Power System Dynamics
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Lecture - 31
Direct Method of Transient Stability Analysis

Friends, we shall study today direct method of transient stability analysis.

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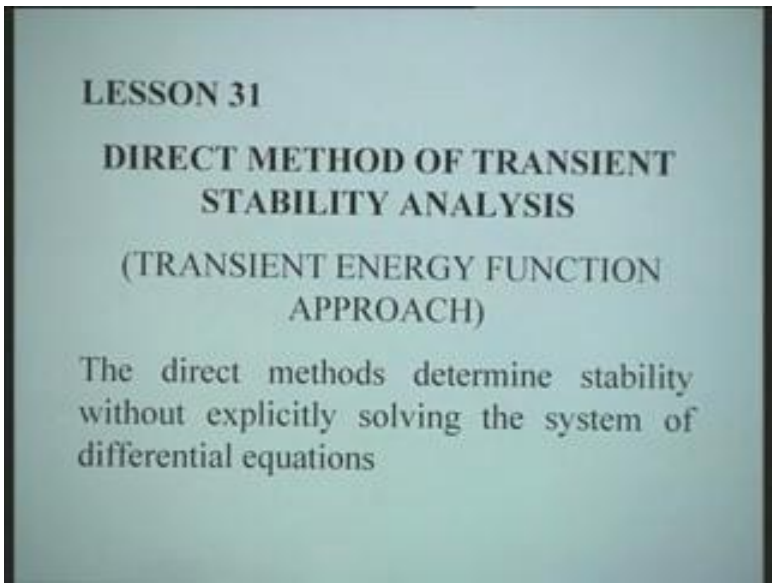

Over the years efforts have been made actually to assess the transient stability of the system by applying some direct methods. As you know I that to assess the transient stability of the system one has to solve a set of differential algebraic equations and for a large system large system the number of equations are large in number and it is a quite a time consuming process, even if we resort to dynamic equivalence right the dimension of the model is quite large. Here, we will concentrate upon one approach which is which is known as the transient energy function approach transient energy function approach.

In direct methods when we tried to determine the stability one can assess the stability of the system without explicitly solving the system of differential equations or we can say that we can partly partly avoid or partly avoid the solution of differential equations we may have to solve the differential equations for a short interval.

For example, as you know that if you take a single machine infinite bus system then for this we can apply the equal area criteria of stability to assess the stability of the system. However to find out the critical clearing time we do have to integrate the equations from initial operating condition up to the fault clearing time because because we know that if we if you obtain if you obtain for a given system given system the critical clearing angle then we have to find out corresponding critical clearing time okay and therefore we may have to integrate this differential equation from initial operating condition up to the critical clearing time okay.

It means actually we may not have to integrate the differential equation or solve the equations for the complete time period and in that case we have partly partly solving the differential equation for the part of the period okay. Now in direct methods in fact they are mostly at the reach level and and as yet actually the application in practical large system is is not reported because in practical large system as we will see that when we try to apply the direct methods there are some problems.

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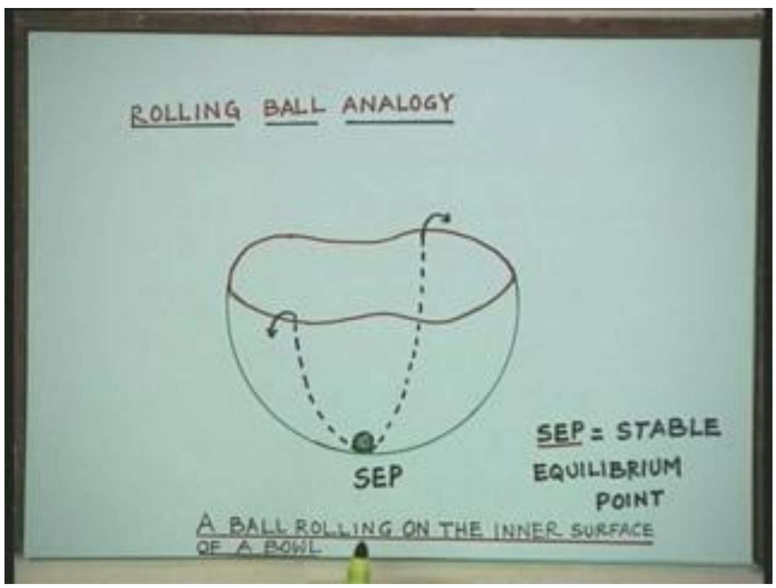

Now to understand the basic concept or basic philosophy of applying the direct methods we make use of a simple rolling ball analogy. In this rolling ball analogy let us say a physical system where you have a bowl, this is a bowl this bowl has uneven or non-uniform rim okay here is a ball a small ball which is lying here in this bowl okay.

Now the the the ball occupies this lowest position in the bowl and this position is called the stable equilibrium point okay. Now when this bowl is perturbed let us say that we we apply some force for a short time so that this bowl is perturbed and when you perturb right you are injecting some kinetic energy to this bowl okay. Therefore, this bowl will roll inside this surface of this bowl it will roll along some path the path along which it rolls will depend upon the direction in which the disturbance is applied right.

Now now if suppose the kinetic energy injected is large enough so that the ball comes right up to the rim right and it crosses over this rim right then we say that the system is unstable because once it crosses it enters into the unstable region and it cannot return back to the stable operating condition okay but suppose if the kinetic energy injected is such that such that it climbs a certain height in the a bowl in the bowl and and returns back right then the system is stable okay therefore, here if you see here that for the stability of this bowl and stably of stability of this ball in this bowl right.

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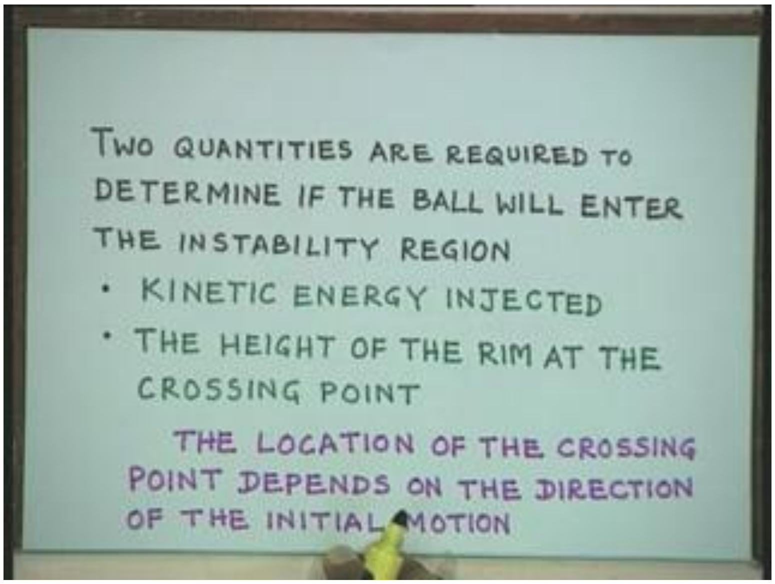

There are two important points to be noted, one is the kinetic energy injected one is the initial kinetic energy which is injected to the ball okay, second is the height of the rim at the crossing point height of the rim at the crossing point then the location of the crossing point depends on the direction of the initial motion, the location of the crossing point depends on the direction of the initial initial motion, it means when you when you give some disturbance right then the ball is going to move over a certain trajectory inside the bowl okay and depending upon the direction in which it is made to move right the height is going to be different because I have assumed that the rim is non-uniform uneven and therefore if I want to find out whether this this ball which is lying inside the bowl right forms a stable system right I should know what is the kinetic energy which is injected to the system and what is the height of the rim?

Now as you know till that suppose when the ball is perturbed right as it moves along along then it acquires potential energy potential energy which is which depends upon the position of the ball or actually height of the ball with respect to the stable equilibrium point, this is the potential energy and and when it is moving it looses kinetic energy.

Now suppose suppose when it just comes to the particular point on the rim rim and the speed of this rim speed of the ball become 0 right it means at this particular point the ball has acquired the potential energy only it has the total potential energy, it does not have kinetic energy.

Now in case the ball comes up to this particular point and it does not have any kinetic energy then the forces will act on this ball to bring back to the stable equilibrium point and therefore if I want to assess the stability I must find out what is the energy or what is the what is the potential energy with the system, with the system can provide or we can say another way that what is the capability of the system to observe the energy imparted to the ball and the energy imparted to the ball depends upon what is the what is the maximum potential energy which it can it can acquire therefore from the stability point of view, we can consider like this that I can define a energy function.

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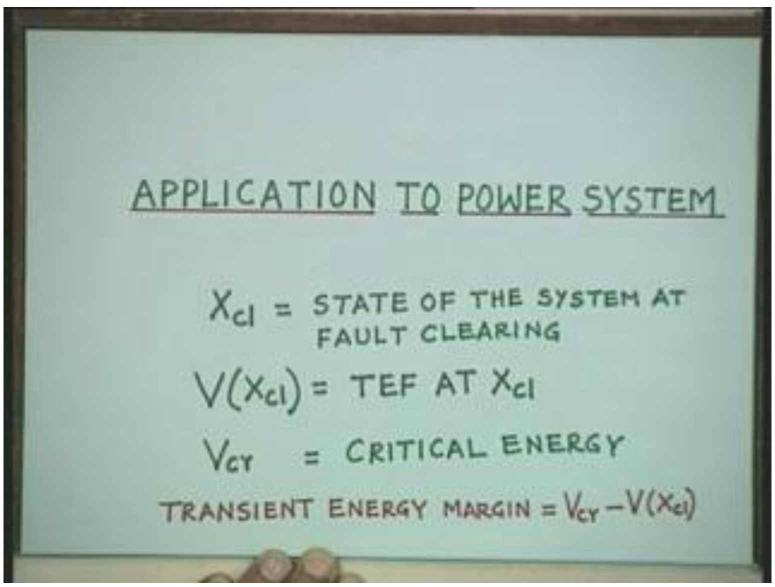

I can define a energy function and let us say the energy which this system acquires energy which the system acquires is denoted by a symbol V. We will denote this energy by a symbol V or V is a energy function right now let us denote the X_{cl} as the state of the system at fault clearing when I say state of the system means for a single machine infinite bus system, the state can be described by the two two state variables one is delta another in omega speed speed and angular change therefore this delta and omega these two are the state variables for a single machine infinite bus system. Now if I know what is the state of the system at at the fault clearing time right then I call this X_{cl} as the state of the system at fault clearing time and let us add V which is the function of the state of the system as V_{cl}, V of V at X_{cl} okay.

I will denote this as a transient energy function is called X_{cl}. Now we find out a critical energy V_{cr} denote the critical energy and the the difference that is the critical energy minus minus the energy at at fault clearing time that is V which is the function of X_{cl} right this difference is the transient energy margin a very simple concept right

Now if I again look back at this rolling ball analogy right the critical energy is the energy correspond corresponding to the potential energy potential energy which this system can provide right at the crossing point okay and since, since actually the the disturbance can make this ball to move in different directions right therefore critical energy is different for different directions of motion right.

Now when I look actually, now a practical power system then we can apply this analogy to a practical power system also how that suppose the system is operating in the steady state condition and a fault occurs in the system okay. Now the moment the fault occurs during this fault on period right. The rotors of the synchronous machines accelerate okay and acquire some kinetic energy.

Similarly, the angular position of the rotor is also change therefore we can say that the the machines in the system acquire some kinetic energy, the total kinetic energy plus some potential energy. This potential energy it depends upon the angular position of this rotor with respect to the stable operating angular positions okay now if the system is capable of observing the kinetic energy which you have injected into it right then the system will be stable if the stable is cannot absorb that kinetic energy completely right then the system becomes unstable this is a very simple concept.

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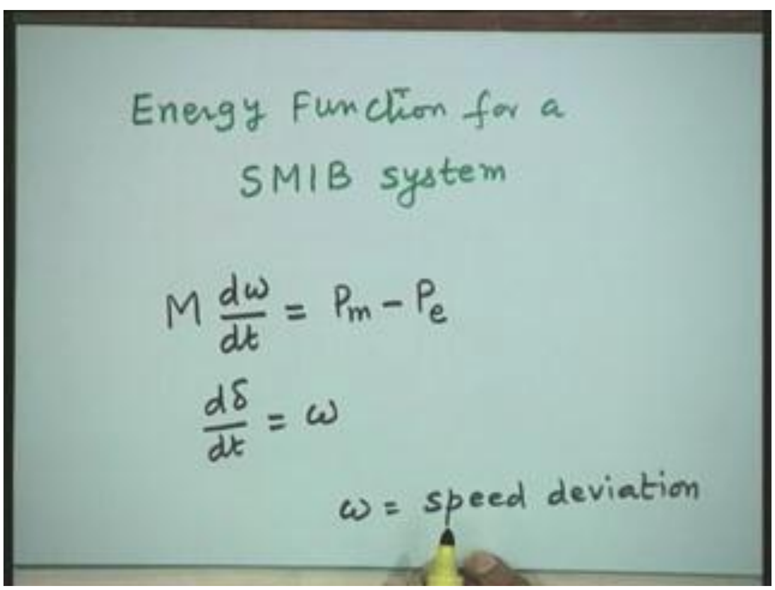

Now here we will first concentrate upon the application of this transient energy function approach to a single machine infinite bus system and we will also show that the equal area criteria which we very much familiar right can also be interpreted in terms of, in terms of energy functions. Therefore, now our first step is to define a transient energy function how we define the transient energy function.

Now to define the transient energy function I will start with with a simple system and that is the single machine infinite bus system that is we will find out the energy function for a single machine infinite bus okay once you understand the basic concepts, we will extend this concept to a multi-machine system. The basic swing equation of a single machine infinite bus system I shall write down in this form M d omega by dt equal to P_m minus P_e. Another equation I am writing here is d delta by dt equal to omega I am writing in this form which is slightly its it appears slightly different from what we have been using.

Now here this omega is the deviation of the speed from the synchronous speed therefore instead of writing delta omega I can write down simply omega for the sake of simplicity that this the omega is the speed deviation okay. Now here here I am not stating that the speed deviation is in per unit or speed deviation actually in radians per second okay it will the value of this if you want to be in per unit then appropriately the M has to be chosen right. Now the energy functions energy functions are always computed for post fault system that whenever we compute the energy function or define energy function right the energy functions are always constituted for post fault system.

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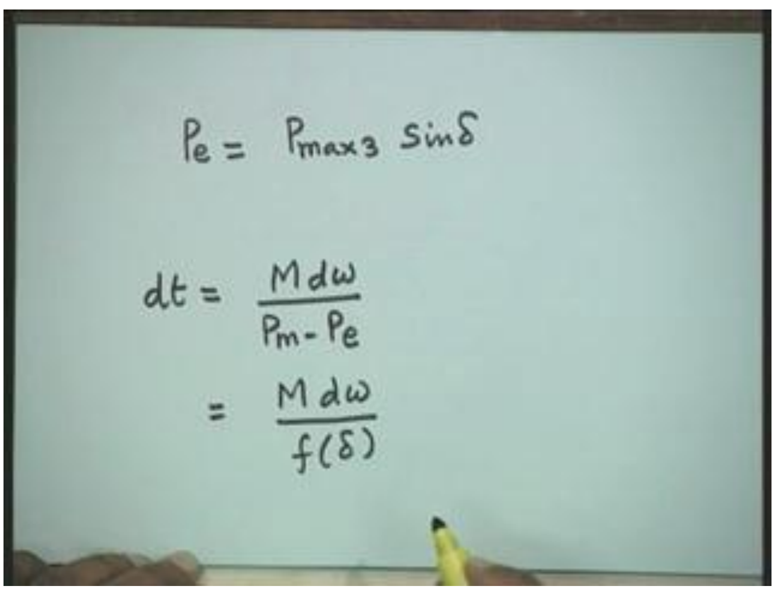

Now for the post fault system, let us define that P_e is equal to P_{max3} sin delta here actually the the three power angle characteristic we will define that P_1 equal to P_{max1} sin delta P_2 equal to P_{max2} sin delta and for post fault condition it is P_{max3} sin delta therefore the in this equation P_e is equal to P_{max} sin delta okay.

Now what we do here is that using this equation these two equations this equation and this equation okay we can write down here dt dt can be written has M d omega divided by P_m minus P_e that is suppose I write down here dt I just consider actually this small pertubation and let us say that this d omega and dt can be separated right. So I can write dt equal to M_d omega by P_m minus let us write down this denominator in the form of a function I can write down this as M_d omega by a function this is basically a function of delta of delta, for the second equation for the second equation we can write down dt equal to d delta by omega.

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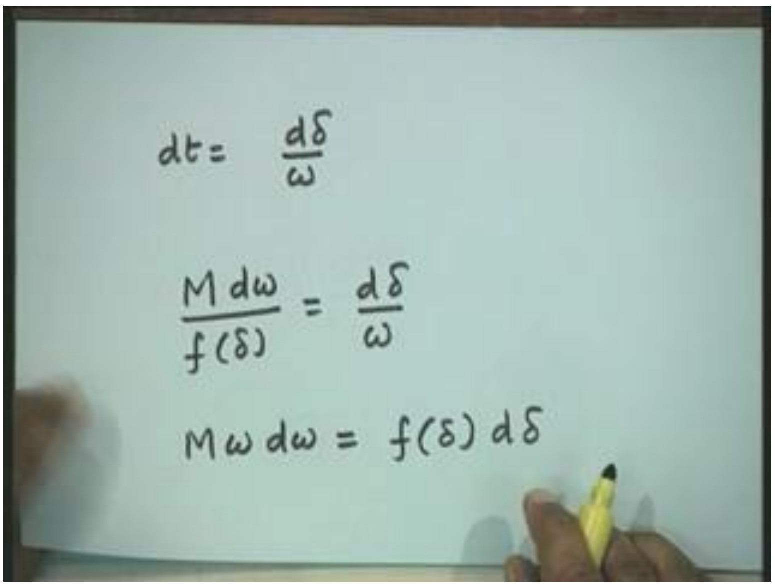

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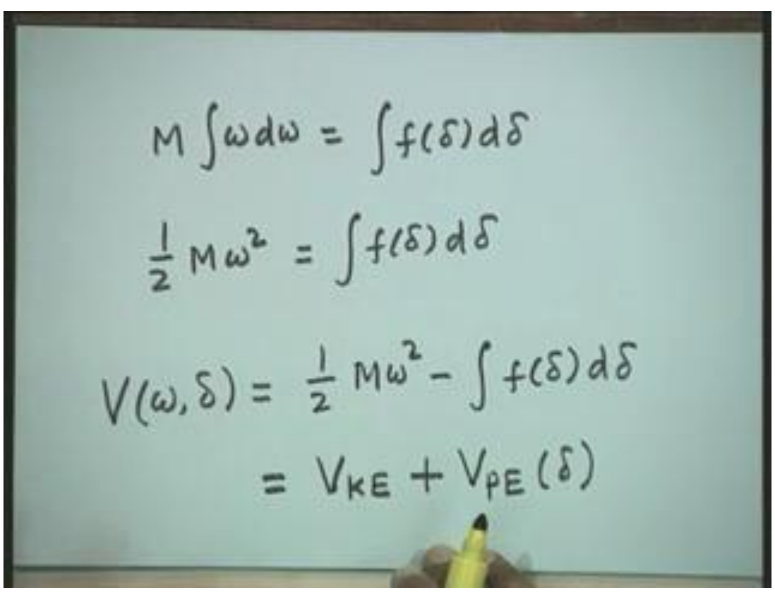

Now if I take this second equation dt equal d delta by omega now we can write down here M d omega divided by f delta equal to d delta by omega because dt equal to this dt equal to this therefore they are made equal okay or we can say here M omega d omega equal to f delta d delta okay. Now you integrate both sides of this equation then we can write down here as M integral of omega d omega equal to integral of f delta, d delta which can be written as 1 by 2 M omega square equal to I will just say integral of f delta, d delta. Here here at any instant of time at any instant of time the the 1 by 2 M omega square is equal to integral of f delta d delta because these are equal.

Now so far omega is finite this quantity is finite right and therefore this is also going to be a finite. The energy function is defined we use this definition V omega delta is defined as 1 by 2 M omega square minus integral of f delta d delta this is what is the basic definition of energy function here. Now we will identify that this this part of this energy function represents the kinetic energy therefore we call this as V_{KE} while this portion or this part of the energy function right it depends upon the angular portion delta is a function of delta only and therefore this is called, this part of the energy function is called V_{PE} and we denote especially or we put delta along with it so so that this is this is a potential energy and it depends upon delta.

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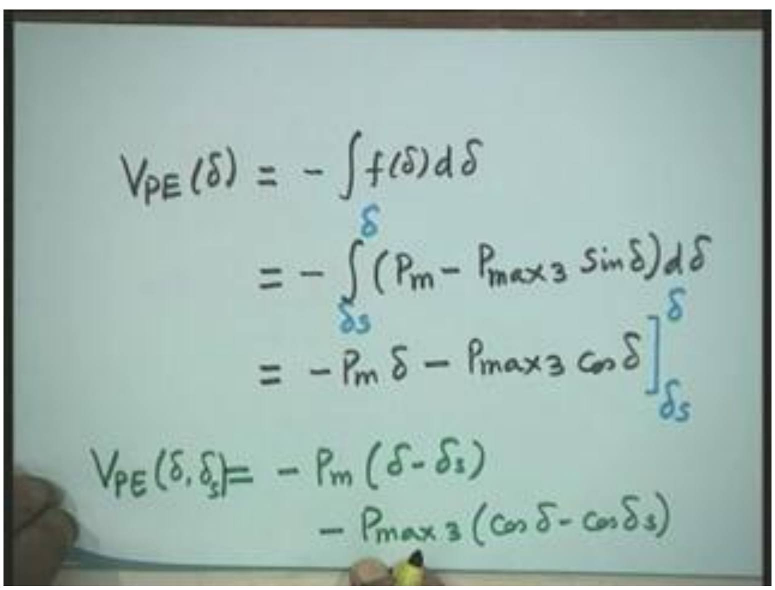

Now here if you clearly you can say try to write down the expression for V_{PE} delta right then this V_{PE} delta can be considered to be because in this equation suppose you see V_{PE} delta is basically basically the negative of integral of f delta d delta right therefore I can write down for this single machine infinite bus system the value of V_{PE} delta V_{PE} delta can be written as as negative of integral of f delta d delta okay and what is your f delta f delta we have denoted as P_m minus P_{max3} sin delta therefore now here you can put here is minus P_m minus P_{max3} sin delta d delta, is it okay? or its value you can write down as you integrate with respect to we are integrating with respect to delta it can written as minus P_m delta. Now when you integrate this quantity sin delta will have a minus cos delta that is integral of sin delta is minus cos delta okay and therefore this continues to be minus minus P_{max3} cos delta okay.

Now here we do one interesting coordinate transformation that is we would like to we would like to you can say express this potential energy such that for the post fault stable equilibrium point its value is 0 that is we want to we want to express this V_{PE} delta that is the potential energy in such a way so that its value is 0 for stable post fault stable equilibrium point.

Now suppose in this expression if I integrate this equation if I integrate and you put the limits let us say that I will put the limits here from delta s to some angle delta and you may say that is the integration is performed. So that we do it from some angle delta s to delta okay then we can write down this as V_{PE} is now function of delta and delta s okay this can be written as minus P_m delta minus delta s minus P $_{max3}$ cos of delta minus cos of delta s that is this expression, this expression we have written while in general I have not put any initial and final limits. Here, what we are doing is that we want to we want to you can say have the coordinate system in such a fashion, so that this V_{PE} is 0 for post fault stabler stable equilibrium point.

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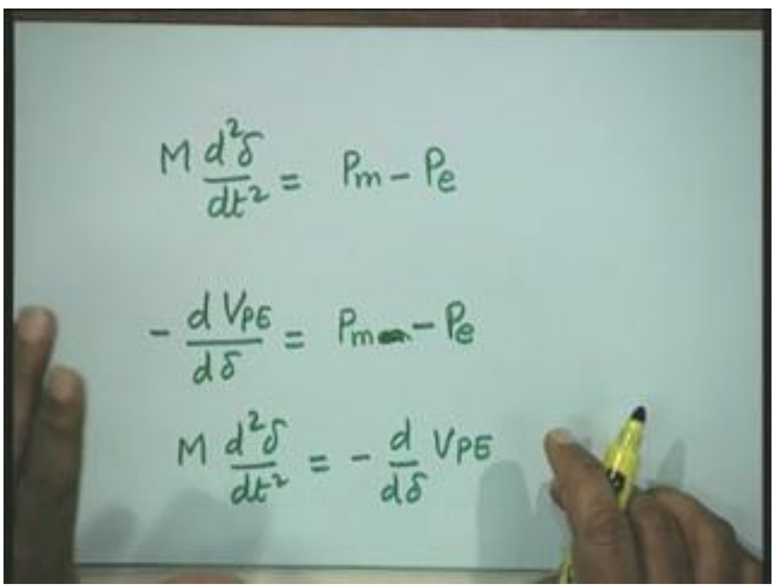

Now for example in this equation if I put delta equal to delta s if I substitute in this equation delta equal to delta s then this quantity is 0 right, yes if delta equal to delta s then V_{PE} delta delta is equal to minus P_m minus P_e because there is a sorry, see I putting delta equal to delta s so that this term is 0 and this term is also 0 cos delta is minus cos delta has become 0 that is instead of making the the our coordinate system such that the potential energy is measured for delta equal to delta naught or delta equal to 0 we even like to have to coordinate system in such a fashion. So that at the post fault stabler stable equilibrium point the potential energy is 0 right.

Further, we will show that show that this the the the total energy function the because when I talk about the energy function this is also called the transient energy function which is sum of kinetic energy and potential energy right. For the post fault condition we will show that this quantity is constant for the post fault system rights the transient energy function is a constant term. Now to show this thing we can proceed in a simple fashion like this that you you have your basic equation M times d_2 delta by dt_2 equal to equal to P_m minus P_e okay.

Now because the the way we have define the way we have defined actually the potential energy the we can write down here that d negative of negative of V_{PE} d delta is equal to P_m not P_{max} P_m minus P_e because when you you have seen here actually, when we define this V_{PE} you just look at this equation the definition of V_{PE} is V_{PE} delta is equal to minus integral of V f delta d delta. Okay therefore, if I take the negative negative and take the d d by d delta of this quantity then it is equal to f delta okay that is I can say now here that in this equation in this equation we can write down M d_2 delta by dt_2 can be written as minus d by d delta of V_{PE} okay.

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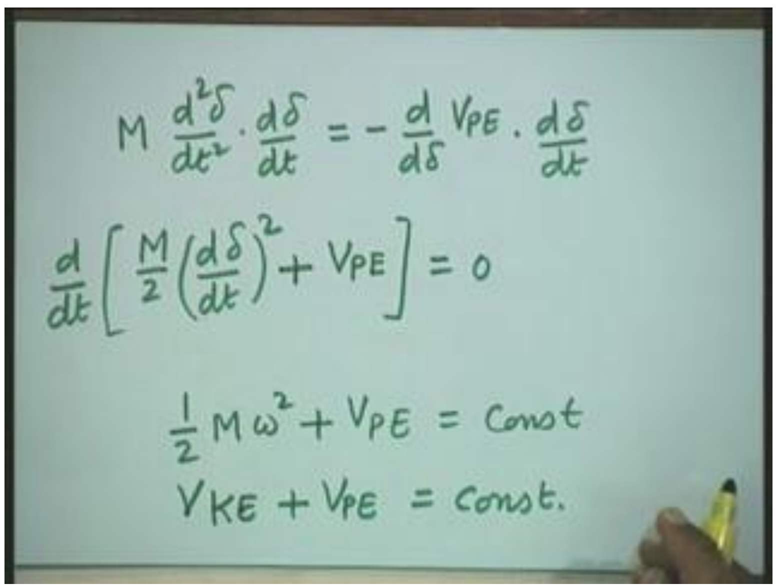

This is this is by definition we multiply both sides of this equation by d delta by dt that is you write down this as M times M times d_2 delta by dt_2 d delta by dt equal to minus d by d delta V_{PE} into d delta by dt okay. Okay, now this term can be identified you can then take this side or take on this on this side this can be identified as M by 2 this is identified as d by dt of M by 2d delta by dt whole square plus plus what will be the quantity here V_{PE} d by dt that delta you can take it out and therefore d by dt is here it is plus V_{PE} and this quantity is 0 that is you take this equation okay and you write in this form and therefore if you if you take the derivative this expression you will get the upper equation right and therefore this term is 0 the meaning of this is that the expression within the bracket is constant right derivative of this complete expression becoming 0 means this this term this is within the brackets is a constant term or we can say here that 1 by 2

M what is d delta by dt omega omega square plus V_{PE} is constant okay or I can say here that the energy function which has two components V_{KE} plus V_{PE} equal to the constant.

Now this is very important thing actually physically I want to just explain that when the system is disturbed okay and let us say that the fault is cleared you are looking for the post fault system, post fault system energy function is is constituted for a post fault system right. Then in the post fault system whatsoever the energy which is injected right when the system is in motion what happens is that the the total energy is a portion between V_{KE} and VPE right.

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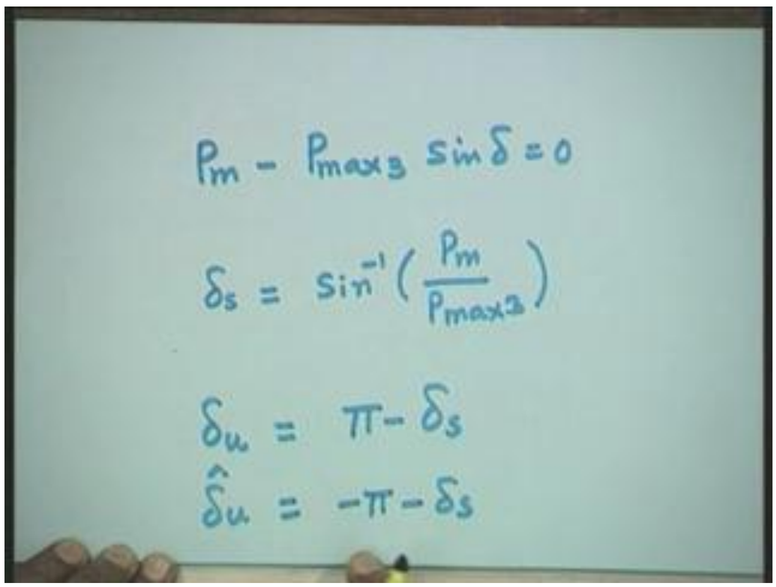

Therefore suppose a situation comes where V_{KE} become 0 then this total energy will become potential energy only right but this total remains same now here I want to just tell you one more point here that is any system any system you will find stable equilibrium point another is the unstable equilibrium point stable unstable equilibrium points for a single machine infinite bus system right under the post fault condition post fault condition we can find out the stable equilibrium point by equating this term P_m minus P_{max3} sin delta equal to 0 right. So that the stable equilibrium point that is delta s is equal to sin inverse P_m divided by P_{max3} this is the stable equilibrium point.

Now the we have unstable equilibrium point there will be two unstable equilibrium points which will satisfy this equation, one unstable equilibrium point I will denote as delta u this will be phi minus delta s, another will be we will denote as delta u say cap as minus phi minus delta s that is you substitute in this equation delta u equal to phi minus delta s this equation is going to be satisfied. Similarly, you substitute delta u hat as minus phi minus delta s then also this equation is going to be satisfied.

Now these two points are called unstable equilibrium point okay and the critical energy, critical energy that is V_{CR} which we call right is the potential energy at this unstable equilibrium point right. Now just to demonstrate how we can plot actually the we plot a graph for potential energy and show actually that how the the potential energy varies as you vary the angle delta.

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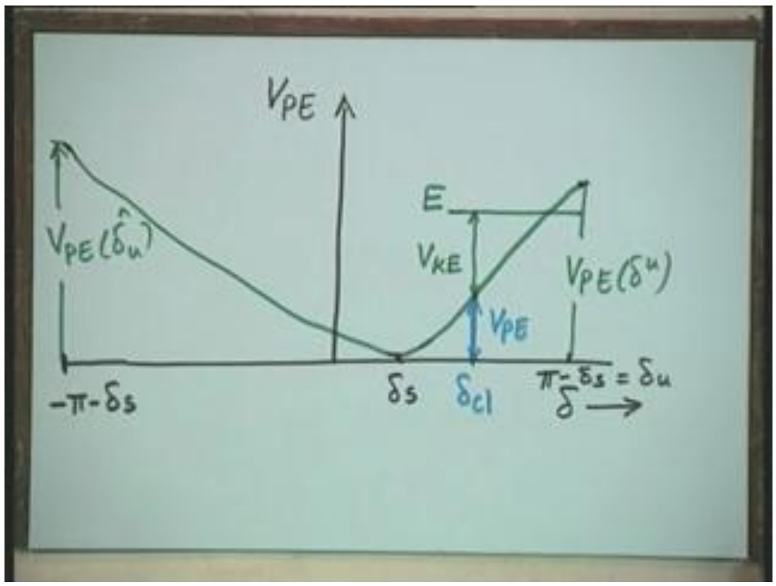

Now to plot this graph let us start like this on this we will on this axis we will put delta and here I am putting V_{PE} is a potential energy as per our as per our definition of the energy function the energy function the potential energy is 0 at delta equal to delta s. Let us say this point is delta s okay now and let us say that this point is phi minus delta s that is delta u when I say that there is another unstable equilibrium point it is on this side that is minus phi minus delta s.

If we now plot the potential energy as a function of delta then this plot will come something like this I will use a different color. Let me use of this form therefore this is the value of V_{PE} this is the value for V_{PE} at delta u of this quantity is V_{PE} at delta u at okay this is the value of V_{PE} at delta u and at delta s which is the stable equilibrium point this quantity is 0.

Now if the total energy if the total energy is equal to some quantity lets say E some quantity say E and let us say that this is the delta that is if angle at fault clearing that is the fault clearing angle then at this point this will represent this represents the potential energy we can call it V_{PE} and if the total energy is say E, I am using the word say total energy E E is the total energy then this portion will be your V_{KE} at this stable equilibrium point the the total energy will become now how much 0 V_{PE} is 0 and the the the total, if E is constant therefore at this point it is going to be only kinetic energy that the stable equilibrium point when the system comes actually it has a large maximum speed delta omega the speed derivation is large at the stable equilibrium point that is why it keeps on oscillating right.

Now suppose the total energy which is which is associated in the post fault system which I have told you that this is a constant and if this is less than this V_{PE} corresponding to delta u my system is stable, okay conceptually the simple the the subject is very simple here that if we can find out the critical energy and the critical energy in this particular system is corresponding to is the is the potential energy corresponding to the unstable equilibrium point and if I know what is the total energy which is injected into the system right and this total energy remains constant this total energy has two parts the potential and kinetic and and actually as the system moves right you will find actually that the the total energy is a portion between the potential energy and kinetic energy right and therefore condition for stability is that the potential energy at unstable equilibrium point which is the critical energy is more than the total kinetic total energy injected or it is the total kinetic energy which is injected which at any other angle is actually a portion between the two parts.

Therefore, whenever if you want to find out whether the system is stable or not right what is to be done is that one has to find out what is the critical energy and one has to also find out what is the total energy injected into the system right and if we can find out actually critical energy without actually solving the set of differential equations right then to find out the total kinetic energy injected one can one may have to integrate the equations from T equal to 0 up to the time T when the fault is clear only for a short time during fault condition you may have to integrate to find out what is total kinetic energy injected.

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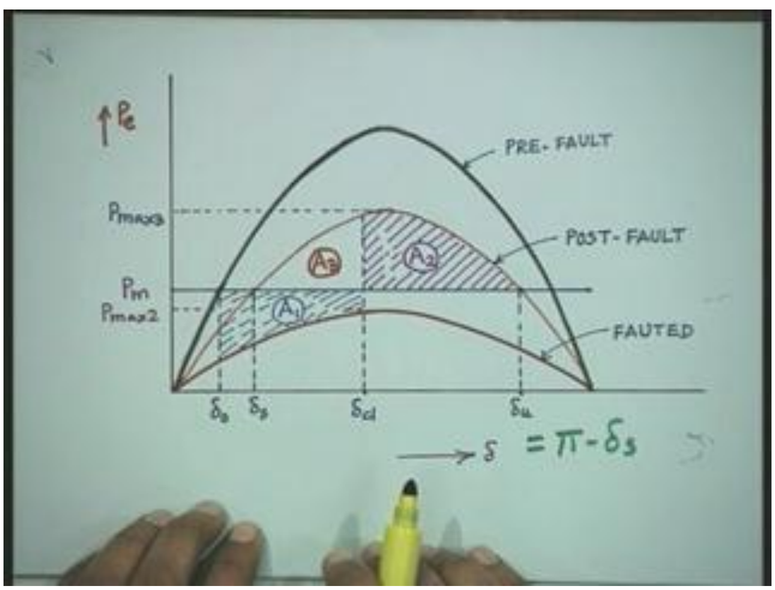

Now we will establish here the relationship between the equal area criteria of stability and the and the transient energy function approach for assessing the transient stability. We will establish that the equal area criteria of stability is same as the transient energy function approach right this

T_E as I have just told you that transient energy function is is function of delta and omega and it has two components, the kinetic energy and potential energy.

Now to illustrate this that we have we consider the same machine infinite bus system and this is the diagram which is drawn and it shows the three power angle characteristics the pre fault this is the post fault this is the faulted one okay. The the peak value of this post fault power angle characteristic is one is P_{max3} the peak value of during the fault is P_{max2} okay and the mechanical input is P_m okay this is I think we are all very well familiar with this.

Now, let us say that we are operating at initially at an angle delta naught on the pre fault power angle characteristic and the fault occurs and which drive the system on this post fault power angle curve and let us say that fault is cleared at delta equal to delta cl so that now you shift your operating point from from faulted curve to post fault curve and you move on this. In this diagram you can see that the unstable equilibrium point is shown here where where the post fault power angle characteristic intersect with P_m that is this is your unstable equilibrium point its value is equal to phi minus delta s and delta s the stable equilibrium point is the intersection of P_m with post fault power post fault power angle characteristic okay.

Now to establish this what we do is we have to look for the 3 areas the area this is area A_1 okay, this is the second area A_2, this area A_1 is is actually the area which represents the kinetic energy I will explain this represents the kinetic energy, we will show that it represent kinetic energy this area A_2 right is a part of the potential energy okay and our our stability criteria is that A_2 should be greater than A_1 right a simply criteria is A_2 should be greater than A_1.

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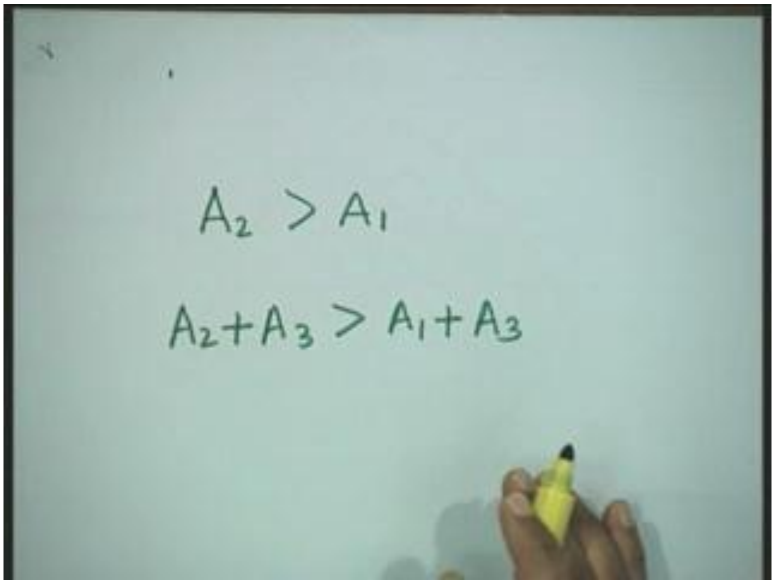

Now here we when try to apply the transient energy function approach, we look at this third area also this area A_3 this area A_3 is bounded by the post fault power angle curve, mechanical input line and the angle delta s and delta cl that is this area is A_3 okay therefore if we our stability criteria is that A_2 A_2 should be greater than A_1 this is our equal area criteria of stability.

Now what you do is that you add you add this area A_3 to both sides of this equation. So that I can say A_2 plus A_3, A_3 there is any change because I am adding the area A_3 to both side of this equation therefore A_2 plus A_3 if it is greater than A_1 plus A_3 right then my system is stable. Now this A_1 plus A_3, A_1 plus A_3 if you look here this area when you are at this point delta cl right this area which is A_1 it represents the kinetic energy that is 1 by 2 m omega square where omega is the speed corresponding to delta cl while this A_2 represents the potential energy V_{PE} corresponding to delta cl okay and this this sum must be equal to this total okay, in fact this area that is A_2 plus A_3 this represents the the critical energy.

We can establish now by obtaining the or by by deriving the expression for A_1, A_2 and A_3 and then we will show that A_2, A_3 represents represents the critical energy that is the potential energy corresponding to delta u and A_1 plus A_3 represent the transient energy or transient energy at delta cl which has 2 components one is the critical energy one is the potential energy right. Now this can be easily proved, I will just do partly because you can just write the expression for this 3 areas A_1.

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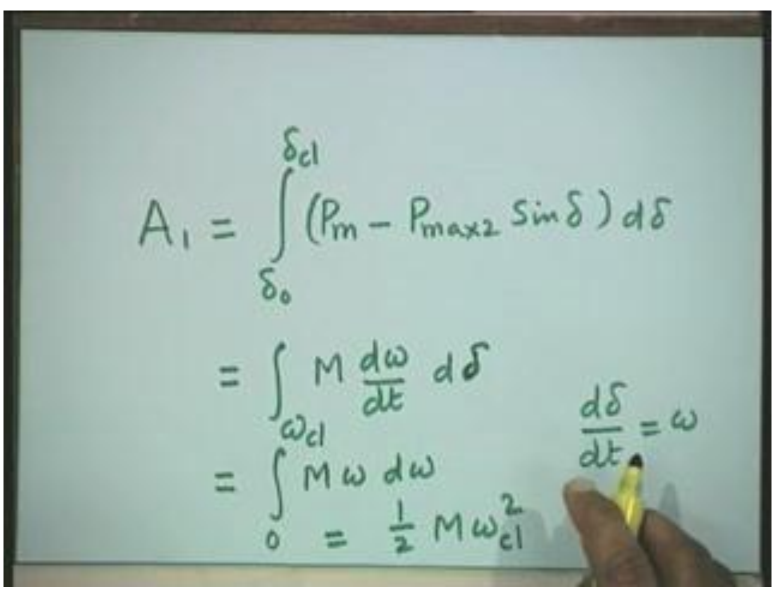

Now A_1 you can write down the let us just write down what is A_1, A_1 is $A_1$is you can find out A_1 by integrating this expression that is P_m minus P_{max2} sin delta from delta naught to delta cl, d delta okay. Now here what we do is this quantity this quantity is equal to M, this quantity is basically your M times d omega by dt. During the this is this is what we are doing is we are trying to find out the area and this represents the the integration over the angle delta cl to or delta naught to delta cl right that is when the faulted condition and this expression is always equal to M times d omega by dt okay and you put this as d this is what we are replacing here, this is replaced by this is d delta.

Now now d delta by this d delta by dt is equal to omega as per our definition so that I can write down this expression as as M omega d omega, where d delta is replaced by omega into dt and this quantity is your 1 by 2 M omega square. Now here your are integrating from 0 to omega cl right because at delta equal to delta naught the speed deviation delta omega cl or omega is 0 right therefore this quantity can be written as therefore A_1 represents the kinetic energy that is you can say that this is the kinetic energy imparted to the system.

Similarly, you write down A_2 and A_3 and establish establish that the the A_2, A_3 is actually the potential energy corresponding to the unstable equilibrium point delta u right and and the the transient energy transient energy corresponding to this fault clearing angle delta cl is this kinetic energy plus the potential energy which is equal to delta which is equal to or which is proportional to your area A_3, if I suggest you to write the expression for A_2, A_3 and establish that A_1 plus A_3 represents the transient energy function right which is corresponding to corresponding to the clearing angle delta cl and the remaining the expression that is A_2 plus A_3 represents the critical energy.

Okay with this let me summarize that today what we have done is that we have studied the basic concepts of direct method of transient stability analysis, we have devoted our time to transient energy function approach and the studies are confined to a single machine infinite bus system. Thank you!