Power System Dynamics
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Lecture - 30
Dynamic Equivalents for Large Scale System (Contd…)

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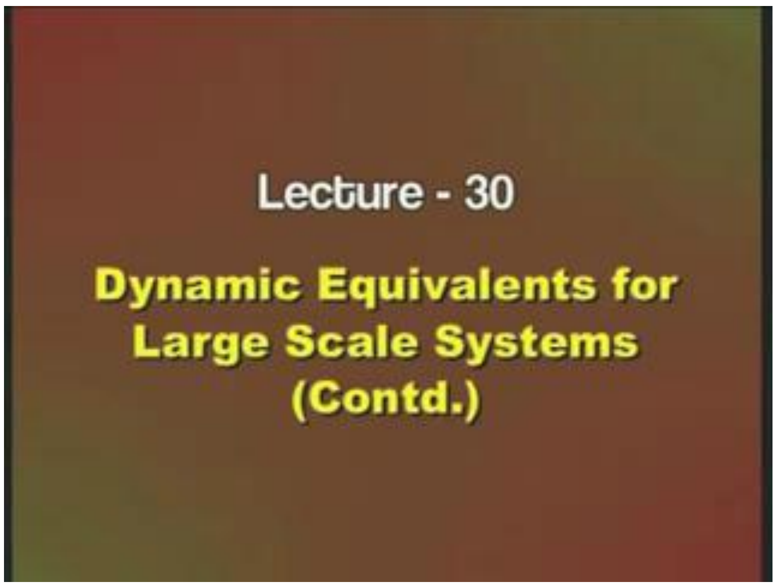

Friends, we shall continue with the study of dynamic equivalents for large scale systems. So far we have studied the basic concepts of dynamic equivalencing and also we have learnt about the algorithm which can be used for identifying the coherent group of generators. Now the next step will be that we have to do network reduction right and then aggregate the models of of the generators which form which form a coherent group that is when I say that if you have once we have identified the group of generators which belong to 1 coherent group then these generators have to be replaced by an equivalent generator okay. Therefore, the first step is that we have to first do network reduction okay. Now for performing this network reduction let us again look at the definition of the coherent group.

Our definition shows that a coherent group of generating units for a given perturbation I am emphasizing this word given perturbation is a group of generators oscillating with the same angular speed and with terminal voltages in a constant complex ratio, therefore we will make use of use of the definition of coherency to do network reduction okay.

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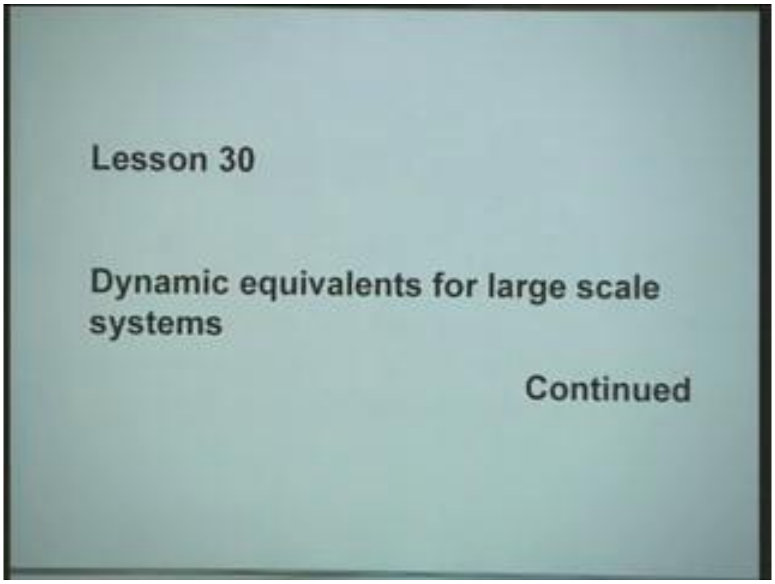

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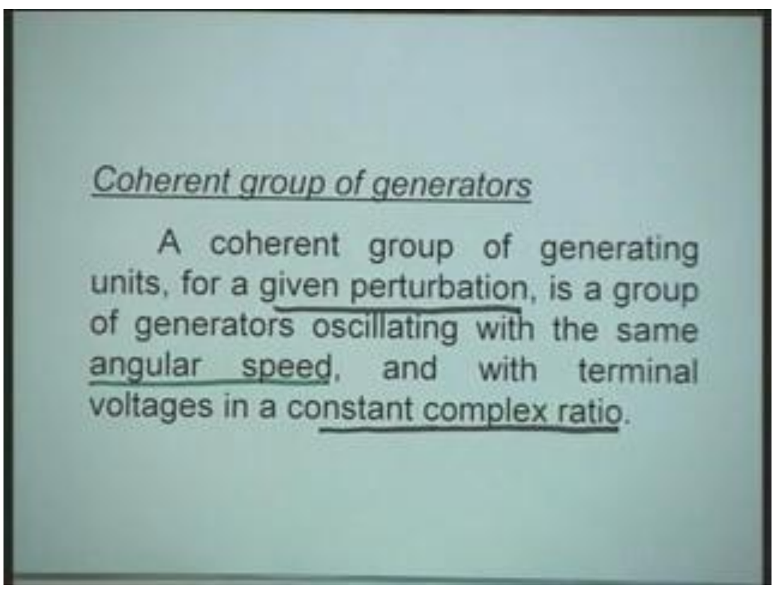

Now here there are 2 aspects, 1 is that the machines in the coherent group will have same angular speed and second is that the terminal voltages in a voltages of this machines will be in a constant complex ratio okay. Therefore, for network reduction we will be using the second, the second important component of this definition okay what we really do is that you have say group of machines which are coherent okay and they are connected to their own individual buses therefore now what we have to do is that we have to reduce the number of buses and retain only 1 bus. So that all the machines which are coherent are connected to 1 bus this is what is the what is the main thrust of network reduction.

Now to illustrate this concept what I will do is that we will consider that a system which has say N generators and N buses okay and let us say that the generator N minus 1 and N they are they form a coherent group okay. We have just start actually we will take that there are 2 generators which are numbered as N minus 1 and N they form a coherent group and we will replace replace the the 2 buses, bus number N minus 1 and bus number N. So that we will reduce the number of buses okay.

Now in this process what is normally done is that we retain retain 1 bus of this group with the same voltage because different buses will have different voltages right therefore let us say that we we retain bus number N minus 1 and the voltage of that bus will be retained as it is right therefore when we do this simplification or network reduction. Let us see that what will be the changes in the network model okay.

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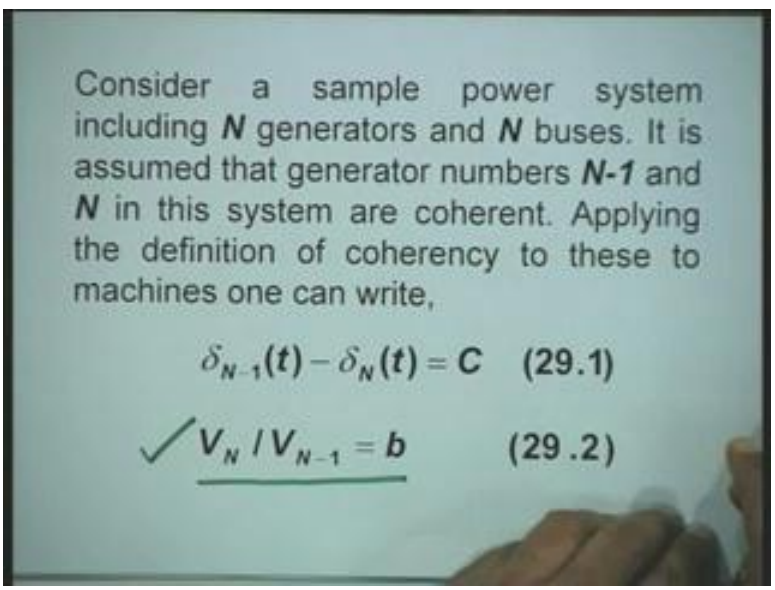

Now to perform this network reduction we will use this condition that is the voltage of N th bus and voltage of N minus 1 th bus right the ratio of these 2 voltages is a constant equal to b and this b is a complex number that the voltages maintain maintain a constant complex ratio okay therefore, we will make use of this condition. However, we all know that for identifying the coherent group of generators we have used this first condition first condition that is delta N minus 1 t minus delta N that is the for all time instants over the complete period right the 2 swing curves were identical if the this difference is a constant and it is it is less than some tolerance literature survey shows actually that that the tolerance of 4 degrees or 5 degrees has been considered therefore now we will use this condition to obtain a reduced network okay.

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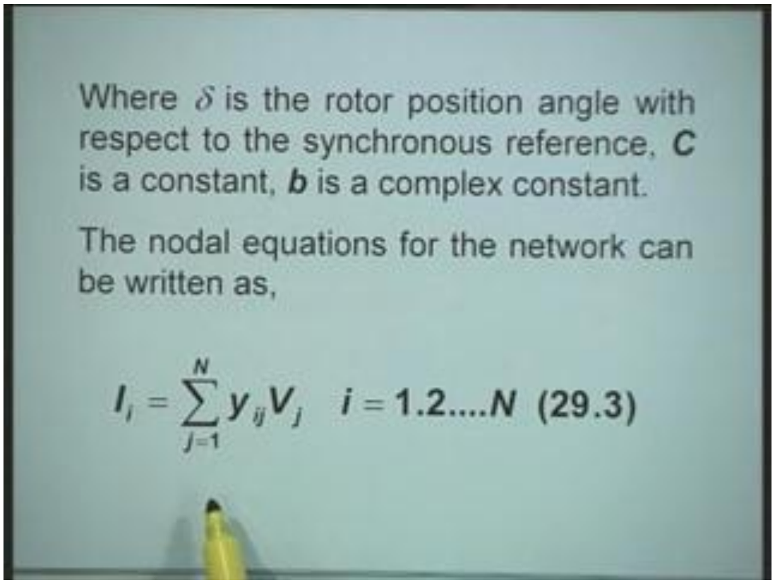

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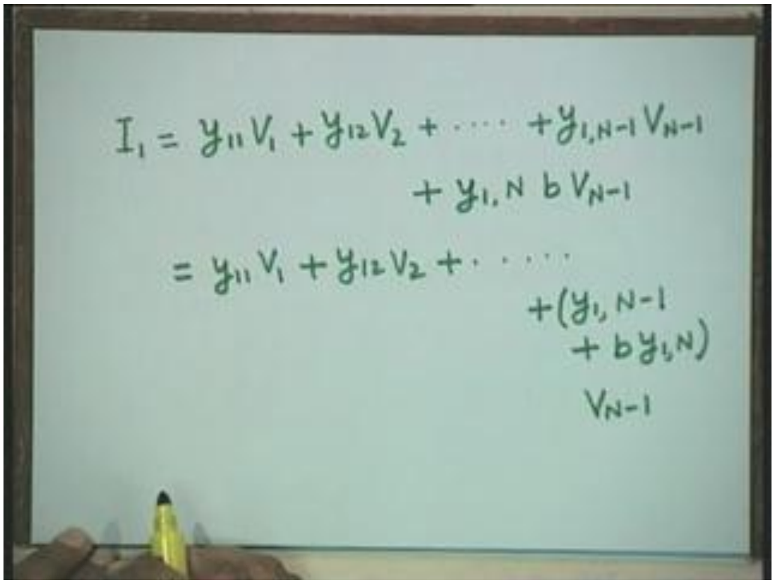

Now we when you obtain the reduced network the process is very simple we start with the actual network model. Let us say that this is the network model that is I_i is equal to summation of j equal to 1 to N $y_{ij}V_j$ right. This forms our steady state network model, where I_i is the current injection at bus I, V_j is the voltage of j th bus okay and y_{ij} are the elements of by bus matrix okay this is a very straight forward this has to be no problem. Now in this when you write actually you will have say N equations where i is varying from 1 to N equations you can write down. Now in these N equations wherever you have V_N we can replace this V_N by b into V_N minus 1 okay.

Now if you perform this exercise let us see what type of equations we will get we will let us write down I_1, I_1 can be written as y_{11} V_1 plus $y_{12}V_2$ y_1 N minus 1 we write up to N minus 1 term first N minus 1 V_N minus 1 plus $y_{1\ N}$ voltage is now replaced V_N is replaced by b times V_N minus 1, okay. Therefore this equation now can be written as I_1 can be written as $y_{11}V_1$ plus $y_{12}V_2$ last term will now be equal to y_1 N minus 1 plus plus b times b is a complex number into $y_{1\ N}$ into V_N minus 1 okay. Therefore, when I replace the voltage of n th bus by b into V_N minus 1 I get the equation for $I_{\ 1}$ as y_{11} V_1, y_{12} V_2 and the last term will now be the y_1 N minus 1 plus b times y_1 N into V_N minus 1.

Therefore what I see here is that now in this in the in the by bus of the network right the the last column first thing is that this y bus is going to N minus 1 into N now and the last column is going to be different otherwise y_{11}, y_{12} these elements up to y_1 N minus 2 they will be same okay.

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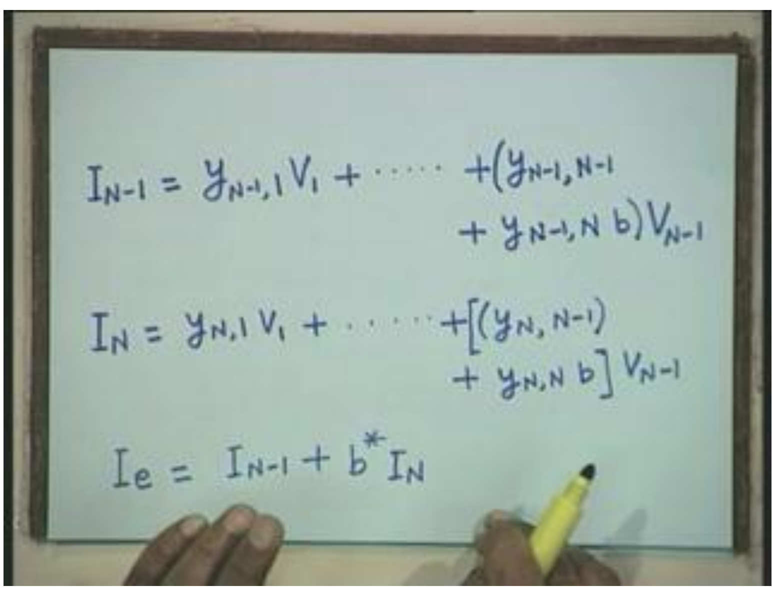

Now similarly, you can write down for all the buses okay. Now suppose now I write down for say corresponding to N minus 1 th bus. Let us write down the equation for N minus 1 th bus that is you can write down here first let us write down the current injection will be I_N minus 1 okay this will be now y_N minus 1, V_1 plus I am not adding these terms lets us write down this last term here plus what will be this thing now you will have here y_N minus 1 what is the value of now this 1? N minus 1 plus plus y that is what is plus plus I am just putting the alphabet first plus yN minus 1 is going to be y_N minus 1 N into b and this sum multiplied by V_N minus 1 okay. Now now you write down the last term that is I_N I_N will be now y_{N1} V_1 that is on the last term here y_N y_N, N minus 1 okay plus plus $y_{N, N}$ into b into V_N minus 1 okay because V_N is replaced it is very simple. Now the number of equations still are equal to N although although the number of elements in the row have been reduced from N to N minus 1.

Now what we do is that we we add these 2 equations but before addition what we do is that we multiply this I_N by b star I will tell you why we do this but you multiply this I_N by b star and add these 2 equations and we will call this a very symbol I_e as I_N minus 1 plus b star that is b star stands for complex conjugate of b into I_N that is that is on this left hand side when I perform this addition that is I multiply this last equation by b star add it to I_N minus 1 right then I get on left hand side the current term which is I_N minus 1 plus b star I_N.

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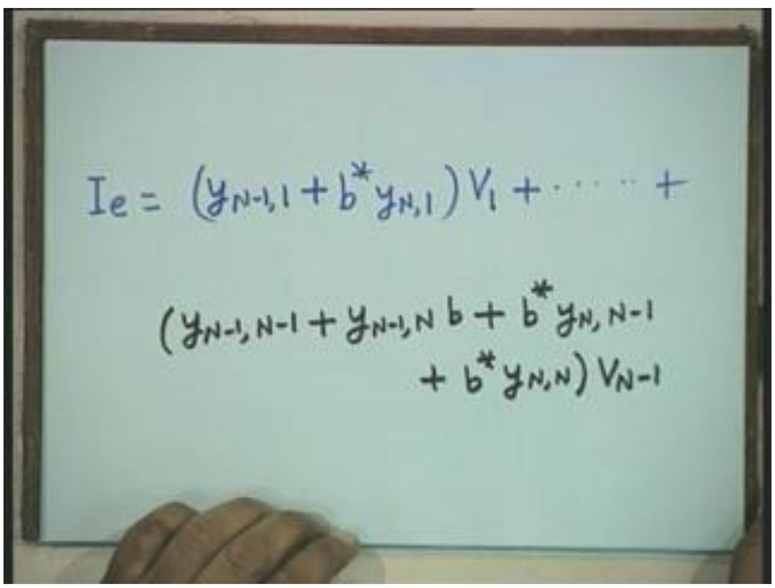

Okay then then you can write down I_e equal to I will write down only 2 terms let us say, I_e will be equal to y_N minus 1 1 plus plus b star $y_{N, 1}$ this multiplied by V_1 okay. The last term it can be written here as it is a then the term that is you can write down here as y_N minus 1, N minus 1 plus y_N minus 1 N into b plus b star into y_N N minus 1 plus b star $y_{N, N}$ okay into V_N minus 1 okay, b star $y_{N, N}$ into b yes very correct therefore it is going to be I will put b here b into b star $y_{N, N}$ okay. Therefore now, what we have obtained is the N equations not I am sorry N minus 1 equations that is number of equations which we were having earlier were N now we have we have reduced them to N minus 1 equations okay and the in the last equation that is the N minus 1 th equation the current injection I_e, I_e is equal to I_N minus 1 plus b star I_N okay. Now to understand why we have done this particular operation multiplied by b star and added to the term I_N minus 1.

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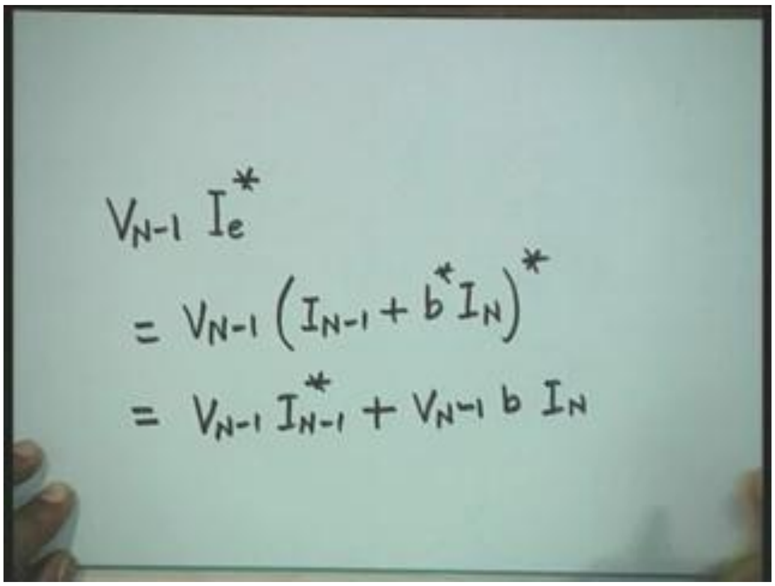

Now let us see what is the complex power injection at N minus 1th bus by doing this equivalency that is the voltage is V_N minus 1 if you multiply this by I_e star what does it show, this product shows the complex power injected at the equivalent bus okay. Now this can be written as V_N minus 1 into into I_N minus 1 plus b star I_N whole star if you write you expand this term then it can be written as V_N minus 1 I_N minus 1 star plus V_N minus 1 b I_N okay, what does this expression represents what does this expression represent.

It represents the some of the some of the complex power injected in the original network at N minus 1 th bus and at N th bus therefore by doing this operation we are retaining the total complex power injected at N minus 1 th bus equal to the total complex power injected at N minus 1 and N th bus because this is the requirement actually because when we are trying to to reduce the network, we have to ensure actually that this condition of power invariance is maintained okay and that is why we have performed this operation here where, we multiply this last equation by b star and added to the equation corresponding to N minus 1 th bus, okay.

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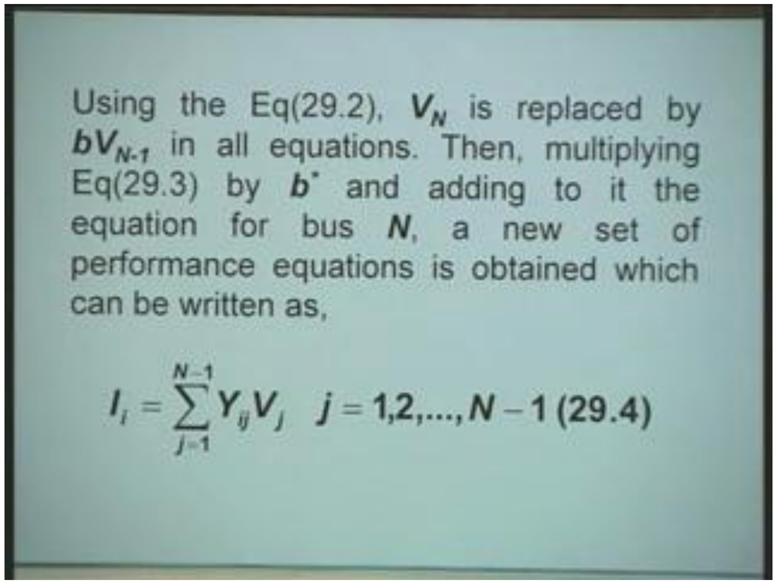

Therefore with this operation performed we get we get now the reduced network, it has now N minus 1 buses N minus 1 buses okay and therefore I can write down the equation of the reduced network or the reduced network model as I_i equal to Y_{ij}, V_j, j varying from 1 to N minus 1, j equal to 1 to N minus 1 okay.

Now this Y_{ij} this this bus matrix which is a N minus N, the dimension of this Y bus is now N minus 1 into to N minus 1 and its elements are modified with respect to the elements of the original network okay. This is the first step which is called the network reduction.

Now in case suppose you have instead of 2 generators which form a coherent group, suppose there are there is 1 more generator which forms the coherent group you can perform additional similar operation and you can keep on reducing the number of buses right and while doing this network reductions process what is to be done is that we retain the voltage of 1 of the buses at the original level that is when I have done this reduction process the voltage of voltage of N minus 1 th bus was retained right but suppose you have instead of 2 machines you have suppose say 4, 5 machines which form a coherent group then you can select 1 of the machines machine bus admission retain that bus at its voltage right.

So that now what we can say is that the the we have a network that equivalent machine is connected to the bus which is retained and the equivalents machine we have to find out what will be the parameters of that equivalent machine okay. The network reduction part we have seen is a very simple process it can be done.

Now in order to obtain equivalent model of the machine okay first we will consider the classical model of the machine that is let us say that we represent the generator by a classical model and then we will find out actually that what will be the equivalent machine which will represent the classical model of the system I think this point is already covered. Now to obtain the equivalents machine we again look at the at the dynamic model of the system.

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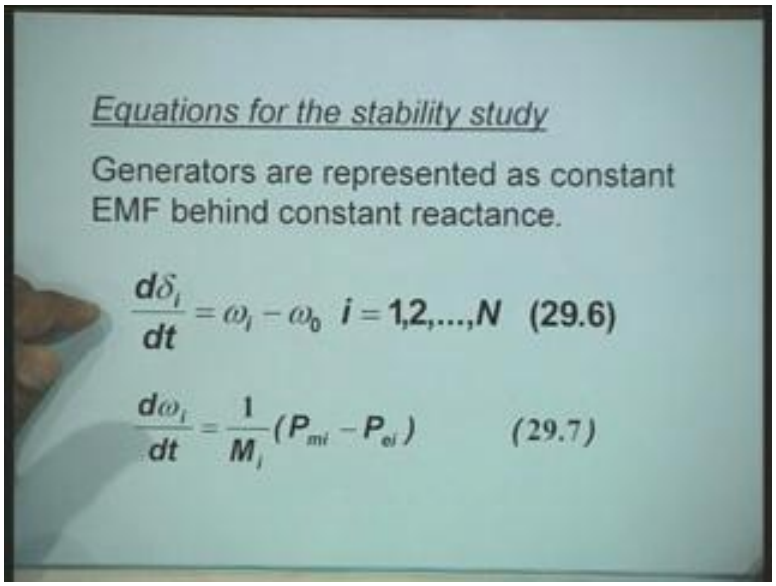

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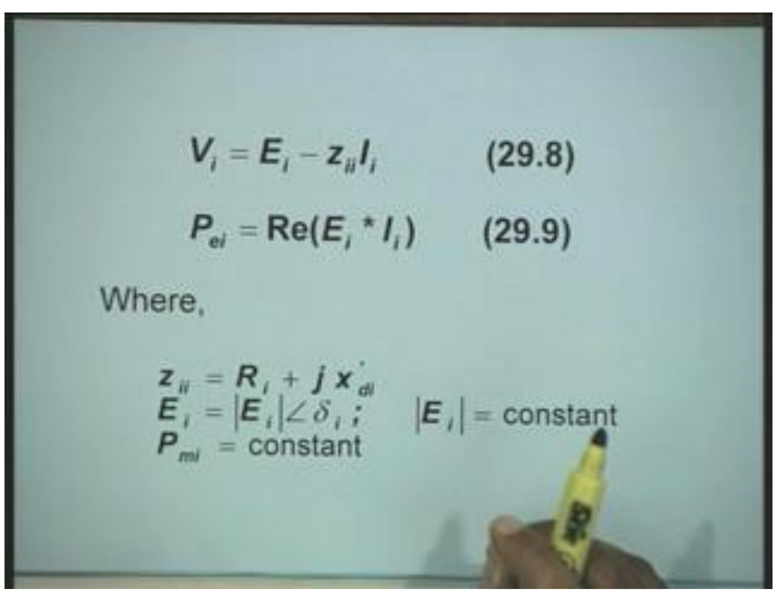

The dynamic model of the system is written in this form that is for Ith machine d delta I by dt is equal to omega I minus omega 0 which can be written as if you use in per unit system or the speed is represented in per unit this can be written as delta omega into omega 0 okay, d omega I by dt equal to 1 upon M_i P_{mi} minus P okay where these terms are appropriately you know dimension d that is dimension of all these terms have to appropriate that we all know it there is nothing now here is the very important point that if I represent a generator by equivalent.

Okay therefore, we know that the the generator model considering the classical representation is of this form that terminal voltage V_i is equal to internal voltage E_i minus z_{ii} I_i this is what is the classical model for a synchronous generator that is the terminal voltage is equal to the internal voltage minus its own impedance that is the impedance behind I am sorry the impedence which is equal to z_{ii} in case you neglect the resistance then this becomes the sub-transient direct axis of sub-transient reactors okay and the the power injected is the real part of E_i star into I_i this is just to repeat actually there is nothing new this we all know it okay where our definition for this z_{ii} is R_i plus j times xd_i prime E_i is magnitude E_i angle delta I, P_{mi} is a constant and the magnitude of E_i is also constant because it is a classical model right we represent the generator by constant voltage behind direct axis transient reactance.

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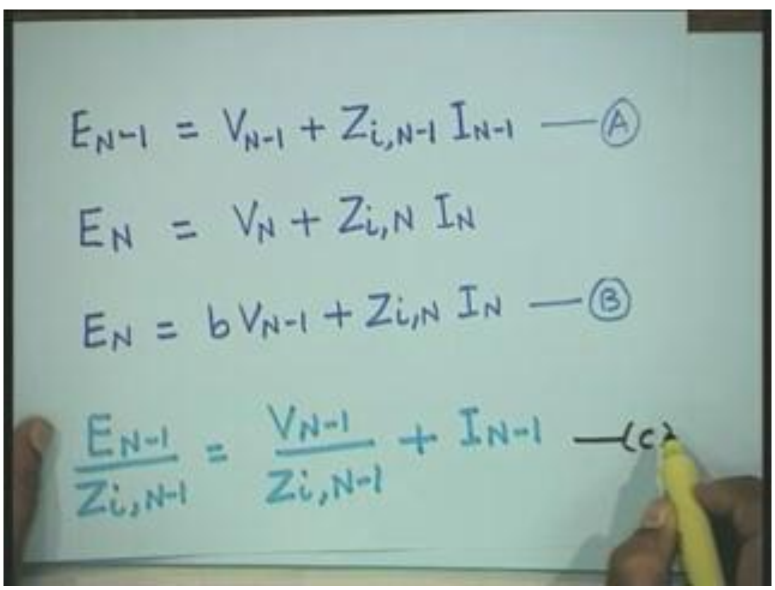

Now here to obtain the equivalent equivalent let us first write down the 2 equations using the original voltages for the machines N minus 1 and N that is if I write down for machine number N minus 1 right I can write down E_N minus 1 that is the voltage right. This can be written to be equal to V_N minus 1 plus impedence of this machine right.

Now let me call this as a there are different symbols are used but I will use I to show that it is the internal impedance Z_i N minus 1 into I_N minus 1 that is let us look actually that system is not reduced we have actually the N machines okay and then the the equation of the classical model looks like this that the internal voltage is equal to the terminal voltage plus this drop. Similarly, I can write down for N th machine as V_N plus $Z_{i,N}$ I_N.

Now what is what should be the next step because we have these 2 equations and the and 1 condition which we know is the relationship between V_N minus 1 and V_N we can use that relationship so that in this equation what you do is you first let us first replace V_N by b times V_N minus 1. So that you can write down this E_N equal to b times V_N minus 1 this term remains same okay. Now you divide this equation I will call this equations say for reference say A and I will denote this equation as B okay, you divide this equation A by Z_i N minus 1 all through so that you can write down now E_N minus 1, Z_i N minus 1 equal to V_N minus 1 divided by Z_i N minus 1 plus I of N minus 1 okay.

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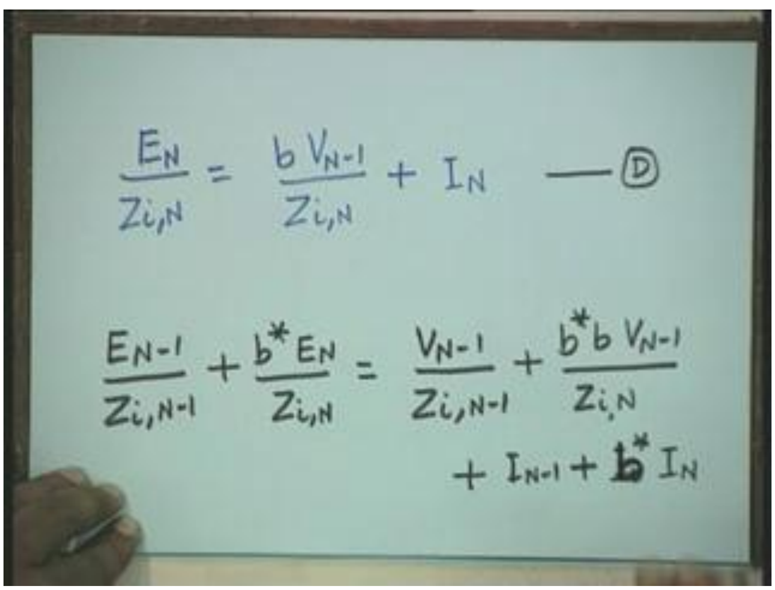

For the second equation we can write down E_N divided Z_i N equal to b times V_N minus 1 divided by Z_i N plus I okay. We shall number this equation as C okay and number this equation as D okay the next step will be that you multiply this equation D by b star and add to equation C okay then what will you get now, when you do this operation you will get E_N minus 1 divided by Z_i N minus 1 this remains same to this we are adding a term multiplying by this by b square the whole equation that is b star not b square b star put E_N divided by Z_i N okay, on right hand side we will get the that is this second equation I am multiplying by b star and I am adding it to equation C therefore this comes out to be equal to V_N minus 1 divided by Z_i N minus 1 okay plus b star b V_N minus 1 divided by Z_i N plus I_N minus 1 plus b star b star I_N okay. Therefore next what should we do this is okay I_N is multiplied b star this term here see here, the this is our equation which remains as it is okay to this I am multiplying this the whole equation by b star so that the first term is okay b star E_N by Z_i N_1 here here I have not multiplied here V_N minus this V_N minus 1 upon Z_i N minus 1 his remains same here b star into b V_N minus 1 upon Z_i N right here I_N minus 1 as it is plus this is multiplied by b star into I_N what is the problem there is a slight mistake. You this is to be called as b star okay this is just a slip okay. Now now what we can do is here we can easily see 1 very interesting thing here that this is the equivalent current that is the equivalent current that is to be injected at the particular bus V_N minus 1.

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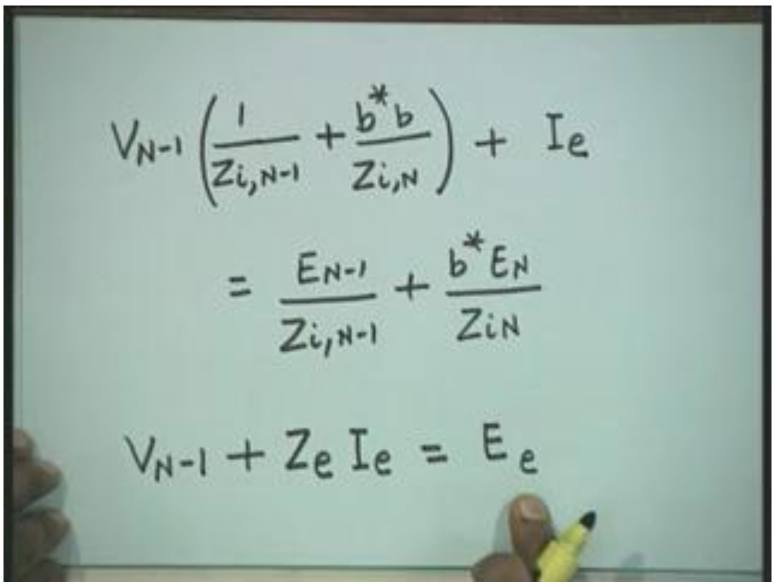

Therefore, now we can write down this whole thing this whole thing can be written as you, we write down this first this term V_N minus 1 I will take it out okay then coefficient of V_N minus 1 we can write down here as 1 upon Z_i N minus 1 plus b star b divided by Z_i N okay plus I_e plus I_e equal to equal to this term E_N minus 1 divided by Z_i N minus 1 plus b star E_N Z_i N okay.

Now the equivalent generator equation if you write down the generator equation of the equivalent generator then the equivalent generator equation is written in the form V_N minus 1 okay plus let me write down Z equivalent Z is Z_e into I_e must be equal to E equivalent that is if I try to write down the equivalent generator. The equivalent generator internal voltage is E_e equivalent impedence is Z_e and equivalent current is I_e therefore equivalent generator model when I put in the classical form is V_N minus 1 plus Z_e into I_e equal E_e okay. Therefore now you can identify identify from this equation what is this Z_e and what will be the E_e, Z_e can be simply obtained because you divide divide this whole equation by this coefficient.

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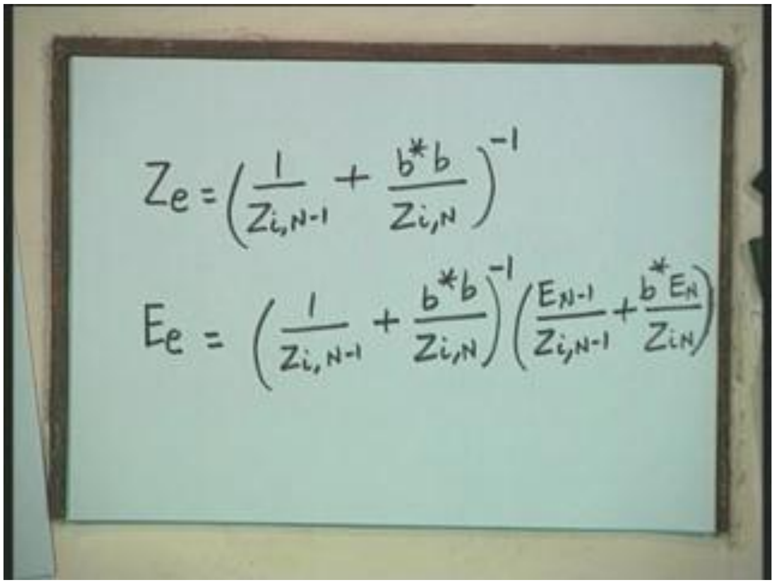

So you write down V_N minus 1 is equal to this reciprocal of this plus I_e plus equal to reciprocal of this term into this whole thing and therefore the Z_e can be identified that is the equivalent impedence Z_e, equivalent impedence Z_e can be identified as as 1 upon Z_i N minus 1 plus b star b divided by Z_i N inverse right and equivalent voltage, internal voltage E_e is identified as 1 upon Z_i N minus 1, N minus 1 plus b star b divided by $Z_{i,\,N}$ inverse multiplied by E_N minus 1 divided by $Z_{i,\,N}$ minus 1 plus b star E_N divided by Z_{iN}.

Okay that is you can see that all the terms on this side of this equation are known to you because the generator equivalent generator is obtained obtained by replacing the 2 generators 1 corresponding to the bus N minus 1 another corresponding to bus N and the equivalent generator what we have done is that we have retain the voltage of N minus 1 th bus intact right what we have done is that we have obtained, what should be the equivalent impedence and what should be the equivalent internal impedance.

Now suppose suppose now instead of you can say having 1 machine, 2 machines forming a coherent group, if you have more than 2 machines then this process can be repeated and once you repeat this process actually you can get the equivalent generator representing the coherent group therefore this exercise has to be done.

Now, so far what we have done is that we have considered considered the classical model but suppose, if I do not want to go for classical model if it is a classical model then in this classical model actually I am not representing the excitation system I am not representing actually the synchronous generator dynamics or stator equations in that form right. We are assuming that the flux linkage flux linkage is constant okay that is what is the classical model but in case you want to have a more detailed representation right then the procedure is that you replace each component of the synchronous machine by equivalent that is what is to be done is you replace the turbine governor models that is each machine has a turbine governor and suppose there are few machines which form a part of the coherent group then you have to obtain an equivalent turbine governor model okay.

Now for obtaining the equivalent turbine governor model the the procedure is very simple because here here the machines which are coherent all the machines have the same same rotor speed okay. Therefore, if you I look actually the turbine governor model then input is speed deviation and output is the mechanical power output is it not therefore each each of the turbine governor model input is delta omega because speed deviation is same for all these machines and therefore the total mechanical output I can get simply by adding adding all the transfer functions because total mechanical power will be equal to the transfer function of machine 1 plus machine 2 plus machine 3 multiplied by delta omega right and therefore these transfer functions can be aggregated to obtain an equivalent model.

Now suppose you consider the rotor dynamics okay. Now since this group of machines are coherent coherent right therefore all the swing equations you can add together because omega is same d omega 1 by dt is same as d omega 2 by dt. Therefore, all these swing equation can be added together and when you add this swing equations what will be the equivalent inertia constant it will be some of the all the machines inertia constants of all this machines therefore this is a equivalent inertia constant. Now when I want to replace say the excitation system models by an equivalent, now here what is the input to the excitation system can you tell me, suppose I try to write the transfer function model of excitation system what are the inputs.

We have V reference and terminal voltage, we have V reference and terminal voltage now all these machine have the same terminal voltage because they are connected to the same bus do you understand this point all the machines have the same terminal voltage right therefore the input to the excitation system of each of the machines will be same terminal voltage and therefore now you can obtain an equivalent excitation system right because because the input is same and you can aggregate all these models. Similarly, if you take the power system stabilizer input to the power system stabilizer is going to be delta omega speed deviation and since the speed deviation is same for all the machines therefore you can find out the total stabilizing signal by adding by adding the transfer functions of all these individual machines and considering the same input as delta omega.

Now this exercises this exercise where you replace the turbine governors by equivalent turbine governor excitation system by equivalent excitation system the power system stabilizer by equivalent power system stabilizer right. For this for this what we have do is that we first assume the model of the equivalent component and then try to fit the transfer function characteristic with the transfer function of the total system right.

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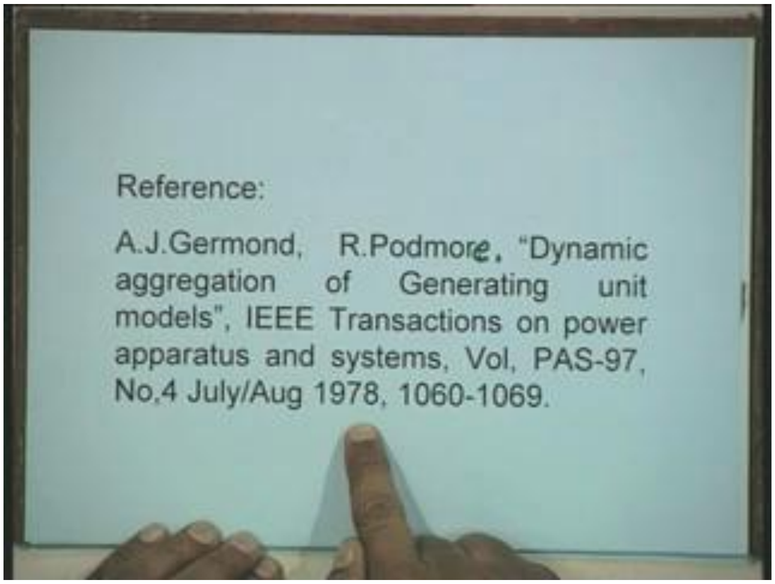

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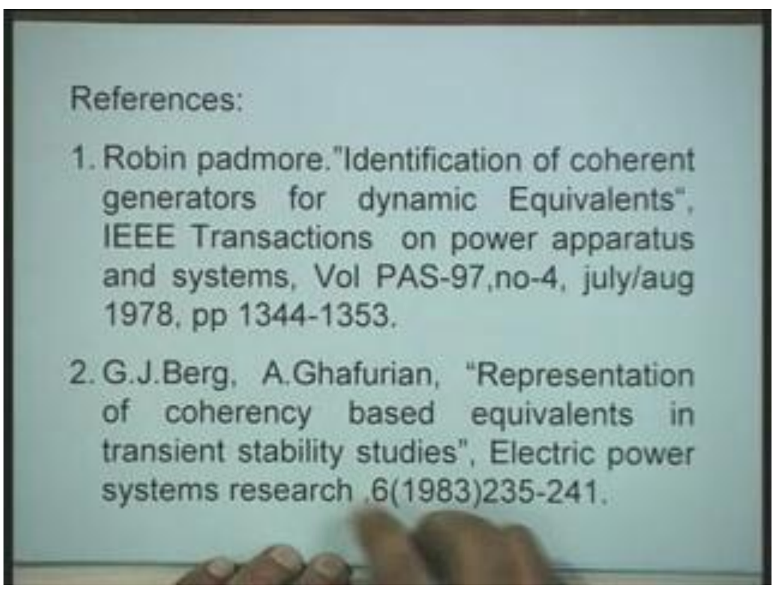

Now the best reference for for studying how how the individual models can be aggregated together is given in this reference paper that is by A. J. Germond not this one, yes A. J. Germond Germond and R. Podmore dynamic aggregation of generating unit generator unit models title is very straight forward, dynamic aggregation of generating unit models IEEE transaction on power apparatus and systems volume, PAS- 97, number, 4 July/ August 1978, then pages are 1060 to 1069 that is when dealing with the dynamic equivalencing I started with these 2 references.

The first reference was Robin Podmore as the author identification of coherent generators for dynamic equivalents therefore this paper addresses how to identify the coherent group of generators. The second paper written by the Berg and Ghafurian representation of coherency based equivalents in transient stability studies here he is dealing mainly with the network reduction and and representation of the equivalent generator by a classical model, while in this paper in this paper actually which is which is actually considered to be companion paper of this these 2 papers are put together practically they have appeared in the same year and in the same issue of the IEEE transaction right they dealt with the 2 aspects 1 aspect is the how to identify the coherent group of generators and then second aspect is how to aggregate them.

Now just to inform you that this these papers were written out of out of the EPRI project report that is EPRI has sponsored some research project and at the end when the research project was completed these 2 papers gave the information about the techniques which they have developed. Now just to give you 1 slight information that the in these research papers to substantiate substantiate the algorithm which they have developed they have applied these to 2 large systems, 1 large system is the system of Michigan system another is called WSCC system in Michigan system the number of buses is 1027 and the number of generators were 295 okay and the for identifying the coherent group of generators a 3 phase fault at a particular bus was considered of 12 cycle duration in the first case and the equivalent model which they obtained at the end right had 50 generators.

Original system had 290, 95 generators and the equivalent had only 50 generators and out of this 50, it was seen actually that the 24generators or 24 coherent groups had only 1 generator and remaining coherent groups were 26 do you understand what is the meaning of a group having only 1 generator that means that machine has its own independent dynamics, it is not similar to other machines right and the the studies were done actually that if you perform now the transient stability analysis using the equivalent the time required was 55 seconds and if you do for the full complete system the time reported is 341 seconds.

Therefore this is the reduction which 1 gets in in computing the transient stability of the system using dynamic equivalent similar results are given for WSCC system. Now I want to 1 point out 1 more point here that is a last point that is the coherency behavior depend upon the location of the disturbance because when I discussed the definition of the coherency we have written actually a particular location fault or the disturbance at a particular location and generally what is done is that suppose in a particular system I am interested in knowing the dynamics that becomes my location for the fault and I find out the equivalent for the whole thing okay.

Now some times you come across a situation where you find machines say hydro-machines and turbo generators may be identical or may form a coherent group in that case generally what is done is that machines the hydro-machines are represent by equivalent hydro machine and the turbo machines or turbo generator represent by equivalent turbo generators they do not we do not mix up the model of hydro and steam turbine generators.

Okay, with this let me conclude today that we have studied the an an an algorithm for network reduction and also for obtaining the equivalent generator, first considering the classical approach and then broadly we have considered that how do we obtained the equivalent model of excitation system, turbine governors, equivalent power system stabilizer and equivalent rotor dynamics and similarly, for equivalent synchronous machine. Thank you!