Power System Dynamics

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Lecture - 26

Numerical Integration Methods for Solving a Set of Ordinary Non-linear Differential Equations

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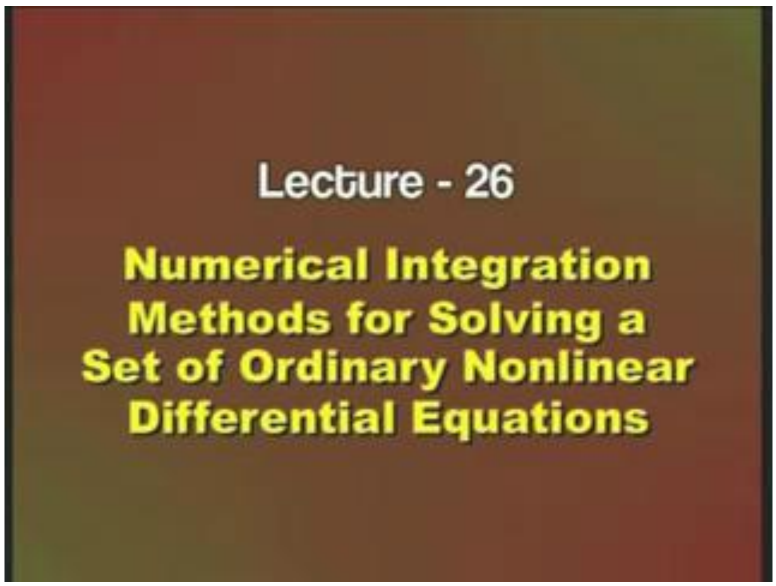

Friends, today we will study the numerical integration methods for solving a set of ordinary non-linear differential equations.

We have developed the mathematical model of the system for analyzing the stability we have developed the mathematical model for synchronous generator, excitation systems turbines and governors and also loads, okay. Now when we want to analyze the transient stability of the system we need to solve a set of non-linear ordinary differential equations along with a set of non-linear algebraic equations and we can say that we have to solve the differential algebraic equations for solving the algebraic equations we have standard techniques available like say technique Newton-Raphson technique and so on.

Now for solving this non-linear ordinary differential equations we have to resort to numerical techniques because there is no analytical solution available for non-linear differential equations. Now the numerical techniques will provide you approximate solution, I am emphasizing this word approximate solution. Here, when I say that we apply numerical technique for solving this problem what we do is that the time duration for which we want to get the solution is divided into small time segments.

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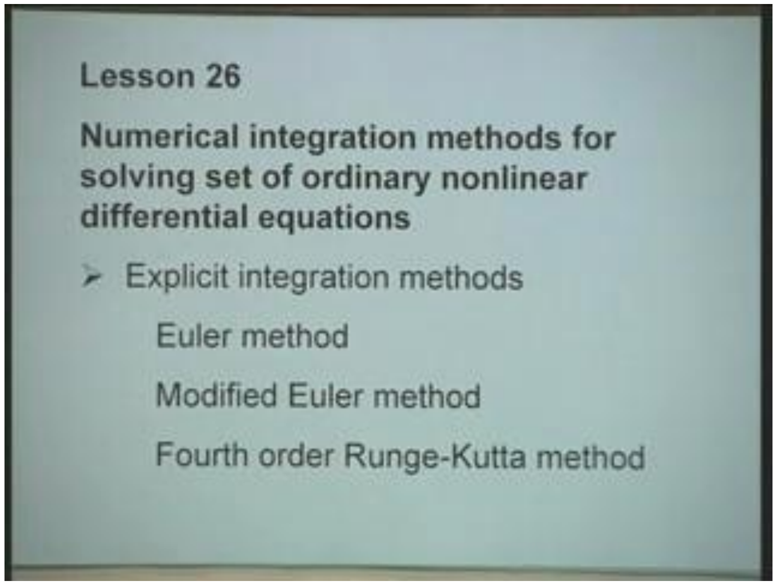

We can call this time segments say staring from initial time t equal to t_0 to t_1, t_2, t_3, t_4 and say t_n and these segments may be equidistant or of the equal length or may be unequal length that is the time step from initial value t_o to t_1 and from t_1 to t_2, these 2 time steps or ah so on may be equal or unequal. Now today we will study these numerical techniques and broadly these numerical techniques are divided into 2 categories known as the explicit integration methods for solving a set of ordinary differential equations, we use the what explicit integration methods and another technique is known as the implicit integration methods.

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We will be in a position to understand what is the difference between the implicit and explicit integration methods and their merits and demerits? Right, as we will discuss the different techniques. We will under the explicit integration methods we will discuss briefly the Euler method, modified Euler method and 4th order Runge-Kutta method. Although number of other techniques are available in the literature right but however we will confine our discussion to these 3 techniques which come under the category of explicit integration methods. We shall discuss 1 simplest technique under the heading of implicit integration method that is called trapezoidal rule that is trapezoidal rule of integration, okay.

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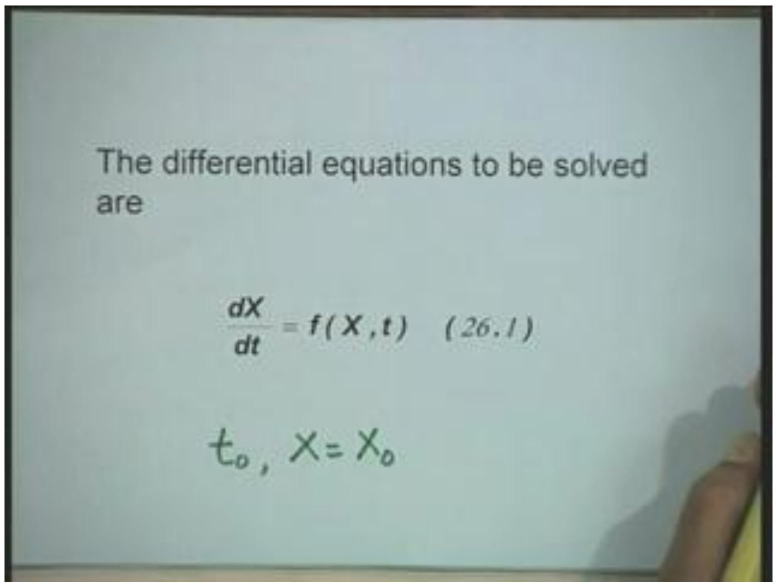

Now to understand the solution of ordinary non-linear differential equation equation. We start with start with the a general differential equation, general non-linear differential equation of the form dx by dt equal to f of x_t where, x is a vector x is a vector it will have its dimension depending upon the size of the problem which we have to solve. For example, in the classical stability analysis the x will comprise of the angles that is the rotor angles delta's and rotor speeds like say delta 1, delta 2, delta n, if it is a n machine system and the speeds like say omega 1, omega 2 up to omega n this will be the state variables.

Then we will come across a set of differential equations which are put in the compact form as dx by dt equal to f of x_t. For solving these equations, we know the initial condition that is as time t equal to 0, time t equal to 0, x is equal to x_0 that is the initial conditions are known to us and in knowing the initial condition we have to find out find out how the x varies as a function of time that is what is the meaning of solving this differential equation that is we want to get the solution for x as function of time.

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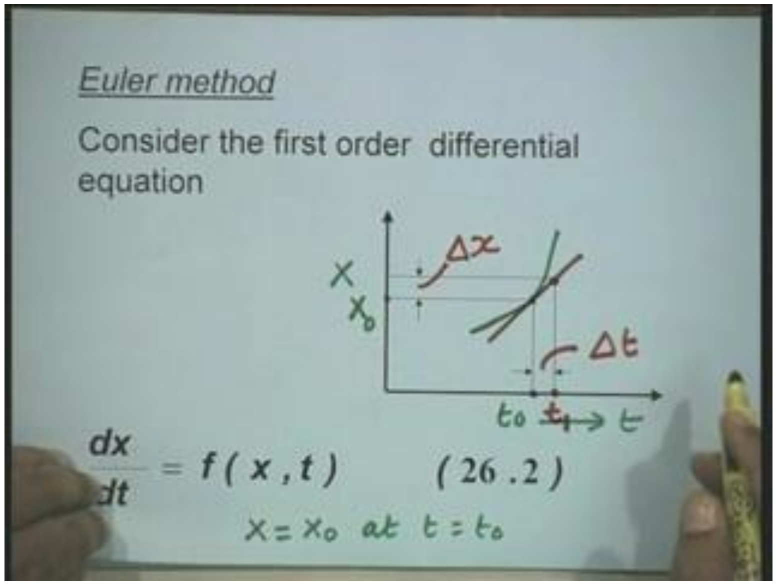

The Euler method is the simplest of all the methods and let us consider considers scalar system that is we have only 1 unknown to be solved to understand the Euler technique we will start first with the scalar differential equation that is dx by dt equal to f of x_t where x is a scalar right. Now to solve this what we know initially is that x equal to x_0 at t equal to t_0 this is the initial condition which is given to us for example, when you solve our swing equation right at time t equal to 0 we know what is the value of delta that is delta equal to delta 0 at t equal to t_0 right, this is the initial condition.

Now to understand how we obtain the solution of this let us assume that the solution for x as a function of time is known to us I will say that this is the exact solution of this that is x varies as a function of time by this very non-linear way. This is the initial point this is the this point is t equal to 0 and corresponding to this the x is equal to x_0 this is the initial operating condition, okay what we do is we we find the derivative of this function that is this function that is obtain the derivative at x equal to x_0 it means after basically finding out slope of this curve at x equal to x_0 and let us say that this line represents the slope at x equal to x_0 right.

Now let us take a small time step and call this time instant as t_1 where, this time step can be denoted by delta t that is t_1 minus t_o equal to delta t okay. Now if this you take a small value of this step right then value of x at t equal to t_1 can be represented by this equation that is first we will find out first we find out what is the changed delta x that is this quantity delta x that is at t equal to t_1 right we find out the value of x that is x is equal to x naught plus delta x, this delta x in the change change when you increase the time by a small step delta t right.

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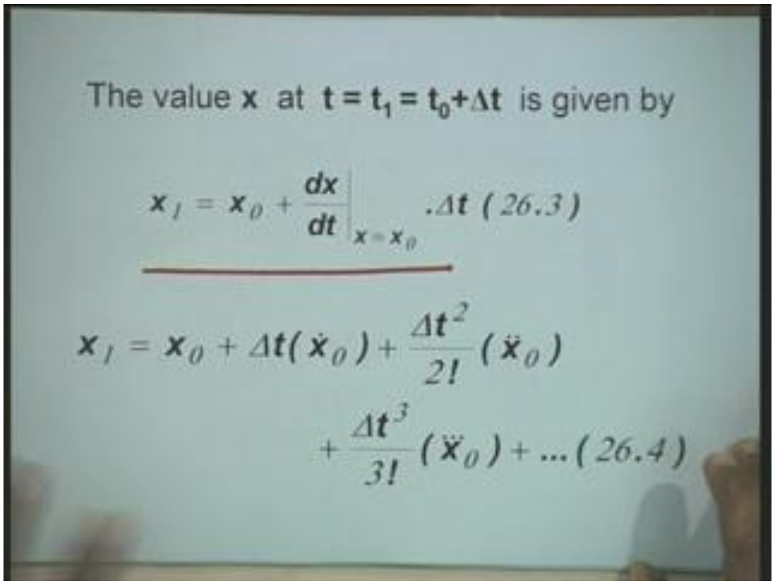

So that now we can write the solution in the form that is x_1 equal to x_0 the initial value plus dx by dt at x equal to x_0 into delta t right. This is this is what is the approach we make use of in the Euler method that is you find out the slope of the curve at t equal to t_0 that is x equal to x_0 right and then find out find out actually the delta x that increment in x when t is changed by time step delta t.

Now this increment is determined by this expression that is dx by dt at x equal to x_0 into delta t. Now here here here this Euler method can be considered to be considered to be the first order method that is suppose you expand this function if you expand this function around initial value x_0 using Taylor series expansion then the Taylor series expansion will give you x_1 equal to x_0 plus delta t into x_0 dot plus delta t square by factorial 2 x_0 double dot and so on that is if we take any function and expand at the initial operation condition using Taylor series expansion then x_1 that is the new value at time t equal to t_1 can be obtained as by this series that is here effectively we are truncating this Taylor series by by considering only the first order derivative okay these higher order terms we are neglecting and therefore obviously obviously the the the accuracy of solution will depend upon delta t, if I take delta t very small very small then naturally the x_1 will come out to be very close to the actual value right.

Now here when we use this Euler's method ,we assume we assume that the slope of this curve is same all through the interval while if you see the actual solution right then during this interval the slope is different at different points right and therefore therefore there is some error that will be incurred actually when you solve using the Euler's method. For example, if you get this point that is x equal to x_1 I will call it as x_1 corresponding to t_1 now if you find want to find out the next step next step that is you want to find x_2 using x_1 right.

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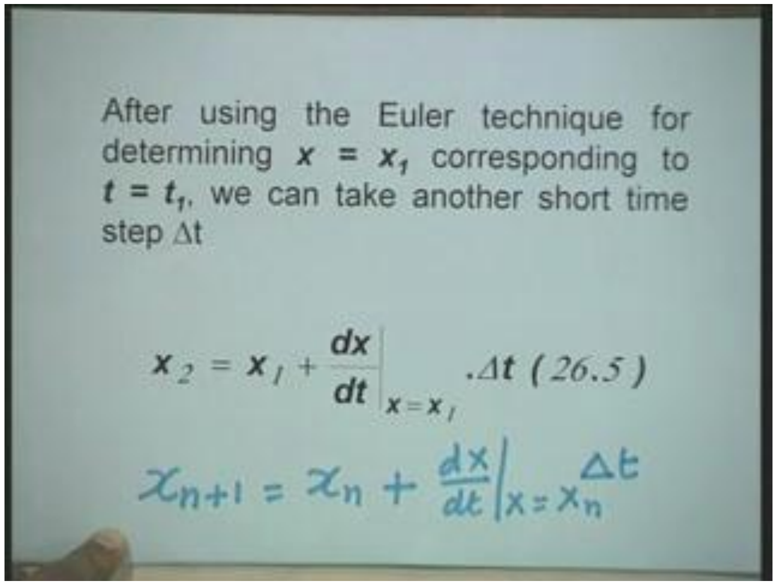

Now what is going to be our approach x_2 will be obtained as x_1 plus dx by dt evaluated at x equal to x_1 into delta t okay. The next step will be that you obtain x_2 as x_1 that is value at t equal to t_1 that is they have becomes the initial value find the derivative at x equal to x_1 okay and multiplied by the time step delta t okay. Now graphically when you see here let us say that the new slope comes out to be like this right then the the new value of x that is at x at t_2 will be obtained by this point, let us say this point this is your t_2 and this is your x_2, this is the point which we get.

Similarly, if you keep on going you will find actually that the the solution which are getting by this technique is is deviating from the actual solution and this solution is going to be very close to the actual solution if you take delta t very very small, this is a very important requirement. Now here this method is called explicit method because the new value of x at anytime is obtained in using the previous value of x that is that is the when you are solving this problem, solving this problem right we find out the derivative this derivative is explicitly computed using the known value of x okay and that is why it is called explicit integration method here in this method there is a problem that in case you take delta t large right the error which is created will propagate and the solution may blow off right and we call this as numerical instability right.

Therefore therefore numerical instability is a problem problem when we use explicit techniques for solving non-linear differential equations therefore in general you can write in algorithms using this Euler method as x_n plus 1 is equal to x_n plus dx by dt, x evaluated at x_n that is you evaluate this derivative at x equal to x_n and this is to be multiplied by time step delta t therefore this is the algorithm a simple algorithm for evaluating the the or for obtaining the solution of first order differential equation.

Now in case we have instead of scalar equation we have a set of differential equations right then the same procedure is adopted because these these equations are coupled these equations are coupled right and therefore when I write the solution this x will be replaced by vector that is you will write x, I can write down here.

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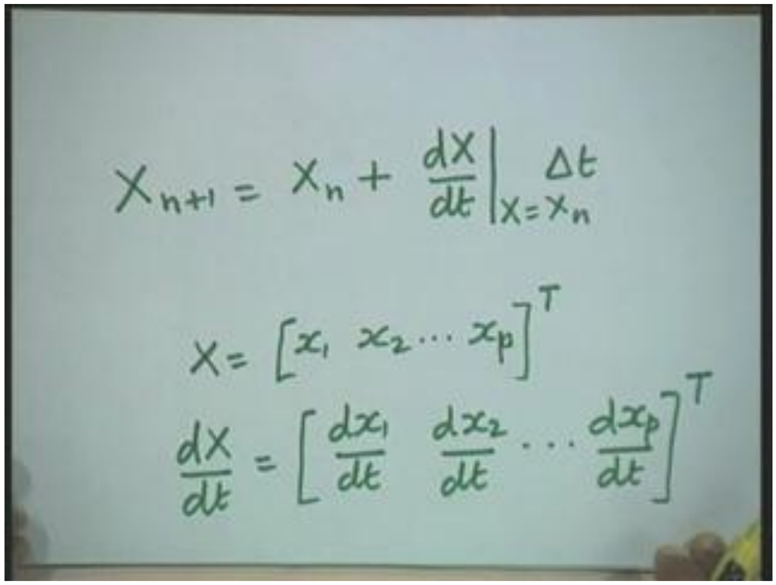

The algorithm will now becomes x_n plus 1 equal to xn plus dx by dt evaluated at x equal to x_n into delta t this is the algorithm when you solve a set of differential equations. Now when I say dx by dt, x is a vector what is the what is the meaning of this now suppose I say here x is equal to x_1 x_2 let us say I will use the word say x_p, we do not want to mix up with this x_n okay let us say they are p components that dx by dt the meaning will be here dx by dt this is transpose here. Okay x is a column vector dx by dt can be written as dx_1 by dt, dx_2 by dt d_{xp} by dt whole transpose it means when I say the derivative of this x means I am taking the derivative of each of the variable dx_1 by dt, dx_2 by dt, d_{xp} this is the way this is to be obtained because when you write down the set of differential equations right, first order differential equations on left hand side we have the derivative terms delta 1 dot delta 2 dot delta omega 1 dot delta omega 2 dot like that.

Okay for our stability analysis right therefore they are all derivative terms this I individually evaluated and put in the top now when I say the dx_1 by dt is function of all the state variables and initial condition right. Now in order to overcome the the the problem of numerical instability this Euler method has been modified modified to reduce the error which will accumulate and propagate right.

So that we can use a reasonably large value of delta t the time step because if you use very very small delta t then to get the solution the number of steps will become very large and it will take more time for getting the solution because at each step you have to do the similar type of calculations.

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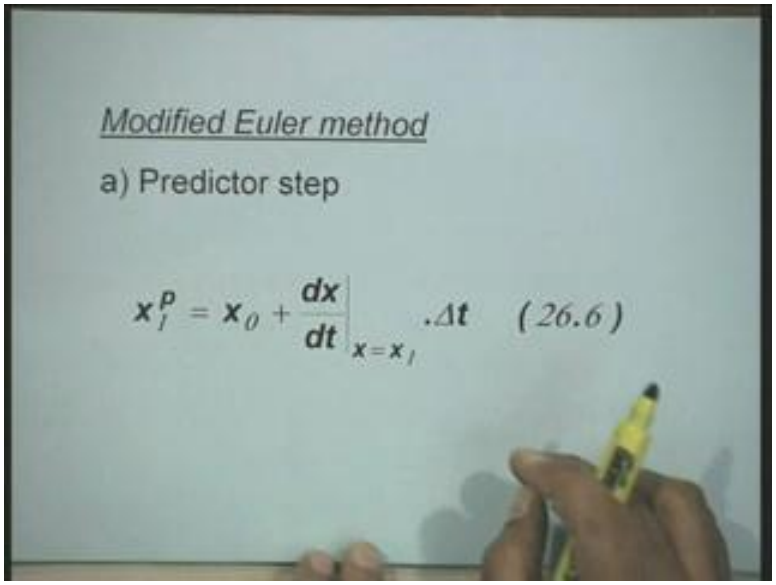

In the modified Euler method the approach goes like this the first step is exactly the same as we use in the Euler method that is we find out the value of x_1 we call this is a predicted value x_{1p} that is the predictor step. The first step this step is called predictor step that is x_{1p} is called to x naught plus dx by dt evaluated at x equal to x_0 into delta t that is you evaluate x_{1p} you call this as the predicted value initial value x_0 plus derivative at x equal to x_0 into delta t.

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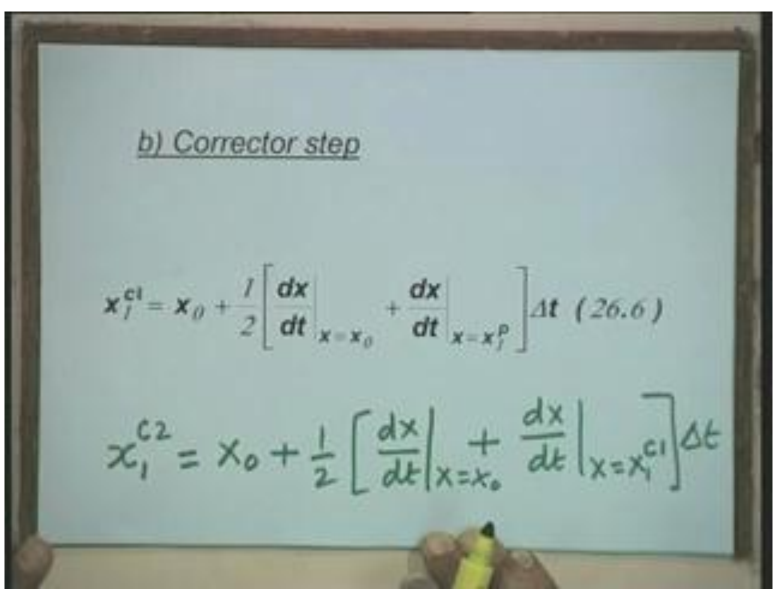

Okay now, using this predicted value this predicted value x_1 is the value at t equal to t_1 is it okay t_1 therefore using this predicted value we will find out the derivative of this function at t equal to t_1 okay and using these 2 derivatives on predicted value. We will find out the corrected value of x_1 okay therefore the approach the algorithm will go like this that x_{1c} is called corrector step x_{1c} is equal to x_0 plus 1 by 2 dx by dt at x equal to x_0 that is what we have already d_1 find the derivative at x equal to x_0 then you find out the derivative of this function right dx by dt at x equal to x_{1p} okay and then take the average of these 2 derivatives where this is the slope of the solution or to the function at t equal to t naught or x equal to x_0 okay this is the slope at x equal to x_1 predicted or we can call it this is the slope at t equal to t_1 okay.

We take the average of these 2 slopes and obtain the a value of x_1 and this is going to be naturally more accurate as compared to the 1 which you obtained by considering only this term because now we are considering the average of the 2 slopes right. Now here 1 need not to stop here you can use this corrected value that is x_{1c} right consider this as a predicted value and further you can find out the more accurate value of x_1 that is what can be d_1 is that you can call this as x_1 I will call this x_1^{c2}, I will just now suppose I use actually 1 second more the second step then I will call this as this equal to x_0 plus 1 by 2. This initial derivative will remain same that is dx by dt at x equal to x_0 plus we will calculate this dx by dt at x equal to x_{1c}, c_1 I will call this as c_1 in that case if I go for 1 more step I will call there this is c_1 use this x equal to x_{1c} okay and then multiply by delta t.

Now in case actually these 2 values are very close you can stop here suppose when you find out the corrected value in the first step then you will find out the corrected value in second step also if suppose these 2 corrected values come out to be very close or 2 conjugative corrected values come out to be very close you can stop here. Now generally generally this second step may not be required you can use a small time step and so that you can stop at the first correction step itself right but if you want to you can say go for further, you know further you can same a sophisticated value or accurate value you can use this steps you know instead of terminating at first corrected value you can go for second, third so and so and since this is can this has to be programmed therefore is nothing very big actually so far uh computations are concerned.

We can always develop a program we will do it because this type of modified Euler method is used very commonly because this is very simple a very simple and more accurate also and the the we can use a slightly larger time step and the problem of numerical instability also is less probability. Okay the next step which is or the next method which is commonly used for solving set of non-linear differential equations is known as the Runge-Kutta method Runge-Kutta method.

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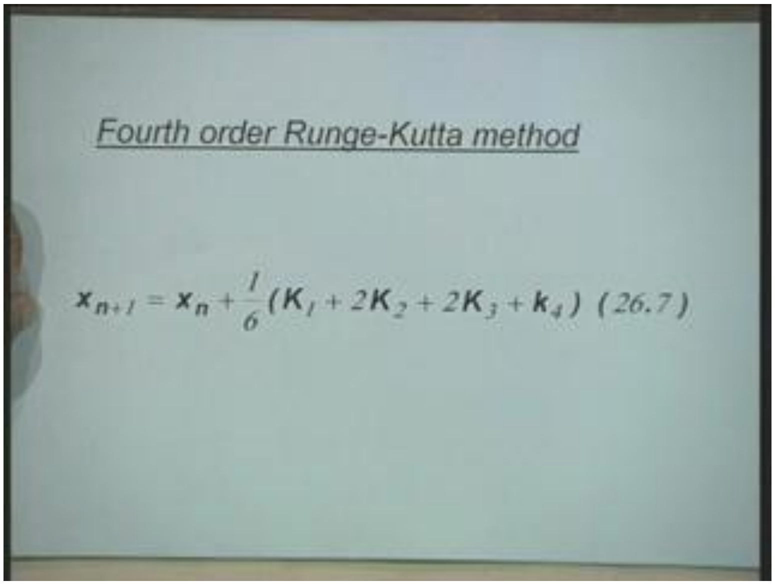

Here I will discuss only the 4th order Runge-Kutta method this is more popularly used because in the literature the second order Runge-Kutta method, 4th order Runge-Kutta method these are discussed right .Now here as we have seen actually that the Euler method is a first order method where we are looking in terms of the Taylor series expansion as if we are we are terminating at first derivative term. Here this 4th order Runge-Kutta method is as if you are using up to the 4th derivative terms actually you will not be evaluating the 4th derivative term 4 derivative terms but it it mimix the Taylor series expansion as if you have taken the the terms up to the 4th derivative.

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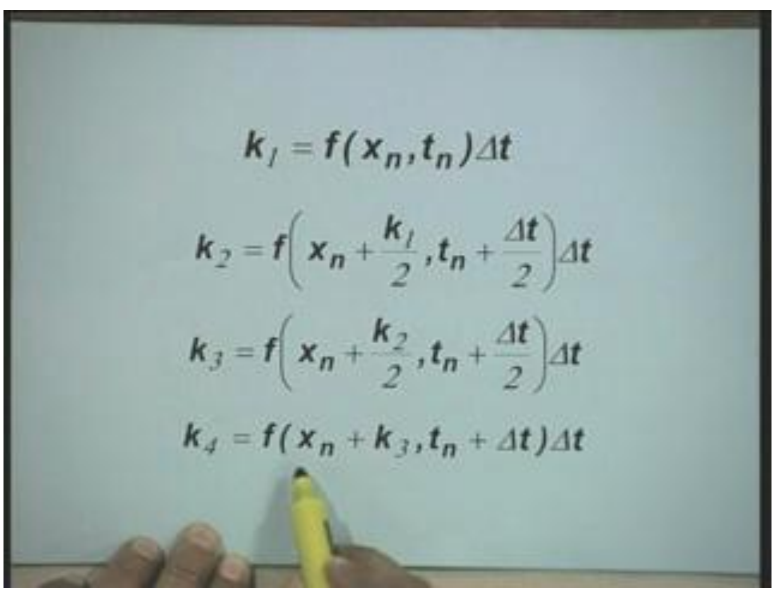

Now the algorithm for evaluating the uh value of x at n plus 1th step is x_n that is the value of the variable at previous step x_n plus 1 by 6 times this term that is x_1 k_1 plus 2 times k_2 plus 2 times k_3 plus k_4 I will explain this k terms that is here what is d_1 is you obtain the value of x_n plus 1 in terms of the previous value plus this term so this is basically delta x okay now these terms k_1 2 times k_2, 2 times k_3 and k_4 are defined as follows here k_1, k_1, k_1 is the f of x_n t_n into delta t that is that is you evaluate this derivative term that is f of x_n t_n at t equal to t.

Suppose that we are starting at t equal to t_0 then this will become f of x_0 t_0 if it is going next step it is becoming f of x_1 t_1 and so on therefore this is by this is a way and this is then multiplied by delta t. Okay then we calculate k_2 k_2 as f of x_n plus k_1 by 2 x_n plus k_1 by 2 and this means actually that the value of this term x_n plus k_1 by 2 is at t_n plus delta t by 2 that is you know in all all the differential equations we come across right the time is is not explicitly available in the equation that is the time is implicit function right.

Suppose if I solve this the swing equation right in my swing equation on the right hand side when I evaluated this function it is function of delta and so on the time is not explicitly visible but it is implicit because delta is corresponding to certain time right therefore here when I say that this is the value of x right it means it is the at the beginning of the interval x_n plus this quantity can be considered to be at the mid of the interval.

So that I can say interpret this total term as at the mid of the and the time corresponding to this will be t_n plus delta t by 2, okay this again you multiplied this by delta t. So that this becomes a increment in x, k_3 we make use of this k_2 and obtain this term derivate that is f of obtain this function x_n same x_n remains same plus k_2 by 2 t_n plus delta t by 2. It means again the this this increment is computed in the last 1 we obtain the function at x_n plus k_3 full not half full that is t_n plus delta t okay then what is d_1 is to obtain the increment delta x we we use this 4 terms and we the waiting given to this k_1 is 1, k_2 we multiplied by 2, k_3 multiplied by 2 and k_4 by 1 and then this is added and divided by 6.

So that it becomes a weighted average right, this is the formula you can see here x_n plus 1 is equal to x_n plus 1 by 6, k_1 plus 2 times k_3 plus 2 times k, I am sorry 2 times k_2 plus 2 times k_3 plus k_4 by 6 it is basically these are all small small increments in delta x and the the increment in a small step delta t is obtained by taking the weighted sum and dividing by 6 that is the weighted average you can call it and here as you can easily see here actually that we have not computed computed the second order derivative or third order derivative or 4th order derivative but this process process gives you the solution, similar to what you will you get if you include up to the 4th order derivatives in the Taylor series expression now I shall illustrate the application of this techniques by taking an example after discussing these techniques.

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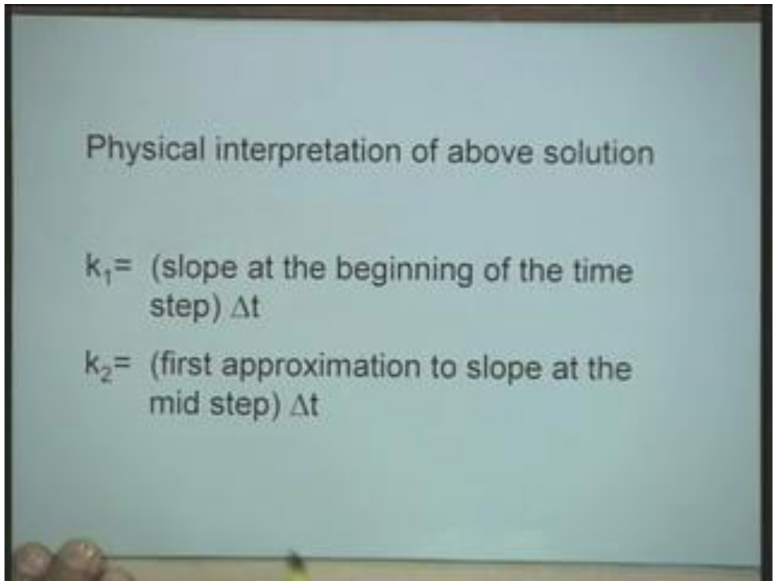

When you physically interpret this 4 constants then k_1 can be considered to be slope at the beginning of the time uh step that is the slope at the beginning of the time step into delta t then k_2 can be interpreted as first approximation to slope at the mid step is a first approximation, first approximation to this to slope at the mid step into delta t therefore we are multiplying slope by delta slope with delta t.

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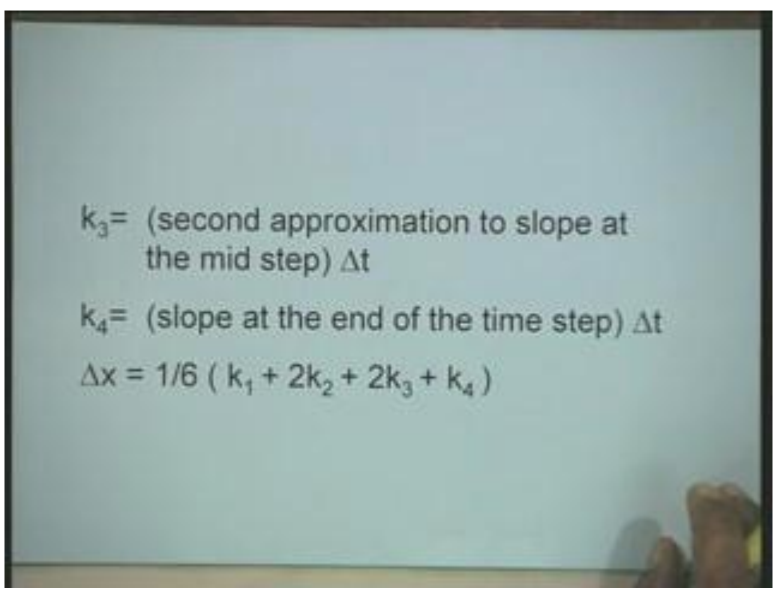

So that this becomes a increment in x okay k_3 will be interpreted as second approximation to slope at the mid step into delta t and finally k_4 the slope at the end of the time step delta t right and then increment delta x is equal to 1 by 6 of k_1 plus 2 times

k_2 plus 2 time k_3 plus k_4.This is what is the 4th order Runge-Kutta method and simpler form of this is called second order Runge-Kutta method right where where actually they use less number of terms. Now I will just discuss something about the numerical stability of explicit integration methods.

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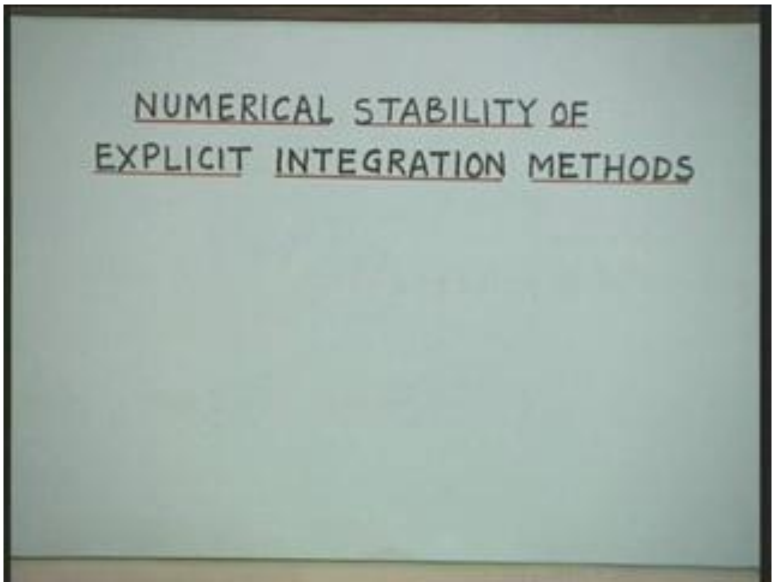

When we discuss this numerical instability the meaning is the meaning is that the the errors propagate and accumulate and solution will blow off and the although the actual solution is possible to get the correct solution but by this numerical technique we are getting a solution which is which is incorrect and that is what is normally called actually the numerical instability.

Now numerical instability can be avoided by choosing the appropriate value of time step. Now in any system when you solve any system when you solve right the system has number of time constants number of time constant. For example, when you consider our uh stability problem then you will have the time constants which will be very short of the order of you can say fraction of second right to few seconds.

Suppose actually I take the the excitation system right then if I take the if I consider the time constants associated with the with the sensing and transducer circuit the voltage sensing and transducer circuit it is of the order of 0.02 second is very small, if I take the open circuit time constant of the field circuit it is of the order of 5 seconds, 10 seconds and so on.

Similarly, if you take actually the times constant the mechanical starting time constant that is t_m one of that is 2H okay that is also a very large because H is of the order of 5 6. So that t_m becomes around 10 and therefore the we define what is called a term steepness of the system which relates the ratio of largest time constant to the smallest time constant.

Suppose largest time constant is 10 second and smallest time constant is say .02 seconds then this ratio determines represents a steepness of the system right. The steepness is defined in terms of another term which is called actually the largest value of the largest Eigen value of the linear system to the smallest Eigen value that is you take a system find out the Eigen value and find out the magnitudes largest Eigen value divided by the smallest Eigen value this will give you the measure of steepness of the system and for the system to be for the the that a solution to be numerically ah stable right the the time step should be related to the smallest time constant right and here we discuss one more important technique where the numerical stability is not a problem, okay and that technique is known as the implicit integration method.

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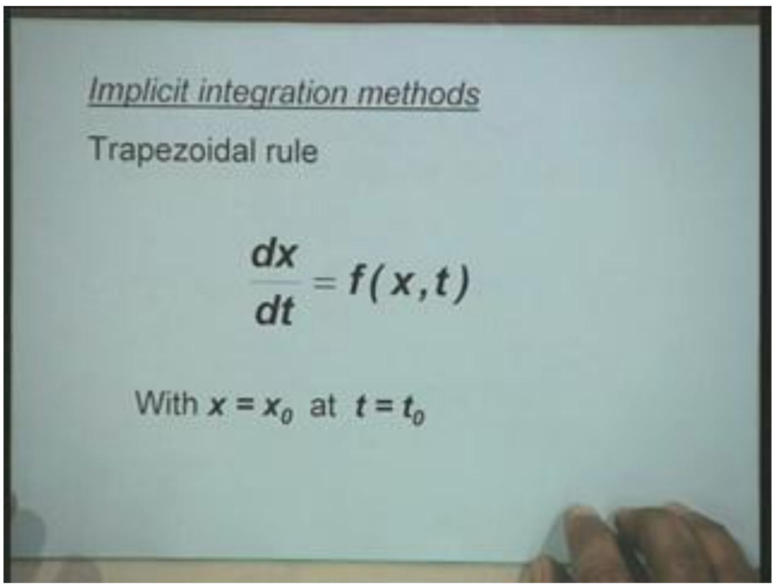

We will first understand conceptually what do you mean by implicit integration method it is a very simple approach. Again we start with the differential equation dx by dt equal to f of x_t x equal to x_0 and t equal to let us say that this is a scalar equation to start with let us consider a scalar equation now I want to solve this equation how do I solve this equation let us say I want to solve this equation the 1 simplest approach would be you integrate both sides.

Suppose I want to get the solution of this equation dx by dt which is a function of x_t you integrate this equation respect to time and I want to integrate this over a time step say t_0 to t_1 dx by dt. This can be written written as integral of f of x_t dt t_o to t_1. Now what is the suppose you integrate and obtain actually this value what is the value of this delta x or if I call this as x_1 minus x_0 that is I am integrating dx by dt with respect to t, so that it becomes a integration of dx only.

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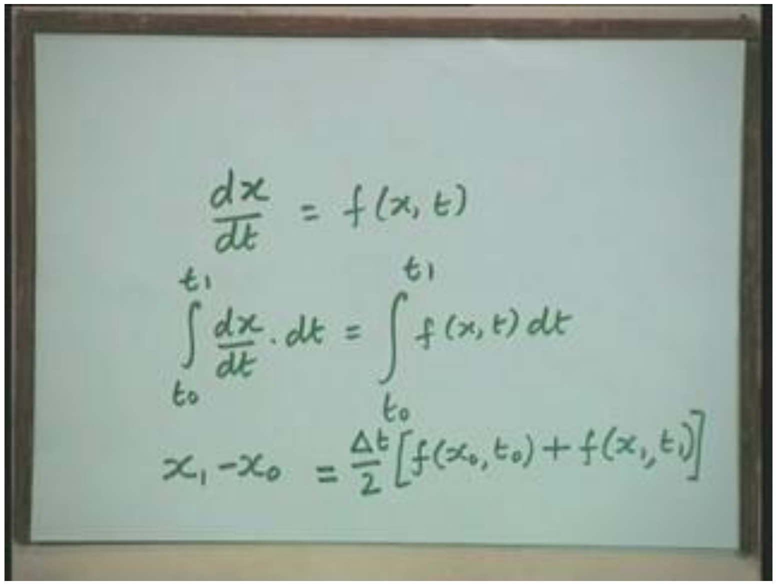

So that you can say that at t equal to t_0 its value is x_0 t equal to t_1 its value is x_1 what is the integral of this function what is the value of this integral therefore, we can say that this is this is area under this curve from t_0 t_1 okay. Now now I can because this area I cannot it find I cannot find it out because I not I do not know how this this function is varying and it is a non-linear function I do not know therefore what we can do is that suppose I take this situation.

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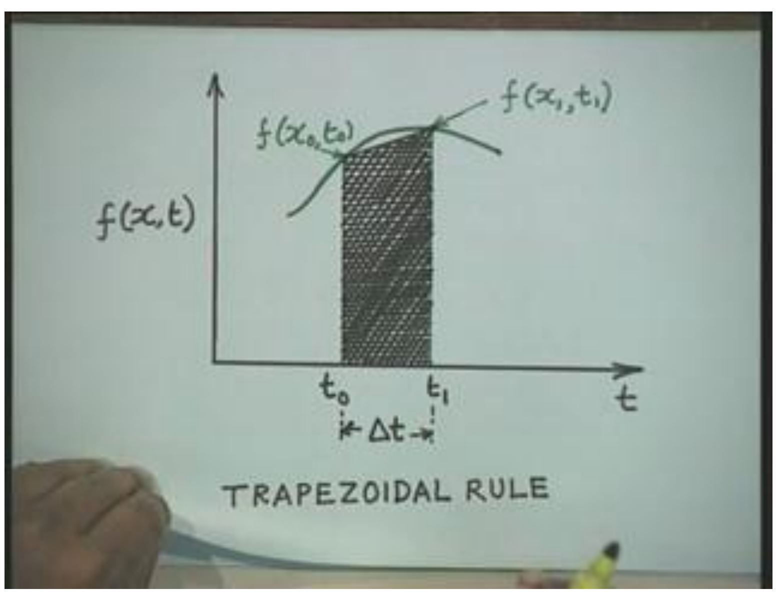

If I plot the function f of x_t as if function of time as I plot this function f of x_t is a function of time and let us assume that this is the this is the actual solution okay. Now as you very clearly told actually that if I am trying to evaluate this integral then it is the area under this curve from t naught to t_1 okay. Now this area under this curve can be approximated by the area of this trapezoid only error which will cause is small error that is you can approximate the area under this curve from t_0 to t_1 the small time step delta t as as f of x_0 t_0 plus f of x_1 t_1 multiplied by delta t by 2 is it not this is very important step actually people sometimes actually fail to appreciate this step which is a very simple step this is the most important step again you can see here actually that area actual area under this curve and the area which is approximated using the area obtained by trapezoid right there is a slight difference and this difference will can be reduced, if I take delta t as small as possible you reduce this delta t this difference will also vanish okay.

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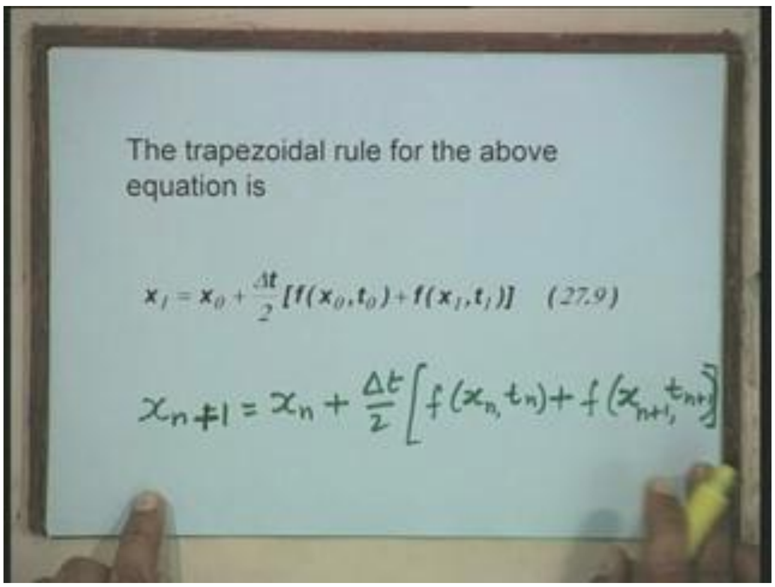

Therefore, I can say now the x_1 can be written as from that equation as because here x_1 minus x_0 let put x_0 on this side you will get x_1 equal to x_0 plus delta t by 2f of x_0 t_0 plus f of x_1 t_1 therefore this is the this is the value of x_1 obtained obtained and next at time t equal to t_1 but the problem here is that this x_1 is expressed in terms x_1 itself. Here, we see you see this step in this step what is happened is that x_1 I have expressed in terms of x_1 itself but this equation is now an algebraic equation this is an this is a non-linear algebraic equation.

Okay and therefore this non-linear algebraic equation can be solved to obtain the value of x_1 using standard equation techniques standard techniques like Newton-Raphson techniques which are used for solving the non-linear algebraic equations because this is a the the algorithm can be written as x_n will be equal x_n plus 1 let us say x_n plus 1 will be equal to x_n plus delta t by 2 f of x_n t_n plus f of x_n plus 1, t of n plus 1. This is the algorithm but you I think you appreciate that you have to solve this algebraic equation, non-linear algerice algebraic equation using again numerical technique.

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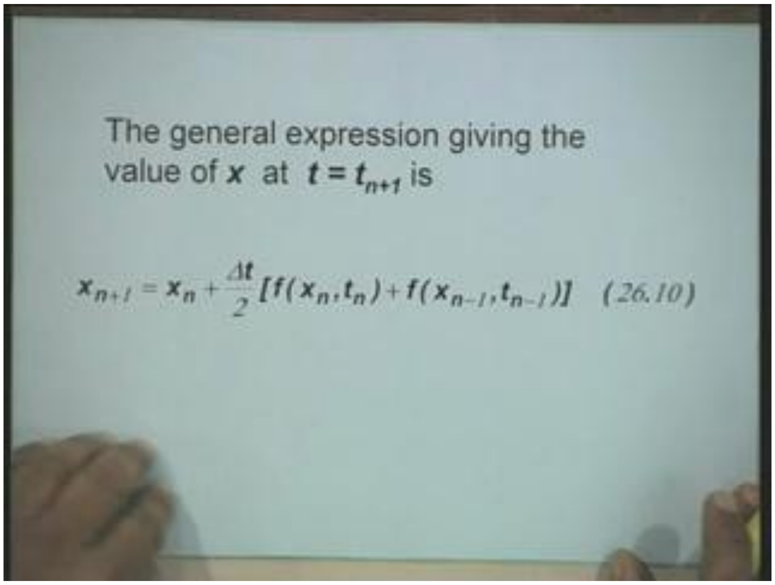

This is what actually I have just written here. Now to illustrate actually this trapezoidal rule of integration, let me take the transient stability problem of a machine connected to infinite bus we will take a classical problem.

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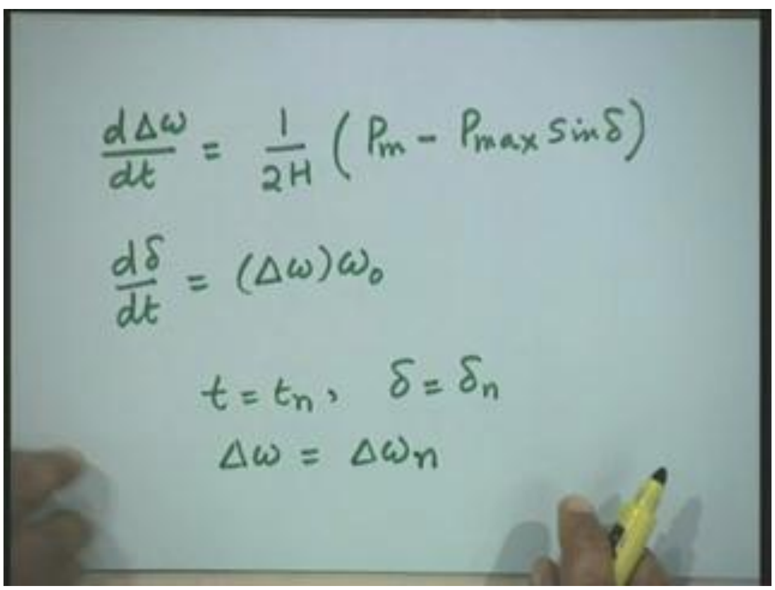

The differential equations which we have to solve can be written as d delta omega dt equal to 1 over 2H P_m minus P_{max} sin theta, second differential equation is d delta by dt equal to delta omega into omega naught. These are the 2 first order ordinary non-linear differential equation.

Now these 2 equations are coupled equations that is you will find actually that in this equation delta term comes right. While in this equation omega term comes delta omega terms come therefore in general you will find that if you take any problem for a stability analysis you will get a set of ordinary differential equations ordinary non-linear differential equations but they are all coupled nonlinear differential equations okay. Now let us say that how we apply the trapezoidal rule of integration and what equations you will get okay

Let us assume actually we know that at t equal to t_n let at t equal to t_n delta equal to delta n and delta omega equal to delta omega n okay that is at time t equal to t_n we know the value of delta delta n and omega as omega n okay. We write down now the 2 first order 2 non-linear algebraic equations by applying the trapezoidal rule of integration that is I consider the first equation first equation I am to obtain this term delta omega n plus 1 this will be equal to delta omega n plus delta t by 2 here I have to write down here f of x_n t_n f of x_n t_n.

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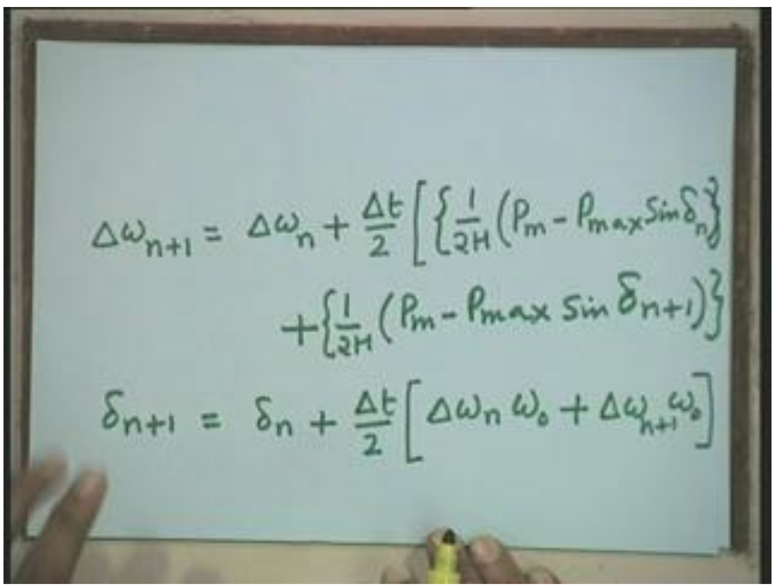

Now in this expression in this expression I will write down here 1 by 2H I am not substituting the values 1 by 2H P_m minus P_{max} sin delta delta n correct. You can close this bracket here plus 1, 1 by 2H P_m remains same P_{max} sin delta n plus 1, you close the bracket here and this is the equation to obtain the value of delta omega n plus 1 in terms of delta omega n and in terms of delta n plus 1 because now delta n plus 1 is coming here.

The second equation we write down here will be delta n plus 1 equal to delta n plus delta t by 2 multiplied by f of x_n t_n. Now in this case it is delta omega n into omega naught plus delta omega n plus 1 into omega naught. Okay now these are the 2 non-linear couple algebraic equations, okay and initial values of these quantities are known delta omega n and delta n initial values are known. Okay and you use the simple simplest technique like technique to obtain the value of delta omega n plus 1 and delta n plus 1 at t equal to n plus 1 the numerical stability of these techniques can be illustrated considering a set of linear differential equations.

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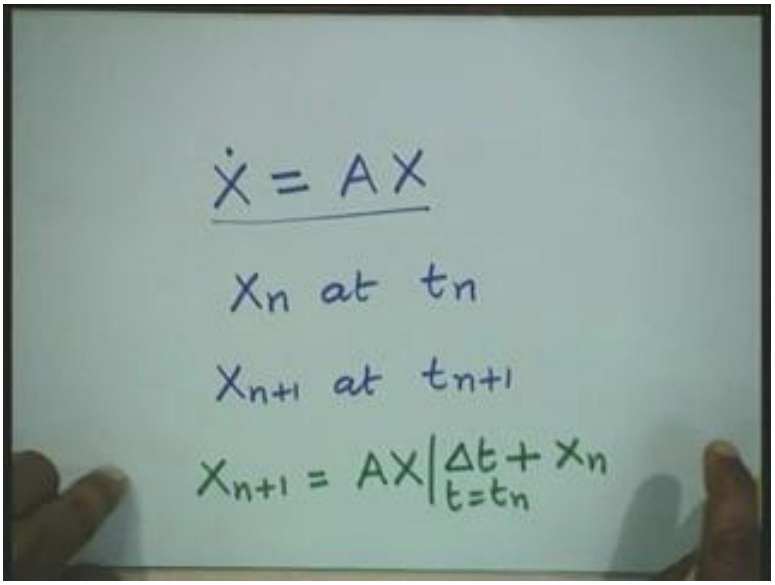

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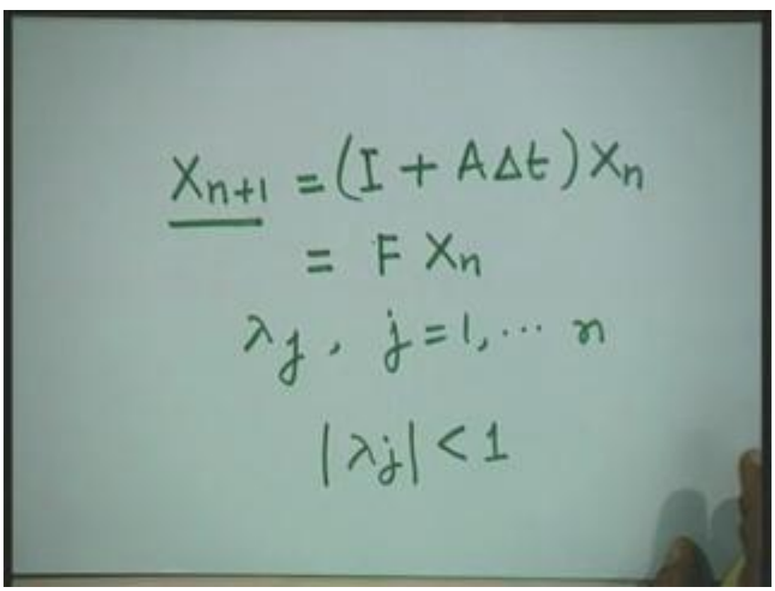

Let us examine these equations written by this in the matrix form X dot equal to A_X. The value of X is known say X_n is known at time t equal to t_n we want to obtain the X_n plus 1 at time t_n plus1. Now this represents the set of linear differential equations and applying the uh Euler's method we can write down X_n plus 1 equal to A_x evaluated at t equal to t_n plus X_n. Now we can write down this equation in the form X_n plus 1 equal to I plus A delta t into X_n which can be written as which can be written as F function F into X_n.

Now this represents this represents the discrete form of this differential equation X dot equal to A_X, now for the system to be stable the Eigen values of this matrix A matrix F not A, matrix F which is equal to A_I plus A delta t must be must lie in a in a unit circle that is the Eigen values, all the Eigen values that is lambda j for j from 1 to n must lie in a unit circle that is magnitude lambda j must be less than 1.

Now here we can see that the the matrix F is function of the time step delta t as the time step delta t increases the the Eigen values of this matrix F may lie outside the unit circle and the and the system response becomes unstable. Now here a small correction when we write down x_n plus 1 equal to A_x t equal to t_n this should be multiplied with delta t. Now with this I conclude my today's presentation by saying that we have discussed the various numerical techniques which are suitable for solving a set of ordinary non-linear differential equations. Thank you!