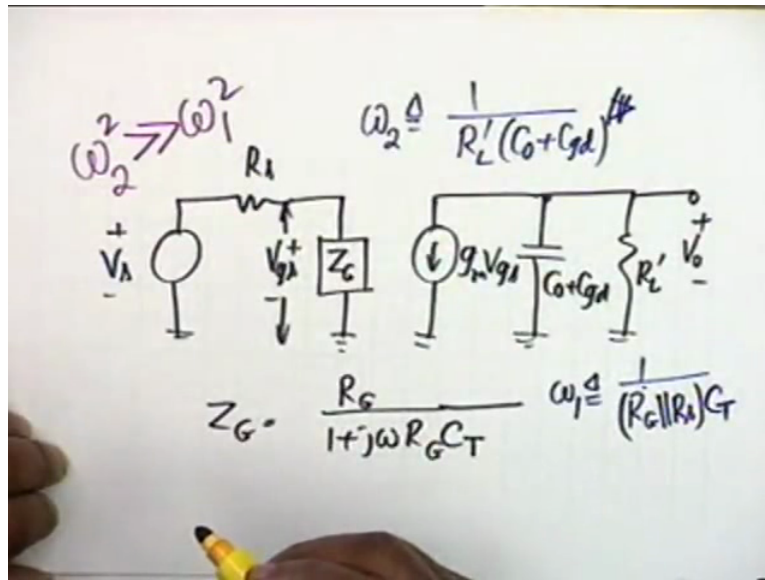
So if I now take Miller effect into account then C gd, I have made a mistake and I am waiting for the correction... C 0 there is a capacitor here CG low and if I want it if I want to analyse this by Miller effect that is avoid this bridge, which creates complications we have to write 2 node equations if I want to avoid this then all I have to do is add 2 C gs, C gd multiplied by 1 + g m R L prime and 2 C 0 I have to add C gd okay, this is the Miller effect equivalent circuit. You want me to draw this more neatly?

Student: Yes sir.

Student: Yes sir.

Professor: Okay, whatever you want.

(Refer Slide Time: 40:25)



V_s , R_s , why do not you draw this as a Z_G , what is Z_G ? It is R_G divided by $1 + j\omega R_G C_T$ multiplied by C_T once again this capacitance total capacitance I shall denote this by $C_T = C_0 + C_{gd}$ $1 + g_m R_L'$, this is V_{gs} and then I have a $g_m V_{gs}$ and $C_0 + C_{gd}$ and then R_L' , this is V_o . Naturally since we have converted 3 capacitor circuit into 2 capacitor one that 0 in the numerator we have ignored okay and there would be 2 poles now, one due to $Z_{sub G}$ and the other due to $C_0 + C_{gd}$ as in the BJT case. And as usual the same relationship would be true that is if we define let me do it in this figure itself, if we define ω_2 as $1 / (R_L' (C_0 + C_{gd}))$ okay and ω_1 as what? $1 / (R_G || R_s) C_T$ not forget this multiplied by C_T .

(Refer Slide Time: 43:45)

$$A_{v_s}(j\omega) \approx \frac{A_{v_{s0}}}{1 + j\omega/\omega_1}$$
$$A_{v_{s0}} = (-g_m R_L') \frac{R_G}{R_G + R_L}$$
$$\omega_1 = \frac{1}{(R_G \parallel R_L) C_T}$$

If we define these 2 cut-off frequencies these 2 frequencies then Omega 2 would be much greater... No that may not happen, what should happen? Mega 2 square let me write it in a brighter colour, Omega 2 square will be much greater than Omega 1 square so that so that our gain expression A vs of j Omega A vs of j Omega will be given by A vs0 approximately divided by 1 + j Omega by Omega 1, where A vs0 will be what the value is, - g m R L prime multiplied by R G divided by R G + R L okay, and Omega 1 is divided by 1 over R G parallel R s multiplied by C T under the usual approximations okay. Since we have done the BJT circuit in detail we did not have to do FET in details, I avoided all the algebraic steps.

Once again it is of interest to find out what kind of error, error may not be the same as in the BJT because the values of the resistances are quite different, they are different magnitude so one has to calculate the error, and if you have to calculate the error then you have to analyse the exact circuit with a 2 node analysis, 2 node equations have to be written and have to be solved, let us see if I have the expression I shall give you the exact expression but let us quickly look at one example.

(Refer Slide Time: 44:36)

Ex

$$\begin{aligned}R_s &= 1\text{K}, R_D = 5\text{K} \\R_L &= 10\text{K}, R_\sigma = 2\text{K} \\R_1 = R_2 &= 100\text{K}, C_{gs} = 3\text{pF} \\C_{gd} &= 2\text{pF}, g_m = 10\text{mS} \\r_d &= 20\text{K}\end{aligned}$$
$$\begin{aligned}A_{vo} &= -g_m R_L' \frac{R_G}{R_G + R_S} \\&= -28.6 \times \frac{50}{51} = -28\end{aligned}$$

In this example we assume that $R_s = 1\text{K}$, R_s was 600 ohms in BJT why have we increased it because FETs can take higher values of source resistance why because FET input impedance is very high very good. R_D is 5 K, R_L it can take larger load also 10 K, R_σ is 2K, R_1 and R_2 both are 100K, C_{gs} is 3 puf, C_{gd} is 2 puf, g_m calculated from I_{DQ} is 10 millimho typical value and r_d is 20 K. Suppose g_m was not given, suppose this was not given, could you calculate g_m from this data so what do you need?

“Professor–student conversation starts”

Student: I D.

Professor: Who is going to give you ID?

Student: I DSS.

Student: I DSS.

Professor: And what else?

Student: VP.

Professor: V DD would have been needed, is not that right? We have not given that because all the parameters are given agreed.

Student: VP.

Professor: VP also that is correct. If g m was not given then you ask for 3 more data; V DD, why is V DD needed to calculate V gs come to the point VDD is needed to calculate V G alright not V gs, V G, V s would be calculated from I D, I D times R Sigma so V G – V s that you have to substitute for expression in I D and solve a quadratic, that you have to exercise your intelligence to reject one of the values; one value shall be admissible the other shall not be okay.

“Professor–student conversation ends”

Now if this is so then A vs0 is given by $-g_m R_L \text{ prime } R_G$ divided by $R_G + R_s$, substitute these values and it calculates out to -28.6 multiplied by 50 divided by 51 , why did I calculate this separately? Pardon me... Why did I calculate this separately? This I will require for the Miller capacitance, Miller capacitance is C_{gd} multiplied by $1 + \dots$ Now if you put this in a calculator and calculate, you will soon discover that you will have to go back calculate the $g_m R_L \text{ prime}$ also so it is better to calculate this in 2 steps and this becomes -28 approximately.

(Refer Slide Time: 48:04)

The image shows handwritten mathematical derivations for corner frequencies ω_1 and ω_2 . The first equation is $\omega_1 = \frac{1}{(R_s || R_G)[C_{gs} + C_{gd}(1 + g_m R_L \text{ prime})]}$, which is evaluated as $16.4 \times 10^6 \text{ rad/s}$. The second equation is $\omega_2 = \frac{1}{R_L'(C_0 + C_{gd})} = 66.7 \times 10^6 \text{ rad/s}$. Underneath the denominator of the second equation, arrows point to C_0 and C_{gd} with labels $\sim 3\text{pF}$ and 2pF respectively.

$$\omega_1 = \frac{1}{(R_s || R_G)[C_{gs} + C_{gd}(1 + g_m R_L \text{ prime})]}$$
$$= 16.4 \times 10^6 \text{ rad/s}$$
$$\omega_2 = \frac{1}{R_L'(C_0 + C_{gd})} = 66.7 \times 10^6 \text{ rad/s}$$

\uparrow \uparrow
 $\sim 3\text{pF}$ 2pF

And Omega 1 the frequency Omega 1 is 1 over R s parallel R G and the capacitance is C gs + C gd 1 + g m R L prime and that calculates out after a bit of calculation to 16.4 times 10 to the 6 rps. It is of interest to know whether this dominates or not that is it is of interest to calculate Omega 2 also, Omega 2 is given by 1 over R L prime then C 0 + C gd, it has not been given what is given is C gd, how much was given 2 puf, C 0 would be comparable to C gd so even if even if this capacitance C 0 + C gd let us say let us assume it to be 3 puf, so even if this is 5 puf even if C 0 + C gd is 5 puf well if it is 5 puf then this calculates out to 66.7 multiplied by 10 to the 6 radian per second alright, 66.6 10 to the 6 radian per second, which is only 4 times approximately but square is 16 times so the approximation is justified. I will strongly encourage you to find out the exact expression also, to find out the exact analysis I have the expression yes.

Student: Sir how do you assume C 0.

Professor: How do you assume C 0? C 0 normally is a stray capacitance and it is of the order of a few puf, you can take slightly higher so long as Omega 2 square is 10 times, we have put a margin here right, so long as Omega 2 square is 10 times it is okay, C 0 can also be measured in the actual varying you can measure and you can reduce C 0 by careful varying okay, right any other questions?

(Refer Slide Time: 50:41)

The image shows handwritten mathematical derivations on a whiteboard. The word "Exact" is written at the top left. Below it, the voltage gain $\frac{V_o}{V_h}$ is given as a fraction. The numerator is $-g_m R_L' \left(\frac{R_C}{R_C + R_A} \right) \left(1 - \frac{j\omega}{g_m / C_{gd}} \right)$. The denominator is $1 + \frac{j\omega}{\omega_1'} - \omega^2 R_L' (R_S \parallel R_G) (C_{gs} C_{gd} + C_{gs} C_o + C_{gd} C_o)$. Below this, the modified pole frequency ω_1' is given as $\frac{1}{(R_S \parallel R_G) [C_{gs} + C_{gd} (1 + g_m R_L')] + R_L' (C_{gd} + C_o)}$.

$$\frac{V_o}{V_h} = \frac{-g_m R_L' \left(\frac{R_C}{R_C + R_A} \right) \left(1 - \frac{j\omega}{g_m / C_{gd}} \right)}{1 + \frac{j\omega}{\omega_1'} - \omega^2 R_L' (R_S \parallel R_G) (C_{gs} C_{gd} + C_{gs} C_o + C_{gd} C_o)}$$

$$\omega_1' = \frac{1}{(R_S \parallel R_G) [C_{gs} + C_{gd} (1 + g_m R_L')] + R_L' (C_{gd} + C_o)}$$

The exact expression for V_0 by V_s by making a 2 node analysis is the following, write it down and try to verify that it is indeed so, $R_G + R_s$ then you have $1 -$ exactly like the BJT $j\omega$ divided by g_m over C_{gd} this is that ω divided by $1 + j\omega$ by ω $1 - \omega^2 R_L' R_s \parallel R_G$ times $C_{gs} C_{gd} + C_{gs} C_0 + C_{gd} C_0$ this is the exact expression. I do have some calculations regarding exact values but I think we can skip that. ω $1 - \omega^2 R_L'$ the only difference is $R_s \parallel R_G$ times $C_{gs} +$ the Miller capacitance that is $C_{gd} (1 + g_m R_L')$, in addition we have a term can you guess what this term is? Exactly like the previous case it is $R_L' C_0 + C_{gd}$ that is it and usually this term is negligible this term is negligible compared to previous one.

Well sure the condition under which this term will be negligible compared to this is that $C_{gd} + C_0$ must be much smaller compared to $g_m C_{gd}$ multiplied by $R_s \parallel R_G$, is that point clear? No okay I want to ignore this compared to this and you know $g_m R_L'$ is much larger compared to 1 so you can ignore this and you know the Miller capacitance swamps the total...

Student: (())(53:10)

Professor: It must close here, where else can it close? It must close here okay.

We can ignore 1 and then $C_{gd} g_m R_L'$ will swamp C_{gs} so you can ignore C_{gs} , what I want is that this term should be small compared to $R_s \parallel R_G$ multiplied by C_{gd} multiplied by $g_m R_L'$, you see what was the purpose of this simplification, to cross out R_L' .

(Refer Slide Time: 53:50)

$$\omega_1' \approx \omega_1$$
$$\text{if } (R_s || R_c) C_{gd} g_m \gg C_{gd} + C_o$$
$$C_o = 1 \text{ pF}$$
$$\omega_3 = 5 \times 10^9 \text{ rps}$$
$$\omega_1' = 14.4 \times 10^6 \text{ rps}$$
$$\omega_1 = 16.4 \times 10^6 \text{ rps}$$

We are finding out inequality so Omega 1 prime make a profound statement Omega 1 prime is approximately = Omega 1 if R s parallel RG multiplied by C gd g m is much greater compared to C gd + C 0 okay this is the condition, no R L prime we have, there is an R L prime here there is an R L prime here so we have crossed that out, we wanted to find out independent of R L prime. You see all that is involved is all that is involved is the interval parameters of the transistor JFET, the external resistors R s, R 1 and R 2 and the stray capacitance C 0 okay, under this condition Omega 1 prime would be = Omega 1.

I have some calculation is, a looming C 0 as 1 puf for the same example as we had taken earlier, assuming C 0 as 1 puf we get Omega 3 which is g m by C gd is 5 times 10 to the 9 rps, A vs0 we have already calculated, Omega 1 prime calculates out to 14.4 times 10 to the 6 rps, what was the value? What was Omega 1? 16.4 times 10 to the 6 it is not too bad. Ignoring the 2nd term instead of 14.4 we get 16.4 it is not too bad okay, in any case in actual circuit you shall have to make adjustments so one can afford to become (())(55:59) circuit calculation electronic circuits in particular because you will have to make fine adjustments and so on and this is a good point to stop, next time we are going to talk about low frequency response.