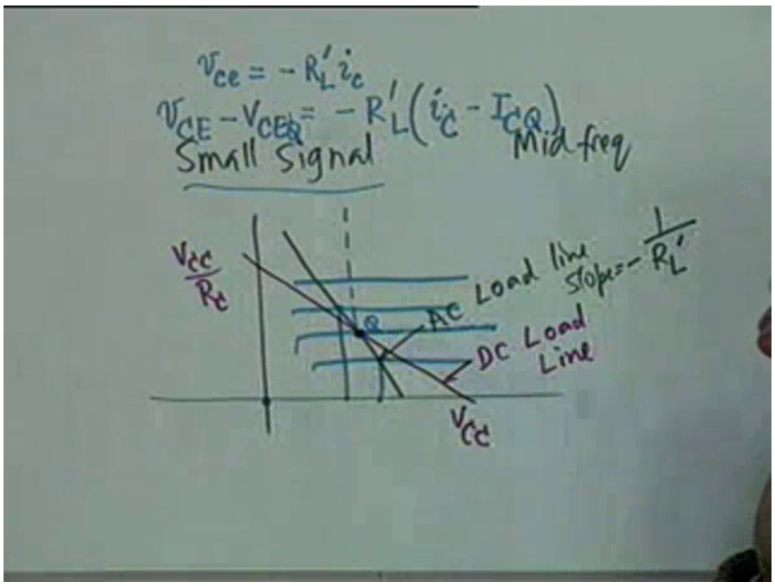
Analog Electronic Circuits
Prof. S. C. Dutta Roy
Department of Electrical Engineering.
Indian Institute of Technology Delhi
Lecture No 09
Small Signal Amplifiers:
Mid Frequency Analysis

This is the 10th lecture and we're going to talk about small signal amplifiers, mid frequency analysis.

(Refer Time Slide: 0:01:13)



Let us first explain the term Small Signal and Mid frequency. As you know the DC operation of a transistor lies on the load line, on the DC load line. And DC load line starts at V C C and ends at V C C divided by R C alright. This is the DC load line and the Q point is somewhere here. And through the Q point you can draw an AC load line, if the AC load line is different from DC load then you can draw an AC load line whose slope is minus 1by R L prime which is equal to parallel combination of R L and R C okay this is known.

And therefore the operation of the transistor the AC operation of the transistor or the signal operation is on the AC load line which virtually means what is the AC load line? The AC load line is described by this V C E is equal to minus R L prime i c which means that when the incremental current is 0 the incremental voltage is also 0. Or I could write this as V C E cap C E minus capital V C E that is the DC part alright equal to minus R L prime I C total collector current minus the DC collector current. And since these are at the Q point we say I C Q and here also we add V C E Q.

It is as if in the operation in the AC operation the axis of the current and voltage they are shifted from this point to the Q point. Is that clear, you see when V C E total V C E is equal to V C E Q the voltage, collector emitter voltage is 0. When the collector, total collector current is equal to the Q point, DC collector current, the current is 0. So it is as if this has been transferred to this point. And AC operation is now on this green line okay. AC operation that is if the incremental voltage. If the incremental collector, I am sorry, incremental base current increases if the base current increases then you will go up on the green line.

If the base current decreases you will go down on the green line. But these variations are small compared to V C E Q or I C Q they are small quantities. That is why it is called small signal operation okay. You also know that characteristics are like this, they are almost parallel lines almost parallel lines and if our operation is let's say limited between this point and this point then the total excursion around the Q point is small enough to consider the device operation is perfectly linear. That is for equal increments of I B that will be equal increments of I C collector current and there would be equal excursion of V C E the collector emitter voltage. So it is not only small signal if it is small signal then it is guaranteed that the operation shall be linear, linear operation okay.

Student: Sir could you please repeat?

Professor: Yes.

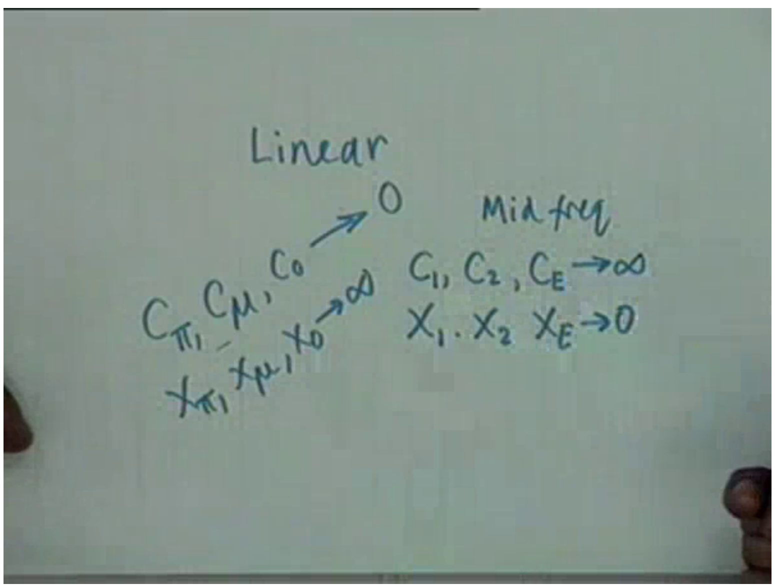
So the point is our AC operation, we're talking of AC operation. AC operation is confined on this AC load line the green line. And since the excursion of base current which is the exciting signal would be small, would be around this point that is it would either go up on the green line or go down on the green line. If the excursion is small then equal increments of I B will cause equal increments of I C an equal swing of V C E and therefore and when the incremental base current is 0 the incremental collector current is also 0. And therefore it is linear system. It can be treated as a linear system although the transistor itself is highly non-linear device. But as far as AC operation around a well-chosen Q point is concerned we chosen I have already told you how to choose it. Then the operation can be thought of as linear and the analysis becomes very simple.

Student: Why is called small signal?

Professor: We're considering only small signal.

Now who prevents you from applying a large signal in a transistor? If a large signal operation is required then we shall have to go into non-linearity's. There is no way out. What we're considering now is either a voltage amplifier or a current amplifier where the excursions on the Q point would be small. If the excursions are large for example in an audio power amplifier then we shall have to go into non-linearity's and we shall have to go into the actual characteristics to be able to analysis and synthesize such a power amplifier okay. That we shall come later on, currently we can find 2 small signals and small signal implies that the operation is linear. These 2 are almost synonymous small signal operation and liner operation. Because of small signal it is linear operation. The other term that we're using is Mid Frequency.

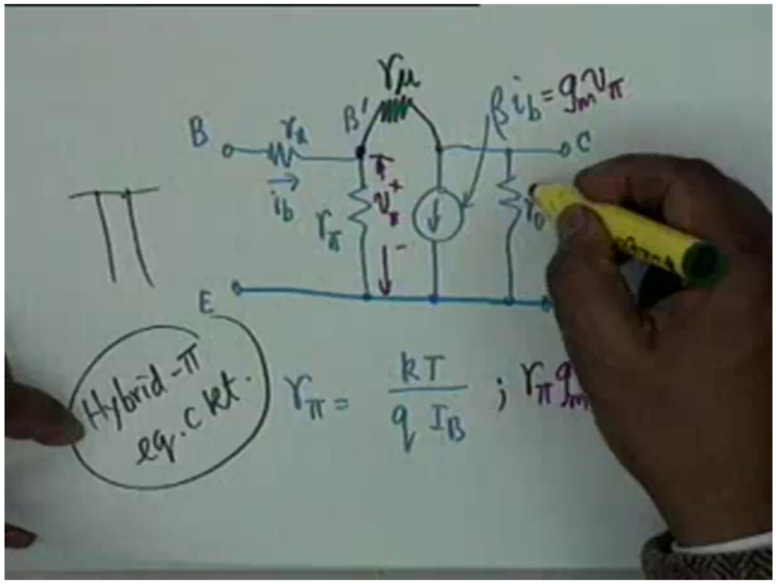
(Refer Time Slide: 0:07:55)



Mid Frequency is the frequency at which the coupling capacitors C 1 and C 2 and the bypass capacitor can be considered as infinitely large. That is that their reactance's approximately R 0. X 1, X 2, X E tend to 0. 1 by omega C tends to 0. The frequency and the values of capacitors is such that they tend to 0 in other words the coupling capacitor act as perfect couplers and the bypass capacitor acts as a perfect short circuit this is what we assume okay mid frequency. We also assume that the transistor internal capacitances while there are capacitances because you have a reverse biased PN junction in the collector region and you have a forward biased junction in the emitter base both of them have capacitance.

In addition the base to collector may have a capacitance, all these capacitances which we shall see C pie we shall call them later C pie C mu and there is also a capacitance C 0 across the output. There are basically 3 capacitances that you have to take care. We assume that at mid frequency they tend to 0. In other words they are reactance X pie, X mu, X 0 tend to infinity so they can be ignored. The 2 sets of capacitances are ignored due to 2 different reasons, one is these are very small and therefore their reactance's can be neglected they are open. These are very large therefore their reactance’s can be considered as short okay. If that is so then at mid frequency we can only work with resistive equivalent circuit okay nothing. We have already seen a very simple equivalent circuit of a transistor.

(Refer Time Slide: 0:09:54)



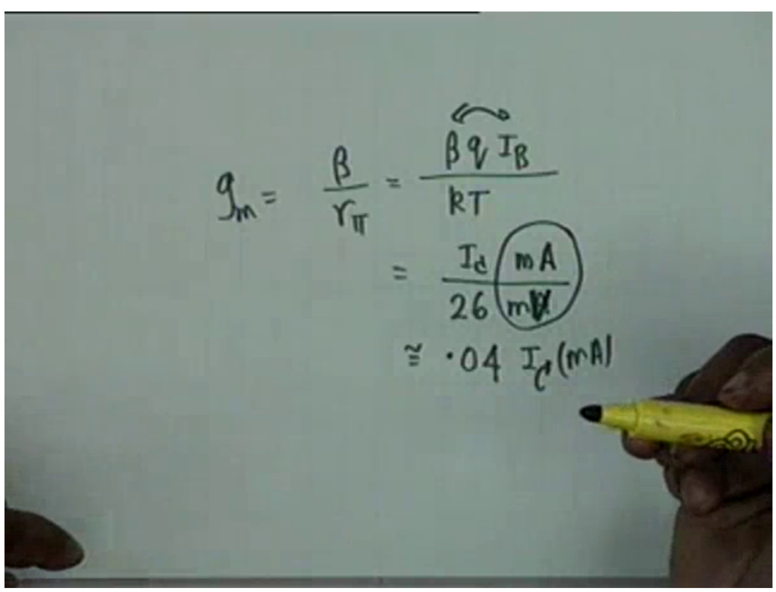
That is we have said that if this is the base and this is the emitter a common emitter and this is the collector, this is the emitter. If the emitter if the emitter is common then we have already come across at various points of time I have kept on mentioning that there is an internal base B prime and that there is a resistance R X which is the base spreading resistance of the order of a 100 ohm or so. And between B prime and E there is this diode resistance R pie. R pie is K T by Q I B the DC I B okay. And between the collector and the emitter there is a resistance which is the Early resistance R 0 which is equal to V A divided by R C you see I C and I B they are related to each other one is beta times approximately.

And therefore I could express this also in terms of I C if I so desire and then you have if this current is I B then you have a current source which is beta time I B okay and we also said that this current source could alternatively be replaced by a voltage controlled current source here it is current controlled. Beta I B is controlled by another current I B if I take the voltage across this V pie plus minus then I could write this as GM times V pie and obviously the relationship between R pie GM and beta is that the product is equal to beta. This is one of the circuits we have we have mentioned again and again during our analysis of DC biasing and device operation.

The only other thing that is needed for an exact equivalent circuit the only other thing that is needed is to add between the base internal base and the collector. Let me do this with a different colour the only other thing that is needed is to add a resistance. And this resistance is called R mu. Between the collector and the base if we add this resistance then this becomes the so called Hybrid Pie Equivalent Circuit. Why it is a hybrid pie, well that it resembles a pie is no doesn't have to be explained. It's like a pie, there is a connection from here to here, there are connections in this direction okay. This this and this makes it a pie. Why it is hybrid should be clear after a few minutes when we discuss what is a hybrid equivalent circuit. So this is the circuit that we shall mostly be using.

Let me recall the values, R X is of the order of 100 ohms, R pie of the order of a K, R mu is of the order of Megs 10s of Megs, typical value is 13 meg and that is why we have not mentioned it so far. We didn't want to complicate life. We have to complicate life if we have a capacitor across it whose reactance is comparable to R mu or other resistances in the circuit we shall see that later. But mostly we shall be fortunate to be able to ignore R mu. If R mu is included life becomes miserable, analysis becomes miserable so is design okay. Then beta is of the order of a 100 alright beta is of the order of a 100 and R 0 is also of the order of a meg R 0 appears due to early effect. What is early effect, that the collector characteristics are not perfectly parallel to the current axis, not current axis to voltage axis? It's slightly inclined and this inclination is taken care of by R0.

(Refer Time Slide: 0:14:20)



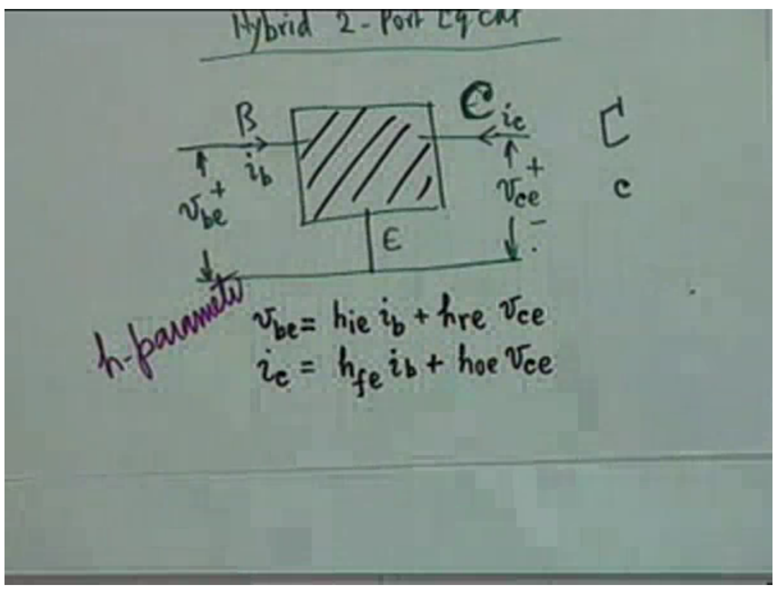
You also notice that gm is equal to beta divided by R pie which I can write as beta divided by what is R pie KT by Q I B. Now beta times I B beta times I B is approximately I C so it is I C divided by 26 milli-amperes at the room temperature.

Student: milli-volt

Professor: Milli-volt okay.

If I C is expressed in milli-ampere if this is expressed in milli-ampere then these two cancelled and I have simply 0.4 approximately times I C where I C is in milli-ampere okay. So if I know the collector current then I know gm. And if I know gm beta is given then I can calculate R pie or if I C and R pie are given then I can calculate beta the 3 parameters are related. And this is the circuit that we shall be using mostly.

(Refer Time Slide: 0:15:47)



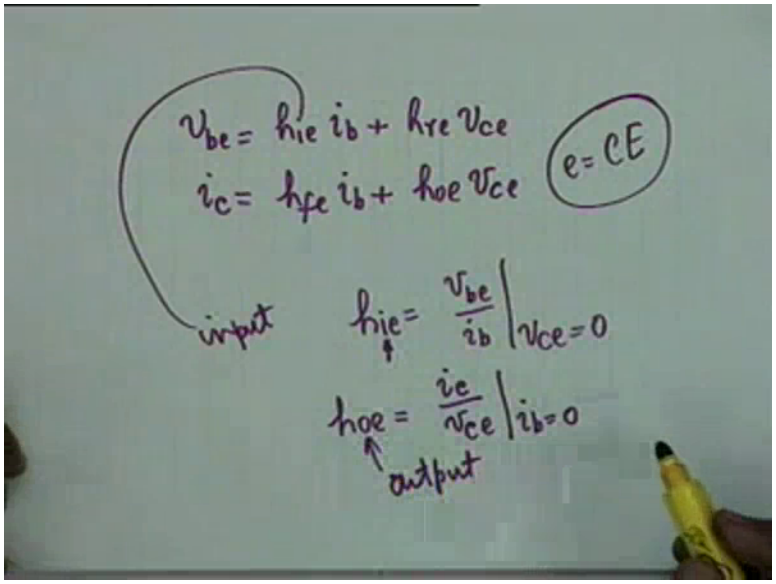
But why is it called hybrid pie, because this is a modification of so called Hybrid 2 Port Equivalent circuit. Now the hybrid pie equivalent circuit to care of a physical phenomenon that is occurring in the transistor that there is an internal base that between the internal base and emitter there is a junction, that's how you calculated R pie and so on. The hybrid 2 port equivalent circuit doesn't take care of any of this, it simply says blindly treats the transistor as 3 terminal device base emitter base collector collector and emitter and it says that if you consider AC equivalent circuit the voltage here is V B E between Base and emitter. The current here is I B the current here is I C and the voltage here is V C E, try to learn distinguishing between capital C and small c okay. Capital C I shall write like this small c I will write like this.

Okay so the hybrid 2 port equivalent circuit treats the transistor as a black box okay. If you want the colour black I will use it. It's a black box okay; I don't know what happens inside, what I can do is to have the terminal voltages and currents. I can apply 2 sources I can measure 2 responses. I could apply 2 voltage sources and measure 2 responses then I get the so called Y parameter. Have you done the circuit theory 2 port parameters not yet okay right? So we will not go into that what I will say is that one of the ways of describing such a black box is to take input voltage V B E and the output current I C okay. These are responses due to 2 sources. One is the base current I B and the other is the collector emitter voltage V C E and since it is linear operation in small signal operation it's a linear operation V B E and I C these two responses you know what are the sources now?

Signal sources are current, a current source at the input I B and a voltage source at the output V C E alright. I can apply only 2 sources measure 2 responses. I cannot apply a current source and a voltage source both here, can I? I cannot okay because that will be a degenerate condition now due to these 2 sources I measure the responses V B E and I C. And since it is linear you know in a linear system super position holds. Therefore the response must be super position of these 2 with appropriate constants alright and this constant is written as H I E okay these are constants plus H R E this constant is written as H R E similarly the collector current is written as H F E I B plus H O E V C E okay. This is the genesis of H parameter equivalent circuit or Hybrid H stands for hybrid.

And why it is hybrid is because the dimensions of the constants are not uniform. You will see later in circuit theory course that the dimensions are all impedances or admittances. This is a mixture, you see what is the dimension of H I E, this is in impedance. H R E is a dimensionless constant. H F E is a dimensionless constant and H O E admittance. So you have impedance, admittance and dimensionless constants. H R E as you can see is a voltage transfer function H R E is a ration of V B E to V C E with I B equal to 0. Similarly H F E is a current transfer function H F E is I C divided by I B with V C E equal to 0. So you have all kinds of things. You have ratios of current, rations of voltages, ration of voltage to current and ratio of current to voltage and therefore this is a mixed set of parameter and it is called Hybrid Set or H parameter description. Now I must explain the symbols the subscripts.

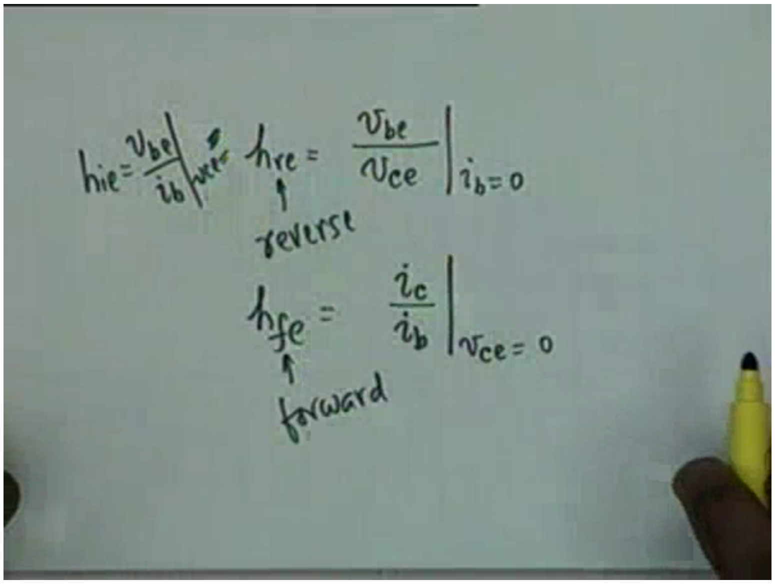
(Refer Time Slide: 0:21:02)



Let me write it again, V B E is equal to H I E, I B plus H R E V C E and I C is equation H F E, I B plus H O E, V C E. The second subscript here E stands for common emitter CE common emitter. If it is a common base then we shall replace them by small B. If it's a common collector then we shall replace the second subscript by small C, is the point clear? We shall mostly consider the common emitter parameters okay, because the common emitter is the easiest to measure and to hand and design and analysis okay. So this second E stands for common emitter, the first I this I stands for input.

As you can see H I E is equal to V B E divided by I B under the condition V C E equal to 0. Naturally V B E and I B are at one single port at the base emitter port. V B E is the voltage and I B is the current that it drives, so this is the input this is the quantity measured at the input terminals and that is why this subscript first subscript I is there. In a similar manner if you look at H O E this is an output parameter because H O E is I C divided by V C E under the condition I B equal to 0. So this stands for output.

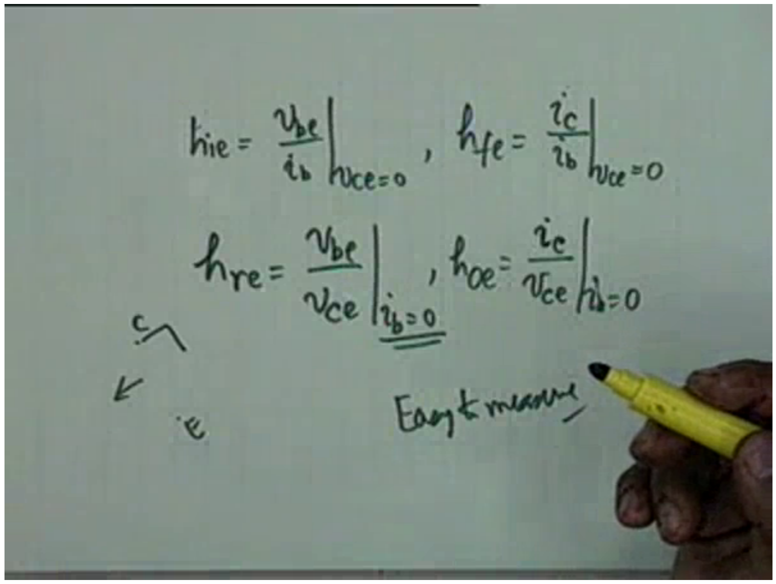
(Refer Time Slide: 0:22:55)



On the other hand the other two parameters, if you look at H R E it is equal to V B E divided by V C E under the condition I B equal to 0. Which means that if we apply a voltage across collector emitter terminals and measure the voltage across the base emitter by making I B equal to 0. That is by making base open circuited no current enters into the base then this is the ratio H R E and you see the transistor normally operates from base to collector. Base is the signal input and collector is the single output. Now here you're working in the reverse direction that is apply a signal at the collector emitter and take the signal out at the base emitter. So R stands for reverse voltage ratio. R stands for reverse.

And if you look at the other constants H F E, H F E as you can see is the ratio of I C divided by I B with V C E equal to 0. Now this is the ratio of 2 currents. What are the currents, the input is the base current and the output is the collector current under the conditions that the collector emitter port is short circuited as far as AC signals are concerned. And therefore this is a forward current transfer ratio, forward from left to right. And this F therefore stands for forward okay. So let me let me put down other 2 also. H I E is V B E divided by I B with V C E equal to 0, no let me write them clearly.

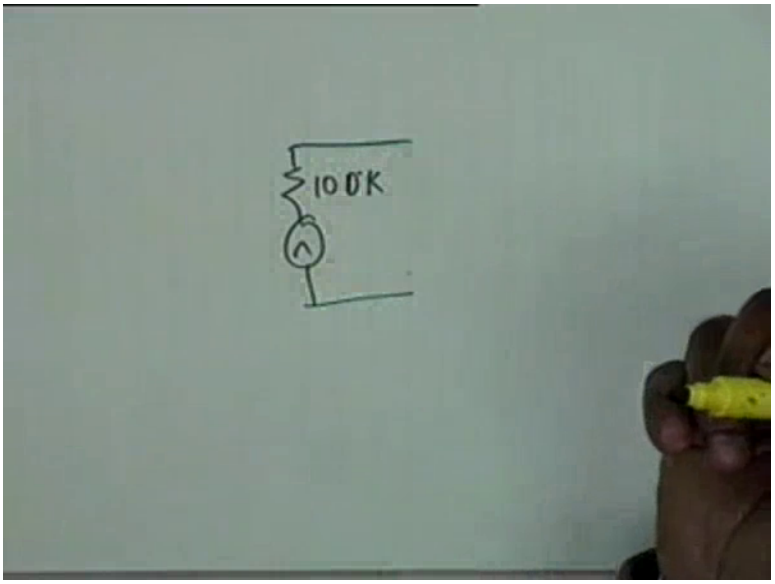
(Refer Time Slide: 0:24:58)



H I E is V B E divided by I B, V C E equal to 0 and H F E is equal to I C by I B V C E equal to 0 and H R E is equal to V B E divided by V C E with I B equal to 0. And H what is remaining? O E is equal to V no I C divided by V C E with I B equal to 0 okay. These are the definitions of the 4 parameters H parameters. Why are you interested in H parameters? If hybrid pie is good enough why are we interested in H parameter that is because H parameters are the easiest to measure experimental? They are easy to measure and why is it easy to measure? Easy to measure because V C E equal to 0 that is the collector emitter junction short circuited means that you have to include air resistance from collector to emitter which is much less compared to what resistance it sees.

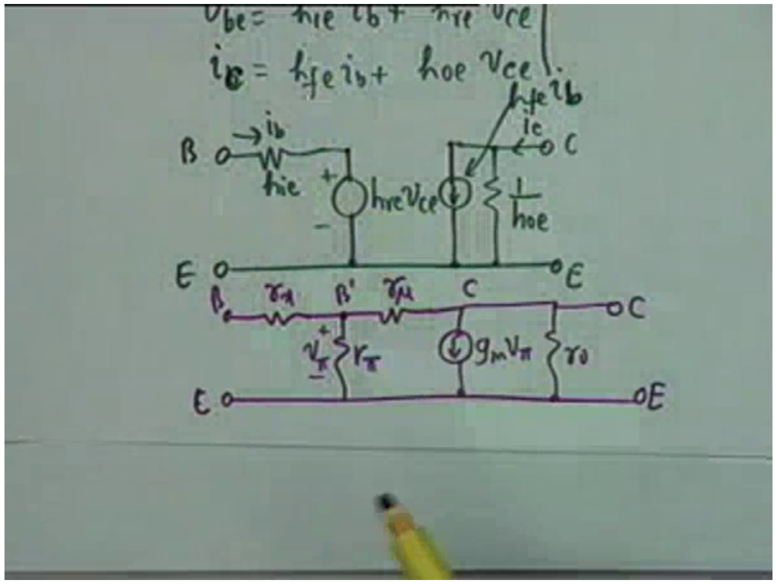
From the collector to emitter what is resistance seen by an outside source, it is R 0 the early resistance which is very very large and therefore between collector and emitter, even if you introduce a 1K resistance it acts as a short circuit isn't it right? It is very easy to short circuit the collector emitter junction, collector emitter terminal because the impedance faced by an external source at the collector emitter terminal is very large. If you have a very large impedance source it's very easy to short circuit alright. On the other hand I B equal to 0 means open circuit alright, open circuit to base. Now you know that if you connect an external source between the base an emitter, what resistance does it face?

(Refer Time Slide: 0:27:22)



It faces R X plus R pie of the order of a K and therefore if you have source of let say internal resistance 100K then virtually this is open circuit agreed? So it's very easy to open circuit a low impedance device and it's very easy to short circuit high impedance device. So experiment a measurement of the H parameters they are very easy. And mostly the manufacturers specify small H parameters rather than the Hybrid pie parameters. And the hybrid pie parameters usually are to be obtained from the H parameter and the relationship is very to derive.

(Refer Time Slide: 0:28:12)



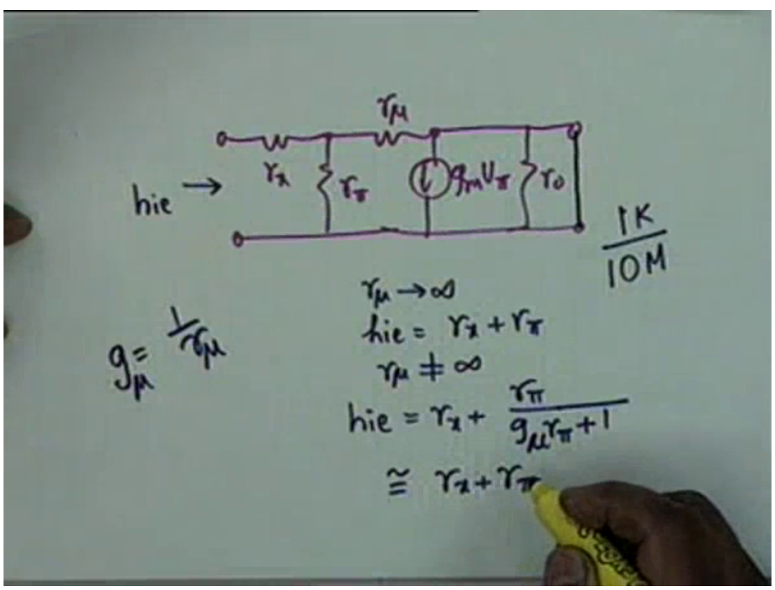
What you do is, first write this relationship that is V B E is equal to H I E, I B plus H R E, V C E and I C I beg your pardon. I C is H F E, I B plus H O E, V C E. If you look at this this relationship one this is obvious that that we can draw an equivalent circuit like this and equivalent circuit like this, between V B E and I B the relationship is there is an H R E this current is I B and then there is a voltage controlled voltage source. So I draw a voltage source which is H R E, V C E alright. The other one is the second relationship I C this is the collector current, this is I C. I C is the sum of 2 terms 1 is, 1 is H O E, V C E and H O E is in admittance, so if I want to write in terms of resistances only I should write 1 by H O E and then a current controlled current source. So I have H F E, I B.

And if you look at this equivalent circuit, there is an equivalent circuit that we have artificially constructed alright from these relationships by using 2 controlled sources. One is a voltage controlled voltage source, takes care of reverse transmission that is from the output to the input. And a forward transmission current controlled current source H F E, I B alright by utilising these 2 sources which incidentally shows that the device is an active device. If these two are not there, there is nothing no activity in the in the device. The presence of a current controlled current source and a voltage controlled voltage source shows that the device is active.

Now how does it differ from the hybrid pie? All that it differs is that if this is absent if this source is absent then you it's exactly the hybrid pie except for the connection between base and collector. So this the hybrid pie circuit has a similarity to the hybrid parameter equivalent circuit and that is why the main hybrid pie, is the point clear? These are the H parameter; this is the H parameter model. Slight modification of this leads to the model that we're interested in, which is derived from physical considerations and that's why that is called a hybrid pie model.

Let me draw a hybrid pie model side by side, what we have is between base and emitter we have R X, R pie then we have R mu, then we have if I call this V pie that this is gm V pie. And then we have R 0. Mind you we're considering only mid frequencies, that's why no capacitor here, otherwise we shall have to complicate things but including a capacitor here between the terminals of R pie which we shall call C pie. A capacitor here between the terminals emitter base prime B prime and C which we shall call C mu and a capacitor here which we shall call C 0 okay. This will make the circuit the hybrid pie parameter equivalent circuit complete. But since we're considering mid frequencies, it is this is enough which is (())(0:32:23). And the relationship between the H parameters and the hybrid pie parameters can now be very easily constructed okay. That is you calculate for example okay let's see.

(Refer Time Slide: 0:32:38)



You calculate H I E from the hybrid pie equivalent circuit, R X, R pie, R mu, gm, V pie, R 0 okay. How do I calculate the the H I E. H I E is the input impedance with the output, yes what we do to the output?

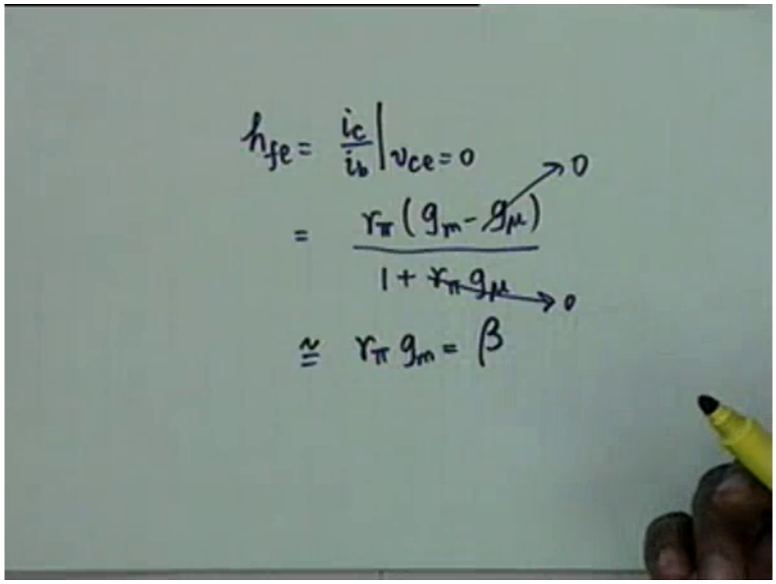
Student: Short circuit.

Professor: Short Circuit.

So I make it short circuited, if it is short circuited you see R 0 goes off, the current gm V pie well it shall be there, it shall be there it goes like this. And the current here, suppose R mu was not there, suppose R mu was infinity, then H I E would have been simply equal to R X plus R pie. On the other hand if R mu is not equal to infinity, one can easily calculate by circuit analysis that H I E, I have the relationship is equal to R X plus R pie divided by G mu R pie plus 1. This is the current relationship okay. If R mu is not infinity, and I shall leave this as an exercise to you to find out that this is correct.

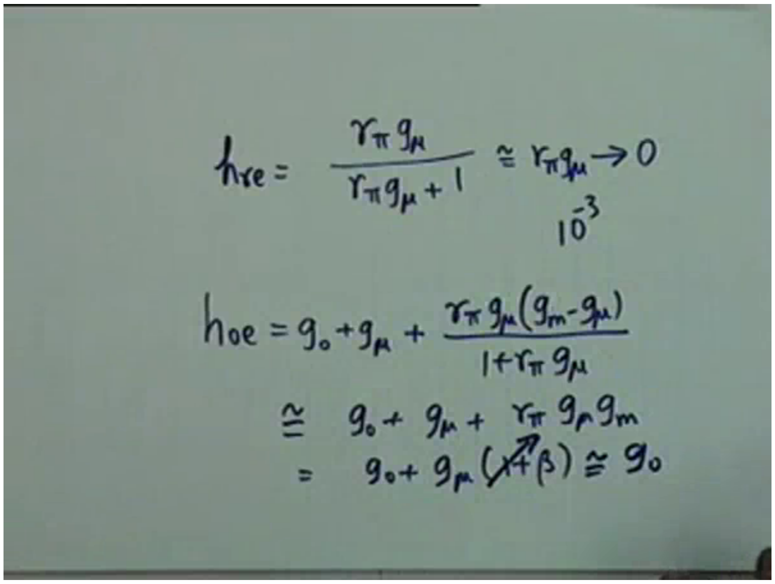
G mu all small g are equal to 1 by r. So g mu is equal to is 1 by r mu. Now usually as you see g mu R pie is R pie divided by R mu and R pie is of the order of a K and R mu is of the order of 10 Meg. And therefore this quantity g mu R pie is very small compared to unity. And therefore this does approximately become equal to R X plus R pie. Even if g mu is not 0 that is even if r mu is not infinity this is approximately correct. H I E is R X plus R pie, so if you are given now the question comes, if you're given H I E a measured quantity alright then how to do calculate R X and R pie. Obviously R pie has to be calculated from the collector current and then you can calculate R X is that correct alright?

(Refer Time Slide: 0:35:27)



You can also show that H F E from this equivalent circuit by applying the definition that is I C divided by I B these are all incremental quantities under the conditions V C E equal to 0. You can show that this is equal to R pie gm minus g mu. Again g mu is 1 by R mu. I leave this to show divided by 1 plus R pie G mu. And once again you see that g mu would be very small compared to gm. Gm is of the order of 0.4 I C in milli-amperes okay. And g mu is very small 1 by 13 Meg and therefore this is approximately you can ignore this and you can ignore this already. We have shown that. Therefore H F E is approximately equivalent R pie gm that is equal to the glorious quantity beta okay R pie gm. So H F E it is, if it is given you can directly equate it to beta no problem. And since you know gm you can calculate R pie. If beta if H F E is given you can calculate gm as 40 times I C okay, 40 times or 0.4 I C if I C is in milli-ampere. Therefore you can calculate the third quantity. These 3 are intimately related.

(Refer Time Slide: 0:37:05)

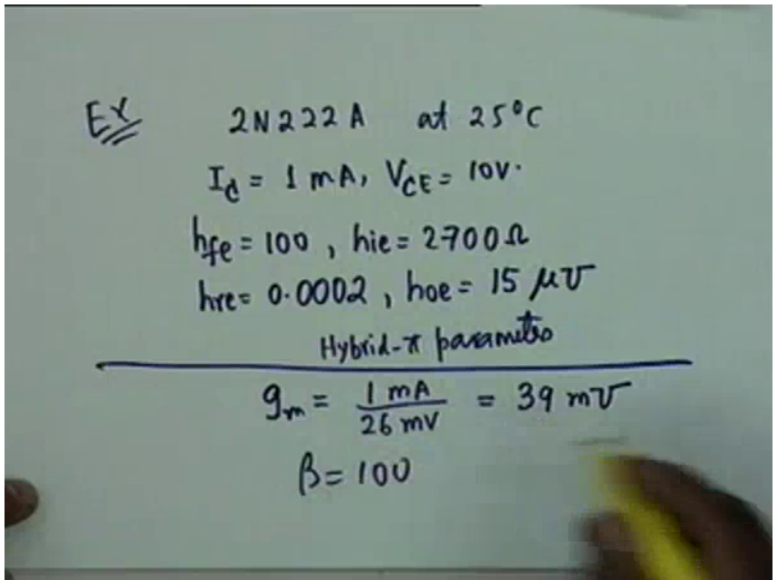


In a similar manner you can show that H R E the reverse voltage transfer coefficient is given by R pie g mu divided by R pie g mu plus 1. This can be measured. And since R pie g mu is much less than unity, this is approximately equal to R pie g mu. And if this is much less than unity this can be approximated to 0. Typical value of H R E, H R E what is the dimension? It is dimension less, it's a product of conductance and resistance, it's dimensionless. And the typical value is 10 to the minus 3. And is almost safely neglected by all electrical engineers except those who are very fussy.

H O E you can show, this is not a very simple exercise for H O E we have to calculate we have to connect a voltage source at the output and find out the current by open circuiting the input okay. And this comes out as g 0 plus g mu plus R pie g mu it's a fairly elaborate expression. As I said I shall check whether you have done this yourself or not, I leave this to you as an exercise. And I have my own ways of checking. You see that this quantity is approximately 0 okay. What is approximately 0? No this is not approximately 0. It is approximately R pie g mu gm and this is equal to g 0 plus g mu 1 plus beta.

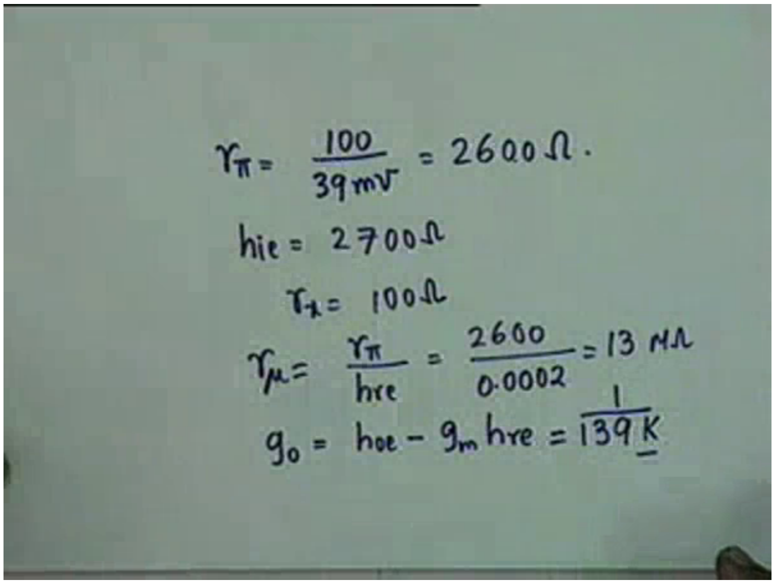
This 1 can be ignored and g mu beta g mu being a very small quantity can also be ignored therefore ultimately it is approximately g 0 which is the inverse of the early resistance okay. So if the H parameters are given one can calculate the hybrid pie parameters and this mostly you shall have to do because of historical reasons the manufacturer’s manufacturers have still not educated themselves. They usually give the H parameter rather than the hybrid pie parameters and as a circuit engineer, we shall have to calculate these parameters from what is given.

(Refer Time Slide: 0:39:41)



Let's take a typical example 2N222A transistor at 25 degrees C has the following operating point. I C is 1 milli-amperes and V C E is 10 volt. At this Q point the parameters measured or given by the given by the manufacturers are H F E is 100 H I E is 2700 ohms obviously. H R E is 0.0002 two times 10 to the minus 4, indeed a small quantity. And H O E is 15 micro more, the question is to find the hybrid pie parameter okay. Find the hybrid pie parameters, the first thing that we do is find out gm. Gm is I C, 1 milli-amperes divided by 26 milli-volts so this is not exactly faulty but 39 milli-volts okay. So you know gm. And H F E has been given and therefore H F E is beta therefore beta is 100 which means that you can find out R pie.

(Refer Time Slide: 0:41:18)



R pie is equal to 100 divided by 39 milli-volts, please take care of the dimensions, don't make a mistake and this comes out as 2600 ohms. H I E has been given as 2700 ohms therefore R X is equal to simply 100 ohms. If in your calculation R pie comes out as greater than H I E then there is something wrong. You must go back and check the check your calculation first then check manufacturer's specifications. If your calculation is right, manufacturers must have made a mistake and often such mistakes occur, that's why I am warning you. R mu is equal to you can show, that it is R pie divided by H R E okay from those relationship and you can show that this is equal to 2600 divided by H R E 0.0002 and this comes out as 13 meg, can be safely ignored in circuit calculation.

G 0 is H O E minus gm H R E this also you can show from the previous relations that you had established. I am not deriving this. And this comes out as 139K.

Student: No.

Student: 1 by 139.

Professor: This is 1 by 139k.

Alright so R 0 is 139k.

This is a good point to stop.