Power System Dynamics
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Lecture - 28
Dynamic Equivalents for Large Scale Systems

Friends, today we will study dynamic equivalents for large scale systems. We know that the stability studies is one of the most fundamental and important study to be performed for system expansion planning for design of the system, for operation and control also.

Now to solve actually the stability problem what is to be done is that we have to develop the model of the system and then solve these equations using numerical techniques. Further, we require actually large number of stability studies to be performed and since there is no analytical you can explicit solution when has to resort to the numerical techniques.

Now this numerical techniques take lot of time to obtain the solution. Further the size of the system has become very large and therefore when the size of the system becomes very large we will have a mathematical model of very large dimension or very high dimension and therefore that two problems one is actually that the dimension of the mathematical model of the system another is the time which is required by using the numerical techniques and in practical situation, in practical situation it is practically impossible to represent the entire system in detail and therefore there is a necessity to to resort to some equivalency.

Now here the when you talk about the equivalencing the basic philosophy is something like this that whenever a disturbance occur in a particular portion of the system then the that portion of the system which is very close to the disturbance is severely affected, while the system which is away from the disturbance disturbance is is lesser affected right and the system which is very far away may have very little affect on the system and therefore one can one can represent the part of the system which is very close to the location of the disturbance in detail and the system which is away can be represented by an equivalent.

However however, actually the system which is which is not that severely affected but affected to some extent right it has an affect on the dynamics of the machines in the study portion right and therefore there is a necessity to adequately capture the dynamics of the system which is away from the study system right and therefore what we do here is that whenever the system large system stability analysis do you perform right we divide the system into study system, external system and third portion we call is remaining system.

Now to understand what we mean by study system or external system or the remaining system let us just take this small system it has a number of generators you can see here actually in this diagram there is number of buses I am not showing the loads but the number of buses the loads are connected.

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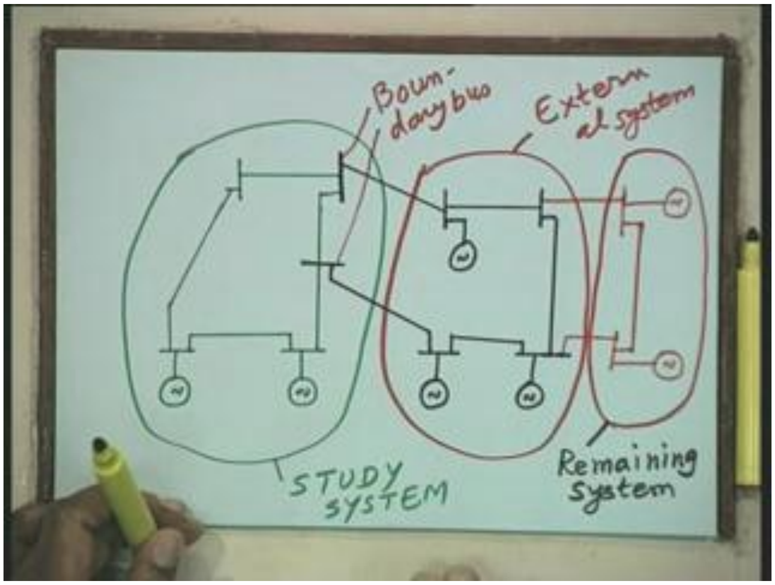

Now suppose I am interested in studying the behaviour of this this portion of the system, let us say this is the portion in which I am interested right it was my interest is to know if there was there is a fault in this portion of the system right what will be the dynamic behavior of the system. Therefore, what we do is that we call this portion as the study system the study system. Now to this study system we have system outside the study system.

Now some portion of this system which is outside the study system has significant influence on the dynamics of the study system. We will different this by we call it is the external system, external system then beyond this external system we have this portion, this can be given the name as remaining system. We will call this as a remaining system, the philosophy here is that we are interested in the behaviour of the of the system whenever there is a fault occurs in this portion of the system dynamic behaviour of the system and to know the transient stability solution of the system then what is to be done is that these external systems should be represented by an equivalent okay, while representing the external system by the equivalent this equivalent should be accurate enough. So that we capture adequately and accurately the the affect of the system onto our study system okay.

Similarly, the remaining system is very far off from the study system and it has very little influence on the dynamic of the study system then this can be represented by approximate equivalent, very approximate equivalent or we can say very simplified model we can represent it okay many times the system can be divided into two parts only study system external system may not have remaining system.

Now here you can just see these two buses at which the external system is connected to the study system right of these two buses these buses where study system is connected to the external system, we denote these buses as boundary buses that is you can say that this bus this bus, these are boundary buses, boundary buses. Similarly for this external system right the the buses for example this bus or this bus where where actually the external system is connected to the study system we call this as a terminal buses these buses will be called at the terminal buses okay.

Now over the years, over the years I can say at least during the last four decades efforts have been made to develop dynamic equivalents okay. Now when we develop dynamic equivalents we have to ensure two things one is that the equivalent is accurate enough so that so that we capture the affect of this equivalent on the study system adequately, secondly that when we obtain the equivalent the process of obtaining the equivalent should be very fast that is there is a mathematical way or there is a methodology which is used for obtaining the equivalent for the system and therefore the time which is required or computations which are required for obtaining the equivalent of the external system should be should be reasonable okay.

Now many times we may require some more time to obtain the equivalent but since we may have to perform perform many stability studies right and if we use this equivalent then whatsoever time you spend in obtaining the equivalent is compensated because actually in practice that you solve the the stability of the study system using the equivalent right and equivalent is to be obtained only once once right and therefore when you do the multiple stability studies that whatsoever time which is required is compensated. In fact the literature on this dynamic equivalents is very worst and we will confine to only few techniques which are quite, we will say popular and used in practice.

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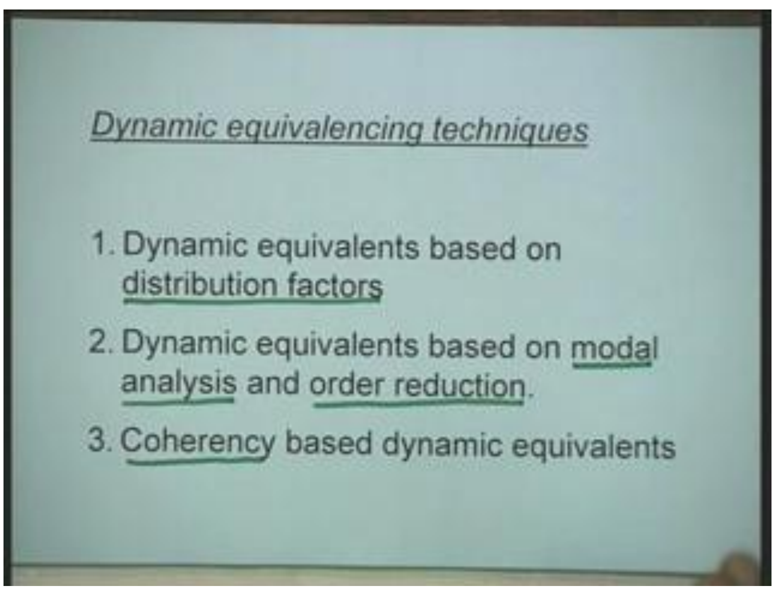

The techniques are classified under the these 3 categories that is the dynamic equivalencing techniques we can say or we can classify broadly in these three categories number one dynamic equivalents based on distribution factors, dynamic equivalents based on distribution factors, the second technique is based on a model analysis and order reduction model analysis and order reduction and the third technique is known as the coherency based dynamic equivalencing technique there are 3 techniques on each technique, you will find lot of literature in the published okay. Recently, people have developed techniques which are called actually the model coherency based techniques that is they take the advantage of the the model equivalencing technique and the coherency based equivalencing techniques right.

The dynamic equivalents which are obtained using the distribution factors right is a simplest of all the techniques in this arrangement what is done is that this whole external system including the remaining system is represented by an equivalent generator connected at the boundary buses that is suppose they are two boundary buses right. At these two boundary buses you connect 2 equivalent generators right and if you have number of loads also then you represent by 2 equivalent loads right.

So that when you perform the stability study you have to perform the study study stability study of this study system for example in this study system. Now you will have four two generators which are there already and two equivalent generators right but but this technique is based upon some heuristic approach it is not having very sound mathematical base right.

Now earlier when the when the when the you know computers are very limited capacities were available then this was the best approach for obtaining the equivalents but now since we have better computing facilities available right therefore another techniques we have also have been considered and they are having a sound mathematical base okay, we will deal with this I will just mention about this model analysis technique.

In this model analysis technique basically the first one has to obtain for the complete system linear model, one is to obtain linear model obtain the with this linear model you obtain the Eigen values of the system okay and then using this information about the Eigen values you eliminate you eliminate certain modes okay and then retain only those modes which have a significant influence on the system dynamic behavior okay.

There may be some modes which decay fast you can just ignore them those modes which are very high you know very low time constant actually and those modes which have high time constant you retain them okay because we are studying actually the stability if this is a slow phenomenon okay because you always know actually in 2 in 2 seconds, 3 seconds and wherever you have actually the time constant of the modes which decay very fast in say fraction of a second you can ignore the the major drawback of this model equivalencing technique is where one is to obtain the Eigen values of large system and then then eliminate or reduce actually the dimension of the system, this is what is now one the very serious problem which comes here is that this reduced order model you cannot you cannot actually correlate each mode with the physical system.

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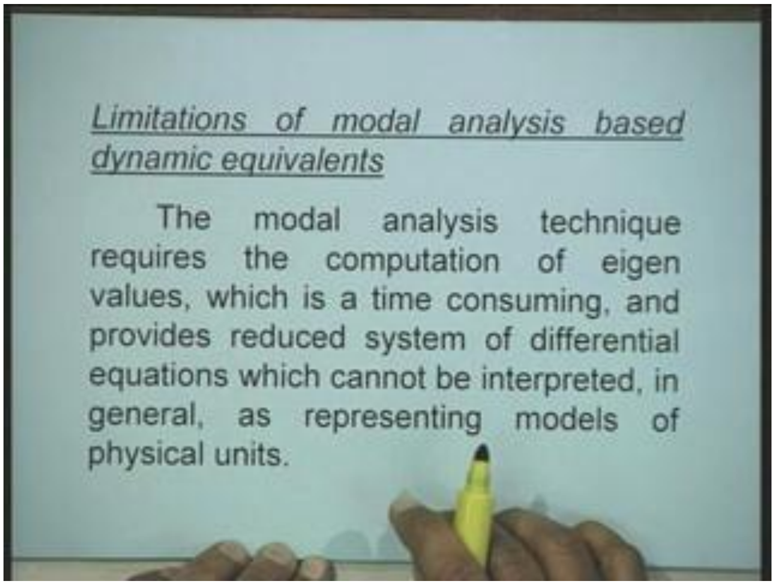

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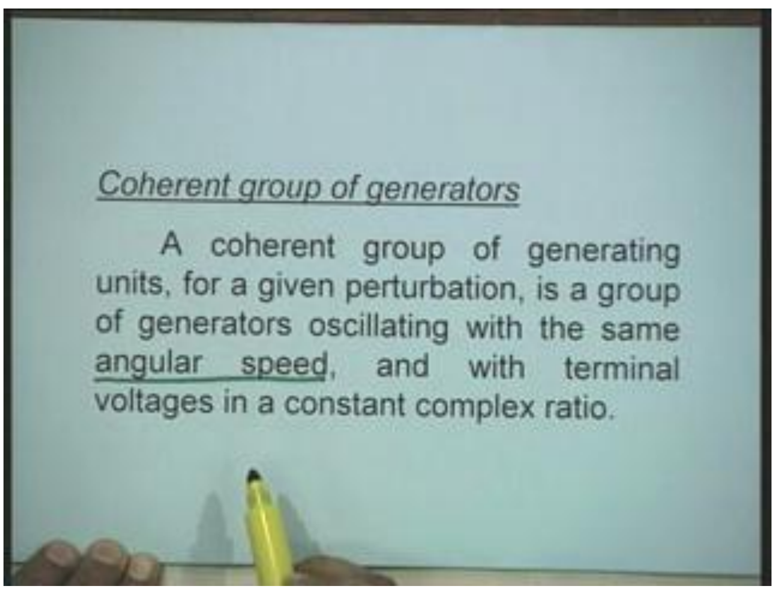

You can appreciate this point actually that there is a flatten physical system right and with this and you have obtained actually this sort of model using this model analysis then these reduced order model right you cannot have one to one correlation ship between the model and the physical system and in view of this we shall be discussing mainly this coherency based dynamic equivalent okay and in this coherency based dynamic equivalents when we study, first let us see what do we mean by coherency based dynamic equivalents. I think I can first tell you what is the definition of coherent group of generators.

The definition is like this, a coherent group of generating units for a given perturbation is a group of generators oscillating with the same angular speed and with terminal voltages in a constant complex ratio, this definition you have to very clearly understand that is in a power system you know large number of generators. Okay whenever a particular disturbance occurs.

Okay then then you will find actually that a group of machines swing together right they all swing together it means where when the when you plot actually the delta versus time out the swing curves then those machines which have the identical swing curves right form a coherent group of machines okay and therefore if we can identify for a particular system the coherent group of machines then we can replace this the coherent group by one equivalent machine right and this is a very you know sound mathematical technique that is yes there is a system where whenever you have perturbation the machines which swing together which have the identical swing curves right they form a coherent group and this coherent group can be replaced by an equivalent machine that is when you talk about the angular speeds they all have the same angular speed okay. However, the terminal voltages of these machines right will maintain a constant complex ratio okay.

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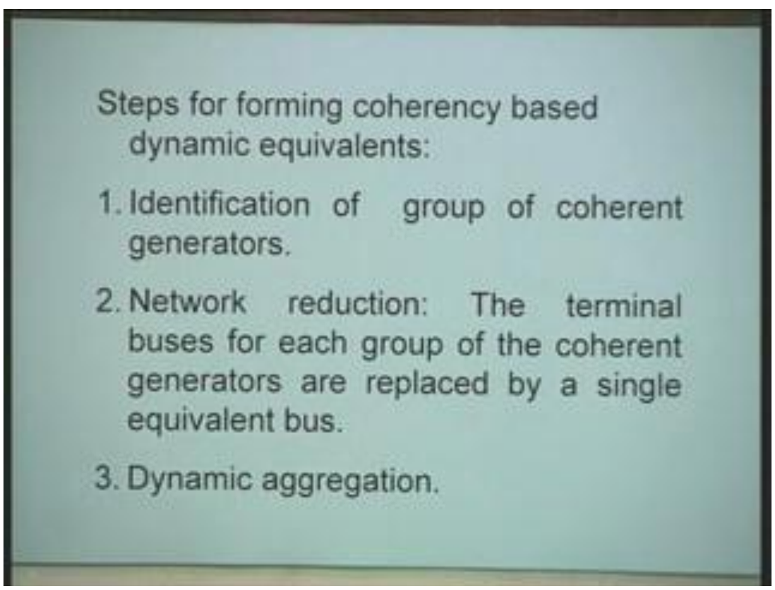

Now when we want to obtain the dynamic equivalents using coherency technique then they are 3 important steps which are involved, the first step is identification of group of coherent generators obviously that before I replace I before by replace the system by equivalent I should know that okay these machines are coherent, these machines are coherent, these machines are coherent, the groups have to be obtained, this is a first step and second step is called network reduction.

The terminal buses of each group of the coherent generators are replaced by a single equivalent bus. You try to appreciate this point that suppose the first step is that we have done some study and identified that okay these machines form a coherent group, okay then each machines has connected to a bus okay, therefore what is to be done is that we have to we have to do network reduction and and obtain an equivalent bus where all the machines are connected okay.

So that once the suppose take an example that if I find that machine number 1,3 and let us say 6 right they form a coherent group then I want to all these 3 machines will have their buses their different buses at which they are connected okay but we want to replace this by one equivalent bus where all these machines will be connected in parallel and then these machines will be replaced by one equivalent, is it clear, this is a second step and the third step is called dynamic aggregation. The dynamic aggregation means once you have obtained a common bus at which all the machines which form a coherent group are connected right then we represent a equivalent machine by what is called a dilute aggregation phenomenon which will have equivalent inertia constant, equivalent excitation system, equivalent turbine governor model, equivalent transient reactance and so on it means the the this is a very important step where where you obtain a equivalent.

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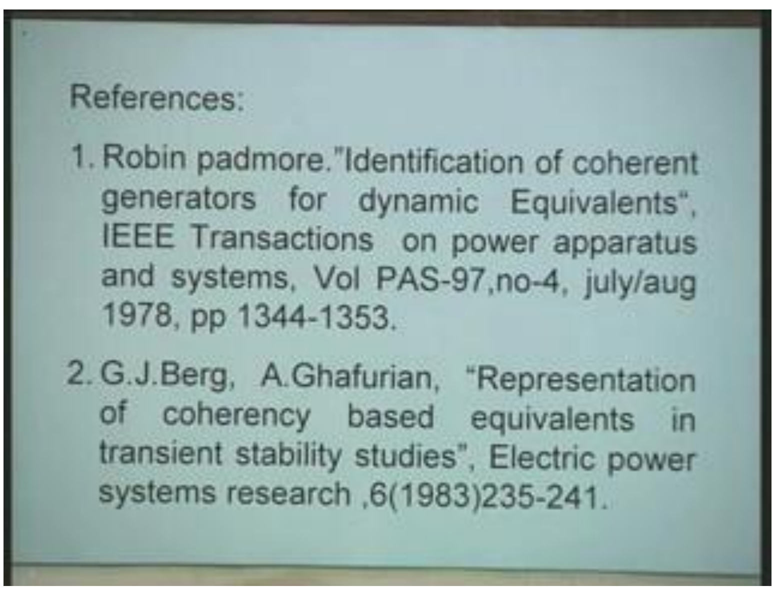

Now because in the sense that these are the three important steps which are involved that is you have to first identify the coherent group of generators and these are the other two steps. Now here to identify the coherent group of generators we have very important reference that reference is the paper is written by Robin Padmore, the title of the paper is Identification of Coherent Generators for dynamic equivalents that emphasis is for identification identification of coherent generators for dynamic equivalents, this paper is appeared in IEEE transaction on power apparatus and systems volume PAS 97 number 4 July August 1978. There is another paper that is G. J. Berg and A. Ghafurian representation of coherency based equivalents in transient stability studies. This has appeared in electric power system research volume 6 which has in 1983 pages are 235 to 241.

Today, we will concentrate on identification of coherent generators for dynamic equivalents okay and then in the next lecture we will talk about the other two aspects that is network reduction and dynamic aggregation and there is one more research paper which deals with dynamic aggregation that I will tell you in the next term.

Now here when we talk about identification of coherent group of generators. Let us see what is the simplest technique, how do we how to identify? can you tell me how to identify the coherent group of generators because you know what we mean by coherent coherent group or what is the meaning of coherency the the response of the class is that we first obtain the swing curves, swing curves of all the machines right and those swing curves which are identical we group into one coherent group and similarly other coherent group, it means we have to first perform a detail transient stability analysis.

You have to take a system and you perform a detailed transient stability analysis okay and then you can group them this is one technique this is a straight forward and accurate technique. However however there are certain facts which have been obtained through lot of research work the facts are like this and these facts can be used to obtain obtain the approximate swing curves for identifying the coherent groups of machines instead of going for the detail transient stability analysis using these facts we can obtain the approximate swing curves right and we can find out the coherent group of machines and these can be used actually to finally obtain the equivalent the the assumptions are the facts are like is their assumptions as well as actually the facts.

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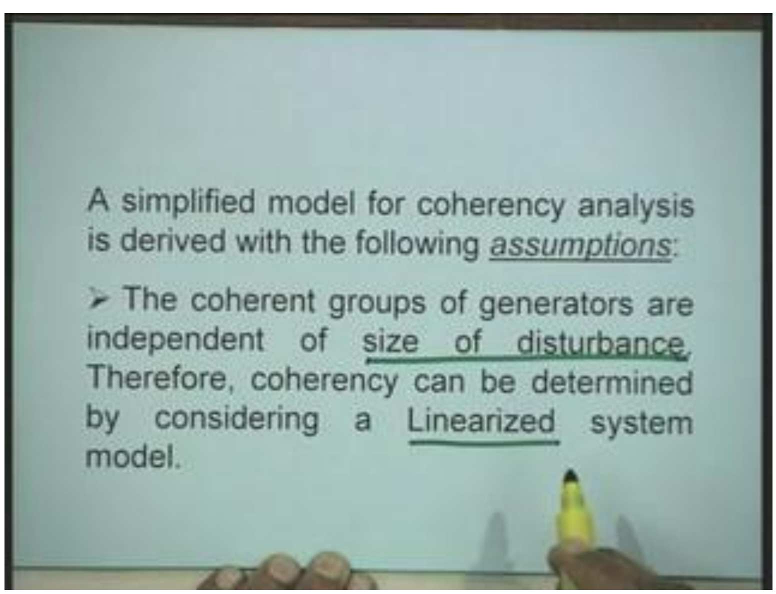

The coherent group of generators are independent of size of disturbance coherent groups of generators are independent of size of disturbance this is a very important point here that is at a particular bus right if a fault occurs whether the disturbance is say .1 second duration or whether it is of .2 second duration because the size of disturbance will depend upon the duration of the disturbance to which the system is subjected, for example if you take three phase fault at a particular bus right if the fault persists for more time then it is a it is a bigger disturbance. If suppose fault persist for very short time it is a disturbance of small magnitude because if you suppose somebody performs the study that okay you consider disturbance and state in the dynamic behaviour considering the different fault clearing time it means you are basically studying the behaviour of the system system with varying magnitude of disturbance.

Now suppose you consider actually the three phase fault or say unsymmetrical fault right the unsymmetrical fault is less severe in as compared to three phase fault right and therefore from that point of view the one very important conclusion here is that it is it is independent of size of disturbance and therefore coherency can be determined by considering a linearized system that is you can obtain the coherency behaviour using a linearized system. Okay this is one very important and on this basis we can get actually the coherency behaviour or coherent groups very quickly.

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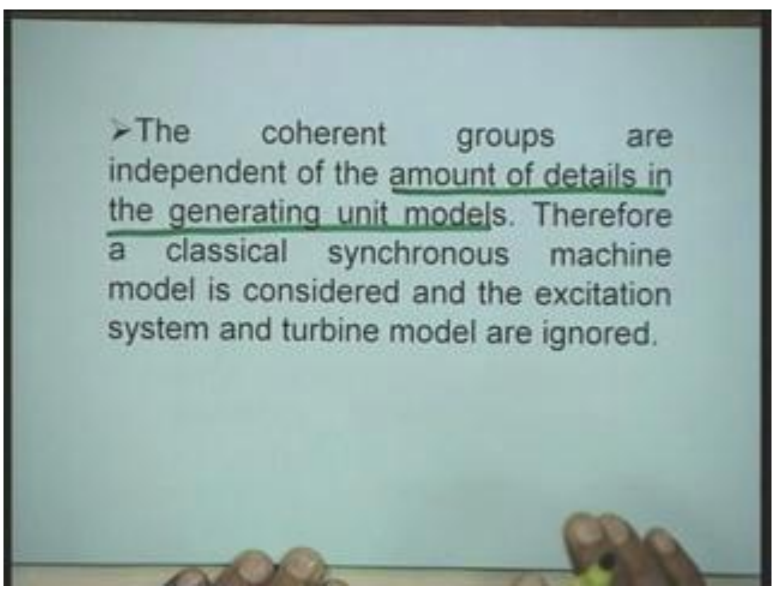

The second assumption is or the fact is that the coherent groups are independent of the amount of details in the generating unit models that is if you look here that is the coherent groups are independent of the amount of details in the generating unit models that is whether I model a generator by classical model or I model using the detailed model considering the amortisseurs field winding, dynamics excitation system, turbine right. The coherency behaviour is same and therefore what can we do, we can use actually for coherency identification as simplified model classical model why should I go for all detail model right and therefore therefore the point is that one can use a simplified model that is therefore a classical synchronous machine model is considered and excitation system in turbine models are ignored right.

Therefore with these two important assumptions right but these assumptions have been verified by doing number of studies okay and the the assumptions are valid to a large extent right okay therefore what we will study here today is that how to identify identify the coherent group of generators making use of these basic assumptions that is we will represent each machine by classical model and we will use a linearized model to obtain the swing okay.

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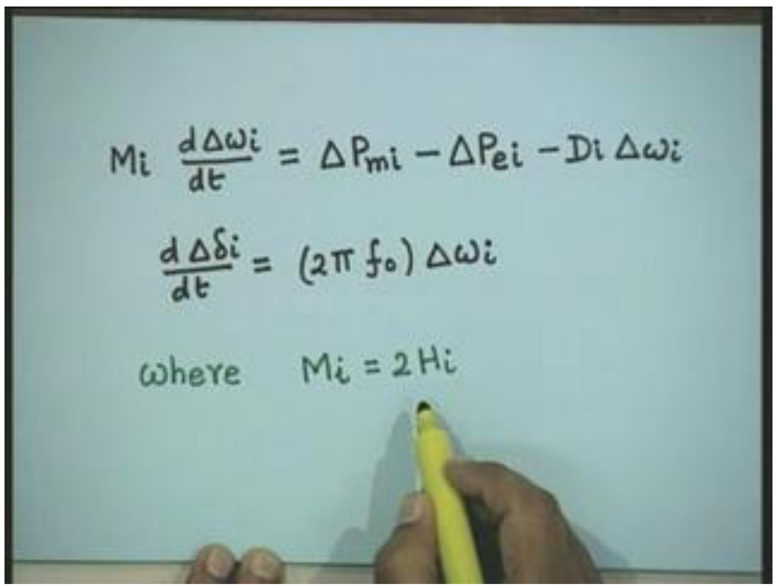

Therefore now if I write down the swing equation for a i'th machine, for an i'th machine considering small perturbation okay this equation is written as M_i d by dt of delta omega i equal to P_{mi} delta P_{mi} minus delta P_{ei} minus D_i delta omega i okay. Now this term is to take care of damping which is present in the system that is D_i delta omega i is a damping term. The second equation is d by dt of delta delta i that is 2phi f o omega 0 into delta omega i. Now here M_i is equal to 2 H_i, 2 H_i and the speed deviation that is delta omega i right they are represented in per unit, okay that is the this is all what you know it.

The next step will be that we write down the the model of the network now here small perturbation model for the network can be represented like this, I can because here you know we will have number of buses okay and we will classify the buses into two categories one is internal bus of the generator internal bus and load buses. Here, we will not resort to any any you can say elimination of nodes nodes will remain where they are okay therefore you can write down the equation here delta P_g delta P_L delta Q_g and delta Q_L as delta P_g divided delta delta delta P_g divided by delta theta I will explain delta and theta delta P_g by delta E, delta P_g divided delta V.

Second term will be delta P_L by delta delta delta P_L by delta theta delta P_L by delta E, delta P_L divided by delta V. The third row is delta Q_g divided by delta delta delta Q_g by delta theta delta Q_g divided delta E and delta Q_g divided by delta V, last will be delta Q_L divided by delta delta delta Q_L divided by delta theta, delta Q_L divided delta E delta Q_L divided by delta V okay, multiply by a vector called correction vector that is delta delta, delta theta, delta E and delta V that is for any network any network right we can write down the equation of this form that this is basically the Jacobian of the network, this is a Jacobian of the network.

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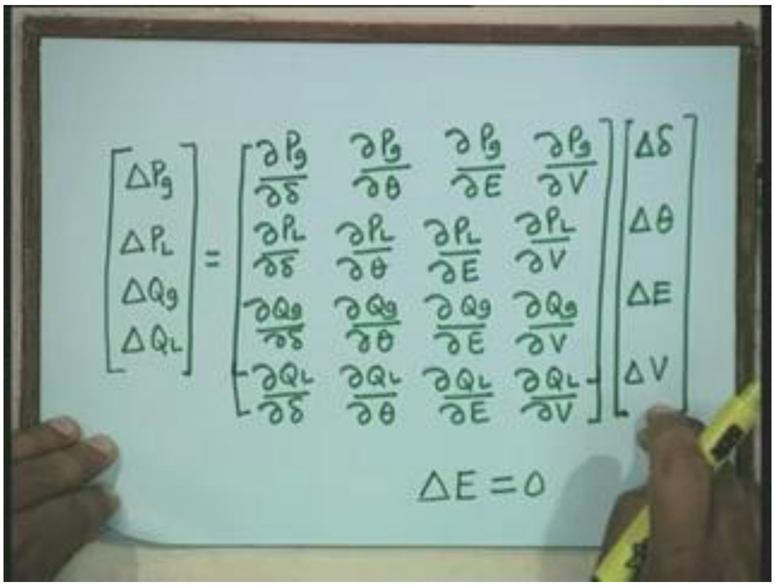

Now here delta because here we will talk about the two types of voltages, voltages these are the internal voltage of the generator are representing by generator by classical model internal voltage of generator I will denote this internal voltage by the symbol E right and this this E is a vector which will have the terms E_1 E_2 up to E_n depending upon the number of the generators V is the voltage of the load buses right delta is the phase angle of internal voltage and theta is the phase angle of voltage of the load buses right delta P_g, delta P_g is the generator injected power okay similarly delta P_L and delta Q_L these are the load bus residuals and delta Q_g is similarly the deviation in generator power okay.

Now here in this model model once we represent once we represent the generated by classical model okay and what about this delta E, if you represent the generator by a classical model then the internal voltage of the generator is constant magnitude of the internal voltage is constant and therefore for classical model delta E is is 0 okay, second point what we do here in this case is that the affect of variation of load bus voltages on the delta P_g, delta P_g and delta P_L is negligible that is the power transfers mainly depend upon actually the angle delta that is we know actually this decoupling there exist a decoupling right and therefore you can simplify this model and we can represent the the delta P_g and delta P_L in terms of these two quantities that is when we simplify this when delta E is anyway 0 and this these generations these these quantities hardly depend upon delta V okay.

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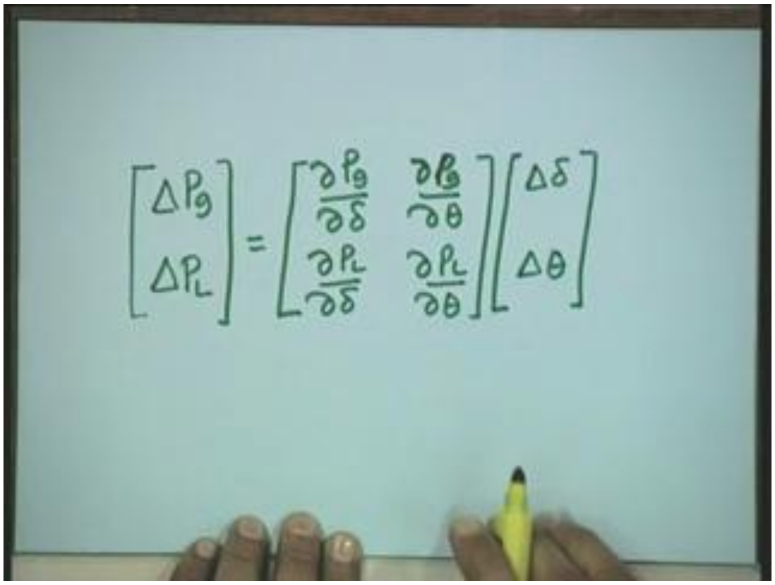

Therefore with this assumption or assuming this decoupling right we can I have a simplified model a simplified model can be written here as delta P_g and delta P_L as as delta P_g divided by delta delta delta P_g divided by delta theta, delta P_L divided by delta delta and delta P_L divided by delta theta that is we will use in our study this simplified model. Now here I wanted to emphasize that this delta P_L that is the the the we call actually the load bus residual power right this delta P_L is also 0 in normal case situations. We will use this term delta P_L to simulate certain type of disturbance to simulate certain I will just explain to simulate certain type of disturbance we will represent this the this term delta P_L will come into picture okay.

Now in order to represent this disturbances because we will come across the different type of disturbances right and we can represent these disturbances in a very simplified manner. Let us first consider actually the a fault occurs in the system and how do we represent the fault. Again since, we are trying to do some simplifications let us say that how do we represent a fault and that is depend the system during fault condition let us say.

Now if you see very very carefully then during fault condition during fault condition since the duration of fault is small may be 2 cycles, 3 cycles, 4 cycles okay the normal fault duration which the system can withstand is of the order few cycles okay. Now during this period the change in delta is small that is when you look on the system as a whole right when the fault occurs the system is subjected to acceleration okay and the delta the the the rotor angles changes but during this period of about .1 second is a short duration of within fault condition the variation in delta is not very large do you agree with this.

If you just try to visualize the criteria of stability right then during the fault condition if you see the see the change in delta it is there but it is not very large further since delta is very has varied over as small range or small variation is there therefore the acceleration rotor accelerations at the insertion of fault and the rotor acceleration at the fault clearing time that is the the accelerating power at the inception of fault and at the instant of clearing the fault there is hardly any significant difference, you appreciate this point that is let us say that I am just taking single machine infinite bus system okay. Let us say when the fault occurs that before the occurrence of fault the mechanical input say .8 per unit.

The moment the fault occurs the electrical output becomes from .8 to let us say .3 okay therefore the accelerating power at the inception of the fault is .5 per unit is .8 minus .3 this is the accelerating power now at the instant of clearing the fault at the instant of clearing the fault right mechanical input is same let us say electrical output instead of being .3 let us it is .31 because they are moving on the during the fault power angle characteristic. Therefore the acceleration power what accelerating power is .8 minus .31 that is say .49.

Therefore, we can make an assumption that okay during this fault the acceleration power remains at the level of 3.5 and will not create any view big errors small error will be there but still and therefore this situation can be simulated by increasing the mechanical input by .5 for that particular period, my mechanical input was initially say .8 I increase in my model mechanical input by that much amount okay and then the system behaviour of dynamics is not going to be affected it remains same therefore in this case I have I have not to I have not to consider obtain the the mathematical model during the fault condition I have to simply increase the mechanical input by that much amount right this is the one way or simply you can say simulating the fault condition right.

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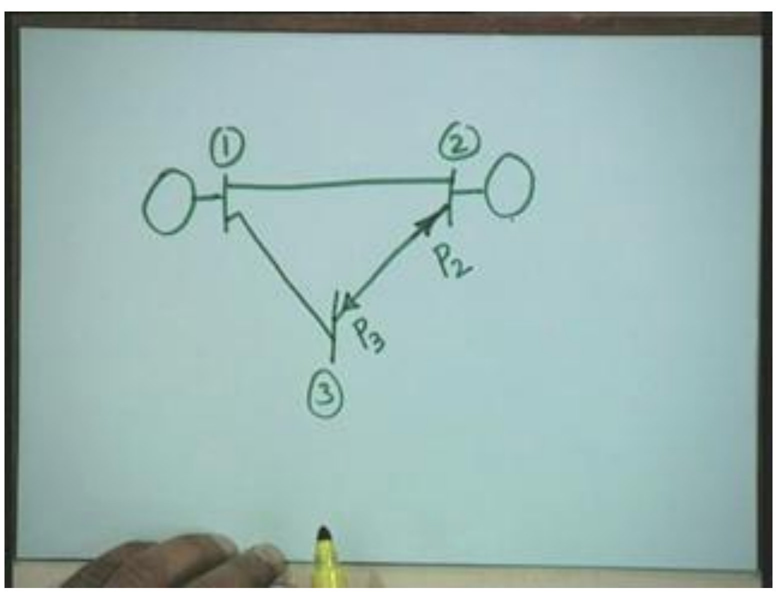

Now suppose you have a situation where where I am just taking an example, simple example. Let us say that you have a system a simple system with 2 generators and 3 buses like this. Let us call this is bus number 1, bus 2 and say bus 3, a simple system. Now suppose I want to consider a disturbance where this line connecting bus 2 and 3 is tripped the line line connecting bus 2 and 3 is tripped one simple way is that okay, I obtained actually the model model right considering this post fault or post tripping situation another way of simulating will be simulating will be that okay you look before the occurrence of fault what was the power which was flowing on this line.

Let us say I am just showing here that after all I am putting opposite air holes. Let us say the power at this point is entering this bus I call as P_2 and power entering at this bus is called P_3 because of this line connection, for example if suppose you consider a lossless line right if P_2 is say 50 megawatt P_3 will be minus 50 megawatt okay then this this particular situation can be represented that is at this bus I inject a power equal to P_3 I inject a power equal to P_2 at this bus therefore this two, these two this condition and this condition they are same okay. Now I can simulate I can simulate the tripping of the line 2, 3 by setting P_3 equal to 0 and P_2 equal to 0 okay.

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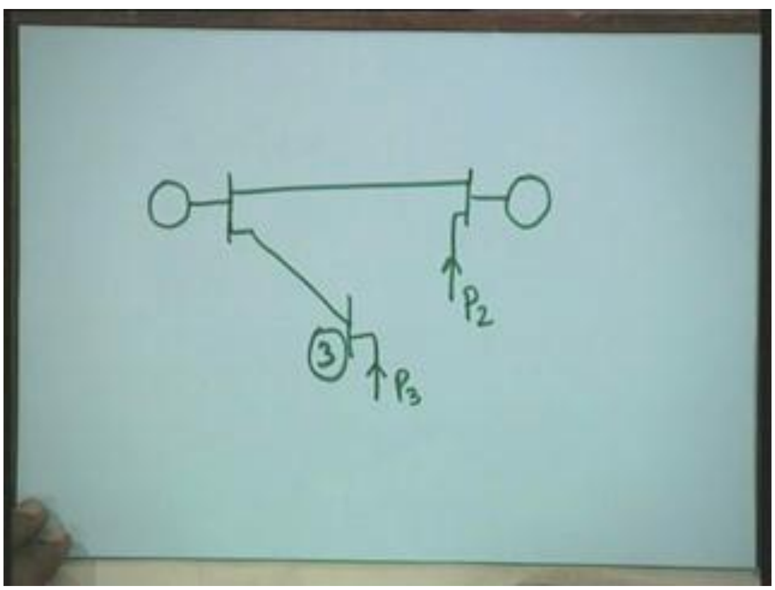

Now once I say that P three equal to 0 means what is the change in the load power it has changed from P_3 to 0 therefore delta P_L will represent actually this this disturbance here that is at this bus number 3 whatsoever is the change in the power right that I put in my model delta P_L delta P_L equal to so much. Similarly, for P_2 I represent actually as change in delta P_L because this delta P_L which I am trying to say is a vector vector where all the load buses are considered therefore here in this particular case this two entries right I will make P_2 and P_3 to represent the line tripping right therefore when you are trying to take a particular fault condition or line tripping condition or even you you consider actually the generator tripping.

You have not to change your model the network model remains same and by just you can say changing the values of delta P_L or delta Q_g, I am sorry Q_a P_g you can change the condition of the or they simulate the condition of right. Now the question basically is that how do we obtain the how do you obtain the swing curve considering this type of situation.

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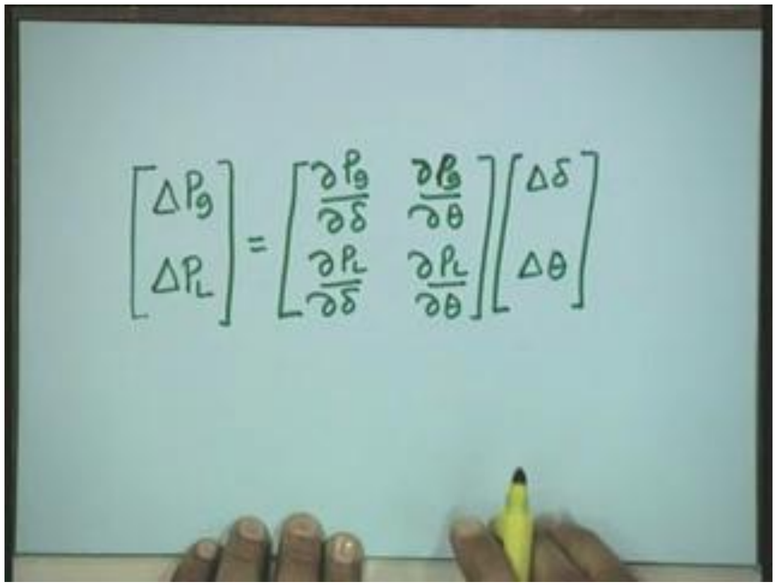

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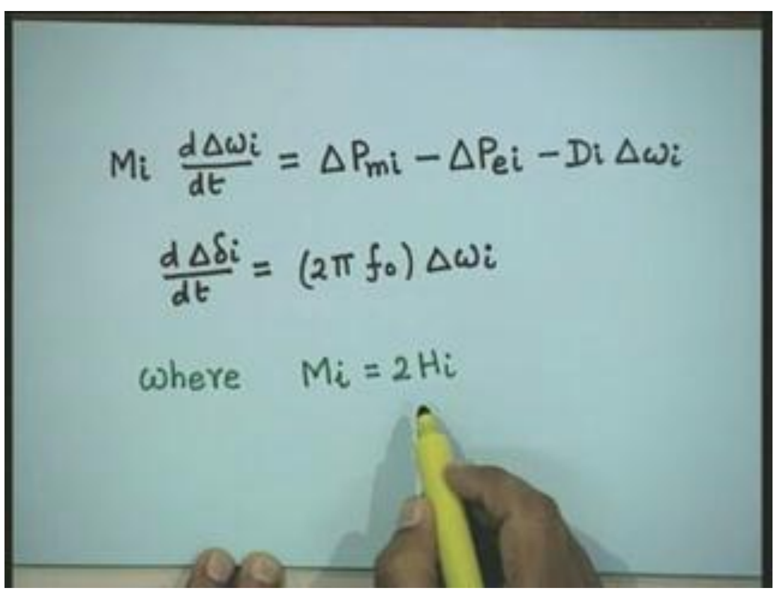

Now here easily see that in this model this delta P_L and can be is is basically to simulate a particular type of disturbance okay. Now to solve this problem again start with this these two set of equations, first is our linearized differential equation, second is the network model okay with this linearized differential network model what we do is that you apply the trapezoidal rule of integration, to convert this linear differential equation into an algebraic equation okay. I will write down this step that is I have the equation of the form which I will integrate both the sides okay.

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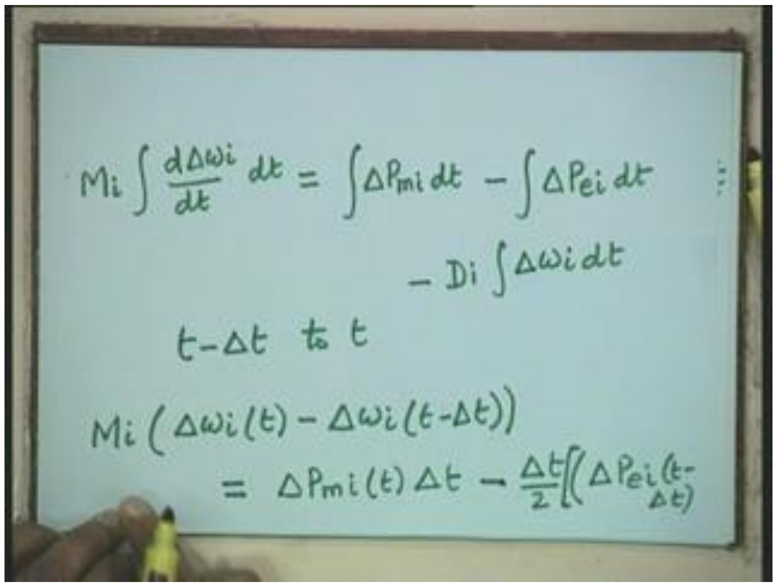

I will have this equation M_i integral d omega i by dt dt this is equal to equal to we have term delta P_{mi} dt minus integral delta P_{ei} dt minus D_i is constant integral integral delta omega i okay. We will integrate this equation over a time interval delta t okay, over time integral delta t applying the trapezoidal rule of integration.

Now following the conventional given in the reference paper I will start like this that when you integrate over the time interval t minus delta t to t_1 can take t to t plus delta t but I am following the same convention which is given in the reference paper okay t minus delta t to t. Let us write down the value of this integral it will be M_i into delta omega i omega i, t minus delta omega i t minus delta t okay so far this term is concerned and I integrate this over this time integral I will get delta omega i t minus delta omega i, t minus delta t.

Now here this term delta P_{mi} this delta P_{mi} is the change in mechanical power okay. Now we use this this term to simulate the type of fault suppose you are considering a fault and you are trying to simulate the fault by increasing the mechanical power by some amount which will be equal to the the accelerating power at the instant of occurrence of fault therefore delta P_{mi} will remain constant will remain constant therefore, what will the integral of this delta P_{mi}.

Of course it is a constant I am saying as a function of t why I am saying the function of t because because when the fault is cleared this value has to be changed when the fault is cleared okay then post fault system is going to be different therefore I am putting this a function of t okay but it will remain whatsoever is there it is stable on particular time this into delta okay minus what will be the integral of this by applying the trapezoidal rule of integration delta t by 2 which is the multiplied by delta t by 2 multiplied by delta P_{ei} P_{ei} t minus delta t t minus delta t plus delta P_{ei} t okay.

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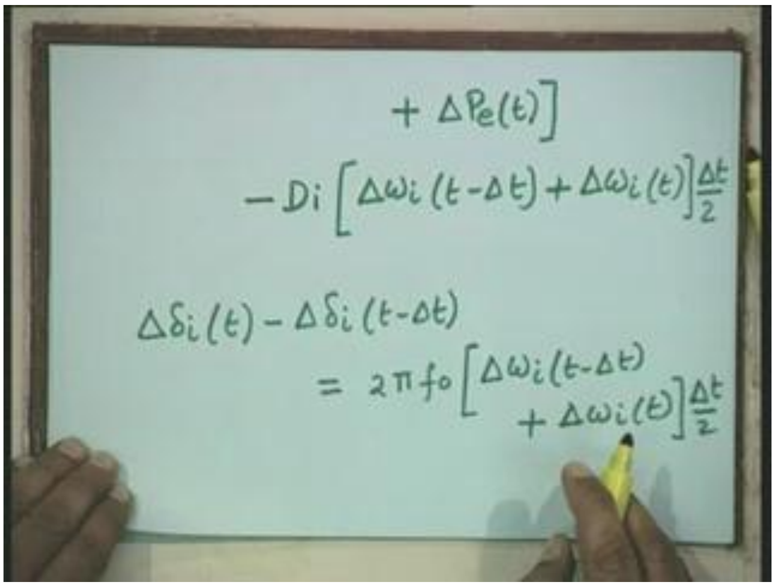

Here you can use this right then one more term is there minus D_i delta omega i, t minus delta t plus delta omega I t multiplied by delta t by 2. Okay that is the first differential equation is integrated over a time interval t to the minus t minus delta t to t and we get actually in the equation applying the trapezoidal rule of integration, this is algebraic equation.

Similarly, you take the second equation when we integrate the second equation, we will get delta delta i t minus delta delta i t minus delta t equal to 2 phi f_0 that remains same you will have the term delta omega i t minus delta t plus delta omega i t into delta t by 2, after this the both these equations are now converted into algebraic equation.

Now these equations along with the the network equations will have to be solved we will do this thing in the next turn afterwards let me sum up what we have studied is we have studied the basic concepts of dynamic equivalents and we have also studied that the dynamic equivalents can be obtained using a linear model and considering a classical representation of synchronous generator okay. Thank you!