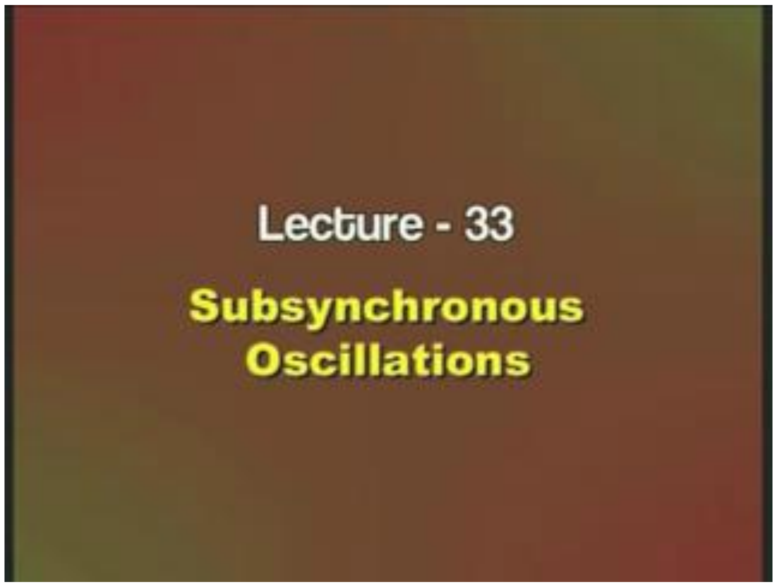
Power System Dynamics
Prof. M. L. Kothari
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 33
Subsynchronous oscillations

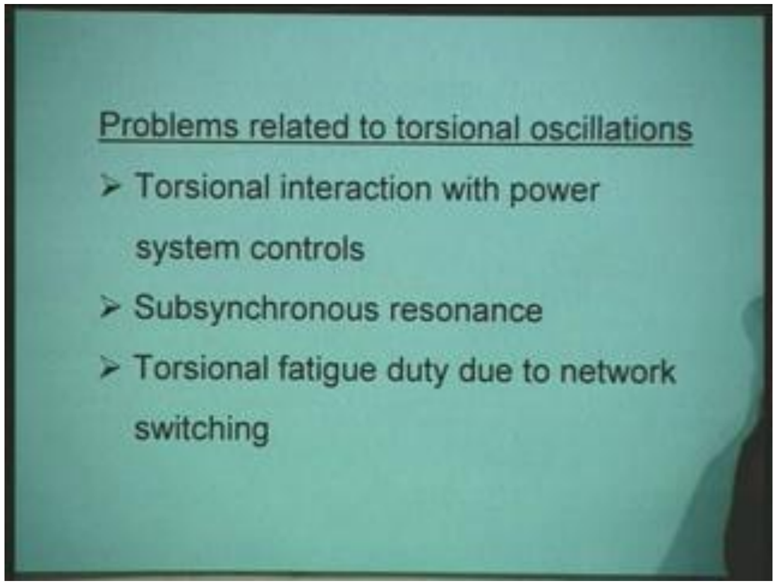
(Refer Slide Time: 00:55)



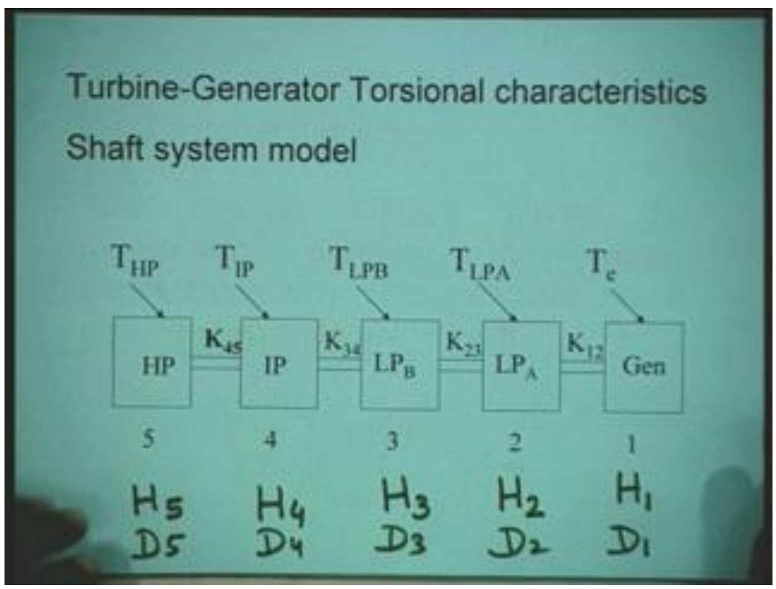
Friends, we shall study today sub synchronous oscillations in the power system. In the analysis of power system dynamics so far we have considered the rotor of the turbine generator system as a single mass system such a representation accounts for the system mode of oscillations, the frequency of this mode of oscillation is in the range .2 to 2 hertz in the system, in reality the the turbine generator rotor is a very complex in the structure mechanically it has several masses mounted on the same shaft. It has high pressure turbine section, intermediate pressure turbine section, low pressure turbine sections, generator and exciter mounted on the same shaft and in such turbine generator rotor when it is disturbed or perturbed, torsional oscillations are produced and these torsional oscillations in the sub synchronous frequency range in certain conditions may interact with the electrical system and such interaction may be an adverse interaction.

In view of this it is necessary to study the sub synchronous oscillations in detail. The problems related to torsional oscillations are torsional interaction with power system controls, sub synchronous resonance and torsional fatigue due to network switching. We shall study all these aspects while discussing the torsional oscillations or sub synchronous oscillations in the system. Before, we address these specific problems the first step in the study of sub synchronous oscillations is to develop the detailed model of the turbine generator shaft system.

(Refer Slide Time: 03:02)



(Refer Slide Time: 03:52)



Now in order to develop the dynamic model of the turbine generator shaft system, we consider a tandem compound of steam turbine which consists of high pressure section, intermediate pressure section and low pressure section B and low pressure section A. All these 4 steam turbine sections are mounted on the same shaft in addition to this the generator is also mounted on the same generator rotor. Here, we will consider the static excitation system and in case in case the system comprises of a rotating exciter then 1 more mass we have to be represented on the same shaft.

Now, here in order to develop the dynamic model of the shaft system or turbine generator shaft system, we will consider that the torques which are developed in the various turbine sections are represented as T_{HP} represents the torque developed in the high pressure section, T_{IP} is the torque developed in the intermediate pressure section, T_{LPB} is the torque developed in the low pressure B section and T_{LP} is the torque developed in the low pressure A section and the air gap torque developed in the generator is represented by T_E.

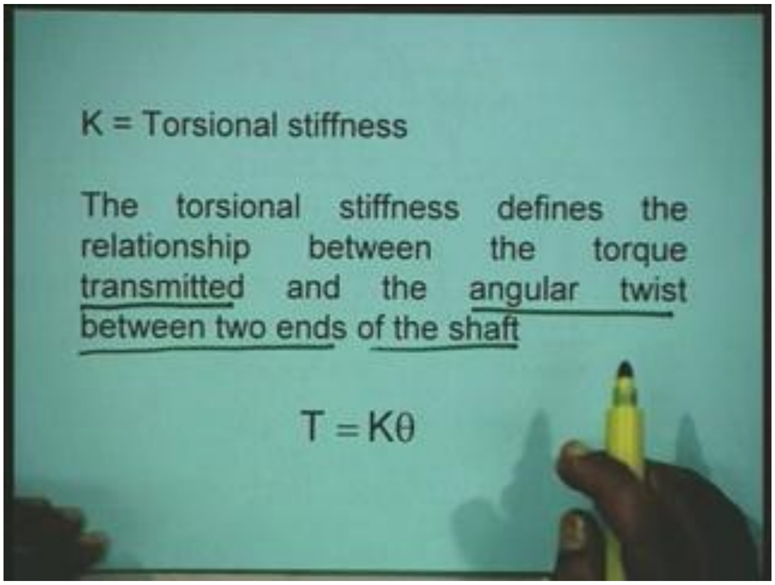
Further, further the the shaft connecting the high pressure and intermediate section similarly, the intermediate pressure and low pressure sections the the shafts have finite stiffness coefficient the stiffness coefficients will be represented by the symbol K_{45} joining the high pressure and intermediate pressure section. Similarly, K_{34} is the stiffness coefficient of the shaft connecting a intermediate pressure and L_{PB} and similarly the K_{23} and K_{12} are the stiffness coefficient of the shaft sections connecting L_{PB} and L_{PA} and L_{PA} and generator sections. While we develop the dynamic model system, we will assume ,we will assume that the that the torque developed in this various turbine sections are constant.

Now this assumption is particularly used for developing the small perturbation dynamic model of the system. Now here each section that is the generator L_{PA}, L_{PB}, intermediate pressure and high pressure section they have their inertia constants. We will represent the inertia constant of generator by the symbol H_1, inertia constant of the generator that is the not generator but steam turbine section L_{PA} by H_2 and similarly, the inertia constant of L_{PB}, I_P and H_P sections. Further, we will assume that there is some damping associated with the oscillation of these masses. Now here in this model the the the various various turbine sections and generator are represented by lump mass. The dampings associated with the these 5 different masses are represented as D_1, D_2, D_3, D_4 and D_5. Now let us understand what causes damping in the turbine generator shaft system.

Now when when the the when the blades of the steam turbine are rotating in the steam which is flowing right therefore during in this process there exists some viscous damping right and similarly the damping is also produced when the generator rotor is rotating therefore the generator rotor also produce some damping because of the damper windings and other phenomena which is exist in the system. Similarly, the when this this system connected to the electrical network then this electrical network also produces some damping.

Now damping is also produced by the mechanical hysteresis of the shafts connecting the various turbine sections therefore there exists some damping in the system. However, the damping produced due to this various phenomena is very small in magnitude, we will develop the dynamic model of the system assuming the certain finite value of the dampings associated with the with the generator and steam turbine sections. Now let us just understand the meaning of this stiffness coefficient.

(Refer Slide Time: 09:52)

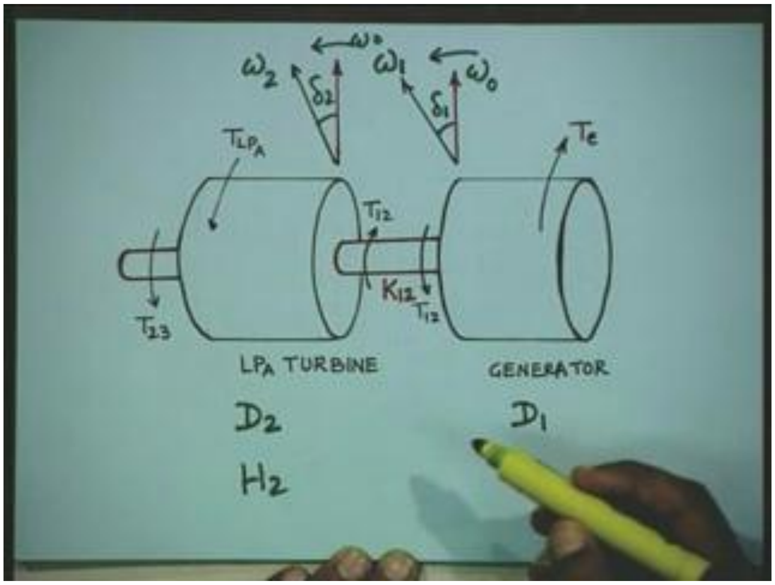


The stiffness coefficient is represented by the symbol K and when we represent the stiffness coefficient of the particular shaft section, we will add additional subscripts. The torsional stiffness defines the relationship between the torque transmitted torque transmitted and the angular twist between the 2 ends of the shaft, this is very important that that when the torque is transmitted by the shaft right then there is a relationship between the the torque transmitted and the angular twist between 2 ends of the shaft and the relationship is represented by this equation T equal to K theta where theta is the angular twist and K is the stiffness coefficient and torque T stands for the torque transmitted by the shaft.

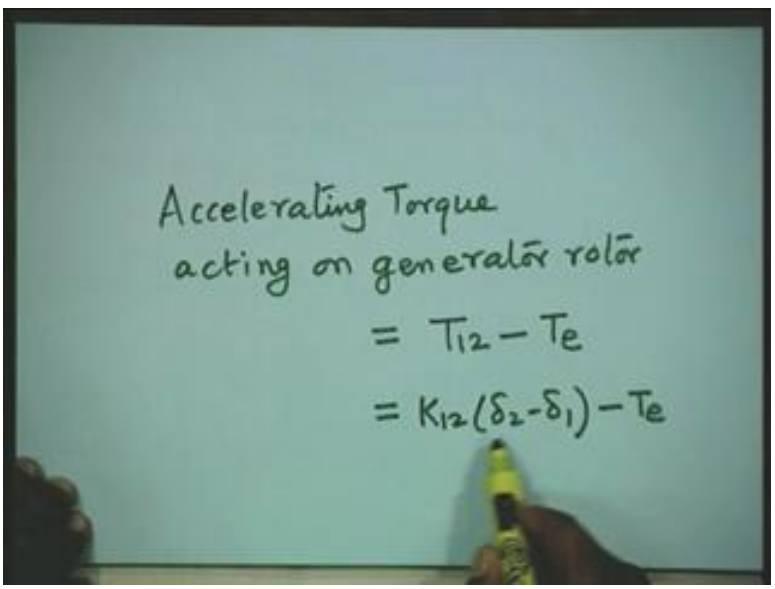
Now in order to develop the dynamic mode of the system, let us consider consider the generator and L_{PA} turbine section in detail the on the rotor of the generator a torque T_e which is the air gap torque is acting. Now this torque acts in a direction opposite to the direction in which the rotor is rotating. Now this generator rotor is connected to the turbine section L_{PA} the shaft connecting the turbine section L_{PA} and generator has a stiffness coefficient $K_{12,}$ the driving torque produced by the shaft is represented as T_{12} acting in the direction of rotation.

Therefore, when we look actually at the dynamics of the generator rotor we have 2 torques acting on the rotor, 1 is the torque transmitted by the shaft that is T_{12} another is the air gap torque T_e. In addition to this there is some damping torque acting on the generator rotor. Now here the position of the rotor will be represented with respect to the synchronously rotating reference frame the this phasor represents represents the synchronously rotating reference frame, the speed of this is omega o. The generator rotor rotates at a speed omega 1 and its axis is displaced with respect to the synchronously rotating reference frame by an angle delta 1.

(Refer Slide Time: 10:59)



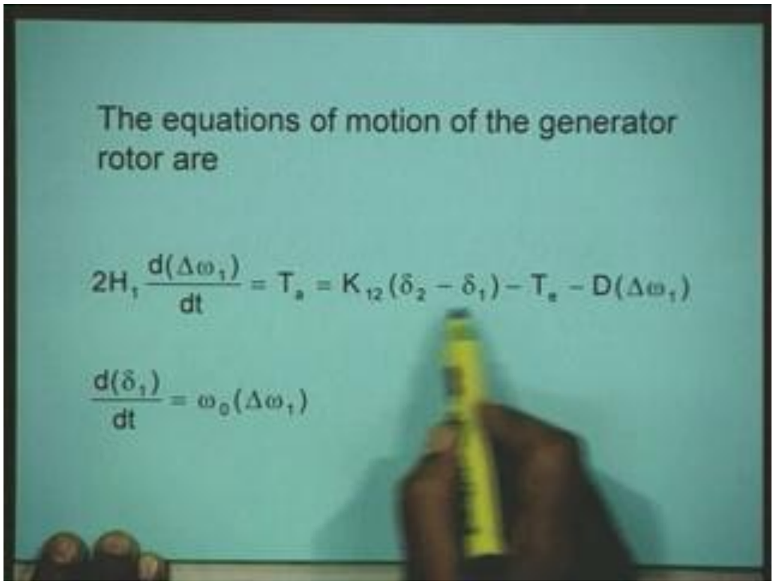
(Refer Slide Time: 13:54)



Similarly, when I talk about the turbine section L_{PA}, L_{PA} then the position of the rotor with respect to the synchronously rotating reference frame is noted by an angle delta 2 and the speed is omega 2, under steady state conditions, under steady state conditions under steady state conditions the omega naught, omega 1 and omega 2 they are all equal. However, there is a finite value of the delta 1, delta 2 and so on and in order to develop or in order to write the equation of motion for the generator, let us write down the accelerating torque acting on the generator rotor the accelerating torque acting on the generator rotor is equal to is equal to T_{12} minus T_e.

Now this T_{12} can be represented can be expressed as K_{12} into delta 2 minus delta 1minus T_e therefore, the accelerating torque which is acting on the generator rotor is this. Now now when we consider the damping of the generator rotor then the damping torque which acts on the system will be acting in the direction opposite to the accelerating torque therefore the net accelerating torque will be this K_{12} will be delta 2 minus delta 1 minus T minus D_1 delta omega 1.

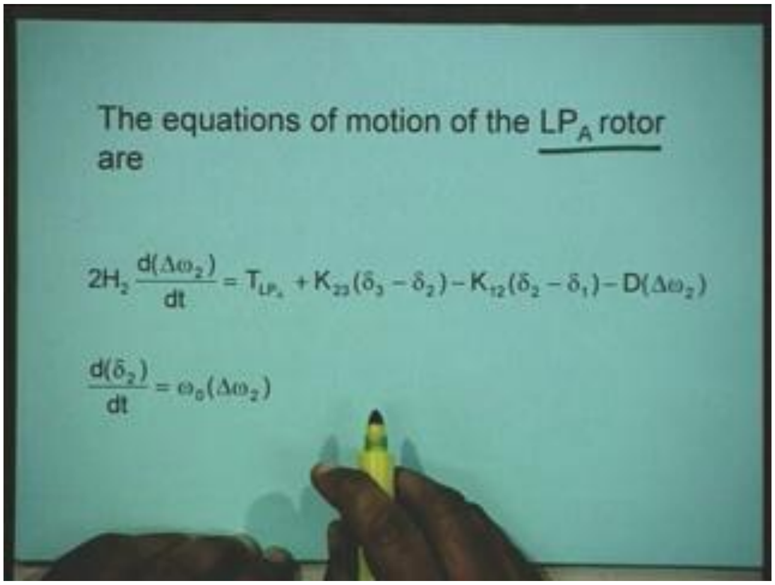
(Refer Slide Time: 15:19)



The equation of the equations of motion of the generator rotor, now can be written as 2 times H_1 d by dt of delta omega 1 equal to T_a, this is the accelerating torque is equal to T_{12} into delta 2 minus delta 1 minus T minus D_1 times delta omega 1 right that this is the net accelerating torque which is acting on the generator rotor. Now in this equation the speed deviation is represented in per unit, the equation d by dt of delta 1 is equal to omega naught into delta omega 1 therefore these 2 equations represents the equation of motion of the generator rotor these are the 2 first order differential equations. Now let us consider the steam turbine section L_{PA}.

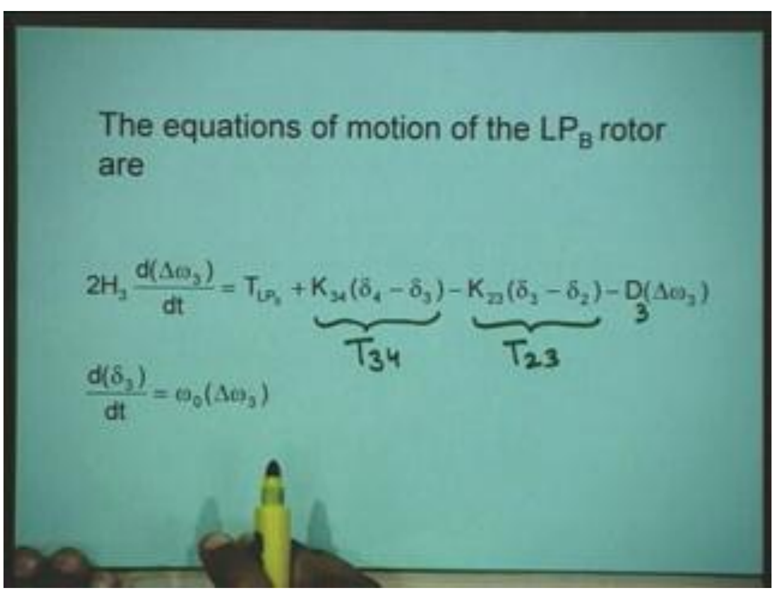
Now when I look at the steam turbine section L_{PA} then we can clearly see that the accelerating torque acting on the steam turbine section L_{PA} can be written as that the driving torque is T_{23} plus the torque developed in the steam turbine section that is T_{23} plus T_{LPA} is the driving torque while the breaking torque is T_{12} and therefore the net accelerating torque will be T_{23} plus T_{LPA} minus T_{12} minus D_2 delta omega 2 and therefore the equation of motion for the L_{PA} rotor that is the steam turbine rotor represented as L_{PA} can be written as 2 times H_2 d by dt of delta omega 2 equal to T_{LPA} torque developed in the steam turbine section plus K_{23} delta 3 minus delta 2 that is this is your T_{23} minus K_{12} into delta 2 minus delta 1 therefore, this quantity is T_{12} and multiplied by D_2 into delta omega 2 this is the damping torque therefore the net torque acting or net accelerating torque is T_{LPA} plus T_{23} minus T_{12} minus D_2 delta omega 2. The second equation the differential equation for this L_{PA} rotor is d by dt of delta 2 equal to omega naught into delta omega.

(Refer Slide Time: 17:19)

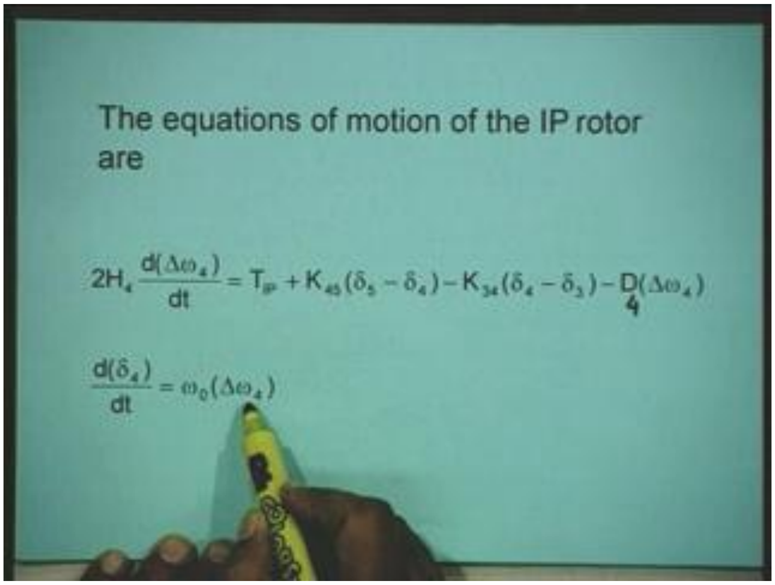


Similarly, we can write down the equations of motion for L_{PB} I_P and H_P sections. The equations of motion for L_{PB} rotor are 2 times H_3 d by dt of delta omega 3 equal to T_{LPB} plus plus the T_{34} which is equal to K_{34} into delta 4 minus delta 3 minus T_{23} which is written as K_{23} into delta 3 minus delta 2 minus D_3 delta omega 3 and the next equation is d by dt of delta 3 equal to omega naught into delta omega 3.

(Refer Slide Time: 18:39)

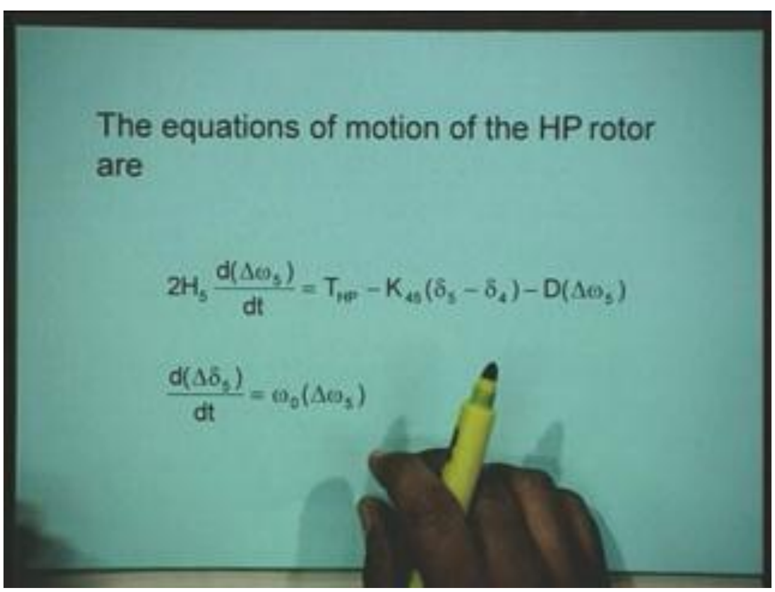


(Refer Slide Time: 19:25)



The equations of motion for I_P turbine section are writing in the same way as we have d_1 for L_{PB} and L_PA sections that is 2 times H_4 d by dt of delta omega 4 equal to T_{IP} plus K_{45} delta 5 minus delta 4 minus K_{34} into delta 4 minus delta 3 minus D_4 delta omega 4 and d by dt of delta 4 equal to omega naught delta omega 4. Our next step is to write the equation for the last turbine section that is the high pressure turbine section.

(Refer Slide Time: 20:12)

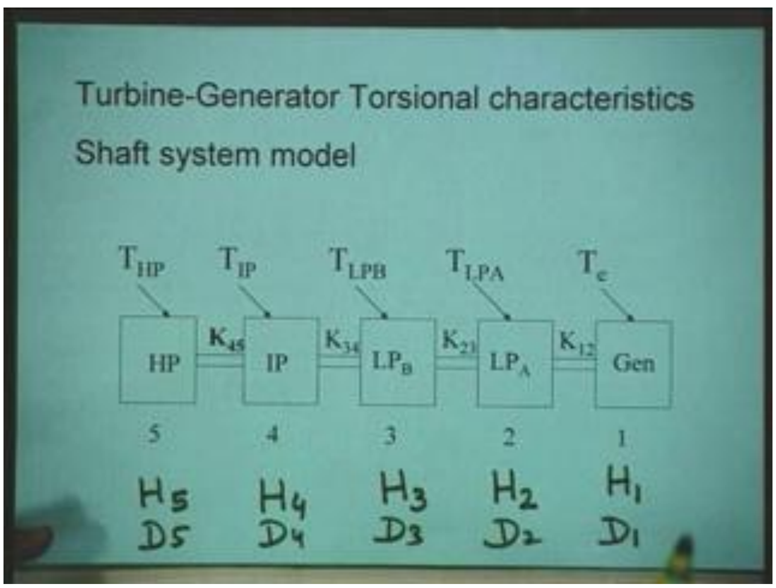


Now when we look at the high pressure turbine section then the there is no shaft on the left of the high pressure turbine section and therefore the net accelerating torque acting on the high pressure turbine rotor is T_{HP} the torque which is developed in the turbine section minus K_{45} minus D_4, D_5 into delta omega 5 and therefore the equation of motion is written here as 2 times H_5 d by dt of delta omega 5that is equal to T_{HP} that is the torque developed in the high pressure turbine section minus K_{45} into delta 5 minus delta 4 minus D_5 delta omega 5 and d by dt of delta 5 equal to omega naught delta omega 5.

Now for this 5 mass turbine generator shaft we have written 10 first order differential equations now these differential equations are non-linear differential equations. The the equations are non-linear because the the air gap torque T_e is non-linearly related to the angular positions of the generator. Now we will attempt a simple problem the problem is like this we consider the same 5 mass system, we will consider this same 5 mass system and we will assume that that the power torque developed under steady state condition in the high pressure intermediate pressure L_{PB}, L_{PA} have certain fixed ratios.

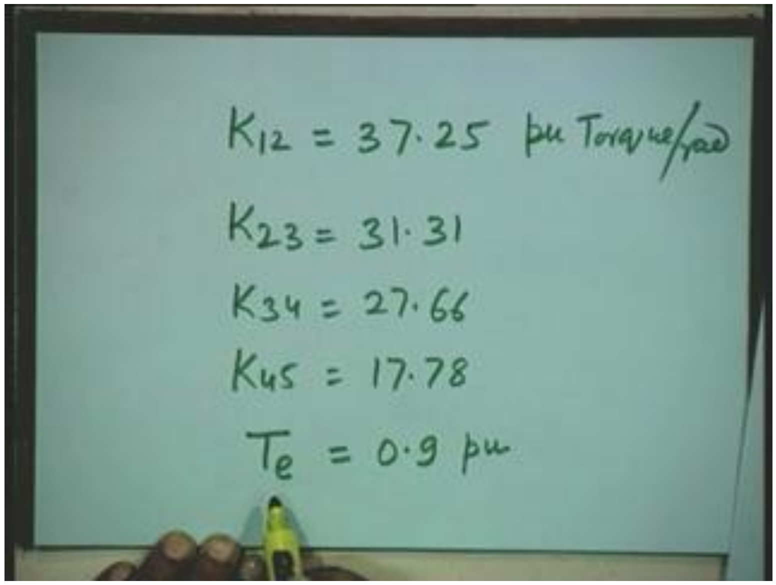
Now for this particular problem our interest will be to find out to find out that if the generator is delivering certain amount of power right then how much is the power which is which is transferred through each of these shaft sections that is the power or the torque transmitted by each of the shaft sections which are connecting the various turbine sections and the generator. We will also find out that how much is the total angular twist which is created from generator to high pressure turbine section.

(Refer Slide Time: 22:12)



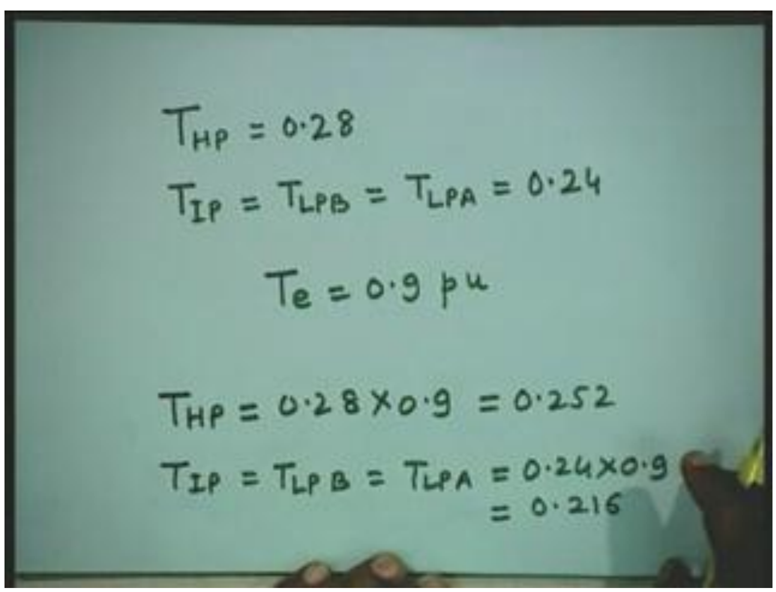
Now to illustrate this we will consider the that this system has the various stiffness coefficients K_{12} is given as 37.25, K_{23} is 31.31, K_{34} is 27.66, K_{45} is 17.78. Now these coefficients are expressed in per unit torque per electrical radian or per radian right however we will assume that the power output from the generator is .9 per unit and in per unit system the air gap torque T_e is also equal to .9 per unit.

(Refer Slide Time: 23:26)



Now to solve this problem we need 1 more information and that information is that what is the fraction of torque which is produced in high pressure, intermediate pressure, low pressure and A and B sections. We will assume here in this say that that the fractions of the torque which are produced in the H_P is 0.28 where the fraction of torque which is developed in T this intermediate pressure this T_{IP} is equal to T_{LPB} also equal to T_{LPA} equal to 0.24.

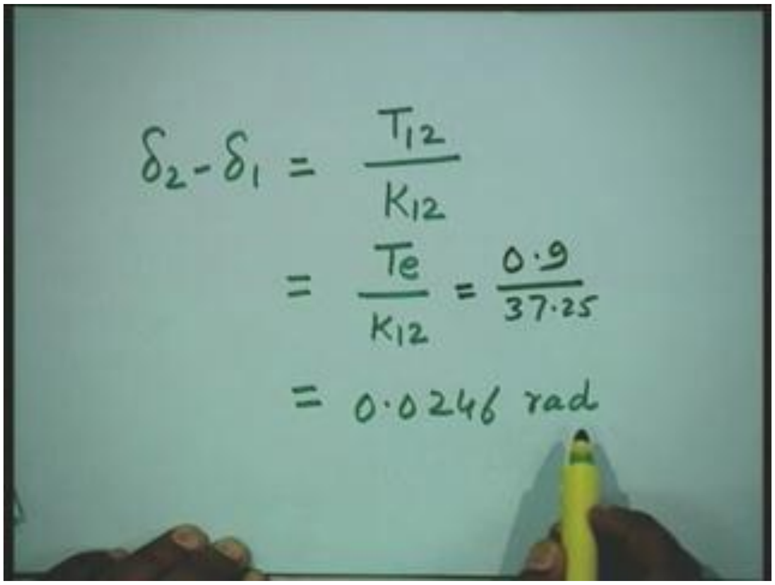
(Refer Slide Time: 24:30)



So that when you add these fractions the sum is 1. Now when the torque which is produced in the by the generator is represented as T_e and its value is 0.9 per unit then the actual actual torque which is produced by H_P, I_P and L_P sections can be obtained by multiplying these all these coefficients are these fraction by .9 therefore, we can say that the the T_{HP} produced is 0 point 2 8 into 0 point 9 similarly T_{IP} equal to T_{LPB} equal to T_{LPA} equal to 0.24 into 0.9, what is this value 0.216 and what is this value 0.25.

The meaning here is that the when the generator is delivering .9 per unit power right the the the torques produced in the I_P, L_{PB}, L_{PA} section will be 0.216 and torque produced in H_P section is 0.252. Now our interest to find out how is the torque transmitted through each of the shaft sections that I want to know what is T_{12} what is T_{23} what is T_{34} and T what isT_{45}. Now to answer this what we have to do is that we make use of our definition for definition for stiffness coefficient.

(Refer Slide Time: 27:10)

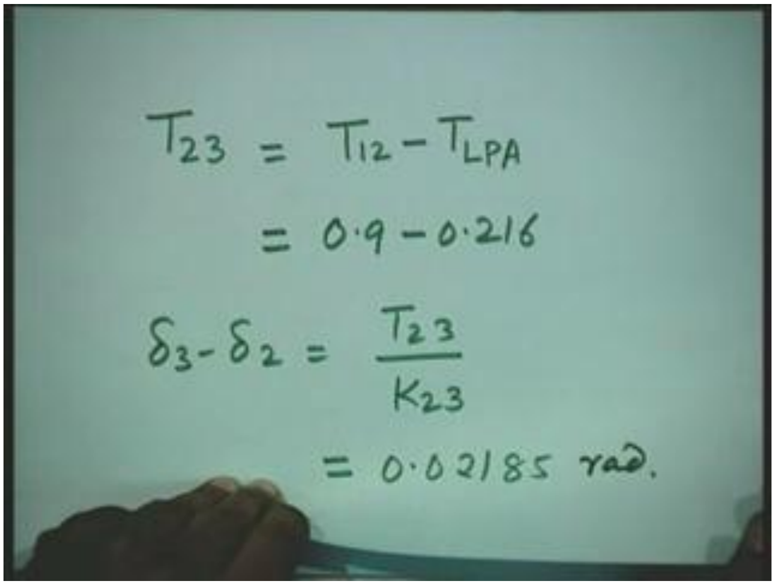


Our stiffness coefficient is that the torque transmitted is equal to the angular twist into stiffness coefficient. Therefore, the angular twist of the shaft section connecting the turbine section L_{PA} and generator is delta 2 minus delta 1 and therefore this delta 2 minus delta 1 is equal to T_{12} upon K_{12}. Now the T_{12} under steady state condition is equal to T_e okay because under steady state condition the net accelerating torque is 0 therefore this angular twist delta 2 minus delta 1 can be obtained as T_e divided by K_{12} and when you substitute the value of T_e equal to 0.9 and value of K_{12} as 37.25, we will get delta 2 minus delta 1 as 0.0246 radian.

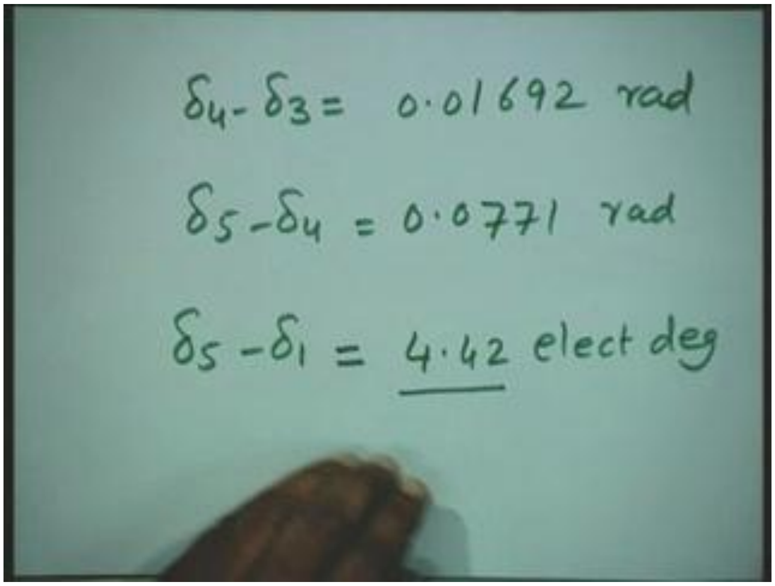
Now the torque transmitted through the shaft section connecting the connecting the turbine section L_{PB} to L_{PA} is T_{12} minus L_{PA},T_{LPA}, now T_{12} is .9 and the torque developed in the turbine section L_{PA} is 0.216 this is the torque which is transmitted and therefore the angular twist is delta 3 minus delta 2 of the second shaft section that is T_{23} divided by K_{23} when we substitute the values of T_{23} and K_{23} we will get the angular twist as 0.02185 radians. Similarly, 1 can find out the angular twist delta 4 minus delta 3 delta 5 minus delta 4 and this come out to be like this and the net angular twist or angular twist of the high pressure section and the generator or we can say the net angular difference between the axis of the generator rotor and the axis of this high pressure stream turbine comes out to be 4.42 electrical degrees.

Therefore when you can you can easily understand here that when when the the steam turbine is delivering certain power that the there exist a exist a certain net or twist in the shafts and the net twist is given by this formula delta 5 minus delta 1 and we can see here actually that the the angular twist is small okay.

(Refer Slide Time: 28:34)

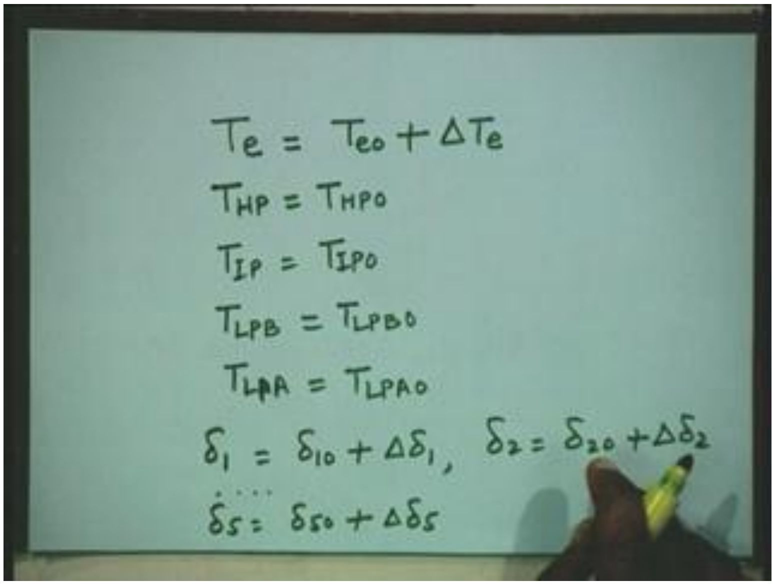


(Refer Slide Time: 29:15)



When it is delivering 90 percent of the rated power its its twist is hardly of the order of few electrical degrees. Now the next step will be to develop the the linear dynamic model of the system now in order to develop the linear dynamic model of the system what we do is that we linearize these differential equations around the nominal operating conditions. Further, while doing this linearization where we will assume that the torque developed in the various turbine sections are constant right.

(Refer Slide Time: 30:58)



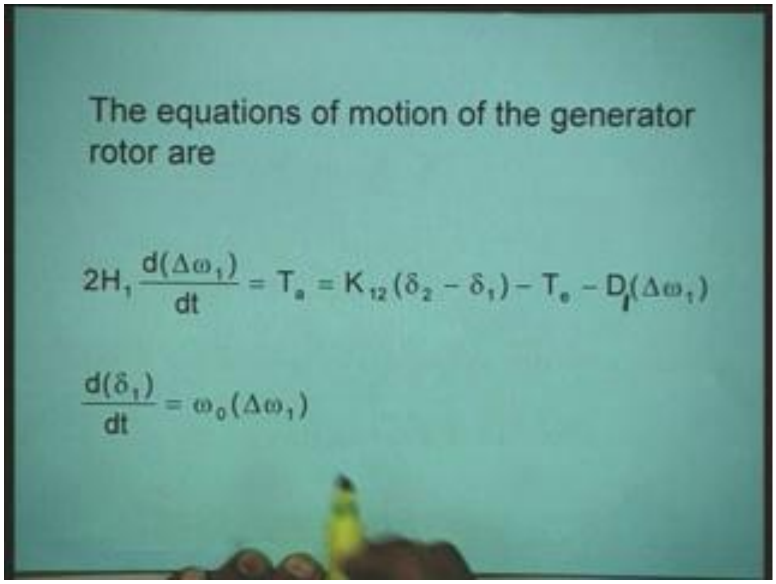
So that we can write down we can write down now that under small perturbation, when the system is perturbed the T_e the electrical torque or the air gap torque is equal to T_{eo} plus delta T_e. Similarly, since we have assumed this the torque developed in various turbine sections constant right therefore the T_{HP} is equal to T_{HPo} there is no change similarly T_{IP} will be equal to the T_{IPo}. Now this o stands for or 0 stands for steady state value of the torques and similarly we can write down that T_{LPB} equal to T_{LPBo} and T_{LPA}, L_{PA} equal to T_{LPA}.

Now these angular positions will represent the angular positions like say angular position under the dynamic condition as delta 1 equal to delta 1 o plus delta delta 1. Similarly, delta 2 will be written as delta 2 o plus delta delta 2 and so on we can write down the delta 5 as delta 5 o plus delta delta 5 similarly when we talk about the speed right. Now under steady state conditions omega 1, omega 2, omega 5 are equal to omega naught okay and when the system is perturbed the deviation in the speeds is delta omega 1 delta omega 2 and delta omega 5. Now using these small perturbations around the operating condition we can write down the dynamic model of the system around the operating condition.

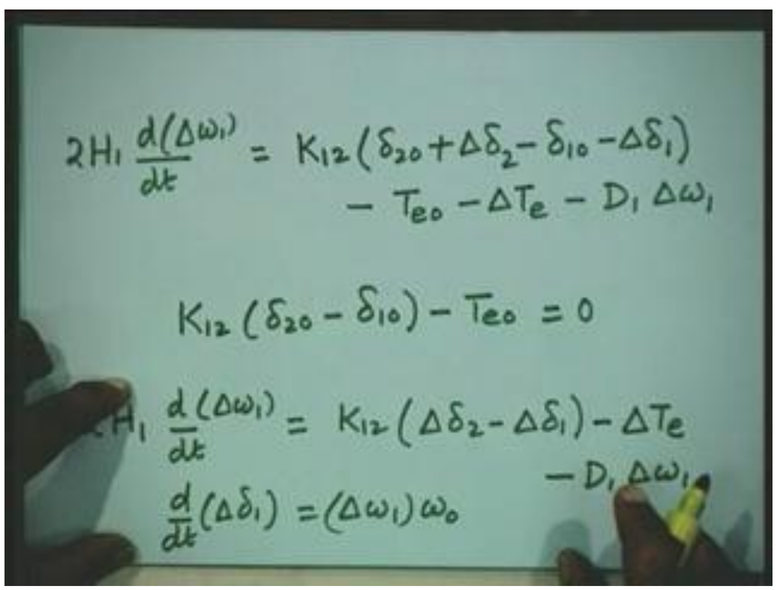
Now suppose I consider this equation this is the equation for motion of the generator rotor. Now here when you substitute substitute delta 2 equal to delta 2 o plus delta delta

delta 1 equal to delta 1 o plus delta delta T_e as T_{eo} plus delta delta T_e and this this term remains same.

(Refer Slide Time: 33:32)



(Refer Slide Time: 34:15)



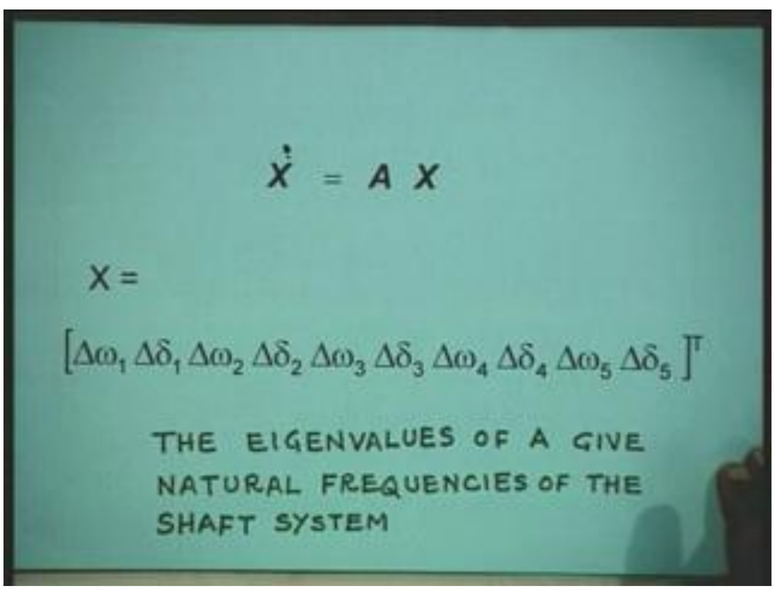
Now with this the when you substitute the quantities we can write down the equation in this form 2 times H_1 d delta omega 1 by dt of d by dt of delta omega 1 equal to equal to the the torque which is acting here on this is the accelerating torque which is acting on this is T_{12} okay now this T_{12} can be written as is K, K_{12} into delta delta 2 o plus delta delta minus delta 1 o minus delta this is delta delta 2,1 okay minus minus T_{eo} minus delta

T_e minus D_1 delta omega 1, I remember that when we had written actually derivative of speed d by dt of delta omega 1 is same as d by dt of omega 1 okay.

Now in this equation under steady state condition under steady state condition K_{12} delta 2o minus delta 1o minus T_{eo} equal to this is under steady state condition therefore therefore for considering the small perturbation around the steady operating condition around nominal operating condition we can write down the linear differential equation as 2 times H_1 d delta omega 1 by dt that is d by dt of delta omega 1 equal to K_{12} delta delta 2 minus delta delta 1 minus delta T_e minus D_1 omega 1. Our second equation when we write down delta 1 equal to delta 1 o plus delta delta 1 the linear equation will come out to be d by dt of delta delta 1 equal to delta omega 1 into 0 therefore these 2 equations are the linear differential equations.

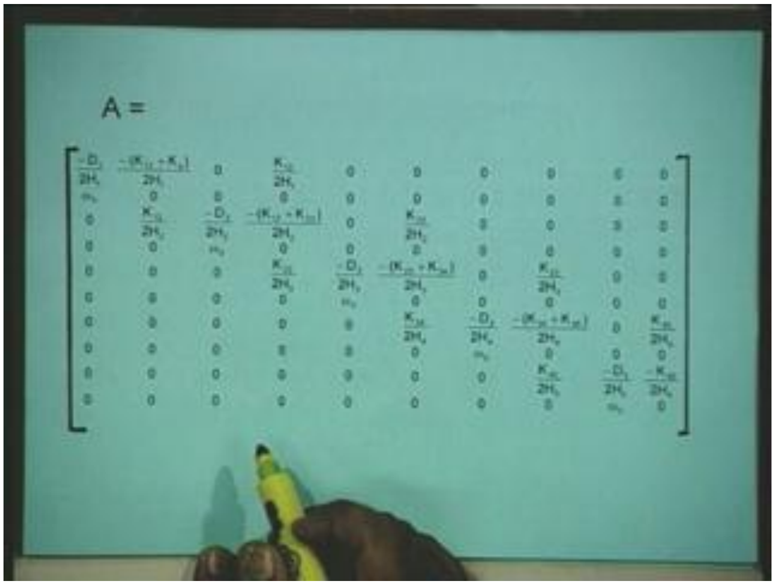
Now here this torque delta T_e is expressed in terms of the deviations of these angles delta delta 2, delta delta 1 and so on it is a function of the the deviation of the angular positions of the various rotors okay therefore, these 2 equations represents the linear differential equations. Similarly, we can write down the or linearize linearize the differential equations associated with the turbine section L_{PA}, turbine section L_{PB} turbine section intermediate from the I_P and turbine section H_P that is all the 9 differential equations can be linearized and once you linearize these equations we will get the dynamic model of the system in the form X dot equal to AX that is we can write down the dynamic model of the complete system in the form X dot equal to AX where X is the state vector written as delta omega 1, delta delta 1, delta omega 2, delta delta 2, delta omega 3, delta delta 3, delta omega 4, delta omega delta delta 4, delta omega 5, delta delta 5 transpose that is X is a vector comprising of these 10 state variables.

(Refer Slide Time: 38:11)

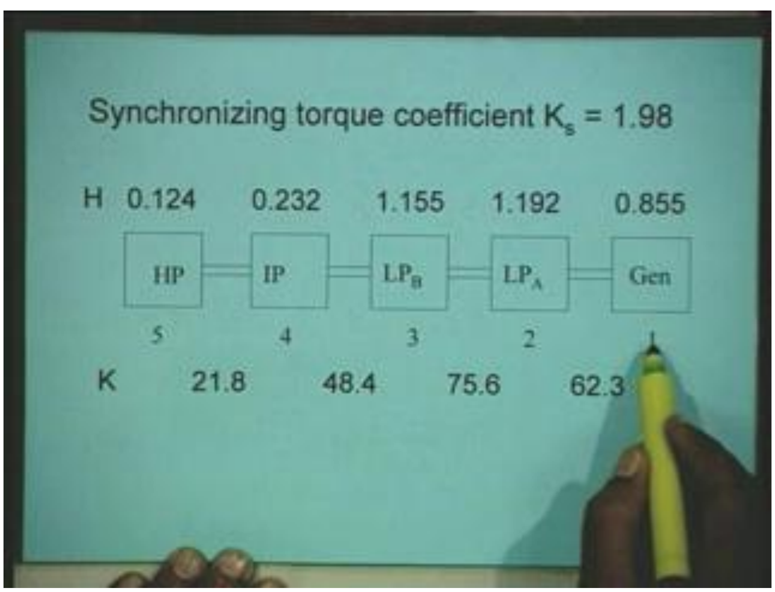


Now once we have obtained the linear dynamic model of the system we can further study the behaviour of the behaviour of the torsional oscillations or we can study the oscillations of the turbine generator shaft. Now to study the behaviour of the system first step is to find out Eigen values of the system and these Eigen values are Eigen values will give you natural frequencies of the shaft system that is when we obtain Eigen values what we will get is we will get the natural frequencies of the shaft system.

(Refer Slide Time: 39:38)



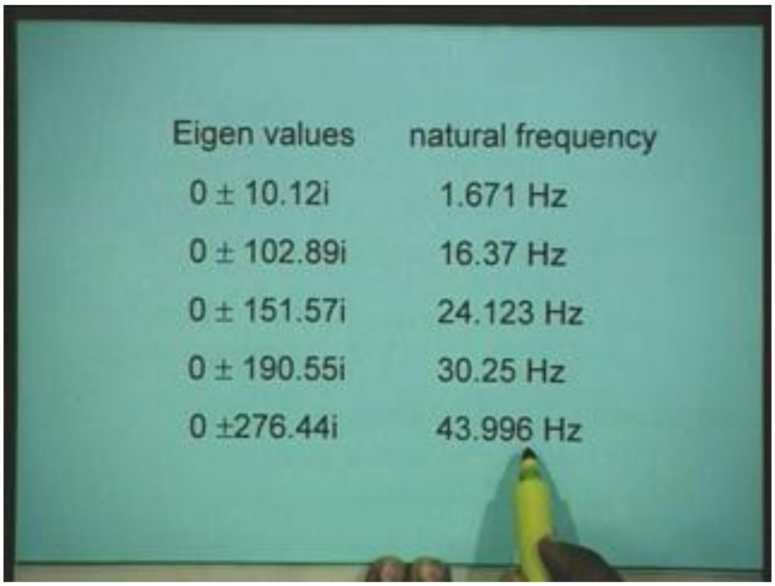
(Refer Slide Time: 40:14)



Now the system matrix A, A is a 9 by 10 matrix and the elements of this matrix depend upon the damping constants, inertia constants, the stiffness coefficients and the synchronizing torque coefficient, okay. Now to understand the behaviour of the or dynamic behaviour of the turbine generator shaft system. Let us take an example, we consider here the same 5 mass system and we represent these masses as 1, 2, 3, 4 and 5 the inertia constant of H_P section is .124, inertia constant of I_P is .232, L_P is 1.155 and L_{PA} is 1 .192 and generator is .855 that is these are the different values of the inertia constants of the 5 masses. The stiffness coefficient of the shafts connecting the various masses are represented as 21.8, 48.4, 75.6 and 63. 62.3 that is in this diagram what I have shown is that these are the inertia constant of the 5 masses and these are the stiffness coefficients. Further, we assume that is synchronizing torque coefficient K is is 1.98 right and this K is depends upon the operating condition okay.

Now the operating condition which we have considered is that the system is delivering 90 percent power okay. Now for this system for this system we develop the dynamic model and write down the a matrix or determine the a matrix of the system. Now in order to in order to simplify out understanding, we neglect here the damping coefficients associated with the system right. When we neglect the damping coefficients and we obtain the Eigen values of the system matrix, the Eigen values this system matrix A which I have just now shown is a real matrix right and since we have assumed the damping of associated with these masses as negligible and with this assumption the Eigen values come out to be all complex conjugate and the real part is 0.

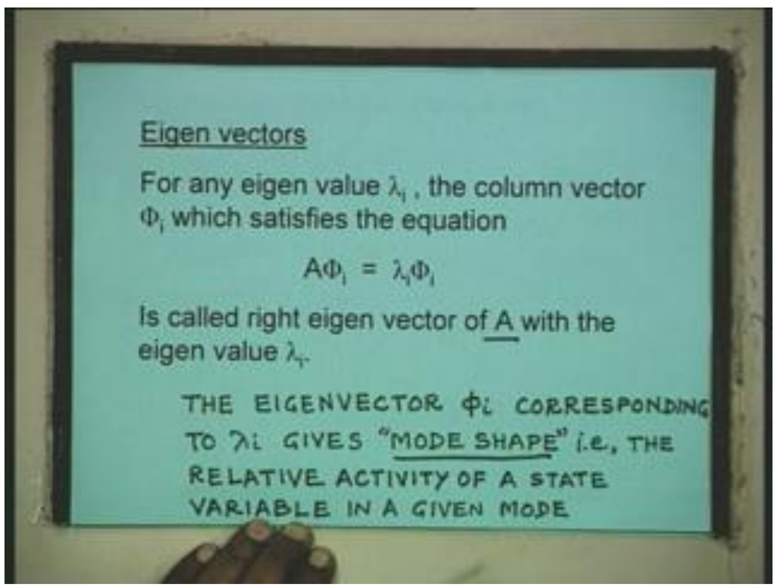
(Refer Slide Time: 42:32)



Now for these Eigen values we can find out the associated natural frequency of oscillation. Now for this first Eigen value whose imaginary part 9.12 the associated natural frequency is 1.61 hertz and similarly, the frequency associated of this second Eigen value is 16.37 for a third Eigen value is 24.123, the next is 30.25 and next is 43.996 hertz, I mean these are the 5 natural frequencies associated with the turbine generator shaft that is whenever this turbine generator is perturbed right then then the torsional torsional oscillations are produced and the torsional frequencies are given like this that is you can see here that we have for this 5 mass system these 5 different frequencies the the frequency 1.671 can be considered to be the system frequency where the the entire turbine generator mass oscillates with respect to the system while other 4 frequencies are associated with the different torsional modes of oscillations.

Now to further understand the behaviour or dynamic behaviour of the system, we analyze the the dynamic model of the system further that is for each of the Eigen values we obtain the Eigen vector and we can easily understand the meaning of the Eigen vectors, we all know what we stand for Eigen vector that is for each Eigen value there exist an Eigen vector and we can understand what the Eigen vector stands for that is for any Eigen value lambda i the column vector phi i which satisfy this equation A phi i equal to lambda i phi i is called the right Eigen vector of the matrix A.

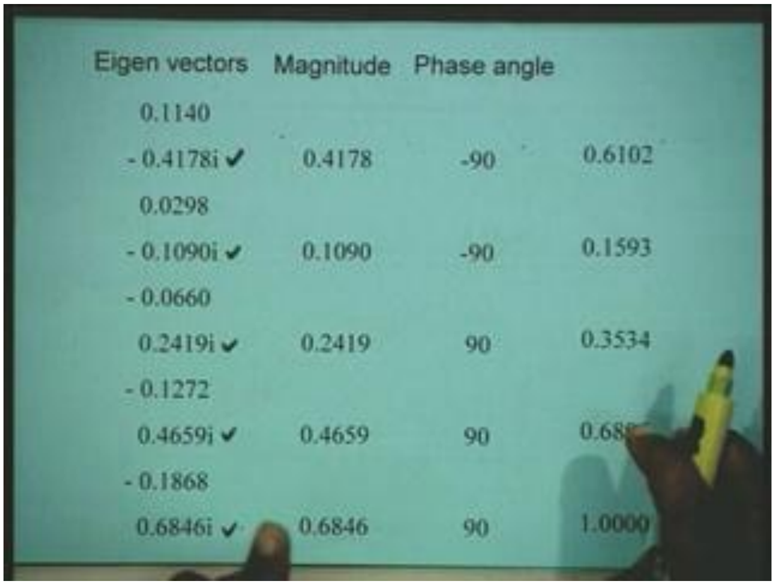
(Refer Slide Time: 44:26)



Now this phi i is associated with Eigen value lambda i if I put i equal to 1, I will get Eigen vector 1 if I put i equal to 2, I will get Eigen vector corresponding to Eigen value lambda 2 and so on now this Eigen vector will have the same dimension as they are the dimension of state vector okay. The Eigen vector phi i corresponding to lambda i gives mode shape, you have to understand this very important this is very important term gives mode shape that is the relative activity of a state variable in a given mode okay.

Now to understand this what we will do is that we will find out we will find out the Eigen vectors corresponding to each of the 5 Eigen values or 5 complex conjugate Eigen values. Now Eigen values of the system matrix A can be easily obtained using mat lab software and similarly, we can find out the Eigen vectors corresponding to each of the Eigen values. Now these are obtained for the system which we have investigated.

(Refer Slide Time: 46:24)

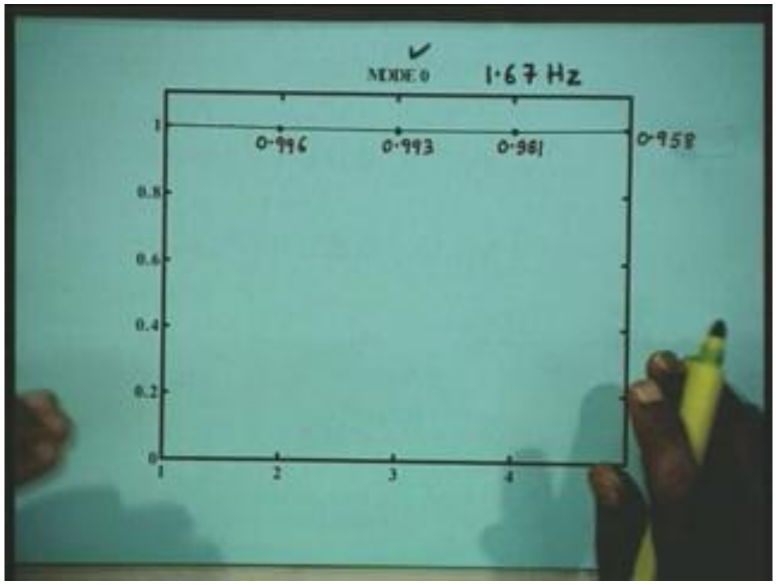


Now here in this table I have shown the Eigen vector, this Eigen vector is associated with Eigen value 0 plus minus 20.21 i that is for this first Eigen value I have obtained using the matlab software the Eigen vector and this Eigen vector has has 10, 10 actually entries or now we can easily see here actually the first these first 2 elements of the Eigen vector are associated with the rotor of generator that is delta omega 1 and delta delta 1. Similarly, the next 2 Eigen values are next 2 entries are associated with with the state variables associated with L_{PB} rotor section and so on. Now here what we have done is the we are considering, we can consider we can consider the either the elements associated with the speed or angle for analyzing the mode shape therefore, we have considered the the elements associated with speed not speed but the angular position these are the angular positions, okay.

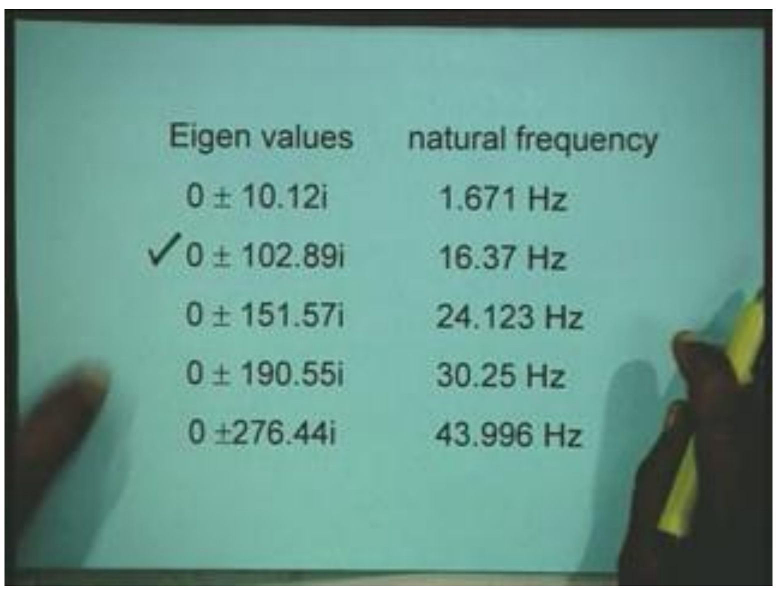
Now next we have plotted the magnitude of these elements the magnitude is same as the .4346 because its real part is 0 and the angles are also shown the angular for example, the angle associated with this element is 90 degrees, angle associated with this is also 90, angle associated with this is also 90 and so on that is all these elements have phase angle equal to 90 degrees. Now when we plot when we plot the magnitudes magnitudes of these Eigen vector elements right what we will do is that those magnitudes which are positive right we will plot actually in the as positive and those which are negative we will plot in the as negative.

I mean in this case we find that the all these elements have the same angular position and therefore they are all positive. Now this diagram shows the plot of mode 0, this mode 0 is the system mode whose frequency is 1.67 hertz and this mode 0 has the magnitudes 1.99 6.99, 3.992 and and .958 that is more or less the the all the all the state variables associated with with this mode have the same displacement okay and this mode this 0 th mode or system mode represents that all the masses rotate rotate together with respect to the system okay or rotate rotors of other machines.

(Refer Slide Time: 48:50)



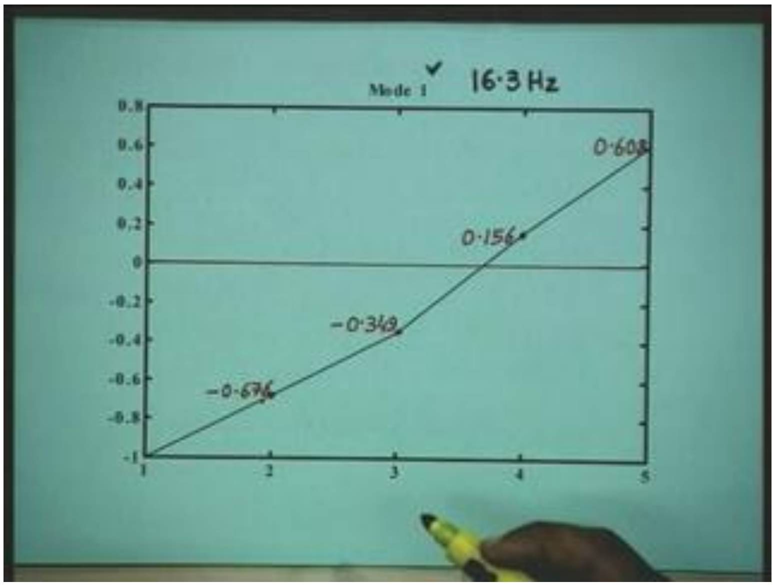
(Refer Slide Time: 50:23)



Now the next we consider the torsional mode, the frequency the Eigen value of the second or the first torsional mode is 102 plus minus 102.89 i and its frequency 16. 37. Now following the same approach for this Eigen value the Eigen vector is obtained, now this Eigen vector you can see here it has again the 9 elements and you have real imaginary real imaginary and so on.

Now this these elements these elements are associated with with angular displacements delta delta 1, delta delta 2, and delta delta 5. The magnitude and phase angle of these elements are written here as .4178 minus 90 degrees because minus is here like that then this vector is normalized when we normalize it the the element of the vector which is maximum magnitude or highest value right, we use this as 1 right and rest are represented with respect to this as the base therefore we find here that the the normalized values of the elements associated with delta delta 1, delta delta 2 and so on are given like this.

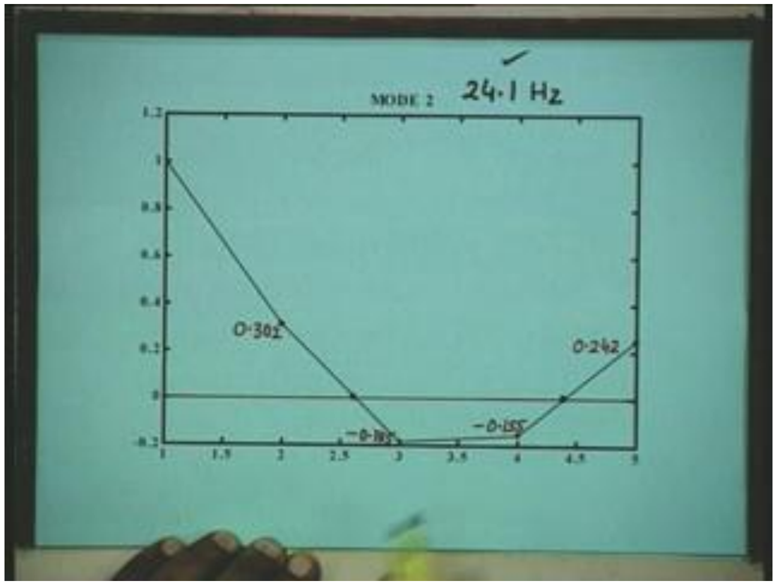
(Refer Slide Time: 52:12)



Now here the angles are minus 90 minus 90 and then they change over to plus 90 plus 90 plus 90 therefore now if I represent these as positive they will become negative right therefore the plot which I have shown here for this it is starting from minus 1 minus .7676 minus .32 that is this is the mass 1 this is mass 2, mass 3, mass 4, mass 5 okay and you show the magnitudes positive or negative this is our 0 line okay therefore this shows where we join all these points by a state line and this gives us a mode shape corresponding to the mode 1 or this is the mode shape of mode 1 whose natural frequency of oscillation is 16.3 hertz.

Now let us understand what we mean by this mode shape the meaning is can be easily understood here that if you look at this diagram right then the displacement of displacement of these these masses 4 and 5 with respect to displacement of mass 1, 2 and 3 are in opposite direction. It means here the mass that is generator the rotor of L_{PA} and L_{PB} sections oscillate with respect to the rotors of I_P and H_P sections that is whenever the generator rotor is perturbed right and the the torsional with this torsional frequency or with this frequency 16.3 hertz in that case in that case the the generator the L_{PB}, L_{PA} and L_{PB} they will oscillate with respect to the other 2 masses and in this mode 1 there is 1 crossing 1 changeover of sign that is you can just see here actually this is a 0 line and there is 1 crossing here.

(Refer Slide Time: 55:37)



Now similarly we have obtained the Eigen vector for the mode 2, this is mode 0, we will call it mode 0, this is mode 1, this is mode 2 that is the frequency of oscillations or torsional oscillations torsional frequency is 24.123 and the corresponding Eigen vector is shown here.

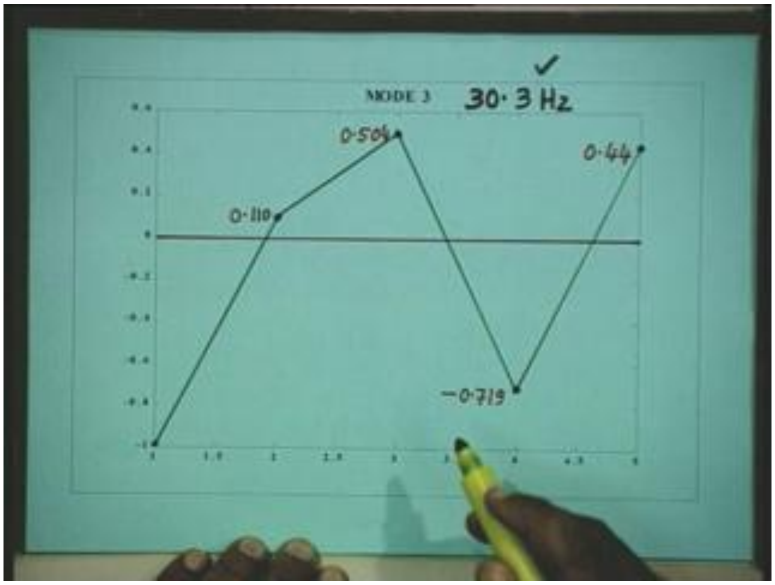
Now if you follow the same procedure as we have done for other 2 cases then we obtain actually the magnitude of the elements associated with delta delta 1, delta delta 2, delta delta 5. These are the magnitudes and these are the associated phase angles you can easily say that phase angle of this is 90 degrees phase angle of these 2 elements minus 90 minus 90phase angle of other these again plus 90 plus 90 it means when we plot plot the the magnitudes of these this displacements right you will find that there will be 2 crossings right and the mode shape is shown here that is this is the frequency 24.1 hertz and the mode shape you can see is like this therefore there are 2 crossings here that is the mode shape will always have 2 crossings okay.

Now in this when you try to analyze here then you can easily understand that the that these 2 masses 1 and 2, these masses will oscillate with respect to the masses 3 and 4 and similarly, the 3 and 4 will oscillate with respect to mass 5 that is whenever it is disturbed or perturbed with the frequency corresponding to this 24.1 right this the shaft will oscillate in such a fashion. So that the mass 5 will oscillate with respect to 3 and 4 and 3 and 4 will oscillate with respect to 1 and 2 right and frequency of oscillation is so much.

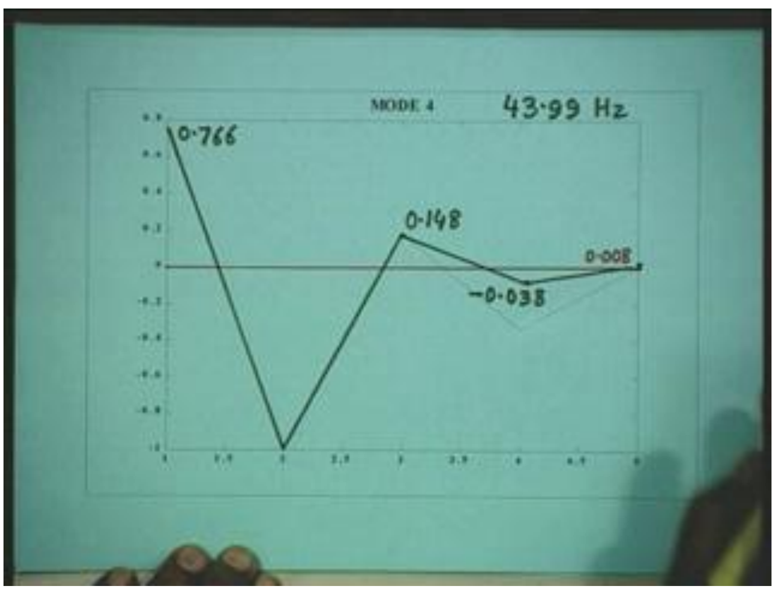
Similar studies performed for the mode 3 whose frequency is 30.25 hertz and this is the Eigen vector again we are considering the elements corresponding to angular displacements. Now these are the normalized magnitudes of the elements associated with with the the speed deviations and you can easily see here actually that we have 3 change over is that is start from positive go to negative then again positive and again negative and this shows the mode shape associated with the mode 3 whose frequency is 30.4, 30.3

hertz and again. Now we can easily see here that when the system is perturbed we will find actually that the mass 1 will oscillate with respect to mass 2 and 3 and 2 and 3 oscillate with respect to mass 4 and 4 and 4 will oscillate with respect to mass 5 and so on right. Similarly, the mode shape for mass 4 is shown and you can easily see that there are 4 crossings.

(Refer Slide Time: 57:26)



(Refer Slide Time: 57:59)



Now with this let me summarize that we have studied the modeling of turbine generator shaft system. We have obtained the non-linear and linear models and we have also

studied the application of Eigen vector concept to study the mode shapes of the system.
Thank you!