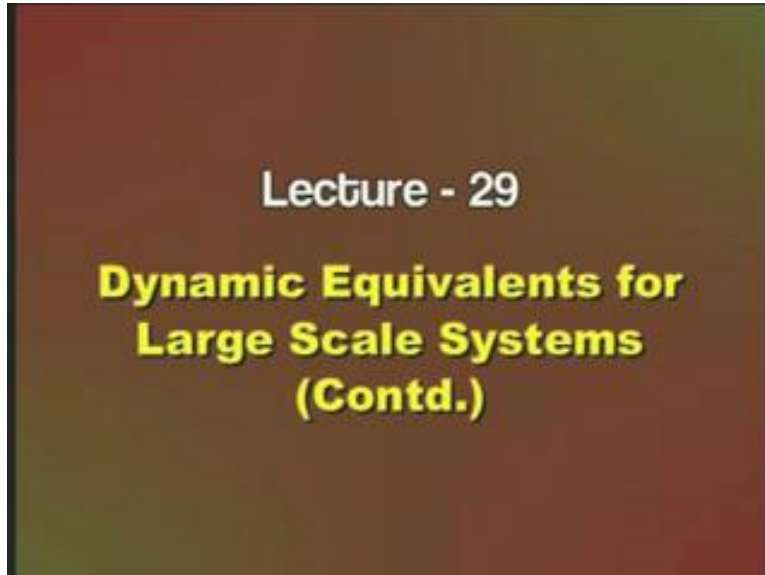


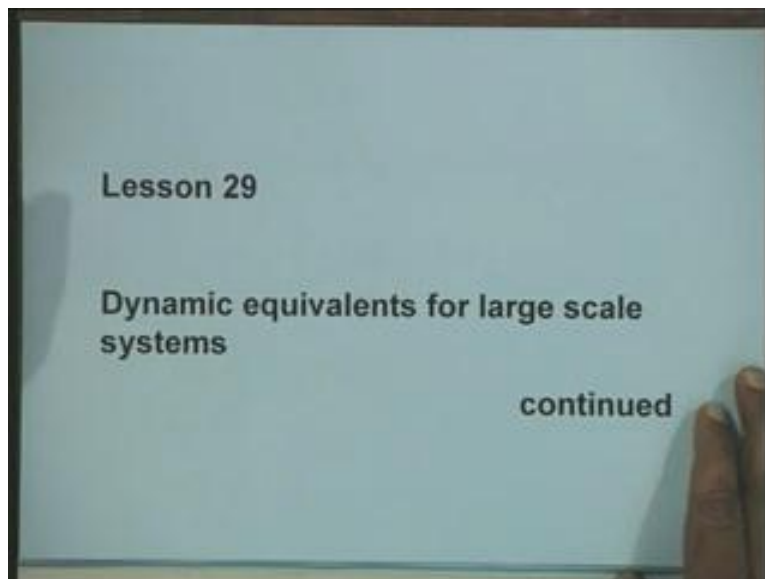
Power System Dynamics
Prof. M. L. Kothari
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 29
Dynamic Equivalents for Large scale Systems (Contd...)

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Friends, today we shall continue our study on dynamic equivalents for large scale systems

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In the last time we have studied the basic concepts of dynamic equivalents, we have also studied the different techniques which are used for obtaining the dynamic equivalents out of these the 3 techniques which we discussed last time when based upon the distribution factors, another based on the model analysis and order reduction and the third one was the based upon the coherency equivalents.

Coherency equivalent is one which is very very simple and it has many merits as compared to the other techniques. First step in the coherency equivalencing is that one has to identify the group of generators which are coherent and for identifying the group of generators which are coherent one is straight away approach is that perform a full transient stability analysis obtain the swing curve and group those machines which have identical swing curves in to an equivalent one.

However, however one can use a simplified approach to obtain the swing curves quickly and then identify the coherent group of generators. Now here the two assumptions on which the simplifications, simplifications can be made are number 1 that the coherency behavior of the synchronous machines is independent of independent of the magnitude of disturbance and second is that is independent of the degree of modeling whether I represent the system by a simple model or I model the system the details the coherency behavior does not change.

Therefore from these 2 points actually we can use a simplified approach for for identifying the coherent group of generators the from the first consideration that the coherency behavior does not depend upon the magnitude of disturbance. We can make use of linear model, second is the since it does not upon the details in which the system is represented we can use a classical representation of the system or classical model right and then obtain the swing curves of the system.

We shall discuss the simple algorithm which is used for identifying the coherent group of generators. The linear form of the swing equations is given here $M_i \frac{d}{dt} \Delta \omega_i = \Delta P_{mi} - \Delta P_i - D_i \Delta \omega_i$, this is the one differential equation out of the two differential equations which represent swing equation of the system. Here, ΔP_{mi} is the change in mechanical power of the i th machine ΔP_i is the deviation in mechanical power $\Delta \omega_i$ is the deviation in speed of the machine.

Now here in this equation the speed is is represented in per unit, second equation is $\frac{d}{dt} \Delta \delta_i = 2\pi f_0 \Delta \omega_i$. Now in this equation M_i is equal to $2 \times H_i$ their network can be modeled model and we can find out actually the the small changes in the network variables or the system variables using this Taylor series expansion. Now here in this in this set of equations which I have written here, we have considering the the generator internal voltages are represented by the symbol E and the load bus voltage are represented by V , the phase angle of the generator internal voltage is δ and phase angle of load bus voltage is θ .

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$$M_i \frac{d\Delta\omega_i}{dt} = \Delta P_{mi} - \Delta P_{ei} - D_i \Delta\omega_i$$

$$\frac{d\Delta\delta_i}{dt} = (2\pi f_0) \Delta\omega_i$$

where $M_i = 2H_i$

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$$\begin{bmatrix} \Delta P_e \\ \Delta P_L \\ \Delta Q_e \\ \Delta Q_L \end{bmatrix} = \begin{bmatrix} \frac{\partial P_e}{\partial \delta} & \frac{\partial P_e}{\partial \theta} & \frac{\partial P_e}{\partial E} & \frac{\partial P_e}{\partial V} \\ \frac{\partial P_L}{\partial \delta} & \frac{\partial P_L}{\partial \theta} & \frac{\partial P_L}{\partial E} & \frac{\partial P_L}{\partial V} \\ \frac{\partial Q_e}{\partial \delta} & \frac{\partial Q_e}{\partial \theta} & \frac{\partial Q_e}{\partial E} & \frac{\partial Q_e}{\partial V} \\ \frac{\partial Q_L}{\partial \delta} & \frac{\partial Q_L}{\partial \theta} & \frac{\partial Q_L}{\partial E} & \frac{\partial Q_L}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\theta \\ \Delta E \\ \Delta V \end{bmatrix}$$

$$\Delta E = 0$$

Now when we represent this generator by classical model then ΔE that is the change in magnitude of the internal voltage of the generator is 0 and further assuming that the change in power flow due to change load bus voltages is also negligible, we can simplify this equations and represent in the simple formula like this ΔP_e ΔP_L equal to ΔP by $\Delta\delta$ $\Delta\theta$ ΔP $\Delta\theta$ or ΔP_L by $\Delta\delta$ $\Delta\theta$ ΔP_L by $\Delta\delta$ $\Delta\theta$ that is these are all the partial derivatives partial derivatives actually with respect to δ and θ . You can write down this equation in a compact form as HGG, HGL, HLG, HLL this is the matrix this is the Jacobian of the system.

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$$\begin{bmatrix} \Delta P_e \\ \Delta P_L \end{bmatrix} = \begin{bmatrix} \frac{\partial P_e}{\partial \delta} & \frac{\partial P_e}{\partial \theta} \\ \frac{\partial P_L}{\partial \delta} & \frac{\partial P_L}{\partial \theta} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix}$$

$$= \begin{bmatrix} H_{GG} & H_{GL} \\ H_{LG} & H_{LL} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix}$$

Now we will make use of the linear differential equations of the system and the simplified network equations. Now in this model this change in load power that is ΔP_L will be used to simulate a particular type of disturbance okay. Now the algorithm for identifying the coherent group of generators is to be derived and the first step is that we integrate the both sides of the differential equations over the integral over the time period $t - \Delta t$ to t , therefore if you integrate both sides of the differential equation we can write down M_i is a integral of t minus integral d by dt of $\Delta \omega_i$ dt . Similarly, integral of ΔP_{mi} dt integral of ΔP_{ei} dt minus D_i $\Delta \omega_i$ dt .

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$$M_i \int_{t-\Delta t}^t \frac{d}{dt} \Delta \omega_i dt = \int_{t-\Delta t}^t \Delta P_{mi} dt - \int_{t-\Delta t}^t \Delta P_{ei} dt - D_i \int_{t-\Delta t}^t \Delta \omega_i dt$$

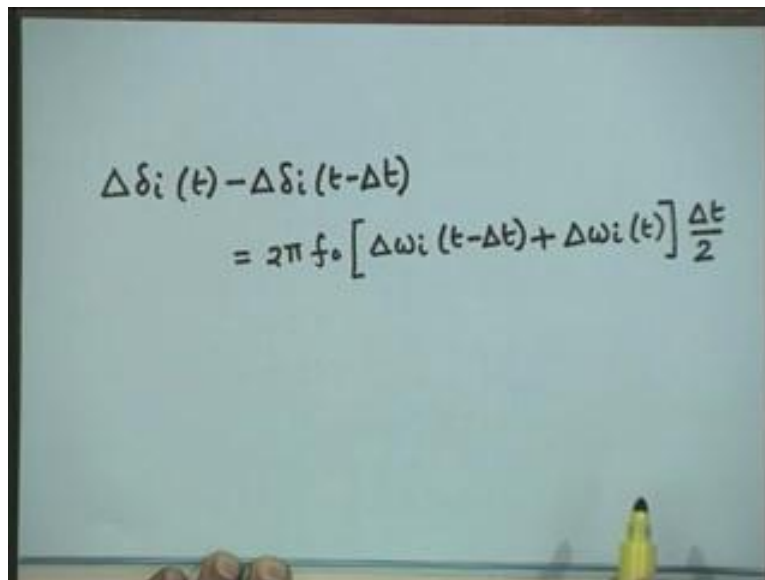
$$M_i [\Delta \omega_i(t) - \Delta \omega_i(t-\Delta t)] = \Delta P_{mi}(t) \Delta t$$

$$- \frac{\Delta t}{2} [\Delta P_{ei}(t-\Delta t) + \Delta P_{ei}(t)]$$

$$- \frac{D_i \Delta t}{2} [\Delta \omega_i(t-\Delta t) + \Delta \omega_i(t)]$$

Now the integrals are taken over a time step Δt that is $t - \Delta t$ to t . Now we integrate and perform this integration function using the trapezoidal rule of integration, when you apply this trapezoidal rule of integration we get equation in this form that is M_i into $\Delta \omega_i(t) - \Delta \omega_i(t - \Delta t)$ that is this integral right will be simply integral $t - \Delta t$ to t of $\Delta \omega_i$ and therefore this will be simply M_i into $\Delta \omega_i(t) - \Delta \omega_i(t - \Delta t)$. In this so far the second term is concerned or this term is concerned ΔP_{mi} is constant this term is there to simulate a particular type of fault. Therefore, since ΔP_{mi} is constant the value of this integral or may be this term is $\Delta P_{mi} \Delta t$.

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$$\Delta \delta_i(t) - \Delta \delta_i(t - \Delta t) = 2\pi f_0 \left[\Delta \omega_i(t - \Delta t) + \Delta \omega_i(t) \right] \frac{\Delta t}{2}$$

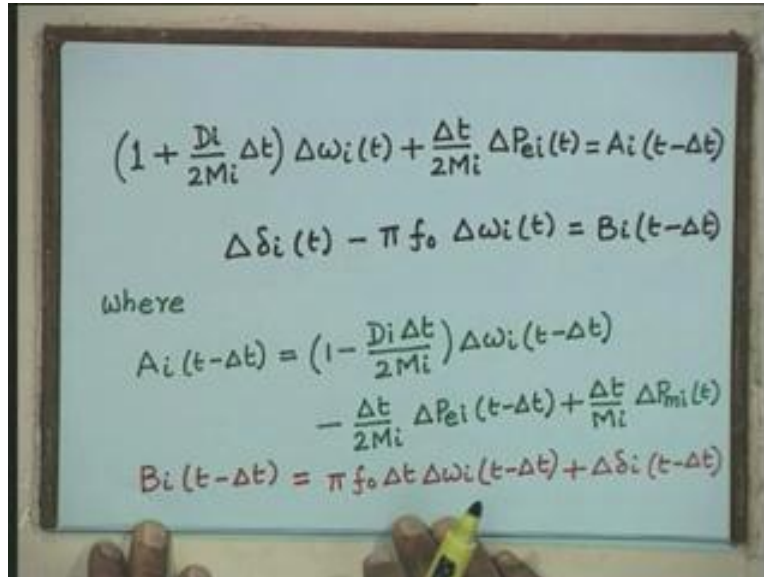
Now performing this integration and this integration by trapezoidal rule of integration we get this expressions. Now in this this equation is a is now an algebraic equation. Similarly, when you perform the perform the integrate similarly when you consider this second equation and integrate this using the trapezoidal rule of integration then we will get the algebraic equation in this form that is $\Delta \delta_i(t) - \Delta \delta_i(t - \Delta t)$ equal to $2\pi f_0$ into $\Delta \omega_i(t) - \Delta \omega_i(t - \Delta t)$ the whole multiplied by Δt by 2.

Now we have now two two algebraic equations, now what what we do here is that we arrange the equation so that all the terms which are corresponding to time t will be kept on left hand side and all the terms which are non or corresponding to time $t - \Delta t$ will be kept on left hand side. When you make this arrangement of the terms the first equation is written in the form $1 + D_i$ divided by $2 M_i \Delta t$ into $\Delta \omega_i(t) + \Delta P_i \Delta t$ equal to $A_i(t) - \Delta \delta_i(t - \Delta t)$.

Now this similarly similarly the second equation is written in this form that is on left hand side we have the terms which are corresponding to time t that is $\Delta \omega_i(t) - \Delta \delta_i(t)$ while on this side the terms are known okay

therefore we represent this right hand side term as $A_i(t - \Delta t)$ and $B_i(t - \Delta t)$ where, $A_i(t - \Delta t)$ is given by this expression which one can derive and satisfies similarly, $B_i(t - \Delta t)$ is written in this form right.

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Handwritten equations on a whiteboard:

$$\left(1 + \frac{D_i}{2M_i} \Delta t\right) \Delta \omega_i(t) + \frac{\Delta t}{2M_i} \Delta P_{ei}(t) = A_i(t - \Delta t)$$

$$\Delta \delta_i(t) - \pi f_0 \Delta \omega_i(t) = B_i(t - \Delta t)$$

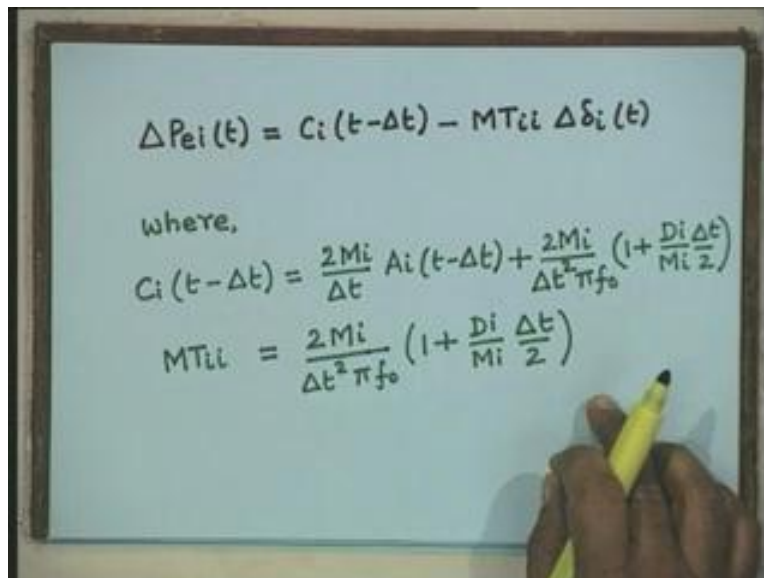
where

$$A_i(t - \Delta t) = \left(1 - \frac{D_i \Delta t}{2M_i}\right) \Delta \omega_i(t - \Delta t) - \frac{\Delta t}{2M_i} \Delta P_{ei}(t - \Delta t) + \frac{\Delta t}{M_i} \Delta P_{mi}(t)$$

$$B_i(t - \Delta t) = \pi f_0 \Delta t \Delta \omega_i(t - \Delta t) + \Delta \delta_i(t - \Delta t)$$

Now the next step is that we eliminate $\Delta \omega_i(t)$ using these equations that is you substitute the expression or the value of $\Delta \omega_i(t)$ from this equation in the first equation.

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Handwritten equations on a whiteboard:

$$\Delta P_{ei}(t) = C_i(t - \Delta t) - M T_{ii} \Delta \delta_i(t)$$

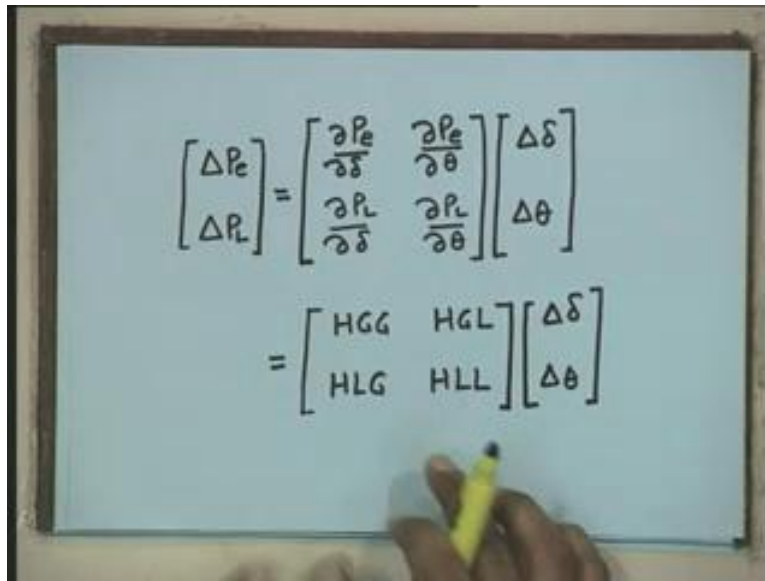
where,

$$C_i(t - \Delta t) = \frac{2M_i}{\Delta t} A_i(t - \Delta t) + \frac{2M_i}{\Delta t^2 \pi f_0} \left(1 + \frac{D_i \Delta t}{M_i}\right)$$

$$M T_{ii} = \frac{2M_i}{\Delta t^2 \pi f_0} \left(1 + \frac{D_i \Delta t}{M_i}\right)$$

When you make this substitution we get an equation ΔP_i equal to $C_i \Delta t$ minus Δt minus $MT_{ii} \Delta \delta_i \Delta t$ that is we express the deviation in power output of i th generator in terms of a term which is non that is $C_i \Delta t$ minus Δt this is its quantity. The value of $C_i \Delta t$ minus Δt is given by this expression this is the total expression that is $C_i \Delta t$ minus Δt is equal to $2 M_i \Delta t A_i \Delta t$ minus Δt plus $2 M_i$ divided by Δt square $p_i f_o$ multiplied by $1 + D_i$ divided by $m_i \Delta t$ it means all the terms on this side of the expression are known to us therefore $C_i \Delta t$ minus Δt is known similarly, MT_{ii} is a quantity which is expressed in terms of non-quantity.

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$$\begin{bmatrix} \Delta P_e \\ \Delta P_L \end{bmatrix} = \begin{bmatrix} \frac{\partial P_e}{\partial \delta} & \frac{\partial P_e}{\partial \theta} \\ \frac{\partial P_L}{\partial \delta} & \frac{\partial P_L}{\partial \theta} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix}$$

$$= \begin{bmatrix} H_{GG} & H_{GL} \\ H_{LG} & H_{LL} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix}$$

Now using this expression and the network model that is this model the ΔP_e ΔP_L Jacobian multiplied by this correction vector it is in this form. Now using this model or these equations along with this equation that is $\Delta P_i \Delta t$ equal to $C_i \Delta t$ minus Δt minus $MT_{ii} \Delta \delta_i \Delta t$ what we do is that we replace $\Delta P_i \Delta t$ in terms of $C_i \Delta t$ minus Δt and $\Delta \delta_i \Delta t$ it means they are eliminating $\Delta P_i \Delta t$. Now when you perform this operation then you can write down the equation in this form $C \Delta t$ minus Δt equal to $H_{GG} \Delta \delta$ plus $H_{GL} \Delta \theta$, where H_{GG} prime is equal to H_{GG} plus MT . Now here this MT , MT is a diagonal matrix diagonal matrix with entries MT_{11} , MT_{22} and MT_{nn} depending upon the number of generators.

Now this equation along with these two equations the whole we can arrange the two equations in the matrix form you write down $C \Delta t$ minus Δt and $\Delta P_L \Delta t$ as H_{GG} , prime H_{GL} , H_{LG} , $H_{LL} \Delta \delta$ and $\Delta \theta \Delta t$. Now this is the equation which can be solved because this term is known because this is expressed in terms of the quantities which are corresponding to the previous time step and this term simulates the type of fault and therefore when we solve this equation we get the value of $\Delta \delta$ and $\Delta \theta$.

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$$\Delta P_e(t) = H_{GG} \Delta \delta(t) + H_{GL} \Delta \theta(t)$$
$$C(t-\Delta t) = H_{GG'} \Delta \delta(t) + H_{GL} \Delta \theta(t)$$

where

$$H_{GG'} = H_{GG} + M T$$

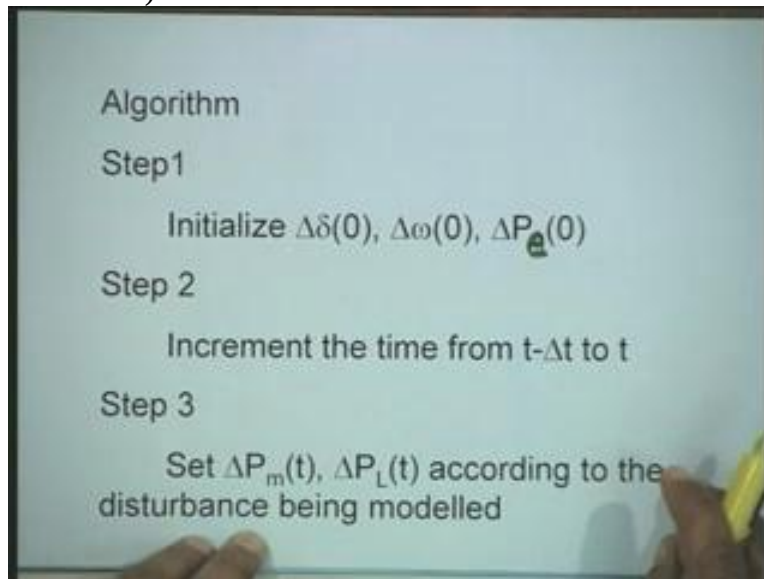
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$$\begin{bmatrix} C(t-\Delta t) \\ \Delta P_L(t) \end{bmatrix} = \begin{bmatrix} H_{GG'} & H_{GL} \\ H_{LG} & H_{LL} \end{bmatrix} \begin{bmatrix} \Delta \delta(t) \\ \Delta \theta(t) \end{bmatrix}$$

Now this this matrix this matrix actually is basically a Jacobian right the elements of this matrix are constant elements of this matrix are constant is a real constant matrix and therefore for solving this equation one simple approach is that you perform triangularization of this matrix that is you perform the triangularization operation of this matrix and once you performance this triangularization operation this is to be perform only once because this matrix is constant then we can find out the value of the change in angles right for in each time step that is first I will start with time as t equal to 0 then I can get the value corresponding to t equal to Δt right and I can iteratively keep on updating the value of these angles δ and θ the let us now summarize the complete

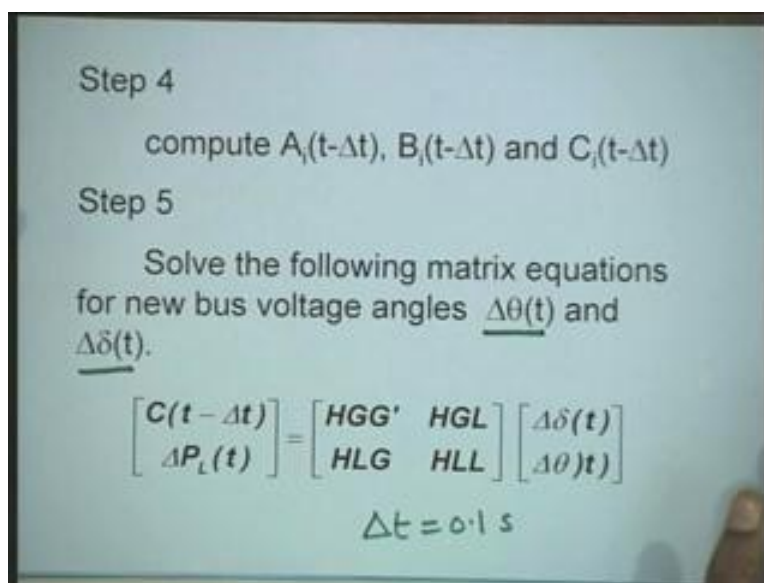
algorithm which is used for for obtaining the approximate swing curves, I am using the word approximate because we have made some assumptions.

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The algorithm is the first step of the algorithm is the even you initialize initialize the delta's, omega's and delta P_e 0 that is the initial values of all the angles, speeds and delta P. Now here the angles speeds and power they are all deviations from the steady state values therefore initial values of these variables will be 0 is it okay then next step is increment the time from t minus Δt to t that is the first the third step is set set delta P_m t delta P_L t according to the disturbance being model.

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Now we have we can consider different type of disturbances, disturbance may be a 3 phase fault, may be tripping of a transmission line, may be load sudden change in load at particular bus or tripping of generator all these disturbances can be modeled model by appropriately choosing the value of ΔP_m and ΔP_l and making some minor changes in the model. Next step is to compute $A_i(t - \Delta t)$, $B_i(t - \Delta t)$ and $C_i(t - \Delta t)$ for all values of i . Suppose you have say n generators, n generators right then we have to we have how many equations will be there for each generator there are two equations and therefore i will be paring from 1 to n not $2n$, 1 to n because when I take the first differential equation that is one equation the constant term is A_i d by i 1 to n then we solve a set of algebraic equations written here that is solve the following matrix equation for new bus voltage angles $\Delta \theta_i(t)$, $\Delta \delta_i(t)$, $\Delta \theta_i(t)$ $\theta_i(t)$ is the change in angle of the bus voltage while $\Delta \delta_i(t)$ is the change in the angle of the internal bus voltage or the voltage behind transient reactance.

This operation can be performed easily easily by the process of triangular resistance of this matrix where we may not have to invert this matrix otherwise this a large has to be inverted the process of triangularization will avoid the of inversion of this matrix. Now one small important point is that when you solve these equations the time step which we have to choose is important right.

Now for solving these equations the time step Δt equal to 0.1 second is considered to be quite adequate even some times you you can choose a time step of the order of .2 seconds but however the steady so that if you take Δt equal to 0.1 second right the results will be quite acceptable. However actually the since the we are solving these equations by using the implicit integration technique and therefore the problem of numerical instability would not come the the Δt should be kept small to obtain the accurate results from the consideration of accuracy.

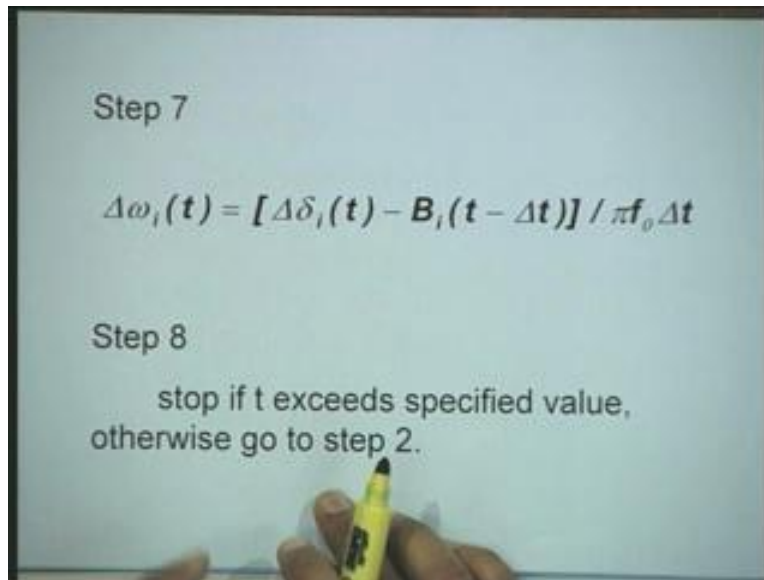
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Step 6
compute $\Delta P_{ei}(t)$

$$\Delta P_{ei}(t) = \frac{2M_i}{\Delta t} A_i(t - \Delta t) - \frac{2M_i}{(\Delta t)^2 (\pi f_o)} \left[1 + \frac{D_i}{M_i} \frac{\Delta t}{3} \right] [\Delta \delta_i(t) - B_i(t - \Delta t)]$$

Now once we have obtained the deviation in angles right we can compute the deviation in generator power ΔP_{ei} can be computed using this equation which was derived earlier even compute that expression for change deviation in generator power output and this is now expressed in terms of the known quantities because A_i t minus Δt is known, B_i t minus Δt is known that is all the terms which are there on the right hand of this expression are known.

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Then in case we are interested in knowing the variation of angular speed as a function of time then we can obtain the value of $\Delta \omega_i$ t in terms of the calculated value $\Delta \delta_i$ t and the known quantity like B_i t minus Δt right that is when you come up to the step 7 we have calculated the values of these parameters $\Delta \delta_i$ t and ΔP_{ei} at time equal to t right and since we will be solving this equation for a certain time duration.

Suppose you want to get the solution for two seconds in the moment the time is equal to the time for which you want to get the solution you stop otherwise you go to step two where to increment the time step and continue the integration process that is step two is increment the time from t minus Δt to t and you continue this process till you get the final solution okay.

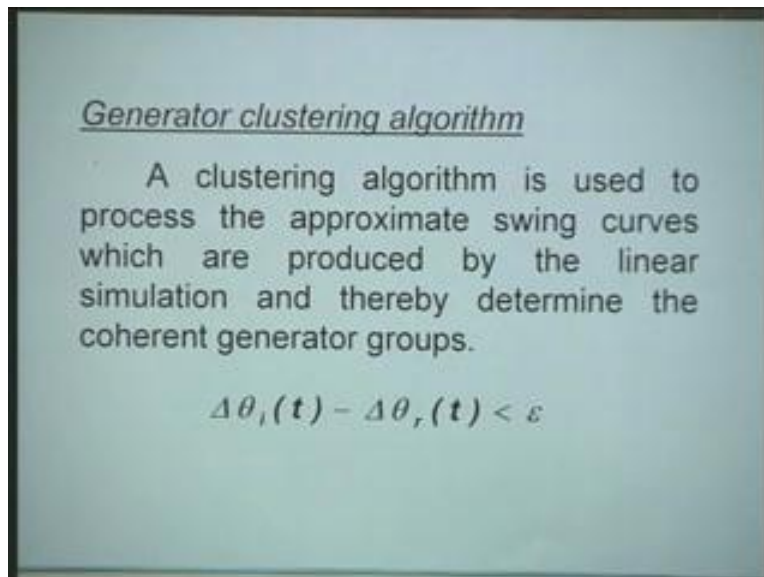
Therefore, the algorithm which we have discussed just now is a simple algorithm which can be used to get the approximate swing curves quickly right and then the next step will be that we have to cluster the swing curves together or you obtain actually the group of generators which have identical swing curves.

Now here we can cluster the generators considering the swing curves corresponding to the bus voltages or considering the swing curves corresponding to the internal generator voltages that is I can cluster or I can examine the swing curves

corresponding to bus voltage that is the delta theta versus T for all the buses or I can cluster cluster the swing curves using the internal bus voltage angles.

Now in practice the first step is that instead of clustering with respect to the internal angle we prefer to first find out the swing curves corresponding to the bus voltages right and then the moment you find out that okay these are the generators where the bus voltages, swing curves are identical okay then you form equivalent group. Now this approach is helpful in network reduction and once we have done this second step of that is called network reduction where we put the all the generators on the same bus and then we check whether these generators form the coherent group or not. Generally it is found actually that when you when you cluster the cluster or the the the swing curves corresponding bus voltages then obtain the coherent groups using the using the delta theta versus T curves okay. Then it is found actually that the machines **machines** are coherent with respect to the angle also okay.

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Therefore the clustering algorithm is an important step and the algorithm goes like this what we do here is that suppose you have a group of machines or a set of swing curves which are to be clustered therefore you start actually with the one let us start with one swing curve and consider that this is the reference curve, then take the swing curve 2 and compare the swing curve of the second generator with this reference in case these two swing curves are identical then the generator 2 belongs to that particular cluster otherwise, you put the generator in to second cluster then you go to the third generator you compare with the compare with cluster one and cluster two if it belongs to either of the two you put it there otherwise you form a third cluster.

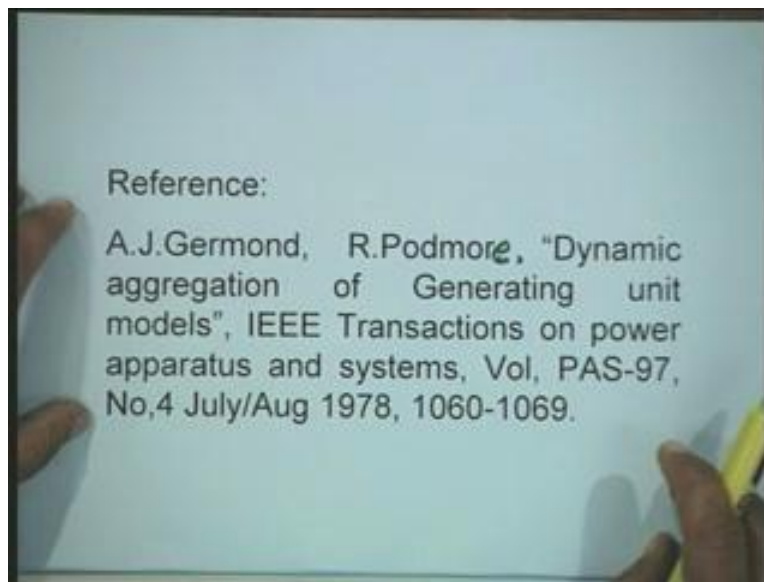
The algorithm goes simply like this that delta theta i t minus delta theta r t should be less than equal to when where is the i is the index, r is the index of the reference reference swing curve right. In case this this difference is is less than prescribed value over the

complete time interval that is that is for the complete time interval of set to seconds over with the swing curves are obtain then the then the the generator whose swing curve is corresponding to θ_i qualifies to be put in along with reference generator otherwise, you form like another cluster.

Now when you perform this step you will be in a position to put the the swing curves in different clusters and that will give us that which these are the groups of generators which form one cluster this is second cluster is a third cluster and once we have identified these groups we have to go for the next step to find out the equivalent machine.

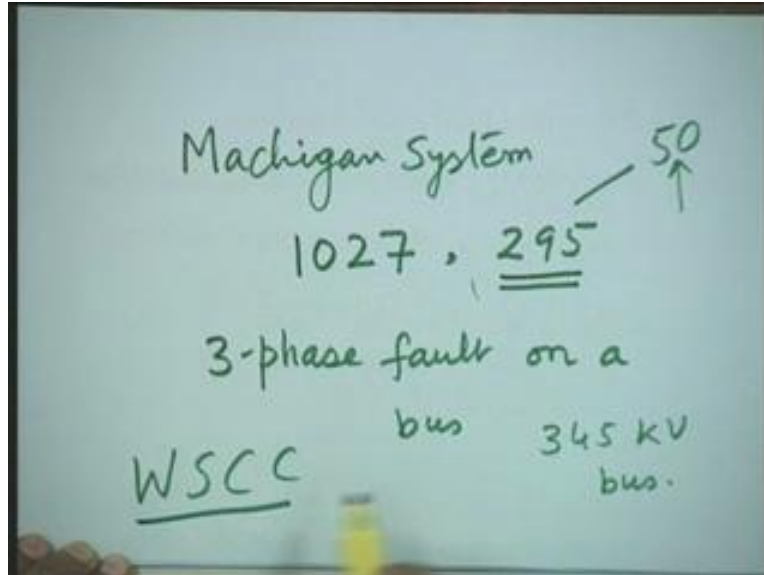
Now at this stage I would like to give some results of some studies which have been conducted and reported in this important reference that is in this reference which is very important reference paper written by A.J. Germond and R. Podmore spell is R. Podmore dynamic aggregation of generating unit models I EEE transactions on power operators and system volume PAS 97 number four July August 1978 pages 1060 and 1069.

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Now this is one reference paper which tells us how to how to aggregate aggregate the the generator models and obtain an equivalent 1 okay. Now before that naturally one has to first perform a exercise to identify the coherent group of generators. The earlier reference paper which I had given which was written by Podmore in that reference paper the two case studies have been presented in the one case study is done for power system is called Machigan system. In this system the system considered is having 1027 buses, 1027 buses and the number of generators in this system were 295, were 295 derivatives.

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Now for this system for the system the the algorithm which we just now described was applied to find out the coherent group of generators for identifying this a 3 phase fault, a 3 phase fault on a bus on a bus was considered that is the bus which is considered in this typical system is a 345 KV bus okay. The the duration the duration of the fault considered in this case is 12 cycles there is no fault consider here for this study 12cycles on 60 hertz bases how much time it comes out to be 12 by 60 seconds which comes out to be .2, .2 second, it's a quite large time.

Now up and the the swing curves approximate swing curves are obtain applying the algorithm which we have discussed here and the result show that these 295 generators can be grouped in to total 50 equivalent generators that is when the equivalence is applied then these 295 generators can be replaced by 50 equivalent generator further out of these 50 it was seen that 25 groups having only one generator that is when you put the all the machines in to different clusters what is found was that you can put all the swing curves of the 295 generators in to total 50 clusters and in in 24 different clusters was seen that there is only one generator that belongs while the other generators were grouped in to number of group together or you can easily say 295 minus 25 that is minus 24 not 25 then this remaining is 271 they were definitely but put in to only 26 cluster and therefore each cluster must be having large number of generators.

Therefore, consider it that the once you reduce from 295 to 50 then naturally the the model dimensions is going to reduce significantly. Now further the time which is required because the result the result which are reported in this research paper so that the time which is required to obtain the equivalent is 180 seconds just to get the equivalent and then once you solve the problem transient stability analyses solve using the equivalent the time required is 55 seconds while if you solve the complete transient stability problem then the time required is something like 341 seconds.

Now we can see actually that the the time which required to get the solution has drastically reduced from 341 second to 55 second. Now these figures are corresponding to a typical computing system which was used right it but relative timings will still remains, the second case study which has been reported in this research paper the system belong to this WSCC, is western system coordination committee we call it where the number of generators considered were or number of buses in the system were 1308 buses you see this dimension of the problem 1308 buses and it had 397 generators, 337 generators for this particular system again a 3 phase fault was considered, 3 phase fault.

The duration of fault considered in this particular case is 83 cycles on 60 hertz system instead 12 cycle in the previous case it is 8 cycles and the time, integration time Δt which is chosen is a again equal to 8 cycles fault duration of the early 8 cycles integrations stable also up 8 cycles and for clustering for clustering the swing curves, the tolerance used here is in the previous case it was 5 degrees in the present case for the WSCC system, CC system it is a 4 degrees and when this exercise was perform right it could be replaced by 87 equivalent generator but the original system had 337 generators, while the equivalent system has now got eighty seven generators and out of these 87 generators you 44 generators or groups 44 groups are having only one generator.

Further yes further it was seen actually that these 44 generators which form group of single generator they were the generators closed to the disturbance, close to the disturbance and the same thing was observed for Machigan system also in the since actually that the machines which are very close to the disturbance right may have to be may retain their independent swing curves while you machines are away they will behave generally more in a coherent fashion okay.

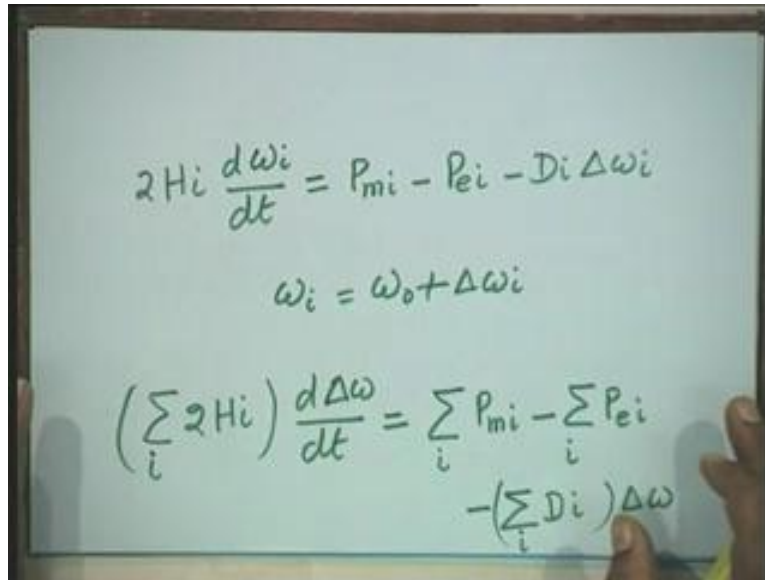
Now the next step for our study will be that okay we have identify now identify the coherent group of generators. Now our requirement is that you want to find out the parameters of the equivalent generator when I say the equivalent generator means there what is the equivalent inertia constant, what is the equivalent model for the turbine and governors right what is the equivalent excitation system for this because each machine has its own excitation system we want to find out what the equivalent of this. Similarly, each generator synchronous machine right you have to find out equivalent synchronous generator similarly if it power system stabilizer we want to find out the equivalent power system stabilizer right.

Now this exercise is quite a quite a complex and involve exercise and we I will just tell you today that how do we obtain the equivalent equivalent rotor dynamics, first rotor dynamic let us see it now the equation or the swing equation for each machine written like this $2 H_i d_i \Delta \omega_i$ here also we can put $\Delta \omega_i$ right or we can put ω_i is equal to ω_0 plus $\Delta \omega_i$.

So that this equation now the when you just look at the definition of coherence right the all the machines which are coherent right the the the speeds of the rotors that is $\Delta \omega_i$ the speed deviation will be same for all the rotors right and therefore if I have

something like n machines then I can write these n such equations okay then I add all these equations.

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$$2H_i \frac{d\omega_i}{dt} = P_{mi} - P_{ei} - D_i \Delta\omega_i$$

$$\omega_i = \omega_0 + \Delta\omega_i$$

$$\left(\sum_i 2H_i \right) \frac{d\Delta\omega}{dt} = \sum_i P_{mi} - \sum_i P_{ei} - \left(\sum_i D_i \right) \Delta\omega$$

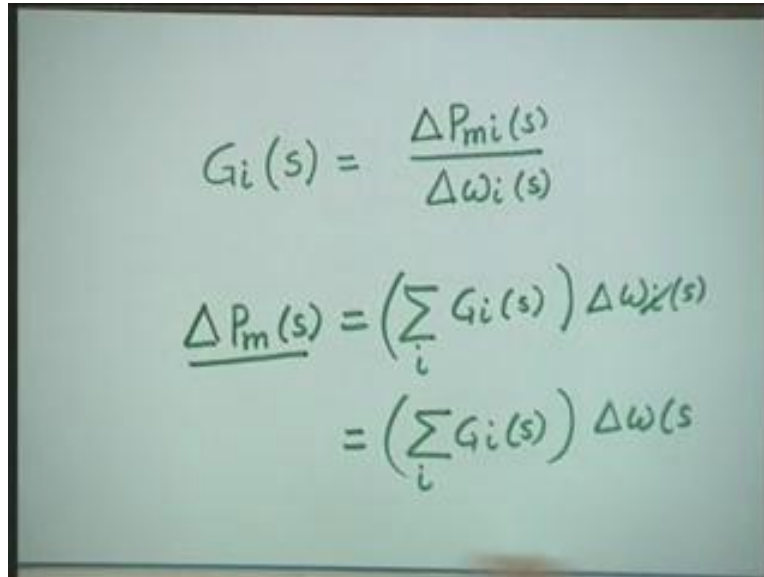
Now since the speed deviation is same for all the machines right I can write to the equation of the form summation to H_i for all i 's multiplied by $d \Delta\omega$ by dt equal to summation or all i P_{mi} minus summation P_{ei} minus summation d_i or i $\Delta\omega$. Now I am dropping this subscript I here right because all the machines which are forming a coherent group they have the same speed deviation and therefore you can see here that equivalent machine will have inertia constant which is equal to sum of the inertia constants of all the machines.

Similarly the equivalent damping constant will be equal to the sum of the damping constants of all the machines and the equivalent mechanical input will be sum of all the mechanical input that is total mechanical input. Similarly, the electrical output is sum of all the electrical output therefore this is one thing which we see that when I try to find out the rotor dynamics of the equivalent machine then that equivalent machine has total inertia which is the the sum of all the inertias and similarly the damping damping that is damping term d_i this the total damping is the sum of damping of individual machines.

Now we will discuss one more important aspect suppose we want to find out the equivalent equivalent model of the turbine governors that is each machine has its own turbine and governor right. Suppose you have a group of machines which form a coherent group I want to find out the equivalent turbine model similarly, the equivalent governor model.

Now we can write down the the transfer function of of of governor turbine to the let us say let us say the governor and turbine together, output of the turbine is going to the change in mechanical power input is speed deviation right.

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$$G_i(s) = \frac{\Delta P_{mi}(s)}{\Delta \omega_i(s)}$$

$$\Delta P_m(s) = \left(\sum_i G_i(s) \right) \Delta \omega_i(s)$$

$$= \left(\sum_i G_i(s) \right) \Delta \omega(s)$$

Therefore, if I represent the transfer function of i th machine turbine governor G_i s this can be written as change in mechanical power output ΔP_{mi} s divided by $\Delta \omega_i$ s but this is the transfer function, transfer function of so i th turbine governor system output is mechanical power input is speed deviation.

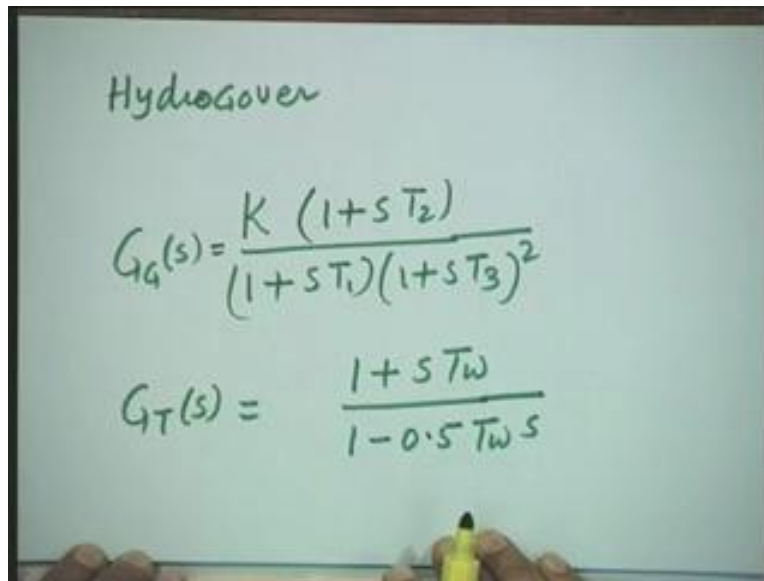
Now since since all these machines have the same speed deviation, same speed deviation. So that I can write down equivalent mechanical P_m ΔP_m s as summation summation G_i s sum over all i's multiplied by $\Delta \omega_i$ s $\Delta \omega$ s or we can drop this I that is you can say that the total mechanical power the equivalent mechanical power can be written as as the sum of the transfer functions of the individual generator, transfer generator I am sorry individual turbine transfer functions that is we can see here ultimately that I can just write down here summation over i G_i s multiplied by $\Delta \omega$ s.

Therefore now question arises is that how do I find out the equivalent equivalent transfer function which will represent the equivalent of all the individual individual governor turbine transfer functions put together. Obviously you can seen here actually that that the equivalent transfer function is the sum of the transfer functions of individual turbine governor model okay. Now here generally what is done is that we come across two type of machines one is the steam turbine and steam turbine governor another is hydro turbine and hydro turbine governors.

Suppose in your study you find that in one cluster or in group of coherent machine there are few machines which are turbo generators, there are few machines with hydro

generators right. In that case when you find out the equivalent we will find out equivalent of all those turbo turbo generators and put by one equivalent machine we find out the equivalent for hydro generators and hydro generator governor in put as one machine, we will not combine the the hydro machines with the steam turbines are driven generators okay.

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Hydrogover

$$G_G(s) = \frac{K (1 + s T_2)}{(1 + s T_1)(1 + s T_3)^2}$$

$$G_T(s) = \frac{1 + s T_w}{1 - 0.5 T_w s}$$

Now just to tell you actually that how do we obtain the equivalent model for this hydro hydro governor hydro governor to this this model which I have considered here that is governor turbine model it can be the speed in to two parts, one is the governor model another is the turbine model and when these two models are combined together will give the total model about the hydro governor model can be put in the general form as K times G_G s, G is stands for governor equal to K times one plus s times T_2 over 1 plus s times T_1 , 1 plus s times T_3 square.

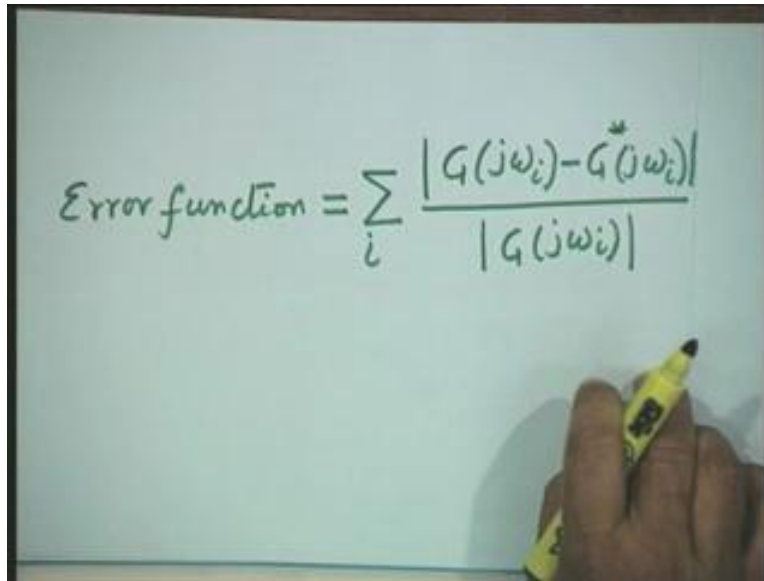
This is the genera form of the hydro governor model and the hydro turbine model if we put it actually G_T s it can be put in the form form 1 plus s T_w , 1 minus 0.5 T_w s this is the model okay. Therefore, what is done is that you put these two together form this is form equivalent model of the hydro governor and turbine then what we are interested is actually that what should be this constants K T_1 , T_2 , T_3 and this T_w right of the equivalent model. When we know the transfer functions of individual models right the approach will be the approach will be that what you do is that.

Suppose you want to you can say study our certain frequency range then you you take discrete frequency range frequencies that is you put s equal to $G \omega i$ takes in discrete values for each value of frequency, you find out the transfer function of individual individual models and add them that is a suppose I want to find out what is the transfer function of say model number one it will be G_1 s, s equal $G \omega a$ substitute that value

of ω_i you can some discrete value then this quantity will come out to have actually its magnitude and phase angle.

Similarly second will also have its magnitude phase angle therefore you can add all these together to form an equivalent magnitude and phase angle at particular frequency okay and therefore for all discrete frequencies you can perform this exercise.

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$$\text{Error function} = \sum_i \frac{|G(j\omega_i) - G^*(j\omega_i)|}{|G(j\omega_i)|}$$

Now let me define an error function equal to summation over all i 's $G(j\omega_i)$ minus $G^*(j\omega_i)$ magnitude divided by $G(j\omega_i)$. Now this error function represents the numerator of this error function is $G(j\omega_i) - G^*(j\omega_i)$ that is this is the transfer function evaluated for discrete frequency ω_i and this is the value of the transfer function of the equivalent turbine generator the difference will come out to be complex number we take the magnitude of this divide by divide by the magnitude of this transfer function $G(j\omega_i)$ for the actual system at the discrete frequency then we do this exercise for over the complete range of frequencies and sum up this function this is define as error function.

Now our objective is to obtain the parameters of the equivalent turbine governor model the equivalent parameters are the gain constant K , the time constants T_1 , T_2 , T_3 and T_w and now in order to obtain the the time the parameters of the equivalent generator what has to be done is that we apply some some technique normally called numerical green technique.

So that this error function is reduced and when this error function is reduced and comes to a minimum value at that will give us the the equivalent turbine governor time constants and density. Now if I plot plot the frequency response of the equivalent equivalent turbine generator model and the frequency response of the actual system right then the these two frequency response should be identical right then we can or I can say

that you plot or it is because of border plot for magnitude and phase angle right and the border plot for the actual system and for the equivalent system they should match closely right that means we have obtained the parameters of the hydro governor and turbine very accurately, this in case now instead of having the hydro governor and hydro turbine if you have steam governor and steam turbine right. We can choose the equivalent transfer function of the turbine governor model right and performing the same exercise that is we define error function and minimize this error function using gradient numerical gradient techniques and hence obtain the parameters of the equivalent governor and turbine okay.

Now let me sum up that today we have discussed an algorithm for identifying a coherent group of generators then we have also discussed how to aggregate the rotor dynamics and obtain the equivalent inertia constant of the machine equivalent damping constant of the machine right and we also obtain the or we also discussed the method which can be used to obtain the equivalent turbine governor model of the system. Thank you!