

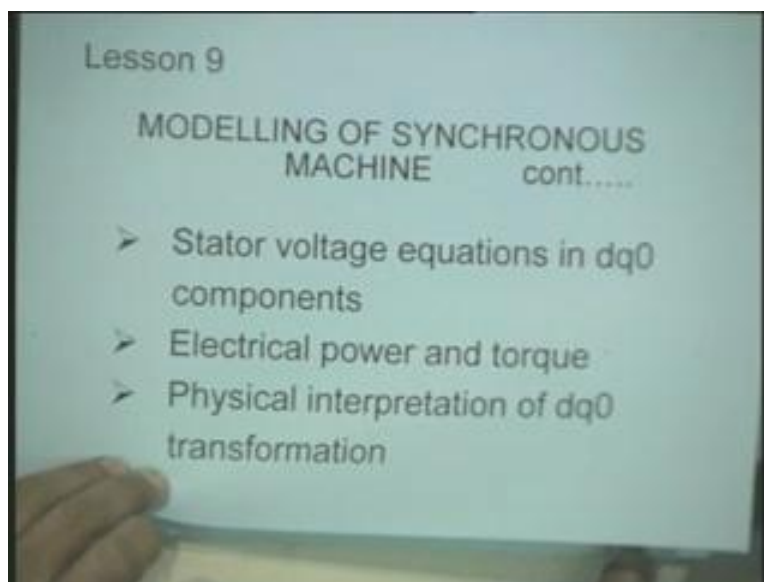
Power System Dynamics
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Lecture - 09
Modelling of Synchronous Machine (Contd...)

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Lecture - 9
Modelling of Synchronous Machine (Contd.)

Friends, today we shall continue to study about the modelling of synchronous machine.

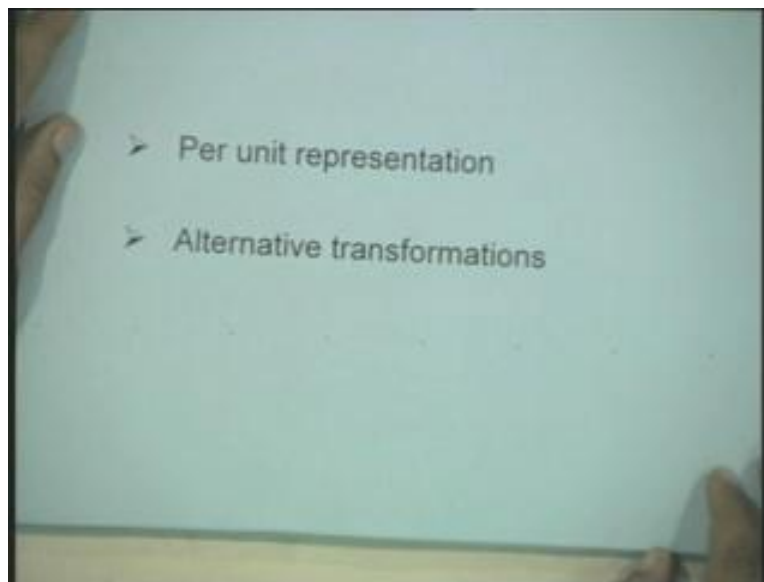
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- Lesson 9
- MODELLING OF SYNCHRONOUS MACHINE cont.....
- Stator voltage equations in dq0 components
 - Electrical power and torque
 - Physical interpretation of dq0 transformation

We have developed in the previous lectures, the basic model for the synchronous machine and we found actually that if we use the model in the original form then the coefficient that is the inductances were all function of rotor angular position. In order to simplify the model the dqo transformation was introduced and with this dqo transformation we found actually that the, the in the mathematical model synchronous machine the all inductances become constant.

Now we will further discuss now the stator voltage equation in dqo components. We will develop the equations for electrical power and torque. We shall discuss the dqO transformation further and its physical interpretation. Any synchronous machine model, we always prefer to use per unit system of representation and therefore, we will address how do we convert the synchronous machine quantities into per unit quantities then I will also discuss an alternative transformation which has all through been discussed in the literature on synchronous machine modelling. Let us again look at the stator voltage equations in terms of, in terms of the dqO components.

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The stator voltage equations were written in the form of $e_d = p\psi_d - \psi_q \frac{d\theta}{dt} - R_a i_d$, $e_q = p\psi_q + \psi_d \frac{d\theta}{dt} - R_a i_q$ and $e_o = p\psi_o - R_a i_o$. Now here, if you look at this first equation we can see here actually the means identify these terms, the first term $p\psi_d$ is the rate of change of flux linkages. Okay therefore similarly, in the second equation $p\psi_q$ and in the third equation we have the term $p\psi_o$ these are, these terms are known as the transformer voltages because they are taking place due to the rate of change flux linking the circuit. Then we have next term as $\psi_q \frac{d\theta}{dt}$ $\frac{d\theta}{dt}$ is the angular speed of the rotor that is ω_R , we call it now here this term. Similarly, this term $\psi_d \frac{d\theta}{dt}$ these two terms are denoted as speed voltages.

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Stator voltage equations in dq0 components

$$e_d = \underline{p\psi_d} - \underline{\psi_q} p\theta - R_a i_d \quad (8.46)$$

$$e_q = \underline{p\psi_q} + \underline{\psi_d} p\theta - R_a i_q \quad (8.47)$$

$$e_o = \underline{p\psi_o} - R_a i_o \quad (8.48)$$

We will further see that in in this equation similarly, in this equation the speed voltage terms dominate over the transformer voltage terms and in many simplifications we may neglect the transformer voltage terms as compared to the speed voltage terms the moment you neglect this then this equation these equations that is the stator voltage equations will become algebraic equations because if you just look here in this 3 equation then we have this as a derivative term and the moment you can neglect this derivative term right the remaining expression that is e_d equal to minus $\psi_q p\theta$ minus $R_a i_d$ $R_a i_d$ this becomes a algebraic equation right.

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Electrical Power and torque

$$P_t = e_a i_a + e_b i_b + e_c i_c$$

$$P_t = \frac{3}{2} (e_d i_d + e_q i_q + \underline{\underline{2e_o i_o}}) \quad (8.49)$$

Under balanced conditions

$$P_t = \frac{3}{2} (e_d i_d + e_q i_q)$$

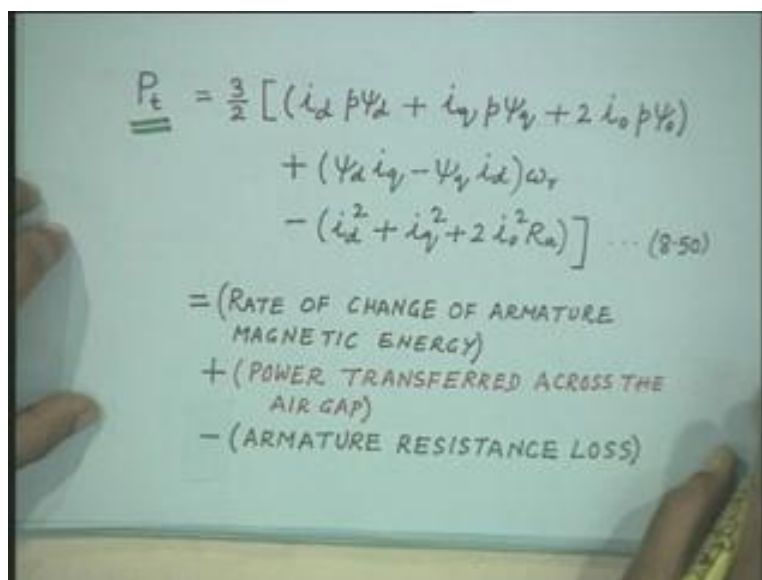
Now we will develop the expression for electrical power and torque in terms of dqO components. The electrical power output we denote it by the symbol P_t here, as can be obtained as product, some of the products of instantaneous voltage and instantaneous currents that is for phase a it is e_a into i_a , phase b e_b into i_b , phase c e_c into i_c right. These are the instantaneous powers you add these 3 instantaneous powers you get the total instantaneous power.

We all know actually that in case the system is having the stator currents and stator voltage very sinusoidally right, then the some of the instantaneous powers come out to be constant. Okay this is denoted by the symbol P_t , now what we do is that we apply dqO transformation. We can apply the dqo transformation on this voltage as well as the current currents and when you apply this dqo transformation then the power output P_t can be written as 3 by 2 times $e_d i_d$ plus $e_q i_q$ plus 2 times $e_o i_o$ that is here the terminal power is expressed in terms of dq components of voltage and currents okay.

Now we will see here actually in this that instead of having the term e_{naught} , i_{naught} we have a term 2 times e_{naught} , i_{naught} when we talk about another type of transformation that is called alternative transformation. We will find actually that the the P_t will be expressed simply as e_d into i_d plus e_q into i_q plus $e_o i_{naught}$ it is something like this the that becomes a power invariant transformation. Here this transformation is not power invariant because you can see there these are 3 by 2 terms then this term is having the coefficient 2 anyway this is resulting because of the type of transformation.

However when we convert these equations into per unit quantities, the per unit quantities will be so chosen, so that this becomes a simple expression and this term 3 by 2 and 3 can be eliminated we will find this thing okay. Now if the system is balanced we do not have zero sequence quantities a balance system this e_{naught} , i_{naught} will be 0 and therefore we can write down the total power P_t as 3 by 2 times $e_d i_d$ plus $e_q i_q$ okay.

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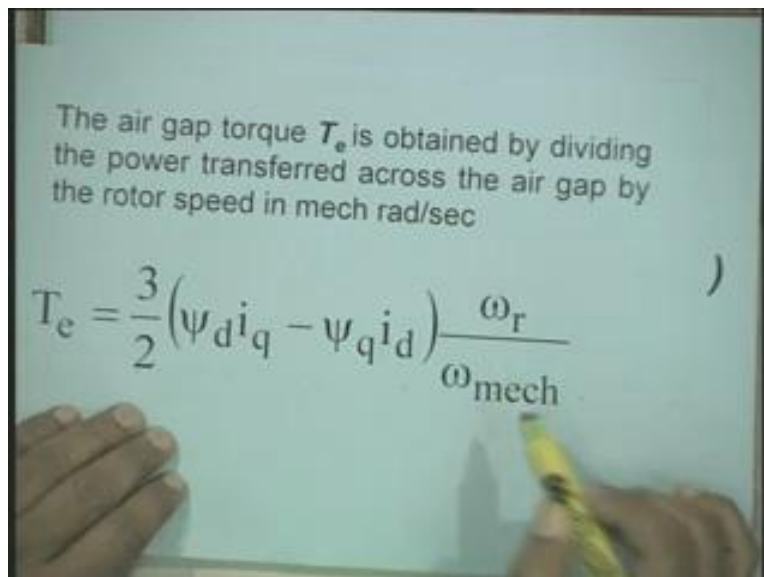
$$\begin{aligned}
 \underline{P_t} &= \frac{3}{2} \left[(i_d p\psi_d + i_q p\psi_q + 2 i_o p\psi_o) \right. \\
 &\quad \left. + (\psi_d i_q - \psi_q i_d) \omega_r \right. \\
 &\quad \left. - (i_d^2 + i_q^2 + 2 i_o^2 R_a) \right] \dots (8-50) \\
 &= (\text{RATE OF CHANGE OF ARMATURE MAGNETIC ENERGY}) \\
 &\quad + (\text{POWER TRANSFERRED ACROSS THE AIR GAP}) \\
 &\quad - (\text{ARMATURE RESISTANCE LOSS})
 \end{aligned}$$

Now to give further insight into the power expression what we do is that we substitute the expression for e_d and e_q , e_d and e_q in terms of the flux, flux linkages okay that is I substitute e_d equal to $p \psi_d \dot{\theta} - \psi_q \dot{\theta} - R_a i_d$ that is in this expression for this power wherever e_d and e_q are present what we do is that we substitute the expression for e_d okay.

Now if we make this substitution and simplify then we can write down the output power P_t as 3 by 2 term remains as it is $i_d p \psi_d$ plus $i_q p \psi_q$ plus 2 times $i_o p \psi_{naught}$ that is in this expression we have this derivative terms. Okay the second term is $\psi_d \dot{i}_q$ minus $\psi_q \dot{i}_d$ into ω_r that the ω_r is your $d\theta$ by dt that is the angular speed of the rotor minus i_d^2 square plus i_q^2 square plus 2 times i_o square into R_a .

Now this 3 terms which we have in the expression for P_t can be identified as the first term can be identified as rate of change of armature magnetic energy, rate of change armature magnetic energy, second term is identified as power transferred across the air gap and third term is armature resistance losses that is we have this 3 terms which can be identified as the rate of change of armature magnetic energy that is in these in the armature there is some magnetic energy stored because of flux linkages okay and this magnetic energy which is stored right is varying at a certain rate right, that magnetic energy is that are increases or decreases it depends upon whether $p \psi_d$, $p \psi_q$ and $p \psi_o$ are positive or negative right and from that point of view this is considered as the rate of change of armature magnetic energy and therefore when we are looking for stability analysis right.

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The air gap torque T_e is obtained by dividing the power transferred across the air gap by the rotor speed in mech rad/sec

$$T_e = \frac{3}{2} (\psi_d i_q - \psi_q i_d) \frac{\omega_r}{\omega_{mech}}$$

The the power which is transferred across the air gap is of concern that is in our swing equation which we had written right the electrical power p_e which we write that is the power transferred across the air gap because that is responsible for producing the torque and therefore now we write down T_e the electrical torque as three by 2 $\psi_d i_q$ minus $\psi_q i_d$

$\psi_q i_d \omega_r$ divided by $\omega_{\text{mechanical}}$ now this torque is written in Newton meters okay therefore we had this expression $\frac{3}{2} \psi_d i_q - \psi_q i_d \omega_r$, we are dividing by this $\omega_{\text{mechanical}}$.

We know that this term was the electromagnetic power that is the power transferred across the air gap. You divide this power by speed that will give you the torque okay and therefore, the expression for torque electrical torque is given by this expression now we know the relationship that the ω_r , the rotor speed in electrical radian per second divided by the mechanical speed in radian per second this ratio is equal to P_f by 2, where P_f is the number of pole pairs.

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$$T_e = \frac{3}{2} (\psi_d i_q - \psi_q i_d) \frac{P_f}{2} \quad (8.51)$$

Okay therefore, we can write down the expression for T_e the electrical torque as $\frac{3}{2} \psi_d i_q - \psi_q i_d \frac{P_f}{2}$ this is the most important expression which we have to keep in our mind. Now here the next step which we will be studying will be the how do we interpret the dqO quantities that this transformation what is the physical interpretation associated with this transformation.

Now we all know that the stator of the synchronous generator when it carries balanced three phase currents it produces resultant mmf wave, this mmf wave is sinusoidally distributed in the space that is the stator of a 3 phase synchronous generator produces sinusoidally distributed mmf wave in the air gap okay. Now any quantity which is sinusoidally distributed can be resolved into 2 sin terms.

Okay now here what is done is that this resultant mmf which are sinusoidally distributed is resolved into 2 quantities one along the d axis another along the q axis okay and therefore what is done is that the armature of the synchronous generator is replaced by 2 fictitious windings, one is along the d axis of the generator synchronous generator another is across the q axis of the synchronous generator and when these windings are

made to carry the current i_d and i_q right and they rotate at the same speed as the rotor it means it means the these two fictitious windings are stationary with respect to the rotor and therefore the mmf produced by this fictitious windings act on constant permeance path and therefore the inductances which we come across are constant.

We will see that under steady state conditions, the currents which are flowing in these two fictitious winding i_d that is direct axis fictitious winding and quadrature axis fictitious winding these two current that i_d and i_q will come out to be constant in magnitude they will be just DC currents, okay this we will just now establish.

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Physical interpretation of dq0 transformations

For balanced steady state conditions, the phase currents may be written as

$$i_a = I_m \sin(\omega_s t + \phi) \quad (8.52)$$

$$i_b = I_m \sin(\omega_s t + \phi - \frac{2\pi}{3}) \quad (8.53)$$

$$i_c = I_m \sin(\omega_s t + \phi + \frac{2\pi}{3}) \quad (8.54)$$

To establish this thing we will start with that let us assume that the armature of the synchronous generator is carrying balance steady 3 phase currents that is system is operating under steady state condition, okay no dynamics is involved let us assume that i_a is equal to $i_m \sin \omega_s t + \phi$ this ϕ is some initial phase angle of the current i_a this is arbitrary it can be 0 also i_b is equal to $i_m \sin \omega_s t + \phi - \frac{2\pi}{3}$ and i_c equal to $i_m \sin \omega_s t + \phi + \frac{2\pi}{3}$ that is these 3 currents i_a , i_b and i_c form balanced balanced set of currents okay.

Now what we do is we apply dqo transformation and when you apply the dqo transformation we will be in a position to write down i_d , i_q and this i_o will be 0 because we are considering the balanced steady state conditions that is in balance system the zero sequence quantities are absent. Now here I will just add one thing that when you talk about the zero sequence currents you always talk in terms of zero sequence current to be the phasor right here the zero sequence term we are putting is as it is the instantaneous component, instantaneous that is we are writing zero sequence as $i_a + i_b + i_c$ by 3 is the instantaneous quantity but this is applicable here.

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Using the dq0 transformation,

$$\underline{i_d} = I_m \sin(\omega_s t + \phi - \theta) \quad (8.55)$$
$$i_q = I_m \cos(\omega_s t + \phi - \theta) \quad (8.56)$$
$$i_0 = 0 \quad (8.57)$$

Now after applying this transformation this exercise has to be done that is in between you have lot of trigonometric simplification to be done after you do this trigonometric simplification the i_d the current in the d axis, d axis fictitious winding comes out to be equal to $I_m \sin(\omega_s t + \phi - \theta)$ i_q equal to $I_m \cos(\omega_s t + \phi - \theta)$ and i_0 is equal to 0. Okay now here what is this θ , θ is the { angular position of the rotor and angular position at any time is equal to $\omega_r t$, ω_r is the rotor speed under steady state operating condition the value of ω_r comes out to be same as the synchronous speed.

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For synchronous operation ,

$$\omega_r t = \omega_s t$$

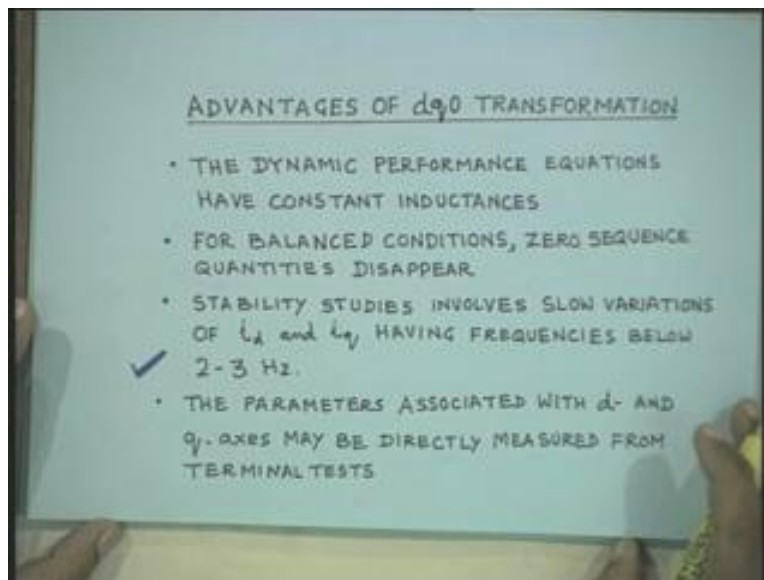
therefore

$$\begin{cases} i_d = I_m \sin \phi \\ i_q = I_m \cos \phi \end{cases}$$

Therefore, now we substitute here ω_r equal to ω_s into t right the moment you make this substitution here in this expression that is θ is to be put as ω_s into t this term vanishes and now we will have the i_d equal to $i_m \sin \phi$ similarly, i_q equal to $i_m \cos \phi$ right and therefore you can easily see here that for a given stator currents i_m is constant it is a peak or it is the amplitude of the phase current as if i_d can be written as $i_m \sin \phi$ i_q equal to $i_m \cos$ therefore what we see here that is i_d and i_q these are constant they are not varying as a function of time therefore they become DC current right.

However, under dynamic conditions when the system is dynamic condition rotor speed is not exactly equal to the synchronous speed right in that case the i_d and i_q will vary but the variation will be a low frequency variation, it is not going to be the 50 hertz or 60 hertz variation.

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Okay now some of the advantages which we get by this transformation can be summarized although we have seen these advantages but I will just summarize the advantages of dqO transformation. The first advantage is that the dynamic performance equations have constant inductances we have seen we have seen the mathematical model which we have written in terms of dqo transformations okay and in this dqo transformation by after applying this dqo transformation, the equations have constant inductance this is the one of the biggest merit of applying the dqO transformation, second is that under balance conditions zero sequence quantities disappear that I am not present the i_d and i_q under steady state conditions are constant in magnitude.

However, when we perform stability studies then stability studies involves slow variations of i_d and i_q the frequencies will be below 2 to 3 hertz that is the frequency of oscillation of the rotor which will come across will be of the order of, order of point 5 to say 3 hertz or we can say that the frequency of oscillation of these currents will be less than 2 to 3 hertz is very low frequency variation this is most important point we have to

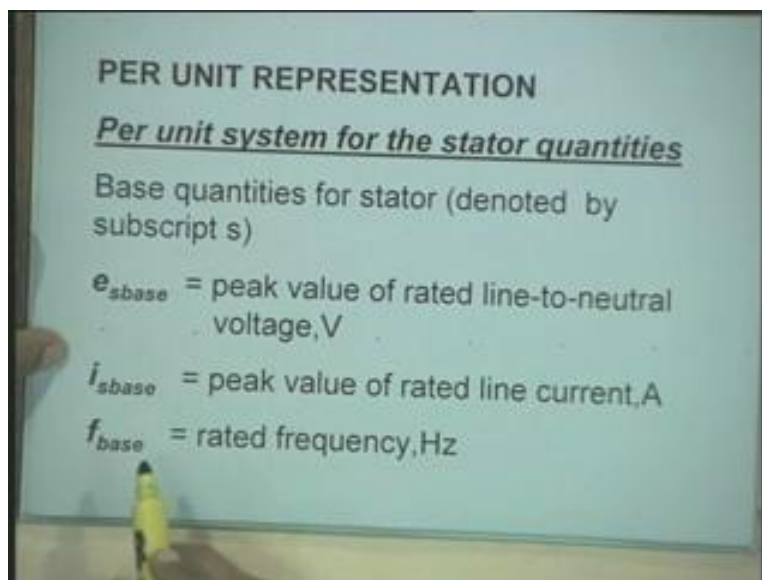
keep in mind and another very interesting advantage which comes is that that d and q axis quantities particularly which will inductance L_d , mutual inductance L_q these can be measured by performing experiments or by making some measurements on the terminals of the machine.

Since all of you must have done experiment in your laboratory to measure the direct and quadrature axis reactance of a synchronous machine. One important test which you perform in on is the slip test right. Now we come to another important aspect so for what we have done is we have developed the expressions for the stator voltage equations, rotor voltage equations. We have also developed expressions for the stator and rotor flux linkage equations we have also developed the expressions for electrical torque and given some meaning to the transformation, physical meaning to represent transformation.

Now in any power system studies per unit system of calculation is the most convenient one and merits of per unit calculations are well known to all of us and therefore now we will study how do we transform, transform our system equations using per unit quantities. Now whenever you transform the actual quantities into per unit quantities we have to choose base quantities.

For example, in any circuit we may choose MVA base and the voltage base then we can find out the base quantities in different parts of the circuit MVA base remains same while the KV base depends upon the circuit voltage that is the transformers which are involved. Now here, we have two parts, one is the stator another is the rotor. Okay therefore first we will define how we obtain the per unit representation of stator quantities.

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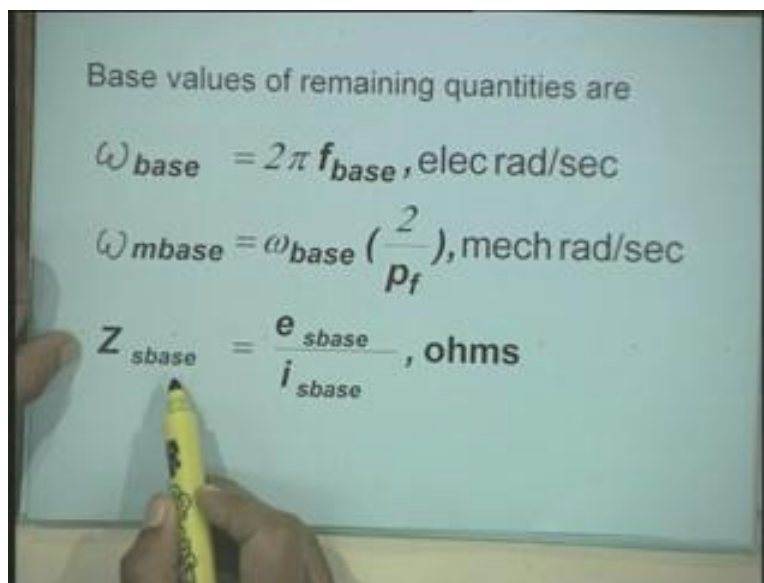


Now to start with here we will define the base quantities, now in any system we can define some base quantities and other quantities can be derived from those chosen base quantities like we start with stator quantity we will use this subscript s to denote that

these are the stator quantities or the base quantities for the stator. We will define e_{sbase} as peak value of the rated line to neutral voltage a slight variation is here that is instead of defining the RMS value as the base value here we are defining the peak value okay.

Similarly, the i_{sbase} that is the current in the stator circuit or the base current in the stator circuit is equal to peak value of the rated current in amperes. The additional quantity which we have to refer here is the base frequency. Generally, we do not require actually referring the base frequency when we talk about the power system network calculations but here this is also an important quantity, therefore these 3 terms are primarily decided therefore decision comes like this that rated frequency becomes my base frequency.

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Base values of remaining quantities are

$$\omega_{base} = 2\pi f_{base}, \text{elec rad/sec}$$

$$\omega_{mbase} = \omega_{base} \left(\frac{2}{p_f} \right), \text{mech rad/sec}$$

$$Z_{sbase} = \frac{e_{sbase}}{i_{sbase}}, \text{ohms}$$

Now with this 3 quantities defined other quantities are defined in terms of the basic base quantities like say omega base becomes $2\pi f_{base}$ okay, omega m base that is the mechanical speed right. The base value of the mechanical speed is written as the base value omega base into 2 by p_f further the impedance base Z_{sbase} is equal to e_{sbase} upon i_{sbase} base but we can define. The base value of the inductance in terms of stator base quantities that is L_{sbase} is equal to Z_{sbase} upon omega base. We can also define the flux linkage or base value of the flux linkage that is ψ_{sbase} equal to L_{sbase} into i_{sbase} . We know that flux linkage can be written as L into i L_{sbase} can be replaced by this quantity the L_{sbase} into i_{sbase} can be replaced as e_{sbase} upon omega s base that is what we do here is this L_{sbase} can be written as Z_{sbase} upon omega base okay and then Z_{sbase} into i_{sbase} can be written as e_{sbase} . So that the flux linkage the base value of the flux linkage can be written as e_{sbase} upon omega s Weber turns.

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$$L_{sbase} = \frac{Z_{sbase}}{\omega_{base}}, \text{ henrys}$$
$$\Psi_{sbase} = L_{sbase} i_{sbase}$$
$$= \frac{e_{sbase}}{\omega_{sbase}}, \text{ weber - turns}$$

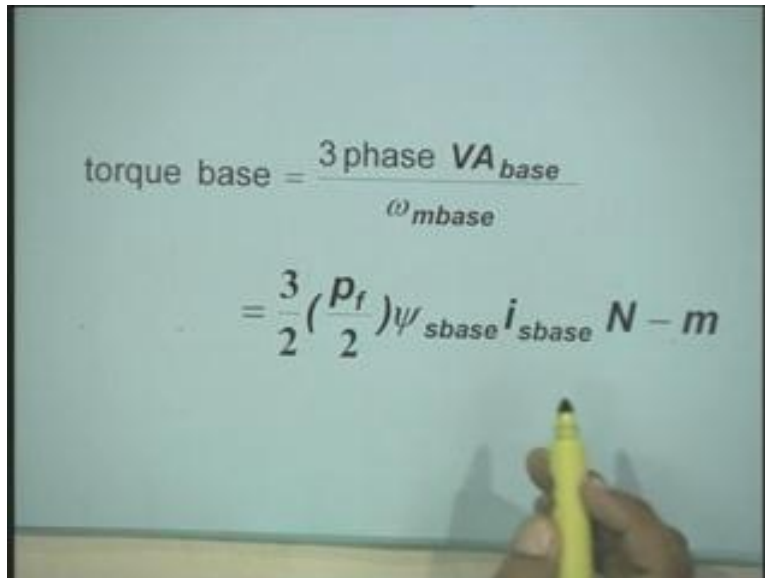
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$$\begin{aligned} \underline{3\text{-phase } VA_{base}} &= 3 E_{rmsbase} I_{rmsbase} \\ &= 3 \frac{e_{sbase}}{\sqrt{2}} \frac{i_{sbase}}{\sqrt{2}} \\ &= \frac{3}{2} e_{sbase} i_{sbase}, VA \end{aligned}$$

Now let us see actually that how we define the three phase VA_{base} volt ampere base because as I told you that your power system computations. We start with the 3 phase MVA_{base} , since we started with the voltage and current bases. Okay therefore we write down the three phase VA_{base} is defined as three times $E_{rmsbase}$ into $I_{rmsbase}$ that is the phase voltage its rms value multiplied by stator current and its rms value this can be written as now 3 rms value is equal to base quantity by root 2 because base quantity is chosen as the peak quantity here.

Similarly, i_{sbase} by root 2. So that what we see here is very very important relationship that is 3 phases VA_{base} comes out to be equal to 3 by 2 times the voltage base into current base. Okay this relationship is very important because we always make use of the three phases VA_{base} for our computations right but the relationship between the quantities which you have chosen as the base quantities and the 3 phase VA comes out to be like this.

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$$\text{torque base} = \frac{3 \text{ phase } VA_{base}}{\omega_{mbase}}$$

$$= \frac{3}{2} \left(\frac{P_f}{2} \right) \psi_{sbase} i_{sbase} \text{ N-m}$$

Then the torque base the torque base can be always written as three phase VA_{base} divided by omega m base. Now when you talk about the base quantities or base power we always say base power is equal base MVA if you recollect in your power system calculations or computations then whatsoever base MVA is there that is equal to base power in megawatts and therefore power divided by the base speed will give you the torque base therefore here the torque base is equal to 3 phase VA base that is omega m okay.

Now when you make the substitutions here in this expression for 3 phase VA_{base} equal to 3 by 2 e_{sbase} , i_{sbase} and this e_{sbase} and i_{sbase} , these quantities can be further expressed in terms of flux linkage base. Therefore, torque is now expressed as 3 by 2 P_f by 2 psi s base into i_{sbase} because we have seen earlier actually the torque is written in terms of flux linkages and currents right therefore we prefer to specify the base or torque base in terms of flux linkages and current base quantities. Okay this is in Newton meters.

Now what till now what we have done is that started initially the definition of base quantities in the stator we have chosen the stator voltage and stator current and the system frequency as the base quantities okay and then we have derived that what will be the base quantities for other variables, other variables like say impedance, base impedance, base torque base flux linkages, base flux base in base inductance and so on.

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Per unit stator voltage equations

$$e_d = p\psi_d - \psi_q\omega_r - i_d R_a$$

Now we start with transforming our stator voltage equations into per unit terms or per in terms of per unit quantities is it not okay now again let us start or stator voltage equation is e_d equal $p\psi_d - \psi_q\omega_r - i_d R_a$ okay this is the equation which we have derived in terms of dq0 components we want to transform this equation in terms of per unit quantities how do we do it you divide this whole expression by voltage base because these all these terms are the voltage terms therefore what you divide you divide this by e_{sbase} .

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Dividing through out by e_{sbase} and noting that $e_{sbase} = i_{sbase} Z_{sbase} = \omega_{sbase} \psi_{sbase}$

$$\frac{e_d}{e_{sbase}} = p \left(\frac{1}{\omega_{sbase} \psi_{sbase}} \psi_d \right) - \frac{\psi_q}{\psi_{sbase}} \frac{\omega_r}{\omega_{sbase}} - \frac{R_a}{Z_{sbase}} \frac{i_d}{i_{sbase}} \quad (8.58)$$

Now when you divide this by e_{sbase} we make use of this relationship that e_{sbase} is equal to Z_{sbase} it is also equal to ω_{sbase} into ψ_{sbase} this relationships we have already derived that is this 3 the voltage can be written as the product of impedance and current can also be written as product of omega and flux linkages okay. Therefore, what we are doing here is that e_d is divided by e_{sbase} p remains as it is we are not touching p right now ψ_d since it is a flux term what we do is that we divide this flux term instead of e_{sbase} i put ω_{sbase} , ψ_{sbase} here because e_{sbase} is same as this quantum.

Similarly, here we put ω_{sbase} , ω_{sbase} okay and in the third term where I have R_a into i_d I put here not in terms of the flux linkages and speed by put errors Z_{sbase} into i_{sbase} because this the denominator terms which I have put here are same as e_{sbase} this can be done okay. Now by doing this what happens is that this e_d becomes now per unit value. Okay that is the direct axis voltage e_d can now it becomes a per unit quantity.

Similarly, the flux linkage becomes per unit, the resistance becomes per unit resistance, now the current becomes per unit current, okay now to distinguish the per unit quantities from the quantities a real quantities in amperes or ohms or watts right what we do is that we will put a super script bar, okay a super bar on the top that is I will denote this as $\bar{e}_d$ equal to $1/\omega_{base}$ $p \bar{\psi}_d - \bar{\psi}_q \bar{\omega}_r - \bar{R}_a \bar{i}_d$ as we will see subsequently the moment we develop a mathematical model of the system in terms of per unit quantities right.

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Expressed in per unit notation,

$$\bar{e}_d = \frac{1}{\omega_{base}} p \bar{\psi}_d - \bar{\psi}_q \bar{\omega}_r - \bar{R}_a \bar{i}_d \quad (8.59)$$

$$t_{base} = \frac{1}{\omega_{base}} = \frac{1}{2\pi f_{base}} \quad (8.60)$$

$$\bar{e}_d = \bar{p} \bar{\psi}_d - \bar{\psi}_q \bar{\omega}_r - \bar{R}_a \bar{i}_d \quad (8.61)$$

We will drop this super bar, drop it because every time writing this bar is not very convenient but when you are trying to derive it first we will maintain this bar and then at the end we will drop it for further use of this model. Now here in this equation p is d/dt and time is in seconds, okay therefore we can also define the base value of time and the definition of base value of time is the time in seconds required for the rotor to rotate by 1 electrical radian, 1 electrical radian per second electrical radian turn per

second electrical radian that when rotor makes one electrical radian and the time which is required to make 1 radian rotation that is called t_{base} that is 1 radian divided by ω_{base} that is when rotor rotates through a angular displacement of one radian the speed is ω_{base} base speed then the that particular time is called t_{base} base time okay.

Therefore, this base time can be written as $1/\omega_{base}$ and if I now use this base time then this term can be replaced by $\bar{p}$, $\bar{\psi}_d$ minus $\bar{\psi}_q$, $\bar{\omega}_r$ minus $\bar{R}_a \bar{i}_q$ that is this expression is now in terms of per unit quantities that is $\bar{p}$ is also expressed in per unit and all other terms are in per unit.

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similarly

$$\bar{e}_q = \bar{p} \bar{\psi}_q + \bar{\psi}_d \bar{\omega}_r - \bar{R}_a \bar{i}_q \quad (8.62)$$

$$\bar{e}_0 = \bar{p} \bar{\psi}_0 - \bar{R}_a \bar{i}_0 \quad (8.63)$$

where

$$\bar{p} = \frac{d}{d\bar{t}} = \frac{1}{\omega_{base}} \frac{d}{dt} = \frac{1}{\omega_{base}} p \quad (8.64)$$

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Per unit rotor voltage equations

Dividing Eq(8.25) to Eq(8.27) by

$e_{fdbase} = \omega_{base} \psi_{fdbase} = Z_{fdbase} i_{fdbase}$, we obtain

$$\bar{e}_{fd} = \bar{p} \bar{\psi}_{fd} + \bar{R}_{fd} \bar{i}_{fd} \quad (8.65)$$

Similarly, you can write down for the q axis voltage and zero sequence voltage. We have just now mentioned that $\bar{p}$ was defined as d/dt where $\bar{d}$ is a per unit current which comes out to be $\bar{p}$ comes out to be one upon ω base into $\bar{p}$ okay and we have seen here actually that one upon ω base into $\bar{p}$ is the it comes out to be equal to $\bar{p}$ that is why we have come to now this model, this model where $\bar{e}_d$ is equal to $\bar{p}$ bar, ψ_d bar minus ψ_q bar, ωR_a bar, $\bar{i}_d$ bar and similarly for $\bar{e}_q$ and $\bar{e}_0$ that is we have developed the stator circuit equations in per unit terms okay.

Now the next step will be to develop the rotor circuit equations also in per unit quantities. Now here to start with what I will do is we will assume the rotor circuit voltage, rotor circuit currents as the base quantities, I am not I am not addressing at present how do we choose that we will see later on but assuming that the for the rotor circuit, the field circuit because in the rotor we have 3 circuits one is the field circuit another is the direct axis amortisseur circuit, third is the quadrature axis amortisseur circuit and therefore for the field circuit a base voltage will be $\bar{e}_{fd}$ base which will be written as ω base into ψ_{fd} base that is ψ_{fd} is the flux linkage in the field circuit a base quantity in the field circuit.

Similarly, we will have $Z_{fd\text{base}}$ into $\bar{i}_{fd\text{base}}$ okay that is as we have done in the case of stator similarly we specify the quantities in the rotor also. Okay but how do we choose this base quantities in the rotor that we will establish in few minutes. Therefore, now following the same approach as we have done for the stator we can write down $\bar{e}_{fd}$ bar equal to $\bar{p}$ bar ψ_{fd} bar, R_{fd} bar, $\bar{i}_{fd}$ bar that is in this equation where actually we have field circuit applied voltage is $\bar{e}_{fd}$ okay the current flowing is $\bar{i}_{fd}$ flux linkage is ψ_{fd} therefore we can establish the equation in the per unit quantity they are very simple as state forward the approach is exactly same you divide by $\bar{e}_{fd}$ base the actual quantities and then wherever you have this R_{fd} , $\bar{i}_{fd}$ divided $Z_{s\text{base}}$ $Z_{fd\text{base}}$ $\bar{i}_{fd\text{base}}$.

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$$0 = \bar{p} \bar{\psi}_{kd} + \bar{R}_{kd} \bar{i}_{kd} \quad (8.66)$$

$$0 = \bar{p} \bar{\psi}_{kq} + \bar{R}_{kq} \bar{i}_{kq} \quad (8.67)$$

Okay and you will get this equation other two equations also in the same form because here we do not have term because they are all close circuits amortisseur closed circuit therefore 0 equal to okay.

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Per unit Stator flux linkage equations

Using the basic relationship $\psi_{sbase} = L_{sbase} i_{sbase}$, then per unit equations for flux linkages are given by,

$$\bar{\psi}_d = -\bar{L}_d \bar{i}_d + \bar{L}_{afd} \bar{i}_{fd} + \bar{L}_{akd} \bar{i}_{kd} \quad (8.68)$$

Now we come to very ah crucial thing actually that is how do we express the stator flux linkages in per unit. Okay because so far what we have done is the stator voltage circuit equation, stator circuit voltage equations, rotor circuit voltage equations. We have expressed in per unit terms therefore now our next step is to express the stator flux linkage equations in per unit.

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$$\psi_d = -L_d i_d + L_{afd} i_{fd} + L_{akd} i_{kd}$$

$$\frac{\psi_d}{\psi_{sbase}} = -\frac{L_d}{L_{sbase}} \frac{i_d}{i_{sbase}} + \frac{L_{afd}}{L_{sbase}} \frac{i_{fd} i_{sbase}}{i_{sbase} i_{sbase}} + \frac{L_{akd}}{L_{sbase}} \frac{i_{kd}}{i_{sbase}} \cdot \frac{i_{sbase}}{i_{sbase}}$$

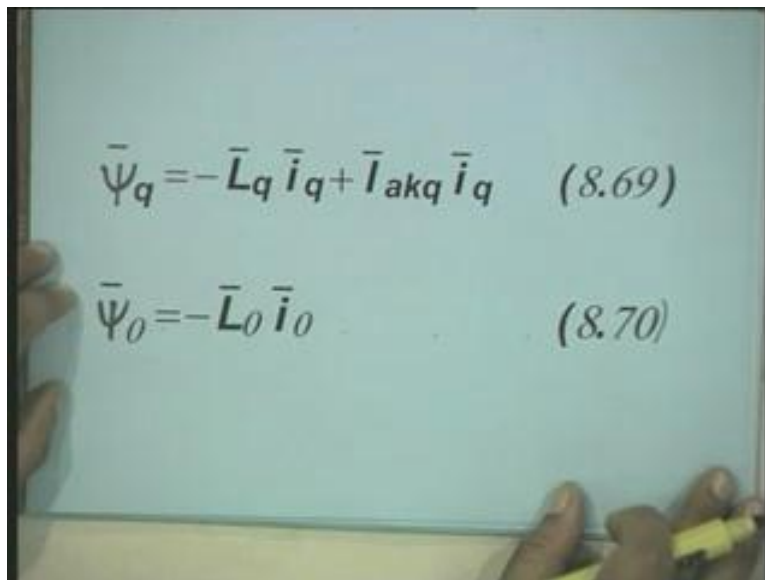
$$\bar{\psi}_d = -\bar{L}_d \bar{i}_d + \bar{L}_{afd} \bar{i}_{fd} + \bar{L}_{akd} \bar{i}_{kd}$$

Now to understand this let me ah rewrite here I have written again ψ_d is equal to minus $L_d i_d$ plus $L_{af} i_{fd}$ L_{akd} , i_{kd} this equation was derived this is the direct axis flux linkages ψ_d equal to minus $L_d i_d$ plus L_{afd} what is L_f this is the mutual inductance between the d axis and field winding. This is actually amortisseur and d axis winding that is why L_{akd} .

Okay now what you do is that you divide by this whole equation by base, when you divide this by base ψ_d will be divided by ψ_s base, ψ_s base is equal to L_{sbase} into i_{sbase} . Okay therefore you can divide the whole quantities by this base quantity right. Therefore you will have L_d divided by i_d divided by L_d into i_d divided by L_{sbase} is base then L_{afd} into i_{fd} divided by L_{sbase} into i_{sbase} .

Now here the problem comes that the per unit value, per unit value of the field current is expressed in terms of field current base not in terms of the stator current base okay therefore what you do is that you multiply this term is okay you multiply this term by i_{fdbase} into i_{fdbase} . Okay then in this term what we can do here is that this i_{fd} divided by i_{fdbase} can be written as per unit current in the field circuit then remaining quantities L_{afd} divided by L_{sbase} into i_{fdbase} divided by L_{sbase} this quantity will be denoted by a term L_{afd} bar that is you can see very interesting thing that we have the now this 3 ratios. Okay we can identity this ratio i_{fd} upon i_{fdbase} as i_{fd} bar right then we have the term L_{afd} by L_{sbase} then i_{fd} base upon i_{sbase} okay.

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$$\bar{\Psi}_q = -\bar{L}_q \bar{i}_q + \bar{L}_{akq} \bar{i}_q \quad (8.69)$$

$$\bar{\Psi}_0 = -\bar{L}_0 \bar{i}_0 \quad (8.70)$$

Now this these are when you divide these two base currents it comes to be a multiplying factor only and therefore what we do is that this L_{afd} will be denoted by a symbol L_{afd} bar to i_{fd} bar. Similarly, you will have L_{akd} bar i_{kd} bar okay is it understood. Now once you understand this thing actually the I have done it for one direct axis plus linkage equation. Similarly. We have to do it for other flux linkage equations that is by doing this exercise we have written the expression for stator winding flux linkage equations that is

the mutual flux linkage this ψ_d is the mutual flux linkage $\bar{\psi}_d$ equal to minus L_d $\bar{i}_d$ plus L_{afd} $\bar{i}_{fd}$ plus L_{akd} $\bar{i}_{kd}$.

Now these terms are defined as L_{afd} $\bar{i}_{fd}$ is defined as now we have just now seen that L_{afd} $\bar{i}_{fd}$ is defined as L_{afd} upon L_{sbase} into i_{fdbase} upon i_{sbase} right. Similarly, L_{akd} $\bar{i}_{kd}$ L_{akq} $\bar{i}_{kq}$ right these terms were defined in terms of the mutual inductances, base quantities, the stator and base quantities in the field circuit. This is very important actually speaking here here actually it is not state forward where we have a base actual quantity divided by base gives you a per unit quantity here that is let me just summarize here that the direct axis flux linkages are written in this form where these quantities are to be defined okay.

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Where,

$$\bar{L}_{afd} = \frac{L_{afd}}{L_{sbase}} \frac{i_{fdbase}}{i_{sbase}} \quad (8.71)$$

$$\bar{L}_{akd} = \frac{L_{akd}}{L_{sbase}} \frac{i_{kdbase}}{i_{sbase}} \quad (8.72)$$

$$\bar{L}_{akq} = \frac{L_{akq}}{L_{sbase}} \frac{i_{kqbase}}{i_{sbase}} \quad (8.73)$$

Similarly, ψ_q is written in this form and ψ_{naught} is written in this form, okay they are the three flux linkage equations in the stator circuit okay and where these terms are defined as L_{afd} $\bar{i}_{fd}$ by this expression L_{akd} $\bar{i}_{kd}$ by this expression L_{akq} $\bar{i}_{kq}$ by this expression do you understand. Now after we have expressed express the stator flux linkages ψ_d , ψ_q and ψ_{naught} in terms of or converted the stator flux linkages equations into per unit quantities, next step is to convert the rotor flux linkage equations.

Now when you convert the rotor flux linkage equations, we will come across the $\bar{\psi}_d$ that is actually this is the flux linkage of the field circuit you can you again start with the original equations, original equations and then divide by the base quantities right and then then these terms may have to be again defined. These terms will not be required to be defined that is L_{ffd} will be simply equal to L_{ffd} is the self-inductance of the field winding right therefore, you divide this by the L_{fdbase} right where this is the rotor quantity divided by the rotor base quantity it becomes a per unit quantity therefore there is no problem here.

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Per unit Rotor flux linkage equations

$$\bar{\Psi}_{fd} = \bar{L}_{ffd} \bar{i}_{fd} + \bar{L}_{fkd} \bar{i}_{kd} - \bar{L}_{fda} \bar{i}_d \quad (8.74)$$

$$\bar{\Psi}_{kd} = \bar{L}_{kdf} \bar{i}_{fd} + \bar{L}_{kkd} \bar{i}_{kd} - \bar{L}_{kda} \bar{i}_d \quad (8.75)$$

$$\bar{\Psi}_{kq} = \bar{L}_{kkq} \bar{i}_{kq} - \bar{L}_{kqa} \bar{i}_q \quad (8.76)$$

Similarly, for the current also there will be no problem but when we talk about this i_{kd} the L_a , L_{fkd} divided by i_{kd} . Okay you will find this similar problem will come right and therefore these quantities have to be expressed in terms of for example i_{kd} is to be expressed in terms of $i_{kd\text{base}}$. Similarly, i_{kq} has to be expressed in terms of its own base right.

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$$\bar{L}_{fda} = \frac{3}{2} \frac{L_{afd}}{L_{fdbase}} \frac{i_{sbase}}{i_{fdbase}}$$

$$\bar{L}_{fkd} = \frac{L_{fkd}}{L_{fdbase}} \frac{i_{sbase}}{i_{fdbase}}$$

Therefore, when you do this exercise you will find that we can express the rotor circuit to flux linkages in terms of per unit quantities where these terms $\bar{L}_{fda}$ bar, $\bar{L}_{fkd}$ bar are to be defined as all of the all this 5 terms I mentioned this 5 terms will appear in those three equations are to be defined in this base.

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$$\bar{L}_{kda} = \frac{3}{2} \frac{L_{akd}}{L_{kdbase}} \frac{i_{sbase}}{i_{kdbase}} \quad (8.79)$$

$$\bar{L}_{kdf} = \frac{L_{fkd}}{L_{kdbase}} \frac{i_{fdbase}}{i_{kdbase}} \quad (8.80)$$

$$\bar{L}_{kqa} = \frac{3}{2} \frac{L_{akq}}{L_{kqbase}} \frac{i_{sbase}}{i_{kqbase}} \quad (8.81)$$

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Per unit system for the rotor

The rotor circuit base quantities will be chosen so as to make the flux linkage equations simple.

- The per unit mutual inductances between different windings are to be reciprocal.
- All per unit mutual inductances between stator and the rotor circuits in each axis are to be equal.

Therefore, now what we see here is that that we have established, establish the rotor circuit equations that is rotor circuit voltage equations, rotor circuit flux linkage

equations, okay in per unit terms while the basic problem here remains is here remains is that how do we choose the rotor circuit base quantities because synchronous machine is having stator and rotor parts, what is the relationship of base quantities in the rotor circuit with the stator circuit base quantities right and for that we have to establish some basic rules and I will just state what rules are to be established is that when we choose the base quantities for the rotor and we want our our intention is our our intention or the objective is we want to simplify the equations make this equations as simple as possible.

Therefore we have 2 major assumptions or requirements but in the per unit mutual inductances between different winding are to be reciprocal. This is one requirement that we would like to choose the per unit quantities in such a fashion, so that the mutual inductances become reciprocal right that is mutual inductance between the d axis field winding d axis winding that is end the the field winding there should be equal therefore if I write down that L_{afd} should be equal to L_{fda} .

Okay another thing which would like to do is that all per unit mutual inductances between stator and rotor circuit in each axis are to be equal that that is another way of simplifying that we would like to make this inductances mutual inductances on each axis equal right with this I conclude my presentation today and let me summarize what we have done today. We have discussed the stator circuit, voltage circuit equations in terms of dqO components. We have also transformed the stator and rotor circuit voltage and flux linkage equations in per unit quantities. Thank you!