

Power System Dynamics
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Lecture – 37
Voltage Stability (Contd...)

Friends, we continue with the study of voltage stability.

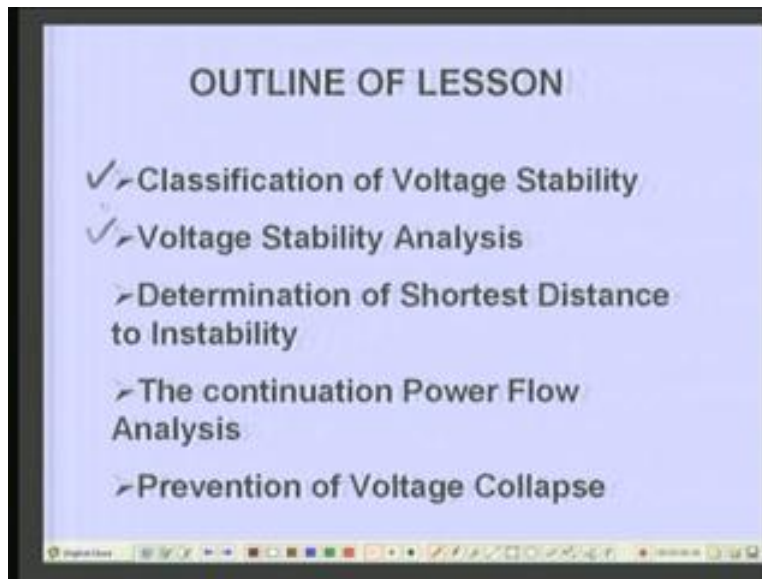
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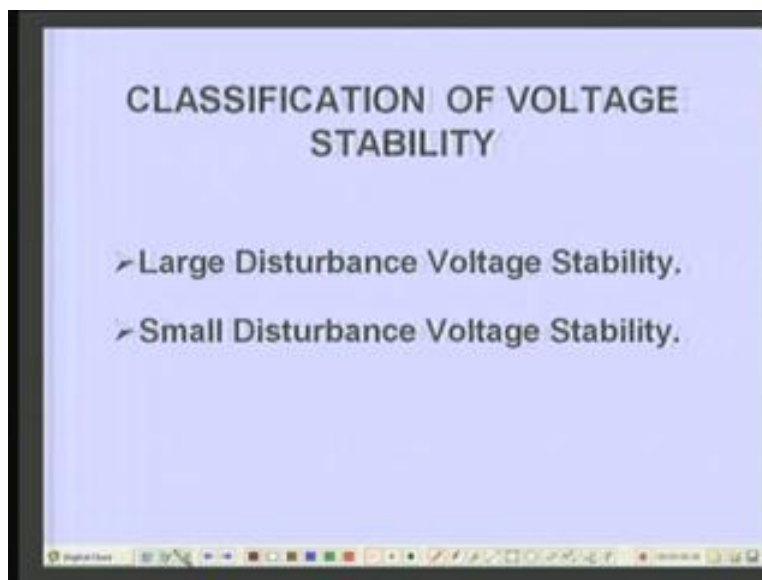
We shall examine today the classification of voltage stability. We will also examine the techniques for voltage stability analysis, under this voltage stability we may have to study determination of shortest distance to instability the continuation power flow analysis and prevention of voltage collapse. Today, we will be addressing the first two points pertaining to the voltage stability.

In the previous lecture, I have discussed about the classification of voltage stability as a large disturbance voltage stability and small disturbance voltage stability. This classification is similar to what is done for for angle stability analysis. The large disturbance voltage stability is concerned with the systems ability to control voltages following large disturbances such as faults, loss of load or loss of generation that is whenever actually the **the the** disturbance is large in magnitude right then we call this as a large disturbance voltage stability problem.

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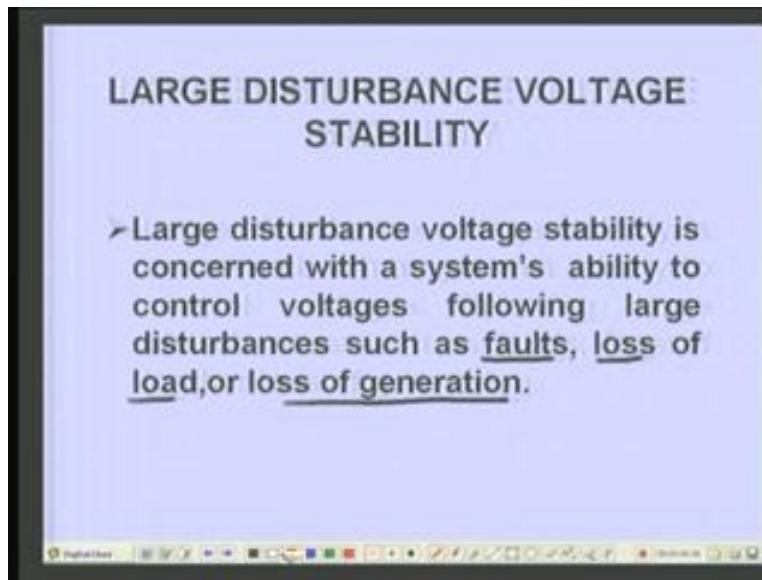
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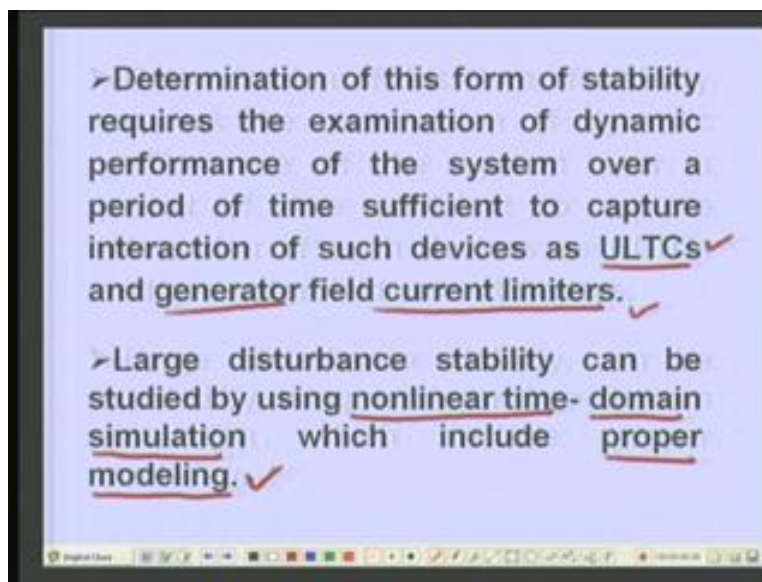
Now when examining the large disturbance voltage stability of a system, one has to one has to model and exclusively include the operation of some of the devices such as under load tap changer generator field current limiters. Generally, what happens is that these devices ULTC they operate slowly, now suppose the voltage is low voltage is at low level then from that stand till the first step changing takes place the time delay is quite large right and therefore when you model the system dynamics or to capture the complete dynamics of the system one has to include the **the** dynamics of some of these devices like ULTCs and generator excitation current

limiters and so on. In fact the large disturbance stability can be studied by using non-linear time domain simulation which include proper modeling.

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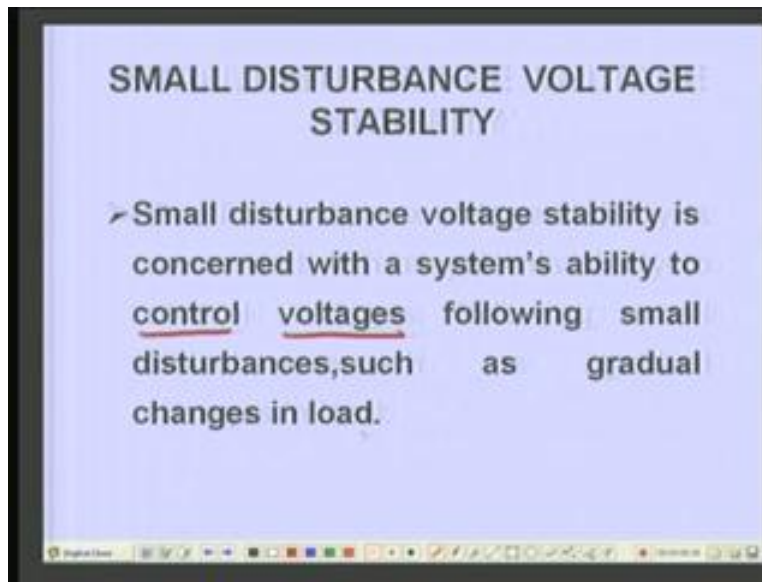
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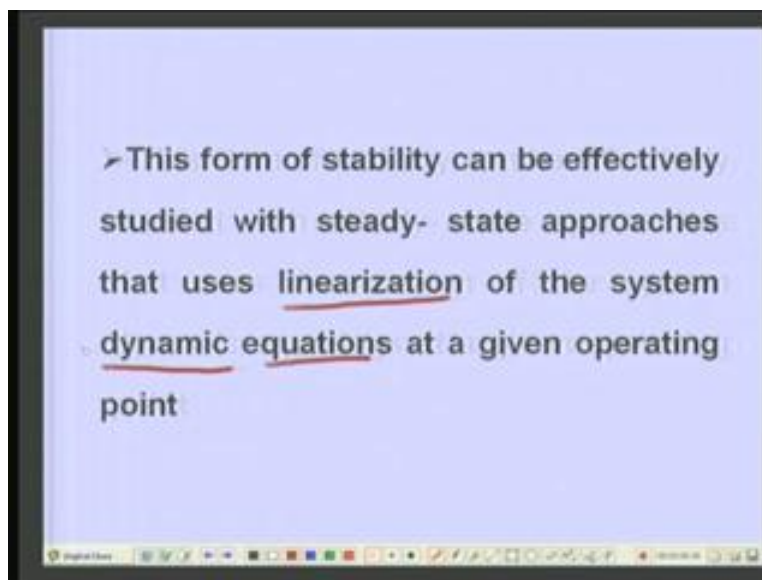
Now when we talk about proper modeling it means it should include the modeling of all the devices when we have discussed the angle stability we we, our thrust of modeling is the modeling of the generator, modeling of excitation system, modeling of controls on the generator and excitation system but we hardly can say concentrate on modeling of loads in the

detail right but here when we talk about the the voltage stability one has to model the devices in more detail because this voltage stability is is relatively a slow phenomena and one has to capture the the dynamics of the slow devices like say ULTCs similarly, the operation of thermostats right.

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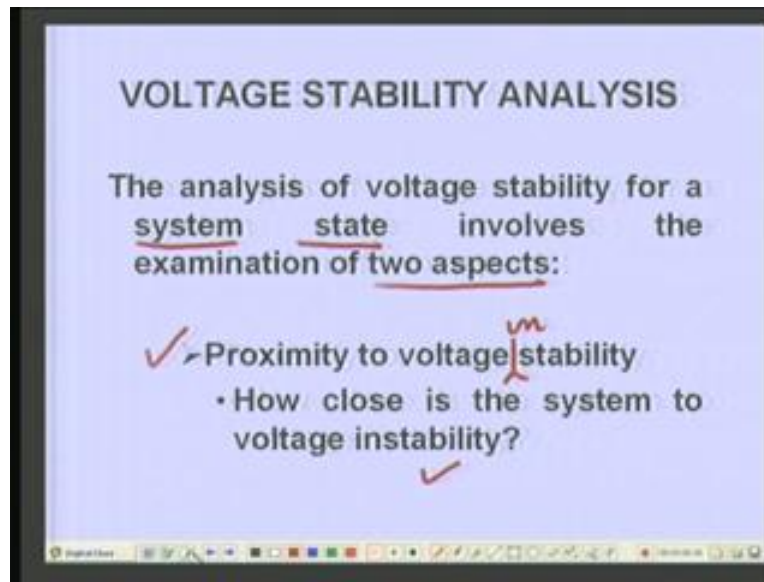
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The small disturbance voltage stability, the definition of the small disturbance voltage stability is that small system voltage stability is concerned with a systems ability to control voltages,

systems ability to control voltages following small disturbances such as gradual changes in load. This form of stability can be effectively studied with steady state approaches that utilizes linearization of the system dynamic equations at a given point that is we may we make use of the **the** linear model of the system for analyzing the small signal stability.

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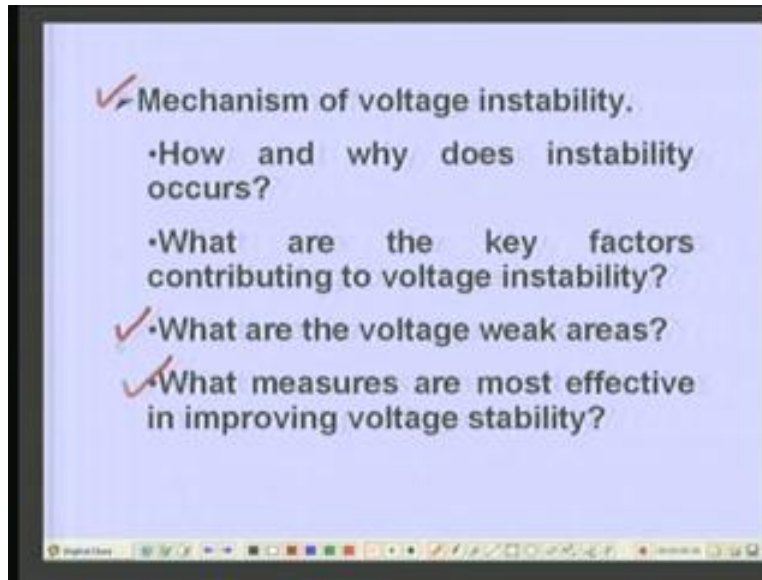
Now when we talk about voltage stability analysis, the analysis of voltage stability for a system state involves the examination of two aspects. Now here when I say we want to analyze the voltage stability is means we have to analyze the voltage stability at one particular operating condition. Suppose the system is operating at a one nominal condition or some condition then you want to you want to find out find out if a certain type of contingency occurs how the system is going to behave, whether the system is going to remain stable or not like that it means basically analysis of voltage stability for a system state for a system state means particular operating state involves the examination of two aspects, one is proximity to voltage stability or proximity to voltage not in stability, voltage instability, proximity to voltage instability that is how **how** much how close we are or how close is the system to voltage instability this is what is the meaning, **how close that is the.**

Suppose I am operating at a particular loading condition and the system becomes voltage instable at a particular loading condition critical load condition then what is the distance between these 2 that is called actually the margin also, same approach is used actually in angle stability also where when we are operating at a certain loading condition right then for a given condition for a given contingency **contingency** we can find out what is the what is the critical clearing time of the system and we know the actual actual fault clearings time.

So that the difference between the critical clearing time and the actual fault clearing time that gives the margin of stability because that is the margin suppose the critical clearing time for a

given operating condition with a given contingency shall come out to be, let us say .2 second, if the actual fault clearing time you say .05 second then we have a margin of .15 second. Similarly, we are also interested in knowing through the analysis that how close we are to voltage instability that is what is the distance to voltage stability you can say.

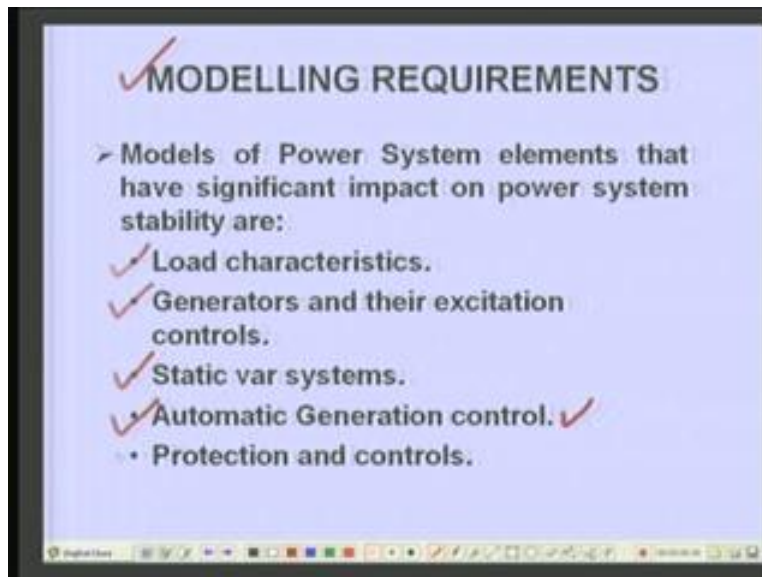
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Then we are also interested in finding out what is the mechanism of voltage instability, the second aspect is to understand the mechanism of mechanism of voltage instability that means how and why does instability occurs what are the what are the parameters or what are the main main reasons for the voltage instability that is what we are interested in knowing through the analysis then what are the key factors contributing to voltage instability what are the voltage weak areas, in a complete system there may be certain portion of the system which will be more prone to voltage instability then we can take corrective measures to make that system strong by injecting or by installing more of reactive power sources, what are the measures or what measures are most effective in improving voltage stability that is what I was mentioning that the analysis should, when we do the voltage analysis our objective is also to find out what are the effective measure that can be used for improving voltage stability.

Now when we perform the voltage stability analysis. First, I will discuss the the large disturbance voltage stability analysis in when you want analysis this voltage stability of the system then the most effective method is the simulation study that is you have to do time domain simulation of the system and when you do the time domain simulation requirement of modeling is important and therefore the models of power system elements that have significant impact on power system stability are as I have discussed the load characteristic, generator and their excitation controls, static var systems and their controls, automatic generation control AGC.

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Generally, we do not model the AGC for transient stability analysis however, since the voltage stability or voltage collapse phenomena is a slow phenomena and therefore, the the slow controls are also required to be model the AGC is a slow control automatic generation control which actually is there to regulate the frequency flows by regulating the the generation of the system then we may also have to see the protection and controls what protections are there in the system and what controls are to be modelled along with actually the complete system.

So that we have complete modeling of the controls protection aspects AGC, then the reactive power sources loads generators everything has to be properly models. When I talk about loads means again we have to model the regulating devices in the distribution system, we have to model the thermostats right and so on when we talk about generators we have to model the over excitation limits limiters. Similarly, we have to model the over load limits of the generator and so on.

Now we go to the dynamic analysis the general structure of the system model for voltage stability analysis is similar to transient stability analysis. This is very general statement that the mathematical model which we develop for dynamic analysis is similar to what we have put transient stability analysis, only difference is that we have some more additional detailed models of the system. The model comprising a set of first order differential equations may be expressed in the following general form that is when you look into the mathematical model of the system we have we have a set of differential equations, a set of non-linear differential equations along with a set of non-linear algebraic equations and this these two sets of equations have to be solved to get the complete time domain simulation.

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DYNAMIC ANALYSIS

✓ The General structure of the system model for voltage stability analysis is similar to that for transient stability analysis.

The model comprising a set of first order differential equations may be expressed in the following general form: ✓

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$$\dot{x} = f(x, V) \quad \checkmark$$

and a set of algebraic equations

$$I(x, V) = Y_N V \quad \checkmark$$

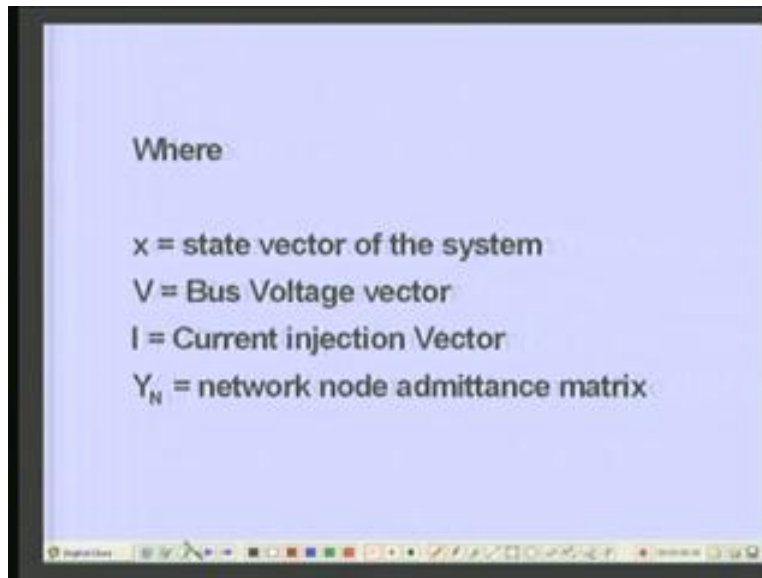
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with a set of known initial conditions (x_0, V_0) ,

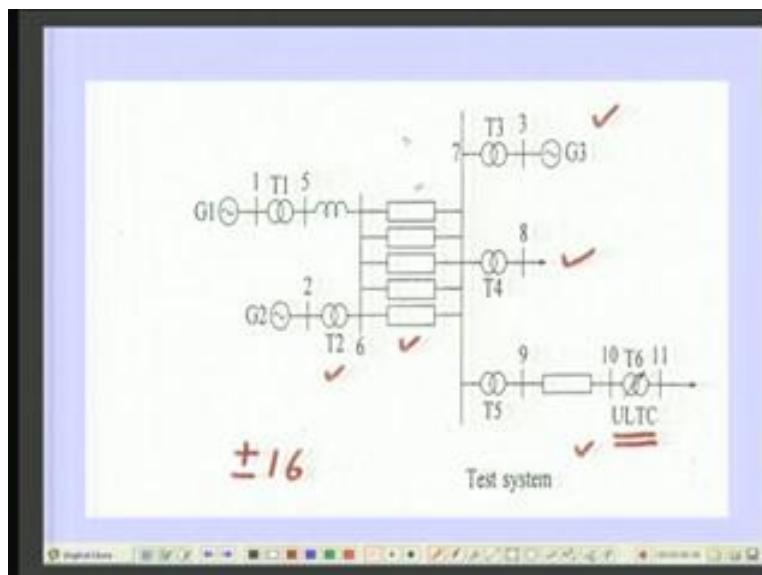
The basic equations of the system differential equations the set of first order differential equations are written as $\dot{x} = f(x, V)$ where, f is a general non-linear function, x is the state vector and V is the bus voltage vector. This is the set of algebraic differential equations the first order non-linear differential equations then we have associated with this a set of nonlinear algebraic equations which is written in the form $i = Y_N V$ where, i is function of state vector and the bus voltage vector. Y_N is actually the network admittance matrix and V is the bus matrix. This set of equations are same as the load flow equations which

we come across where we write down i bus is equal to Y into V bus right. Here, also it is same thing only emphasis is here that yes the current vector is function of the state variables as well as the voltages.

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Now about this Y_N which is the network admittance matrix. Now here when we model the system for stability analysis of voltage stability analysis we do consider the tap changers or or say under load tap changers therefore when you have the under load tap changers operating

as a function of time then then this YN will also change at each time you have actually the change in the tap setting that is YN has to be modified. Therefore whatsoever change that takes place actually in the system have to be accommodated in network admittance matrix. Initial operating condition is determined as x_0 V_0 where, initial state value state vector is x_0 and initial bus voltage vector is V_0 . You already specified that x is the state vector of the system, V is the bus voltage vector, I is the current injection vector and YN is network node admittance matrix already studied.

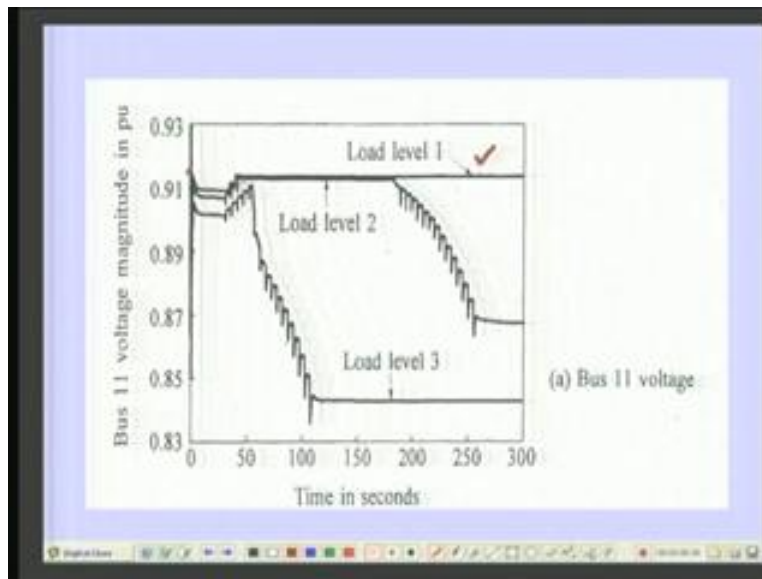
Now to illustrate the voltage stability problem, we will consider this system this system has been discussed by several researchers and it is also discussed by Prabha Kundur in a in this book power system stability and control in chapter 14 on voltage stability this system has been the system comprises system comprises of generator 1, generator 2 and generation they are three generators generator one is connected to this bus 7 through a transformer T_1 and a transmission line there are total 5 transmission lines in parallel ((00:17:36 min)) G_2 is connected through transformer T_2 then there are 2 loads, one load is connected at bus 8 through transformer T_4 and another load is connected at bus 11 through a through a ULTC transformer at bus 11.

Now this system is considered for for understanding the basic phenomena of voltage stability. Here we are talking about the large disturbance voltage stability and the voltage stability is to be analyzed by time domain simulation and therefore all the generators have been modelled in detail dynamics of the generators, dynamic to the excitation system, automatic voltage regulators and their limiters they are all modelled in detail.

Similarly, the loads at bus eight and bus eleven are also modelled in detail the **the** tap changer at bus the tap changer that is under load tap changer on transformer T_6 is such that a first tap changer occurs in a time with the time delay of around 30 seconds and the second tap changer tap change will take place in 5 seconds and this it has total of 16 plus **plus** minus 16 steps in tap changing and the moment actually the all these steps that it reaches the upper limit or lower limit then the tap changer limits are reached for this ULTC is also model.

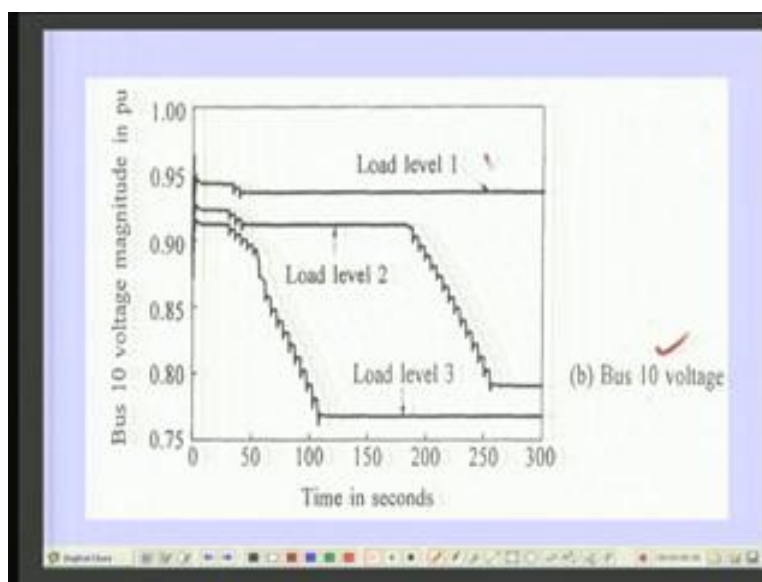
Now with this complete modeling the voltage stability analysis is carried out considering 3 different levels of loads that is the loads at bus 8 and bus 11 total put together are are classified as load level 1, load level 2, load level 3, load level 1 is light load then to a further increase load 3 is a further increased load. Now these three load conditions are developed, now here the when the system is operated under normal conditions a contingency occurs and the contingency considered here is that out of this 5 transmission line, 1 is line tripped right then the complete simulation has been carried out and the the response of voltage at bus 8, bus 11, bus 7 are recorded in addition to the voltages at these places the 4 generator 3, the reactive power generated by the generator 3 is recorded, the field current of the generator 3 is recorded and the voltage at the terminal generator three is also recorded. Now let us examine the responses which are obtained for this particular contingency.

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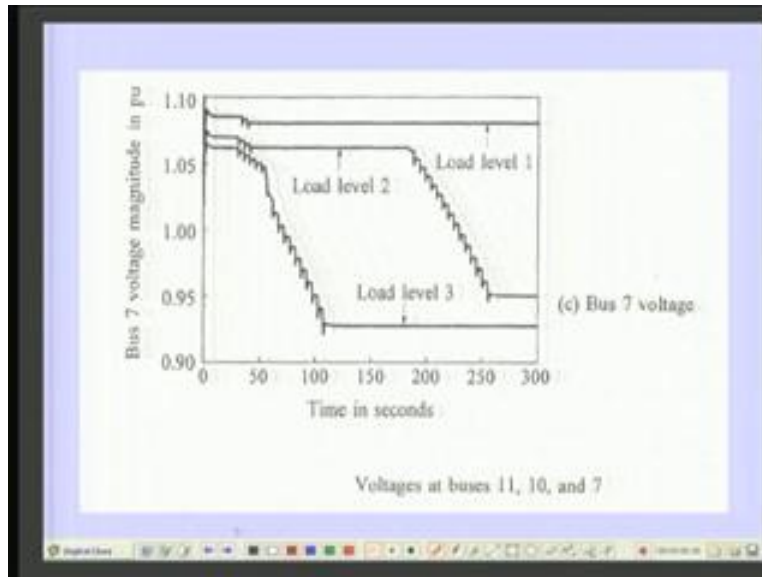


Now here this diagram shows that this is the response for voltage at bus 11, bus 11 for load level 1. Now we can see here that initially the system is operating under normal conditions the moment one line is tripped the loading on other lines increases and it will cause a drop in voltage. Now with this drop in voltage the first thing which happens is that the generator AVR's will come into operation and they bring back the voltage to the original level, if you see actually the responses of voltages at all the three buses that is bus 11 this is the response for voltage at bus 10 in the level 1.

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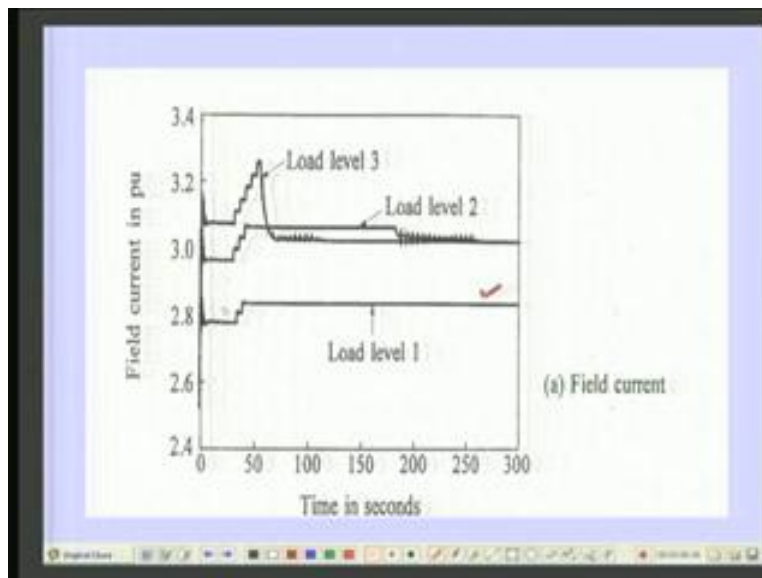


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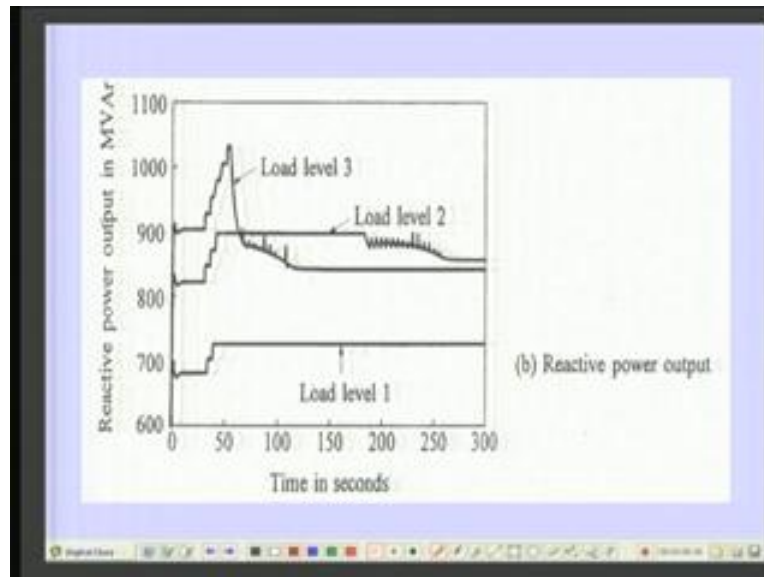
Similarly, this is the response for voltage at bus 7, I told you that the responses for the voltages for these 3 buses, 7, 10 and 7, 10 and 11 are recorded okay this is 11 this is 7 and this is bus number 10, these are recorded right. You can if you could see actually that if loading is at this level then the voltage is regulated to the initial value only for a short time there is a dip in voltage.

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However, if we examine now the response of the generator 3 for the generator 3 the 3 parameters were recorded, one is the field current in per unit. Now for the load level 1 the field current the moment there was a dip in voltage the field current this side you will find that when the voltage is regulated back to the regular value the field current has risen increase but however the field current remains below the below the limiting value right and therefore there is no action of any other controller and the voltages is restored back to the initial value.

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Now you can look at the reactive power output in MVR, the reactive power output of the generator three has increased from the initial value to this value because the moment, the moment one line is taken out additional lines are loaded and therefore additional reactive power demand is posed on the system and the generator reactive power output has increased but still this remains below the limiting condition of the generator, things are fine.

You can see actually the a generator terminal voltage in per unit the generator terminal voltage is regulated to the desired value when the loading condition 1 is there and you have actually the particular contingency. With the same contingency occurring, if we are now at the load level two this is a higher level in that case what happens is that the moment one line is out first the voltage dip take place the generator will regulate the voltage increase its excitation and bring it to the **the** voltage is brought up to the initial value.

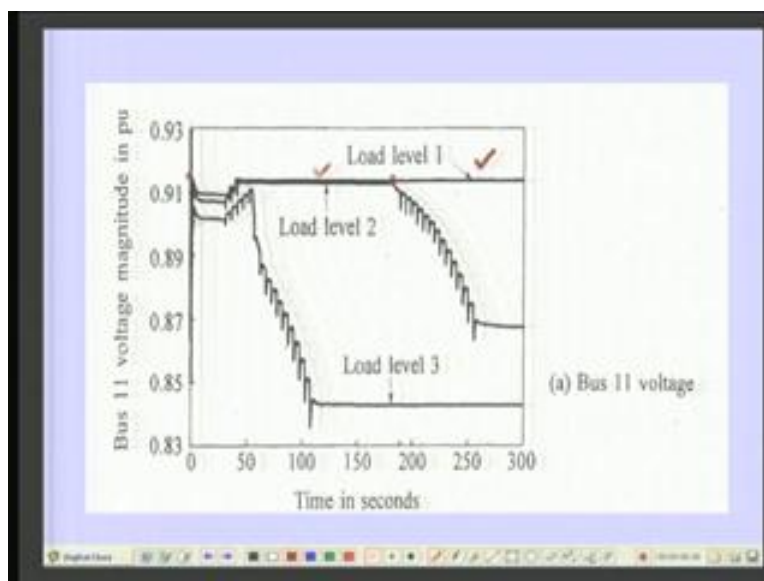
Now this happens and this remains at that level for a time equal to something like 180 seconds. Now during this period if you see the voltages what happens at other buses this is the voltage at bus 10, this is at the lower level then what it was earlier same thing is happening for the voltage at bus number 7. Now when this load level voltages are there the immediate thing which happens is that the the generator **generator** will try to bring the voltages and will try to generate more and more reactive power.

So far the generator three is concerned with this load level 2, load level 2 you can see here the field current has increased to this level initially of course it was at this level with load level, one it has gone up it remains there but at this increase in increased current level since this field current is more than the rated value what happens is that the overload or over **over** current field limiter is there and this the field current will be ramped down. It will made to decrease from this level therefore this field current is made to come down that is you can easily see see this is the level at 2 and it is brought down it is brought down to the level. So that is below the permissible limit.

Now during this period when the voltage is coming down when when the field current is brought down voltage comes down now in during this period the OLTC or you can say under load tap changers will start taking into coming into action because they ULTC will try to restore the voltage at the load points. Now when the ULTC tries to restore the voltage right you will find actually that the more of reactive power flow on the lines because the voltage is restored load will draw more reactive power and therefore more reactive power flows and with this more reactive flowing on the line, you will find actually the that the voltage will further drop.

When voltage drops ULTC will again try to boost it up if you see this, this response is here at this point somewhere at one hundred eighty seconds or so the ULTC comes into operation and at every step when it is trying to raise the voltage, the voltage is coming down and at this point it has reached the upper limit that is the limiting condition of the tap changer has reached. So that the voltage now settles down to a low level of the order of .95 same things happens for the voltages other buses you can see the voltage at bus 10 this is 11 at this particular point has started happening this is the action of ULTC okay.

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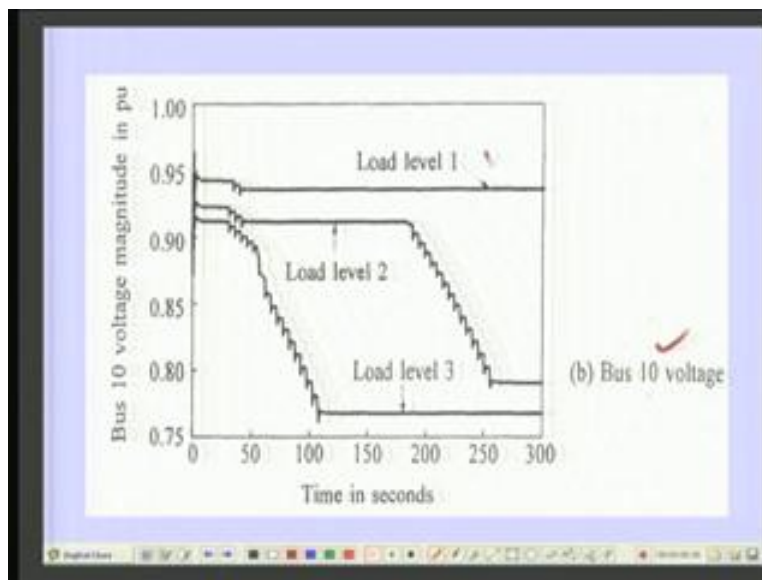


Now we can see the what is happened **happened** to the reactive power output of the generator the reactive power output has increased from this level, this level because this this is the reactive power output at load level 2, the moment actually one line is out this it has increased. Now this is now has gone beyond the limit it remains there and then it is stepped down or it is ramped down comes down like this okay now let us look at the what happens at load the load level 3, now at load level 3 if you just see here this is the voltage at bus 11.

At load level three the moment actually the fault occurs not fault but the contingency takes place, line is taken out the voltage dip takes place this voltage dip is more here because the initial load is more right. The first, the first thing which happens is that the AVR will try to regulate the voltage and bring to the desired value but in this case what happens is that in a very short time the AVR has thrust its field excitation limit and therefore the AVR will no more regulate the voltage then immediately the ULTC comes into action and tries to regulate the voltage but in every step when ULTC is trying to regulate the voltage instead of voltage being regulated it it drops down because for the every time the voltage is brought up more reactive power demand is there and that causes further voltage drop.

So you can see here actually the voltage drops to a very low value of the order of something like this .83 plus or .838 or so 8.845 or so something like that you can see the level brought down to this level.

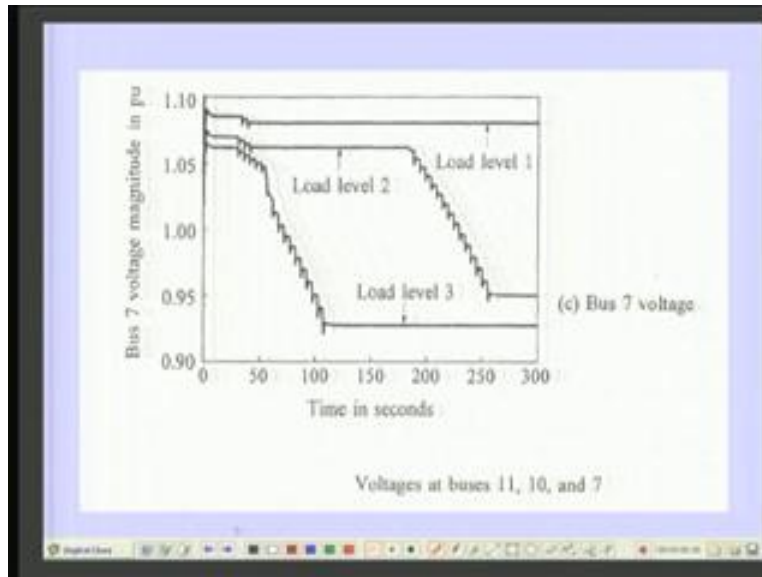
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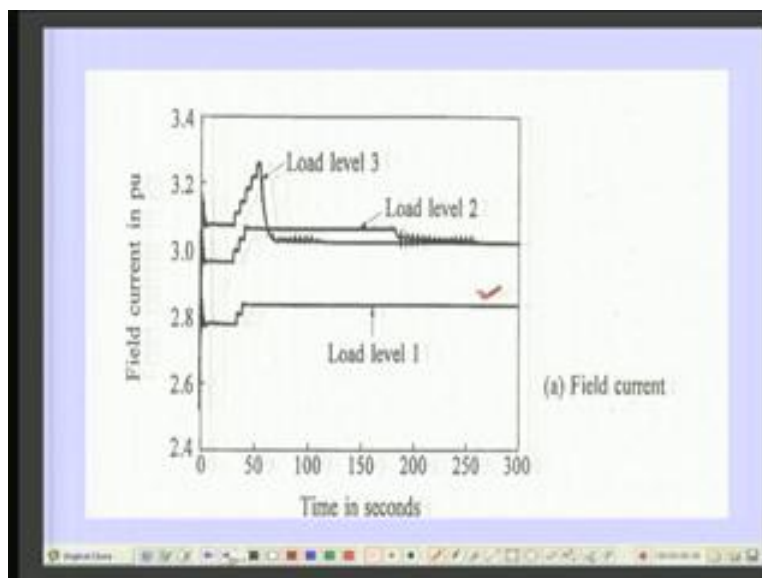
Similarly, if you see the voltage at bus 10 its happening like this comes down this is much less than .81 the voltage at bus 7 is going down to this level. You can just see that is .92 or so the voltage at bus 7 is going down to this level you can just see that is .92 or so. If we examine the field current field current of the generator three then the moment this generator has come into

action the field current is increased to high level then it is going to be ramped down and it comes to this level.

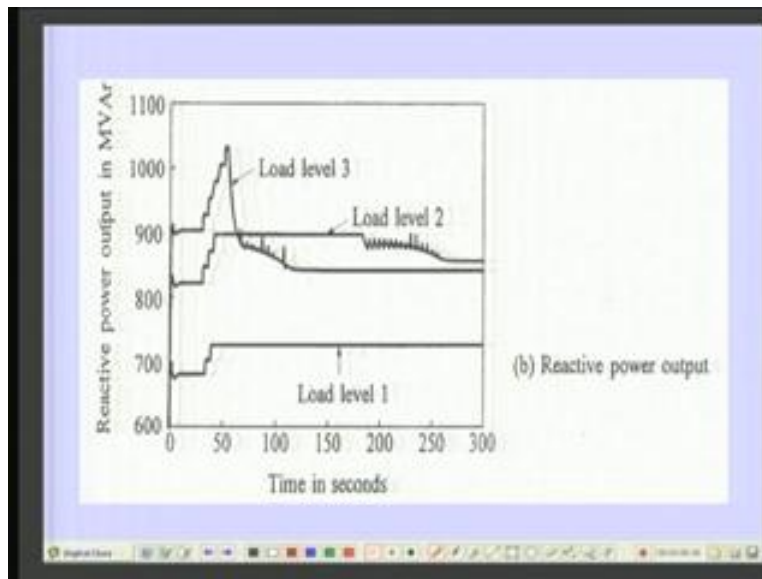
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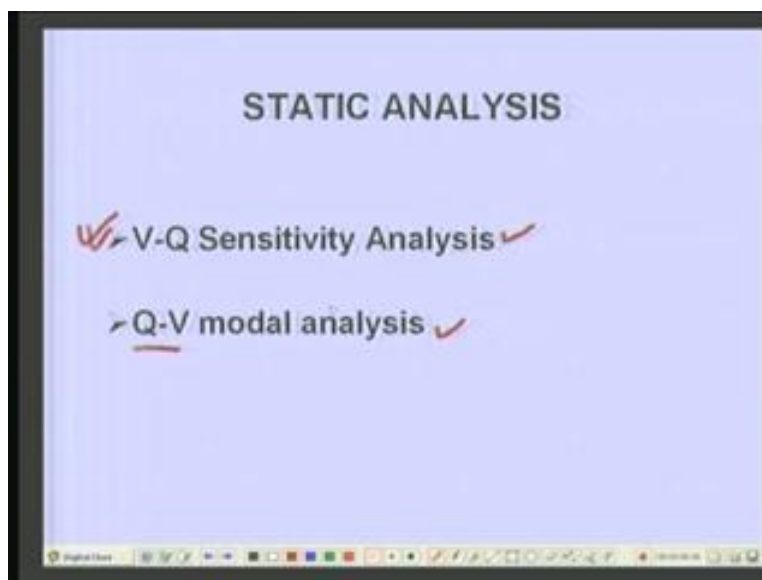


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Now during this period the ULTC comes into action and the voltage, the reactive power demand, reactive power demand you can easily see it goes up very high then it is ramped down therefore, what we see here is actually that the through this example what I wanted to illustrate is that whenever some contingency occurs in the system system gets overloaded. The first action is that the AVR's come into action tries to bring the voltage to the desired value however, in that process the field current may go beyond the required limit.

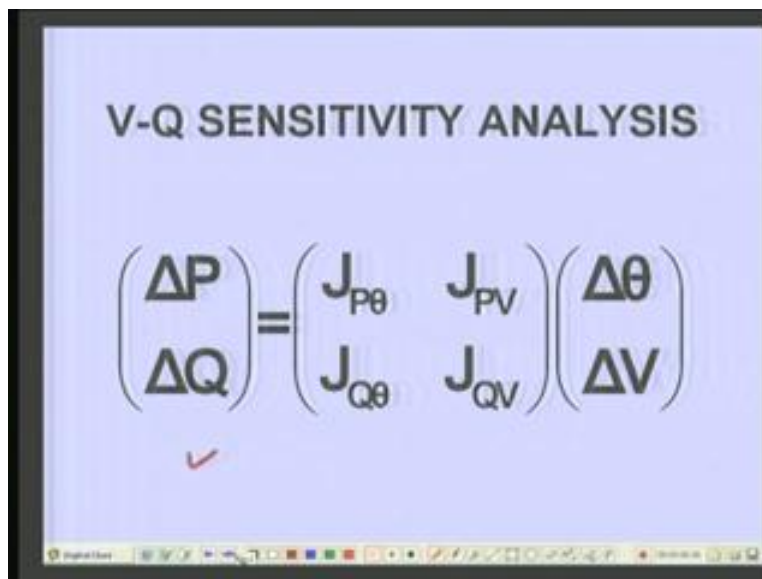
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Similarly the reactive power generated by the generator may go up so that the over excitation limiters will come into action and will ramp down the excitation from that higher value to the safe value. Similarly, reactive power will come down when the voltages come down the ULTCs will start functioning to restore the voltage but in that process instead of succeeding in restoring the voltage, the voltage keeps on going down till the ULTC settled down to the limiting condition. This is the type of phenomena which has been recorded in this particular example. The next thing which we will discuss is the static analysis you know so far the dynamic analysis is concerned, the dynamic analysis is very useful but its very time consuming it requires actually the large number of time domain simulations because for for each operating condition for the each topology and and the for each type of contingency the behaviour of the system is going to be different.

Therefore, one has to do large number of such studies however the **the** dynamic study which is performed gives you clear picture about the voltage stability causes what were the key factors which effected the voltage instability and so on. However, it does not give the information about how much how much my system is stable, how far I am from the from the voltage instability that is the distance from voltage instability is not is not can say determined through dynamic analysis it gives the complete information system is stable or not this is how the system collapses complete information you get it but not this quantitative information that yes what is the additional load which I can put on the system without losing voltage stability right.

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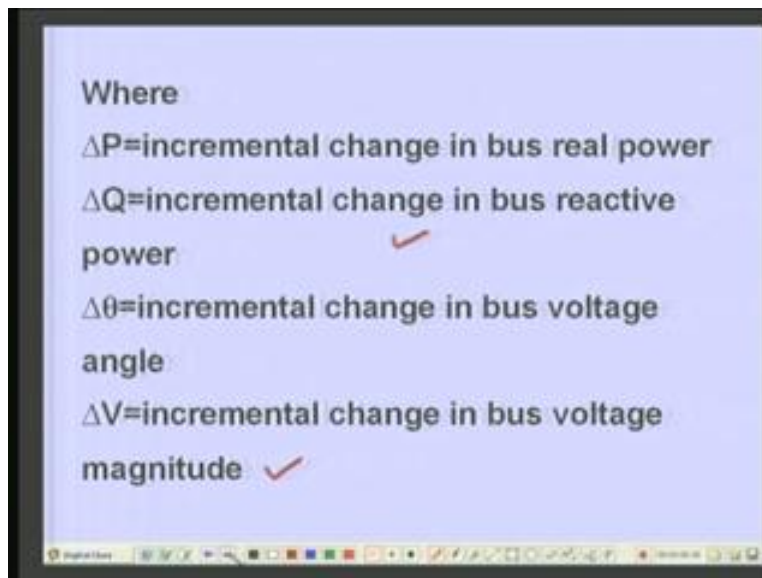
$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{pmatrix} \begin{pmatrix} \Delta \theta \\ \Delta V \end{pmatrix}$$

The static analysis s simpler and give some more insight into the system the static analysis is possible only because **because** the dynamics of the system is slow and therefore for when you perform the static analysis we you perform or we capture the snap shots over a certain time frame and during this time frame we assume actually that the system is practically in the steady state condition and therefore, we can apply apply static analysis tools for determining the

system stability. The two important techniques which have been developed over the years one is called V-Q sensitivity analysis, another is called Q-V model analysis. I have already discussed actually the significance of the Q-V curves, we have also seen actually the importance of slope of these curves for stability and therefore this V-Q sensitivity is on the similar lines and we will study the V-Q sensitivity first.

To analyze this V-Q sensitivity, we develop the steady state model of the system similar to what we do for load flow analysis using Newton Raphson technique where you can down write down this as ΔP ΔQ vector is equal to a Jacobian into error vector see $\Delta \theta$ and ΔV . Now here, we will not call it as error vector as you thing we will say this is the deviation in voltage, this is the deviation in angle, this is the deviation in real power, this is deviation in reactive power therefore, this equation which is which is similar to our load flow equation is used for performing the V-Q sensitivity analysis.

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Here this ΔP is the incremental change in bus real power, ΔQ is the incremental change in bus reactive power, $\Delta \theta$ is the incremental change in bus voltage angle and ΔV is the incremental change in bus voltage magnitude. Now here to obtain or to obtain this V-Q sensitivity what we do is we assume the the deviation in real power at the bus equal to zero that is we make ΔP_0 . Now once we make ΔP_0 then by matrix simplification we can eliminate this term $\Delta \theta$ that is you can we can write down the expression for ΔQ in terms of ΔV only.

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Based on above considerations

$$\Delta Q = J_R \Delta V$$

$$J_R = [J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV}]$$

$$\left. \frac{\Delta V_i}{\Delta Q_i} \right\} \Delta V = J_R^{-1} \Delta Q$$

Now this approach is similar to the bus elimination technique which we make use of in the fault analysis, when you develop the Z bus matrix and so on the similar approach is required that is we can write down here delta Q in terms of delta V and this matrix J_R this matrix J_R is obtained in terms of the 4 elements of the Jacobian matrix J_{qv} $J_{q\theta}$ $J_{p\theta}$ J_{pv} I will advise these people to derive this expression and satisfy that we can obtain the reduced Jacobian matrix J_R , J_R as J_{qv} minus $J_{q\theta}$ $J_{p\theta}$ inverse J_{pv} .

Now using this equation we can write down delta V as J_R inverse delta Q it means this equation relates the the incremental change in bus voltages to incremental change in bus reactive power through this matrix J_R inverse this J_R inverse is a full matrix right and therefore I can find out the sensitivity of ith voltage or sensitivity of voltage at ith bus with with the reactive power change or incremental change in reactive power at ith bus in terms of diagonal elements of J_R inverse because J_R inverse is a full matrix but I am interested in finding out the the, let us say ΔV_i versus a ΔV_Q not ΔV_Q , a ΔQ_i that is incremental change in voltage at bus i with incremental change in reactive power at ith bus.

This ratio is nothing is but the the diagonal ith diagonal element of the J_R inverse right and thus I can find out the the V-Q sensitivity right. Now here our requirement is for this V-Q sensitivity basically you know the QV curves which we have generated from the QV curves one can find out the V-Q sensitivity but however to generate those V-Q curves is a cumbersome process but now we get the information about this the voltage V-Q sensitivity by through this matrix J_R inverse straight away.

Now for the system is stable system is stable when this sensitivity is positive and in case it is low it is more stable, in the sense that in case suppose this this quantity is very large that is if the sensitivity is very large the meaning of this is a small change in mall incremental change in

reactive power at bus I will make a large change in voltage at bus i that will be instability therefore our requirement is it should be positive and small for the instability in case this quantity is negative that indicates that system is unstable.

Now generally instead of taking the inverse of this matrix that is J_r inverse one can find out this information right from this matrix itself because this is ΔQ , $J_r \Delta V$ therefore we get the information through the elements of J_r how these change in ΔQ or change in Q is related to change in V or incremental change in Q is related to incremental change in V but whichever way you determine one can find out the V-Q sensitivity and using this V-Q sensitivity one can make out the voltage instability or find out voltage stability. Another technique which has been developed is called Q-V model analysis.

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Q-V MODAL ANALYSIS

$$\underline{J_R} = \underline{\xi} \underline{\Lambda} \underline{\eta}$$

Where

- $\underline{\xi}$ = right eigenvector matrix of $\underline{J_R}$
- $\underline{\eta}$ = left eigenvector matrix of $\underline{J_R}$
- $\underline{\Lambda}$ = diagonal eigenvalue matrix of $\underline{J_R}$

Now in this Q-V model analysis, we make use of the information about the Eigen values and Eigen vectors of the J_r matrix that is this matrix J_r this is the reduced Jacobian matrix this J_r is used to further obtain the information about the voltage instability using Q-V model analysis. The approach is very simple this matrix J_r can be written as as matrix zeta zeta is a matrix into lambda lambda is also a matrix into eta this zeta is the right. Eigen vector matrix of J_r that is what is do we that take this matrix J_r then find out the Eigen values and for each Eigen value you find out the Eigen vector right Eigen vector and since this matrix will have n Eigen values if it is n by n matrix then there will be n Eigen vectors therefore, you obtain this right Eigen vector matrix making use of n Eigen vectors that is these Eigen vectors will become the columns of this eta matrix, this zeta matrix.

Okay this is the similarly, is eta is the left Eigen vector matrix of J_r we one knows about what we left Eigen vector and right Eigen vector. This lambda is the diagonal Eigen value matrix of

J_r that is J_r suppose has a n Eigen values then this λ is a diagonal matrix the elements on the diagonal are Eigen values of the matrix J_r . Now at this point I want to mention here that in case you neglect the resistance of the system network and the the if the and system is symmetrical then then these Eigen values come out to be real.

Generally you will find actually that we have a real complex conjugate Eigen values in the system but for this particular system, power system where if you the resistances are negligible as compared to the reactance of the network system, if you neglect the resistance right and you will if you do not consider again the the tap changers position because the tap changers will make this matrix or bus admittance matrix unsymmetrical like that but however that effect is small.

Now this is a real the the this Eigen values are real and therefore actually the diagonal Eigen value matrix of J_R is written as λ . Now our interest is again to find out the the how incremental change in Q is related to the incremental change in V so that we make use of this expression. Now here J_r inverse can be written as $\eta \lambda^{-1} \xi$ I am sorry, $\xi \lambda^{-1} \eta$ inverse into η .

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$$\checkmark J_R^{-1} = \xi \Lambda^{-1} \eta \checkmark$$

$$\boxed{\Delta V = \xi \Lambda^{-1} \eta \Delta Q}$$

$$\eta \Delta V = \bar{\Lambda}^{-1} \eta \Delta Q \quad \eta \xi = I$$

Now this is possible because **because** this relationship exists between this right Eigen vector matrix and left Eigen vector matrix that is the $\xi \lambda^{-1} \eta$ is equal to $\eta \xi$ and because of this relationship that is inverse of J_r can be written as $\xi \lambda^{-1} \eta$ one can verify this relationship. Now you substitute the value of J_r inverse in our original equation that is ΔV is equal to J_r inverse ΔQ therefore this J_r inverse I am substituting this expression.

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Since $\xi^{-1} = \eta_i$ ✓

$$\eta \Delta V = \Lambda^{-1} \eta \Delta Q$$
$$v = \Lambda^{-1} q$$

Where

$v = \eta \Delta V$ is the vector of modal voltage variations and ✓

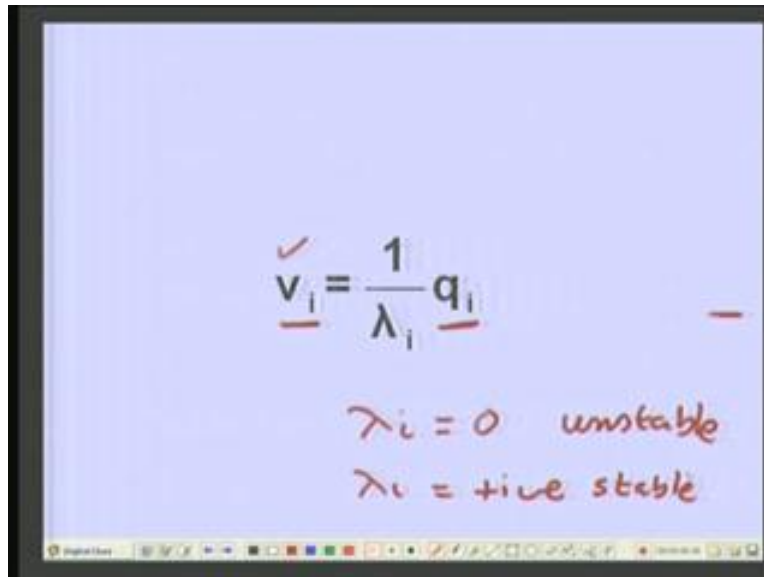
$q = \eta \Delta Q$ is the modal of reactive power variations ✓

So when I substitute for J_r inverse zeta lambda inverse eta we get this relationship ΔV equal to zeta lambda inverse eta into ΔQ this is very important relationship how it is important let us just look into it we can further simplify this. We multiply both the sides of this equation by eta you multiply both sides by eta then you will get eta ΔV equal to lambda inverse eta ΔQ why because eta into zeta inverse is one identity matrix because that is you can say eta into zeta this product is an identity matrix right okay.

So that we have this relationship which I have shown here that eta into ΔV is equal to lambda inverse eta into ΔQ that is when we talk about the Q model Q-V analysis this is the most important expression. Now in this expression we see here actually what is eta eta is actually the left Eigen vector matrix is a matrix suppose, that when this matrix is multiplied with this vector we will get a vector this vector we denote as small v and this vector will be called actually the vector of modal voltage variations this Δv is incremental change in voltage but this product is called modal voltage variations. In fact actually here this v is a vector right and this vector contains the information information or the the of all modes in the system all the modes are represented right Q is represented as eta into ΔQ which is called as modal reactive power variations that is instead of having the vector of vector of reactive powers incremental change in reactive powers.

Now we have got here modal voltage variations is a vector of modal voltage variations this is a modal reactive power variations and therefore we get this information here very interesting expression. Now since this lambda is a diagonal matrix where diagonal elements are the Eigen values of J_r these are the Eigen values of J_r and therefore if I write down here the i th component of the modal voltage variation then this i th component can be written as one by lambda i into Q_i where Q_i is the Q_i is the i th i th reactive power i th modal reactive power okay.

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The image shows a handwritten equation and two lines of text on a blue background. The equation is
$$\underline{V_i} = \frac{1}{\lambda_i} \underline{Q_i}$$
 with a red checkmark above the V_i and a red minus sign to the right. Below the equation, the text reads: $\lambda_i = 0$ unstable and $\lambda_i = +ive$ stable.

Now what we see here is that this V_i is inversely proportional to λ_i right in the sense actually here we get a relationship which shows the relation of the modal voltage vector, modal voltage of i th bus in terms of modal reactive power change at i th bus in terms of Eigen value of the J_r matrix right. Now here suppose λ_i is a positive what does it show positive λ_i shows that the modal voltage variation vector is along with the modal reactive power variation.

In the sense that where the reactive power varies in a positive direction then the modal vector will also vary in positive direction and that shows that system is a stable but suppose if λ_i is 0, 0 that is the case of instability because that means a very small because there is a this becomes V_i becomes infinite when Q_i is small and therefore if λ_i is 0 it is unstable while if λ_i is positive **positive** instable and in case λ_i is small positive it is more stable if large positive it means λ_i zero is unstable λ_i is positive with stable right and if λ_i is negative system is unstable.

Now here I will just give you the relationship between V-Q sensitivity and λ_i Eigen value the V, V the **the** relationship is like this ΔV_k one can establish this ΔV_k divided by ΔQ_k is equal to summation over i , ζ_{ki} , η_{ik} divided by λ_i . In fact this is this gives you a relationship between the 2 approaches, one approach is based upon V-Q sensitivity another approach is based upon the Eigen value right the modal analysis is based upon the Eigen value and as I have told is Eigen value is positive you can see that this this V-Q sensitivity is inversely proportional to λ_i right.

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Example

$$\frac{\partial V_k}{\partial Q_k} = \sum_i \frac{f_{ki} \eta_{ik}}{\lambda_i}$$

Example-14.3

Now one simple example we can consider here the example is that you have a strong system infinite bus system which is supplying a load through a transmission line to a load through to a transformer. At this bus you have a reactive power source where I am injecting the reactive power Q_i and this becomes a simple system therefore this voltage is fixed V_1 this is equal to 1 per unit, its phase angle is 0 and only quantity which is varying is V_2 right. Now this for this simple problem this is solved in example 3 of the example 14.3 in the book power system stability and control by Prabha Kundur.

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TABLE

Q_i (Mvar)	High voltage solution				Low voltage solution			
	V_2	δ (in $^\circ$)	λ	dV/dQ	V_2	δ (in $^\circ$)	λ	dV/dQ
✓ 500.0	1.024	-37.3	17.0	0.059	0.671	-66.7	-39.87	-0.025
	✓	✓	3	✓	✓	✓		
400.0	0.956	-40.01	12.4	0.812	0.706	-50.3	-20.96	-0.048
			1					
306.0	0.820	-48.2	0.52	1.923	0.812	-48.8	-0.95	-1.055
305.9	0.814	-48.7	0.02	50.1	0.815	-48.6	-0.70	-1.434

✓

I will give the results what has been done is that for a particular load load is about 1500 megawatt the V-Q sensitivity analysis and Q-V modal analysis have been applied. Now in this particular case this J_r becomes simply a one by one matrix, J_r becomes one by one matrix and therefore the problem becomes a simpler one however, actually the **the** results are very interesting the results are shown here. The studies are carried out for different values of reactive power injected at bus 2. Suppose it is 500 megawatt MVR is 400, 306 and 305.9 just to show the difference as we go from 306 to 305.9 for this the voltage is computed V_2 in fact for each value of the injected reactive power there are 2 solutions for the voltages, one is 1.024, another is .671 the lower one there are two solutions always because the it is a nose curve the phase angle is computed.

Now you can see here in this diagram this is a lambda i not very clearly shown here. This is lambda and this is d_v by d_Q right. The lambda for this 500 MVR injected is 17.03 and the corresponding d_v by d_Q is small of the order of .059 although when you talk about the other part other solution lower solution system is unstable because lambda is negative and d_v by d_Q is also negative right.

Now through this example what we see here is as the injected reactive power decreases from 500 to 306 there is a drastic change in the value of lambda that is 17 to .52 the larger the value of lambda larger the value of lambda right system is more stable it is similar to a damping ratio in a small signal stability analysis right. Now this 306 and 305.6, if you see here the lambda is changed from .52 to .02 by small change in Q by point one MVR right like this.

Now with this let me sum up what we have discussed today. We have discussed the 2 important aspects, one is the classification of a voltage stability phenomena, then we have discussed the techniques for analyzing the voltage stability of the system. I have illustrated the application of application of a small signal stability analysis through an example and we have seen that how how actually the system functions and how the system voltage collapses then we have studied the two two static voltage stability analysis tools, one is the V-Q sensitivity another is modal Q_u , Q_v approach a simple example we have illustrated the application of these techniques to a simple system. Thank you!