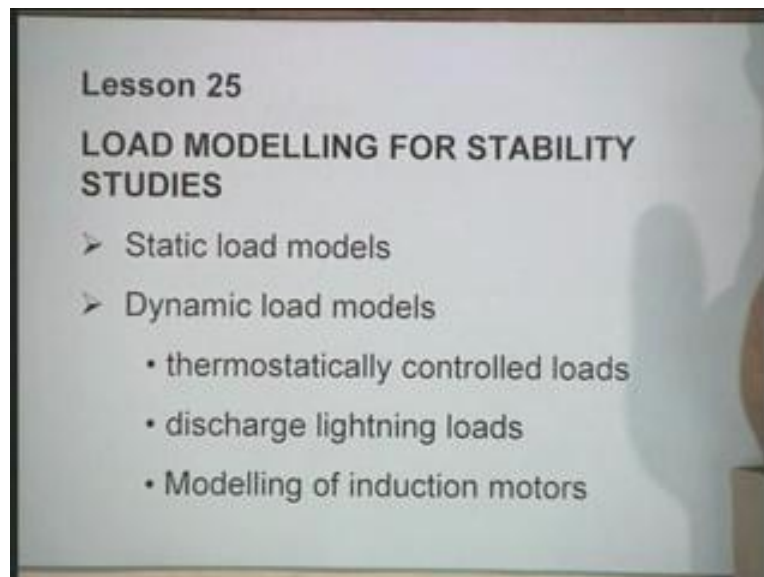


**Power System Dynamics**  
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**Lecture - 25**  
**Load Modelling for Stability Studies**

Friends, so far we have studied the modelling of synchronous generator and steam turbines, steam turbine governor's, hydro turbines and hydro turbine governors. We have also studied the modeling of excitation systems. Now this completes the modelling on the on the generator side right. We have also assumed actually that for power system stability studies the transmission line network is modelled under steady state conditions.

Now we have to model 1 very important component that is the loads in the system. The modelling of loads has has important influence on the stability of the system. Further the modelling of load is the most complicated process, the reason being that whenever we whenever we try to model the load in the power system stability studies right we identified some load buses and at that load bus we have variety of loads which are supplied. The load supplied at the lighting loads, heating loads, cooling loads, motor loads, furnaces and so on right and all these loads have different characteristics.

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Now in our classical stability studies, we represent load at a bus by constant impedance load. However, the studies have shown actually that this representation may give some erroneous results and therefore there has been, there has been over the years efforts being put to develop

develop the models of the system. Now the problem is because the loads are of different variety and this composition of load at a particular bus is also not fixed it varies from time to time is a function of time it is also a function of season, weather conditions and so on and therefore whenever we develop the load models we do a lot of simplifications right. Now today in our study we will look into the modelling aspect of the loads for power system stability studies.

Now traditionally for the purpose of modelling or for the purpose of representing loads in stability studies, the load models are classified into 2 categories, 1 is called static load models another is dynamic load models. The dynamic load models are normally required or essentially required for long term stability study, voltage stability and when you want to study the inter area oscillations right. Now today we will briefly discuss how the loads are represented as the static loads and dynamic loads.

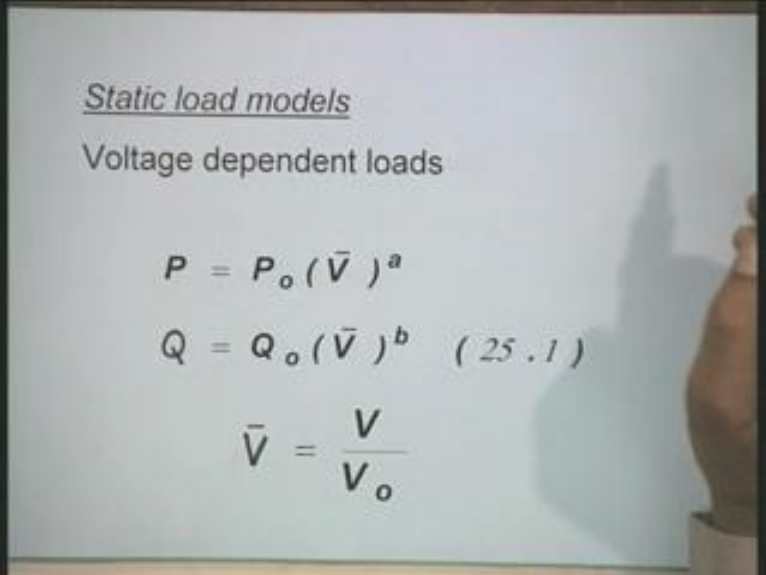
When I talk about the dynamic loads, we have a number of loads which are thermostatically controlled loads when I say thermostatically controlled loads for example most of the heating loads are thermostatically controlled loads right, the temperature is controlled and the thermostat is on and off. Similarly, there are number of examples like refrigerators, coolers right and so on. Then we have another a very big component in our load that is the discharge lighting loads where you have the fluorescent tubes, we have the sodium vapour, sodium vapour lamps and mercury vapour lamps.

Now these are all under the category of a discharge lighting loads. The third most important component in the dynamically dynamic loads is the induction motor loads in any system about 60 to 70 percent load, 60 to 70 percent load is due to motors and the modelling of these motors is a very important and lot of attention has been given to modelling these large size induction of motors for stability studies right because in power plants or actually in some big industries we have large size induction motors and their characteristic affect the system stability as a whole right.

Now first we will address the modelling of static loads when I talk about the modelling of static loads right then we represent the load at any time a function of voltage magnitude and frequency right. Now here the load will be expressed in terms of voltage and frequency using algebraic expressions because it's not a dynamic load right therefore the equations which are used to represent static loads will be algebraic equations okay and since the loads are function of the magnitude of the system voltage and system frequency we will represent these loads as using algebraic expressions in terms of voltage magnitude and system frequency.

Now for representing these load models we will consider, we consider the real power and the active power components of the load separately because the behaviour of the reactive power consumed by a load and the real power consumed by the load following variation in voltage and frequency they are somewhat different. Now let us see what different load models have been used in the literature. A simplest load model considering the voltage variation is shown in this form that is the  $P$  is the real power consumed by a load at any voltage  $V$  equal to  $P_0$  into  $V^a$  where  $a$  is an index  $a$  is some index right.

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Static load models

Voltage dependent loads

$$P = P_o (\bar{V})^a$$
$$Q = Q_o (\bar{V})^b \quad (25.1)$$
$$\bar{V} = \frac{V}{V_o}$$

Therefore we can say where this  $\bar{V}$  is defined as  $V$  divided by  $V_o$ , where  $V_o$  is the nominal voltage at which the power consumed is  $P_o$  right and therefore this  $\bar{V}$  is a fraction of the actual voltage and the nominal voltage or we can say that this  $\bar{V}$  is a per unit voltage expressed in terms of  $V_o$  okay. Therefore  $P$  can be expressed as the nominal value of the real power consumed  $P_o$  multiplied by this  $\bar{V}$  to the power  $a$ , then similarly the reactive power  $Q$  is represented as  $Q_o$  into  $\bar{V}$  to the power  $b$ ,  $\bar{V}$  to the power  $b$ ,  $a$  and  $b$  are the 2 different indices.

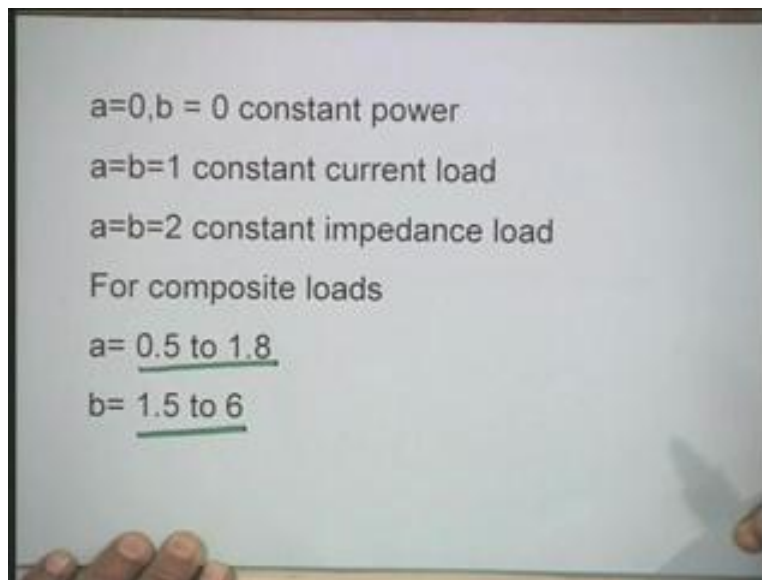
Now here suppose if I take  $a$  and  $b$  equal to 0 both let us say that these coefficients are 0 what does it mean  $P$  equal to  $P_o$  it means the load is independent of voltage right. Now suppose I take  $a$  equal to 1 and  $b$  equal to 1 if I take these coefficients  $a$  equal to 1,  $b$  equal to 1 what does it represent yes therefore here the meaning is that the load draws constant current, constant current right and therefore the power is proportional to the voltage, power  $P$  and  $Q$  which are consumed they are proportional to the voltage.

Now if I take these indexes  $a$  and  $b$ , indices  $a$  and  $b$  equal to 2 that shows that the power consumed is proportional to  $V$  square and that is the characteristic of a constant impedance load. For example in our transient stability studies classical transient stability studies which we discussed we were representing the loads by constant impedance at that time the index becomes 2. Therefore, we have by putting the different values of  $a$  and the  $b$  we can have different representation of loads whether the load is a constant power load, constant current load or constant impedance load.

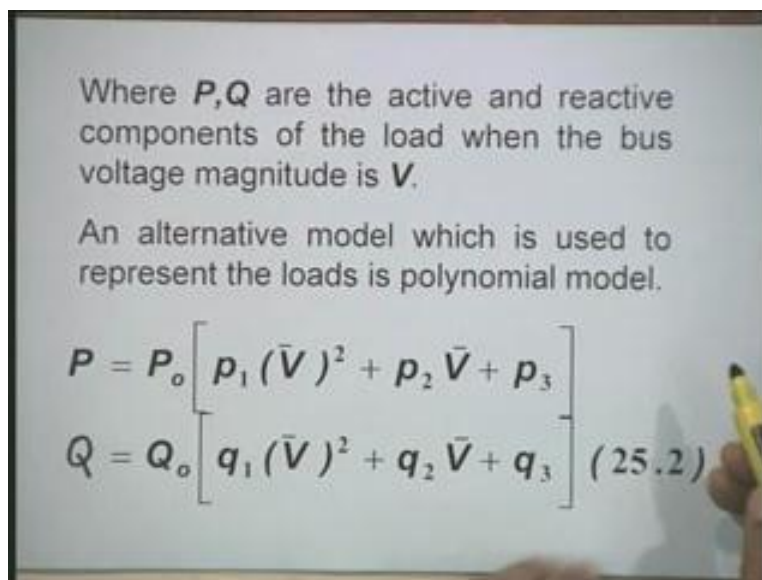
Now in practice the load behave different loads behave in a different way, different devices. I just take the example of say **incandance** incandescent lamp okay. Now this lamp does not consume reactive power right and the power consumed by this will be proportion to  $V$

square,  $V$  square although it is not very linear for it is for a so small variation it is proportional to  $V$  square and therefore this load will behave like a constant impedance load. While there are other loads they have different characteristics and in any at any bus, at any bus we have different devices and therefore we we may divide this load into 3 different categories like say constant power load, constant impedance load and constant current loads right.

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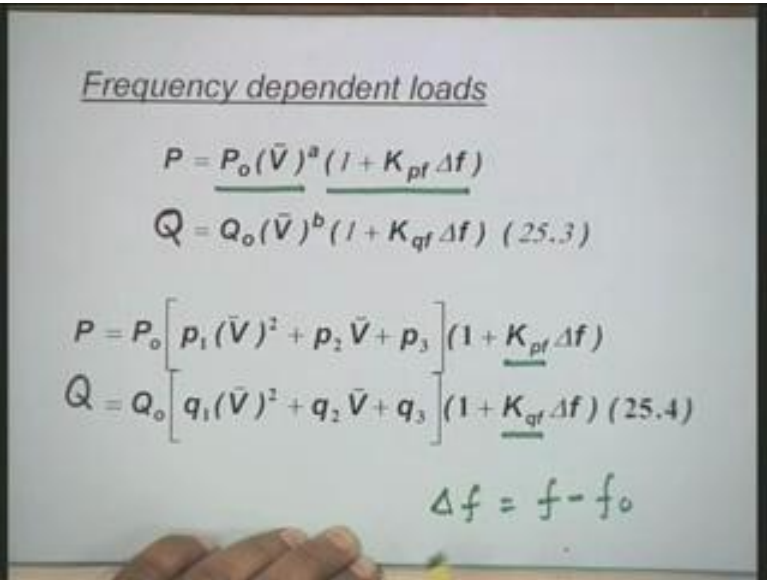


Now if we in case we want to represent a composite load, composite load then these coefficients  $a$  and  $b$  will be different for example for a composite load this index  $a$  varies from .5 to 1.8 similarly  $b$  has a wide variation that is 1.5 to 6 for composite loads. Again it depends upon what is the composition. Now here this the loads right which we represent by static loads they generally have very high response in the sense that when the voltage and frequency changes right the power consume also changes practically, instantaneously, there is no time delay right and therefore at any instant of time they can be represented as static loads and the variation can be represented using algebraic equations. Any composite load, any composite load can now be represented as in terms of 3 components we normally call this as ZIP representation  $Z$  means constant impedance  $I$  stands for constant  $\{p_a\}$  ((00:14:08 min)) current and  $P$  for constant power.

Therefore we can have a load representation by this equation  $P$  equal to  $P_0$  multiplied by  $p_1$  into  $V$  square,  $V$  bar square plus  $p_2$  into  $V$  bar plus  $p_3$  now this  $p_1$ ,  $p_2$  and  $p_3$  these are the fractions of the load which represent the constant impedance constant current and constant power fractions okay. Suppose I take  $p_1$  equal to .5,  $p_2$  equal to say .3 and  $p_3$  equal to .2 that means the 20 percent load is a constant power load, 30 percent load is a constant current load and 50 percent load is a constant impedance load.

Similarly reactive power that is reactive power  $Q$  is represented as  $Q_0$  into  $V_1$   $q_1$  into  $V$  bar square plus  $q_2$  into  $V$  bar plus  $q_3$  because it is multiplied by  $q_0$  okay. The basic problem comes is actually is difficult to know this these fractions  $p_1$ ,  $p_2$  and  $p_3$ .

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Frequency dependent loads

$$P = P_0 (\bar{V})^a (1 + K_{pf} \Delta f)$$

$$Q = Q_0 (\bar{V})^b (1 + K_{qf} \Delta f) \quad (25.3)$$

$$P = P_0 \left[ p_1 (\bar{V})^2 + p_2 \bar{V} + p_3 \right] (1 + K_{pf} \Delta f)$$

$$Q = Q_0 \left[ q_1 (\bar{V})^2 + q_2 \bar{V} + q_3 \right] (1 + K_{qf} \Delta f) \quad (25.4)$$

$$\Delta f = f - f_0$$

The next important aspect of static load modelling is the frequency dependency of loads, this is a very important component very important aspect of load modelling that is the loads depend on the system frequency. In most of the cases the real power consumed by the load increases when frequency increases and real power consumed by the load decreases when frequency decreases

and in our Indian Power System where we do not have enough generation, enough generation right the load depending upon frequency right that is the these loads decide actually the system frequency that is generations are particularly constant when you put additional load frequency drops right and therefore at a lower frequency lower frequency we are in a position to meet the additional loads which comes, although the total power which is fed is same right but the whenever additional loads comes what happens.

Suppose your system is operating at a certain frequency and let us put that I switch on some additional load of say 1 megawatt this additional load will be met from from where it comes because the answer is answer is that under steady state condition under steady state condition this additional load is met by met by 1 component is the release of the existing load plus the increase in the generation, while in dynamic condition we have all the 3 points that release released by the existing load, increase in generation and change in kinetic energy.

But the moment you attain the steady state condition right this change in kinetic energy component become 0 because you attain the constant speed again right this is the way the system you known operates right when you are trying to match the generation with the load. Okay because whenever additional load comes the first thing which happens is the the frequency drops. Okay and initially this additional load is met by kinetic energy mainly change in kinetic energy then the regulators volt the the governors will come into action will increase the generation and when the frequency drops the existing load will start consuming less power they release some power right.

Therefore these are the 3 components and therefore here we are trying to address these thing that if the frequency changes how the real power and reactive power consume vary. Now this equation is shown here shows the variation of real power consumed by the load with the variation of magnitude of voltage as well as frequency that is, if I consider this general equation to represent the real power variation with magnitude of voltage and I multiply this expression by this term  $1 + K_{pf} \Delta f$  right.

Now this term  $K_{pf}$  is the this coefficient is an important it is the characteristic of the system okay. Similarly the reactive power consumed can be represented as  $Q_0 + V^2 \bar{p}_2 + V \bar{p}_3$  into  $1 + K_{qf} \Delta f$ . Okay in a sense that both P and Q are now represented as function of system voltage magnitude and frequency deviation okay.

Now you you suppose if we represent the load, load by this expression that is ah ZIP representation okay. So you can represent P equal to  $P_0 + V^2 \bar{p}_1 + V \bar{p}_2 + V \bar{p}_3$  then this whole thing is multiplied by this fraction this factor that is  $1 + K_{pf} \Delta f$ . Okay similarly Q can be represented as  $Q_0 + V^2 \bar{q}_1 + V \bar{q}_2 + V \bar{q}_3$  into this therefore this becomes a very general representation of a load which now accounts for change in magnitude of the voltage and also change in frequency that is frequency deviation.

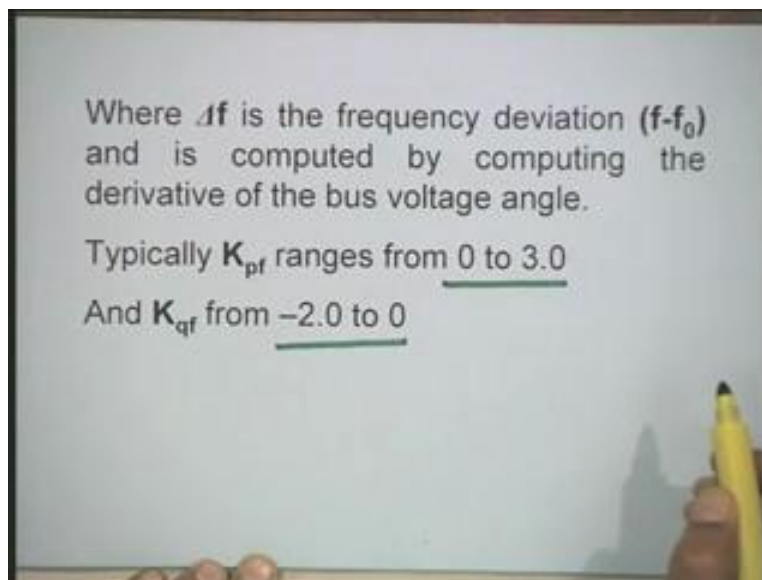
Now here I has to understand what are the values of this coefficient  $K_{pf}$  and  $K_p$   $K_{qf}$ . Now so far the real power consumed is concerned right as the frequency increases the real power consumed

increases and therefore this  $K_{pf}$  will always be positive,  $K_{pf}$  is always positive and this  $\Delta f$ ,  $\Delta f$  is expressed as  $\Delta f$  is defined as the actual frequency of the system minus the nominal frequency  $f$  minus  $f_0$  the  $\Delta f$  is positive when the actual system frequency is more than the nominal frequency right and therefore this coefficient  $K_{pf}$  is always positive but it varies, it is different for different loads it different for different loads. There are some loads where the power consumed does not depend upon frequency, lighting load flow particularly this is a you know the these powers particularly the fluorescent tubes are you know all sodium vapour lamp or mercury vapour they do depend upon frequency but particularly incandescent lamps.

Suppose I have actually a a room heater a rod type room heater then it will not depend upon frequency so far magnitude of voltage is same right but since the loads are composite loads and therefore this is a very important coefficient to be known and is difficult again the estimating this coefficient is a very difficult task actually for power system those who are taking the course on power system control and instrumentation we use the term load frequency constant  $d$  a parameter load frequency constant  $d$  this  $d$  is same as this term  $K_{pf}$  which I call it.

Now what about  $K_{qf}$  that is this is also a positive all through or whether it is always negative or what should be the value of this  $K_p$ . The the response of the audience here is that may be positive or negative but answer is not correct, is always negative, always negative. We can easily understand why why it is negative?

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The typical values of  $K_{pf}$  is from 0 to 3 and  $K_{qf}$  is in the range of minus 2 to 0 minus 2 to 0. These are the typical values of these coefficients this is a wide range depends upon the type of load.

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$$P = P_0 (P_{ZIP} + P_{EX1} + P_{EX2}) \quad (25.5)$$

where

$$P_{ZIP} = p_1 (\bar{V})^2 + p_2 \bar{V} + p_3$$

$$P_{EX1} = p_4 (\bar{V})^{a_1} (1 + K_{pf1} \Delta f) \quad (25.6)$$

$$P_{EX2} = p_5 (\bar{V})^{a_2} (1 + K_{pf2} \Delta f)$$

In the literature there is 1 more way of representing this representing the static loads right that is the static load model can be represented as  $P$  equal to  $P_0$  multiplied by  $P_{ZIP}$  plus  $P_{EX1}$  plus  $P_{EX2}$  where  $P_{ZIP}$  is represented by this expression  $p_1 \bar{V}^2$  plus  $p_2 \bar{V}$  plus  $p_3$  that what we have already seen then  $P_{EX1}$  that is exponential component is represented as  $p_4 \bar{V}^{a_1}$  multiplied by this term  $1 + K_{pf1} \Delta f$   $P_{EX2}$  as  $p_5 \bar{V}^{a_2}$  into  $1 + K_{pf2} \Delta f$ .

In fact the you know the load models which have been used actually and used in analyzing the stability of the system right the the results which are obtained and the actual actual thing which happens in the ground right. Suppose actually for a given operating condition right my stability study shows that I have this much stability margin while in practice you will not achieve that much and that has been the concern actually for the power system engineers because generally generally we do most of this studies of light using the system models.

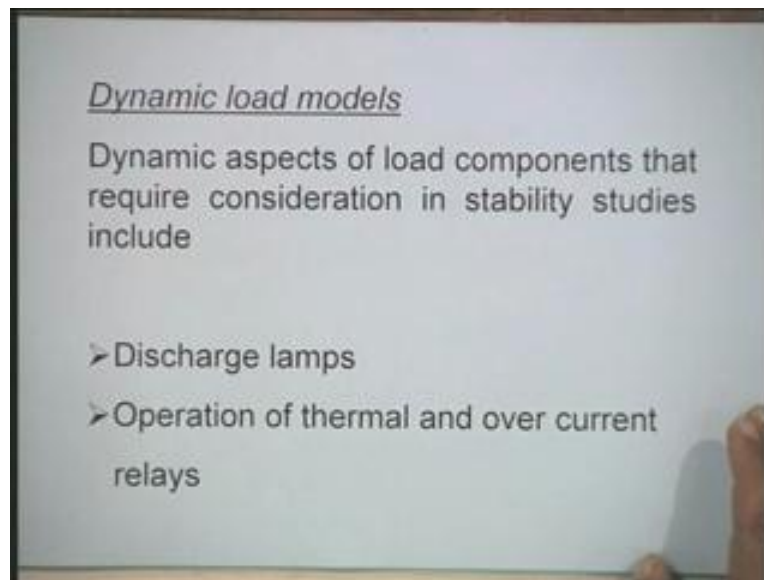
Okay and the when you try to we can see what happens on the field in the field right for the similar operating conditions right sometimes it is seen actually that the the results obtained by offline studies and what happens on the ground there is a lot of variation and that is why actually the all through the efforts are being made to have more detailed model of the system because when I talk about the stability studies stability studies then the the crux of the whole problem is the detailed model of the system right.

As you have seen actually that we have to model all the sub systems in detail synchronous generator excitation system, governing system, turbine system and even we are not  $g_1$  to the boiler model system, boiler models are also required, transmission line models are required, load models are required right and in case there is a big approximation or error in this modelling the results are going to be different or erroneous and that is why lot of efforts are being made to develop um better and better model and to estimate this the these parameters by doing some



experiments in the field right this is because suppose I take this model then it has how many coefficients will be estimated I require this coefficient  $K_{pf1}$  and  $K_{pf2}$  in this I have 2 different coefficient which determine the frequency dependency of loads right. Similarly this  $p_1$ ,  $p_2$ ,  $p_3$ ,  $p_4$ ,  $p_5$  right and these are to be estimated actually for a particular load bus and this is a quite a load task.

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Now we will discuss the dynamic load models, this is all about the static load models and justification of static load models is that whenever the magnitude of frequency and voltage changes right the the new value of power consumed changes quickly in a sense the response time is practically load and therefore we can represent these loads by static models. But there are so many loads actually which need to be modelled in a dynamic way and particularly this dynamic modellings or load modelling is needed when we are going ah interior oscillation studies that is you want study the gear of the system when it is subjected to inter area oscillations is a long term phenomena.

Similarly we are studying the long term or midterm stability or voltage stability of the system right during that period the load models or dynamic load models are also required now here the these aspects should be considered will be the discharge lamps. I have talked about the discharged lamp, the fluorescent lamps, the sodium vapour lamps and the no incandescent lamp is not a discharged lamp, mercury arc or mercury vapour lamps.

Okay in these lamps there is a very important actually that the moment the voltage is around .7 to a .8 per unit this these lamps extinguish and when the voltage recovers within 1 or 2 seconds they are restored back. Understand this is a very interesting thing that in a power system when you are doing the stability studies under disturbance condition the voltage at various buses vary and in case the voltage at a particular point at a particular bus becomes less than about 70 to 80

percent then these discharged lamps extinguish and when the voltage is restored back right then they they are restored back after 1 or 2 seconds because when I switch on a fluorescent lamp it takes 1 or 2 seconds to we will put on right. This is 1 of the important characteristic to be model, then there are many devices which have the over current and thermal relays these are also required to be modelled. Then the thermostatic control of loads particularly these thermostatic control loads are they are mostly heating and cooling loads air air heating, air cooling, refrigerators and some other devices which we can then another very interesting thing which we have to understand is that in a distribution system we have under load tap changing systems ULTC sin distribution systems we have some voltage regulators which regulates the system voltage ULTCs are also for regulating the voltage and then we also have I think distribution system the switch capacitors, voltage control capacitors right.

Now whenever actually the system voltage, system voltage dips or become slow then this ULTCs will try to operate and regulate the voltage. You know that is suppose actually at a particular time the voltage has dipped right then the unload tap changers or under load tap changers right they will operate to increase the voltage but the response time is quite high, the the voltage is slow after about 1 minute they start operating and their operation is completed in about 1 or 2 minutes time, they are slow devices. Then there are many voltage control capacitors right if the voltage is high the capacitors will be switched off and when the voltage is low capacitors are switched on right and further the reactive power which is consumed by these capacitors also depends upon the system voltage and therefore these are all ah the the aspects which have to be considered while doing the modelling.

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$$K \frac{d\tau_H}{dt} = P_H - P_L \quad (25.7)$$

where

$\tau_H$  = temperature of the heated area

$P_H$  = power from the heater

$= K_H G V^2$

$P_L$  = heat loss by escape to ambient area

$= K_A (\tau_H - \tau_A)$

$G$  = load conductance

In fact actually the 1 stability program which has been developed by the EPRI, Electric Power Research Institute USA right they have modelled the thermostatically controlled loads and also the induction motor load in their system they call it the program is called load seal. There are

many you know a companies they have purchase those programs although question is that whether we are fully utilizing those software packages for operating our system that is a different issue but detail detailed stability programs accounting for the load models are available globally.

Now here I would like to discuss actually the thermostatically controlled loads. Let us take the example where where actually the load is to heat a particular space, space heating right and the differential equation which governs the heating process or temperature control is written by this equation that is  $K \frac{d\tau_H}{dt} = P_H - P_L$ . This is very important equation because once I say it is a a dynamic model then these derivative terms will come, we will have to have differential equations to represent certain devices that is a constant coefficient  $K \frac{d}{dt}$  of  $\tau_H$   $P_H$  minus  $P_L$ .

Here this  $\tau_H$  is the temperature of the heated area,  $\tau_H$  temperature is the temperature of the heated area which we want to regulate, we want to control, how do we control is by by increasing or decreasing the heat generated by the heating device. Now here on this right hand side we have the 2 terms  $P_H$  and  $P_L$  the  $P_H$  stands for the power from the heater, this  $P_H$  can be represented as a function of constant  $K_H$ ,  $G$  is the conductance of the system and  $V$  square.

Now since actually we are trying to study the stability affect of these devices on stability because the power consumed here is function of voltage  $V$  square right therefore the the power from the heater is shown as a constant into conductance into  $V$  square. Then  $P_L$  stand for the heat loss by escape of escape to ambient area atmosphere, this is the heat dissipation which can be by which can be represented by a constant  $K$  into  $\tau_H$  minus  $\tau_A$  the  $\tau_A$  is the temperature of the ambient area or we can say ambient temperature where  $G$  is the load coefficient okay.

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$$K \frac{d\tau_H}{dt} = K_H G V^2 - K_A (\tau_H - \tau_A) \quad (25.8)$$

$$\frac{d\tau_H}{dt} = \frac{K_H}{T_I} G V^2 + \frac{1}{T_I} \tau_A - \frac{1}{T_I} \tau_H \quad (25.9)$$

where

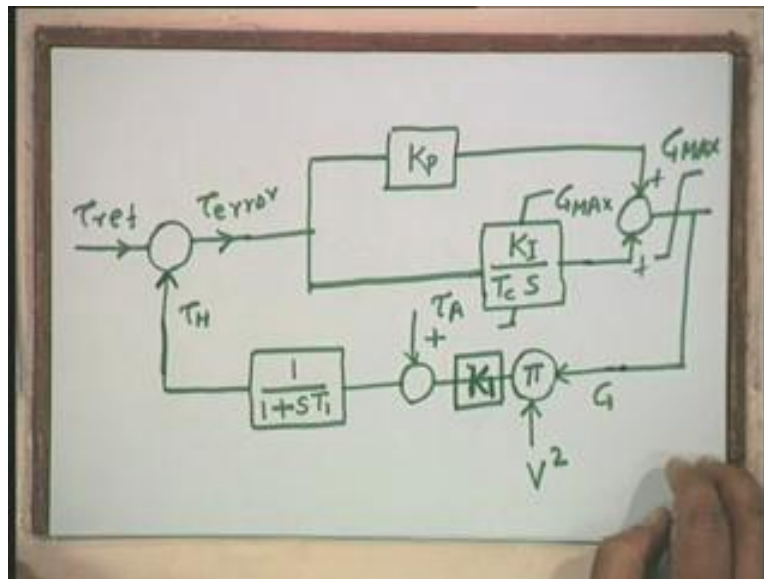
$$T_I = \frac{K}{K_A}$$

$$K_I = \frac{K_H}{K_A}$$

Now this equation, this equation has been put in a form where I can realize the transfer function relating the actual temperature of the system and actual conductance of the system. Now to do this thing what we do that in this equation we substitute for  $P_H$  and  $P_L$  these quantities. Now when you make this substitution this  $K$  times  $\tau_H$  by  $dt$  for  $P_H$  I am putting  $K_H G V^2$  and for  $P_L$  we are putting  $K_A$  into  $\tau_H$  minus  $\tau_A$ .

Okay then you simplify this equation and arrange the terms we will get the term this  $d\tau_H$  by  $dt$  equal to  $K_1$  by  $T_1 G V^2$  square plus 1 by  $T_1 \tau_A$  minus 1 upon  $T_1 \tau_H$ , where these quantities are defined like this  $T_1$  equal to  $K$  by  $K_A$  and  $K_1$  is defined as  $K_H$  by  $K_A$  by just these definitions okay. Now this thing this equation I have now if you take the Laplace transform of this equation we can put in the form of a transfer function I will just show you how it is put in the form of a transfer function.

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In any system, any system we want to regulate the temperature right then we start with the reference temperature let us call it  $\tau$  reference and we want to know what is the actual temperature at any time  $T$  of the space okay. Now this this equation that is see this equation, this differential equation relates actually the time at a any time  $T$  with other parameters of the system this is simple differential equation, now this can be put in this form.

Now if you see here actually this differential equation then we can represent the this differential equation by this form that is  $G$  is the system conductance, you multiply this  $G$  by  $V^2$  square multiplied by  $K_1$  okay to this you add this ambient temperature  $\tau_A$  okay. Then when it passes through this transfer function  $1$  upon  $1 + S T_1$  you will get a term  $\tau_H$ . Now in practice what is  $d_1$  is that this error this is the error signal we call it  $\tau$  error right this will be put into a controller to regulate the conductance of the system. Generally, we use a  $P_I$  controller if you use a  $P_I$  controller you can have a proportional gain setting  $K_P$  here and we put integral

control I am just putting on this side is, it can be it is  $K_I$  divided by some time constant  $T_c S$  and here we have to put the upper maximum limit  $G_{MAX}$ . You can add these 2 outputs again there will be a a maximum limit which has to be put  $G_{MAX}$  and this conductance is the 1 which bring back.

Now this becomes a simple closed loop control system which we can show here to regulate the temperature of a space which is to be heated. Now here  $\tau_H$  is the state variable state variable this is only simple and this is the  $P_I$  control which I have shown. Now depending upon this error it is going to increase the or decrease the conductance right the more the conductance right. The more is the heat which is going to be generated right but since we have using this integral also so that the we want actually that the maximum conductance should be limiting therefore maximum limit either it may be touched by integral integrator or by the sum because here it is plus okay, therefore this is 1 way of we can representing a thermostatically controlled loads.

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Under pre-disturbance condition  $\tau_H$  is equal to  $\tau_{ref}$ .

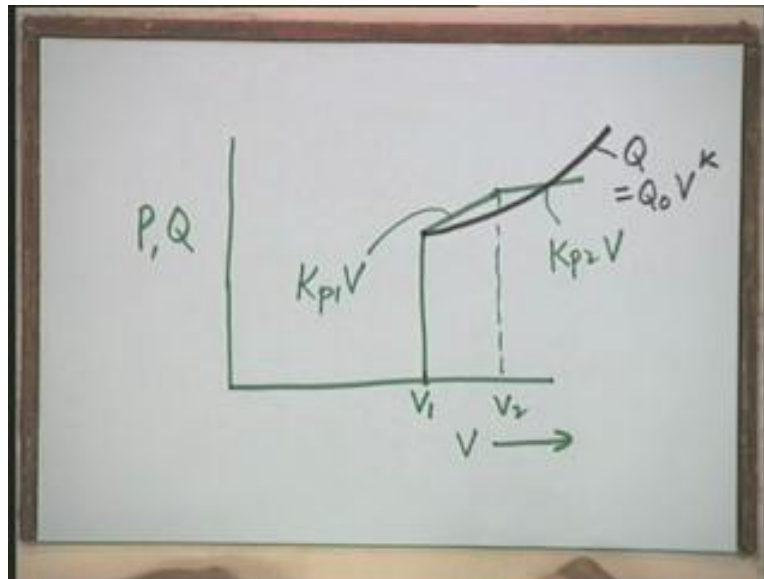
$$\tau_{ref} = K_I V_0^2 G_0 + \tau_A$$

$$K_I = \frac{\tau_{ref} - \tau_A}{V_0^2 G_0} \quad (25.10)$$

Now under steady state conditions under steady state conditions the  $\tau$  reference equal to  $\tau_H$  right. Now if you make this substitution in this equation in this differential equation put the derivative term to be 0 and I make derivative term equal to 0 right then we can write down the  $\tau$  reference equal to  $K_I$  times  $V_0$  square  $G_0$  plus  $\tau_A$  that is the reference temperature setting which is required to attain the required value of  $\tau_H$ , where the constant  $K_I$  is equal to  $\tau$  reference minus  $\tau_A$  divided by  $V_0$  square  $G_0$  okay.

Now with this I will just give briefly introduced actually that how a dynamically controlled or thermostatically control load can be represented. These differential equations are to be written along with the system differential equations.

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So far actually the discharge lamps are concerned their characteristic can also be shown as function of voltage but these characteristics are quite non-linear in nature. I will shown in this axis P and Q consumed by the discharged lamps and on this x axis you put voltage V okay. Now let us first see the power P consumed by a discharged lamp. At a certain voltage up to certain voltage say  $V_1$ ,  $V_1$  the discharged lamp should be off right therefore no power is consumed by discharged lamp etcetera when the voltage becomes  $V_1$  the suddenly these devices are switched on and when the voltage increases beyond this the power consumed that is at a when the voltage is  $V_1$  the power consumed suddenly jumps from 0 value to this value then as the voltage further increases the power consumed may vary linearly and by another segment.

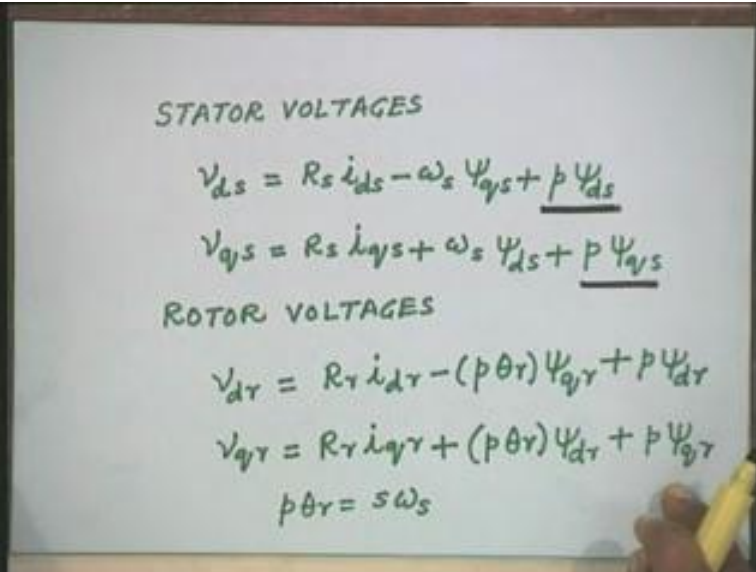
In fact actually you can do ah this non-linear characteristic and represented by a piecewise linearized model that is you can put at this voltage  $V_1$  from  $V_1$  to  $V_2$  it can be represented as some  $K_{p1}$  into V and here it can be represented as  $K_{p2}$  into V okay while this so far the reactive power is consumed. The reactive power variation is a very non-linear in nature for reactive power variation for these devices can be shown this is type of curve we can call this as a Q which is equal to  $Q_0$  the value at this particular point multiplied by V to the power some coefficient K a very non-linear type of variation right and therefore since the system voltage is varying and we want to model these devices this type of model is to be shown that in case the voltage is less than  $V_1$  they are off when it is more than  $V_1$  to  $V_2$ , P and Q have to be shown like this that is the model you have to have that facility where you represent the loads are present as a function of voltage. Okay the next important component for modelling is the induction motor load.

Now induction motor load or modelling of induction motor has has attracted the attention of many researches and many practicing engineers in the past because over 70 percent of the system load is induction motor load. Now for developing this dynamic model for the induction motor the



approach is similar to what we do for a synchronous motor and we represent the model using DQ transformation, DQ transformation I would not go into details about all these transformations but here we have 1 simple advantage actually for so far this induction motor is concerned, the rotor of the induction motor is a cylindrical 1 right and therefore the the a variation in mutual inductances and self-inductances right are different as compared to that of a synchronous motor.

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STATOR VOLTAGES

$$V_{ds} = R_s i_{ds} - \omega_s \psi_{qs} + \frac{d\psi_{ds}}{dt}$$

$$V_{qs} = R_s i_{qs} + \omega_s \psi_{ds} + \frac{d\psi_{qs}}{dt}$$

ROTOR VOLTAGES

$$V_{dr} = R_r i_{dr} - (p\theta_r) \psi_{qr} + \frac{d\psi_{dr}}{dt}$$

$$V_{qr} = R_r i_{qr} + (p\theta_r) \psi_{dr} + \frac{d\psi_{qr}}{dt}$$

$$p\theta_r = s\omega_s$$

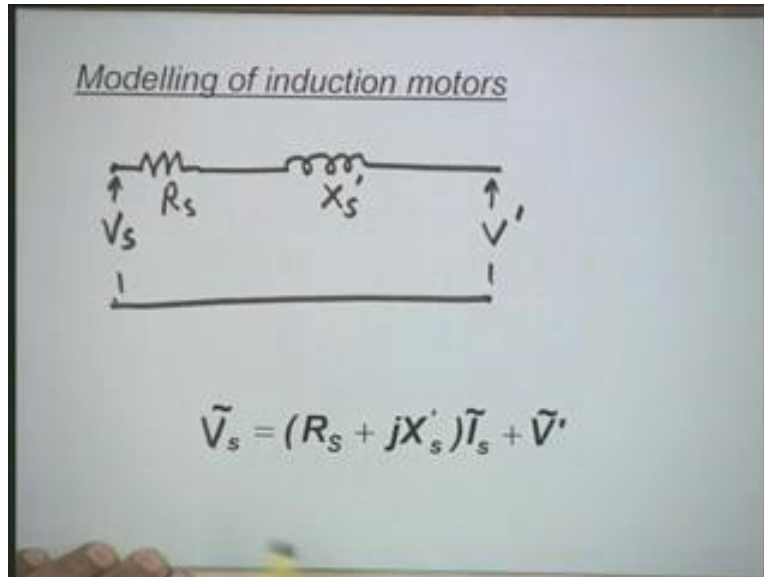
Let us just see so far the stator windings are concerned they will they will not see any difference in the in the reluctance of the magnetic circuit as the position of the rotor changes rotor is cylindrical therefore depend whichever is the position of the rotor right. The stator windings will not see any difference in the reluctance and therefore the self-inductance of the stator winding, mutual inductance between the stator windings are not function of angle this is the advantage, second point is that the mutual inductance between the stator and rotor windings vary vary because because the axis of the rotor and stator windings are having different orientations.

When the axis of the rotor and stator windings are aligned they will have maximum mutual inductance when they are in quadrature it will be 0 mutual inductance that is the only variation and here the stator voltages are written in terms of the D and Q axis components just as we do in the case of synchronous machine but here we also require the transformation for rotor quantities this is the main another difference.

Further for studies the transformer voltages in the stator circuit will be neglected, the stator circuit for example for stability studies in the synchronous machine we have ignored the transformer voltages in the stator voltage equation. Similarly, here also we ignore it however actually the the transformer voltage in the rotor circuit will not be ignored while this so so far rotor circuit is concerned the rotor is at closed circuit therefore D and Q axis voltages in the rotor

circuit are 0 that is  $V_{dr}$  and  $V_{qr}$  were set equal to okay, these are the simple equations which can be easily derived and they are available in the standard literature.

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The stator of the synchronous generator can be simply represented by a model of this form. This is simple model which 1 can arrive at for the stator equation because the stator equations we are not considered the transform voltages right therefore this is an algebraic equation.

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where

$R_s$  = stator resistance  
 $X_s$  = transient reactance of the induction machine

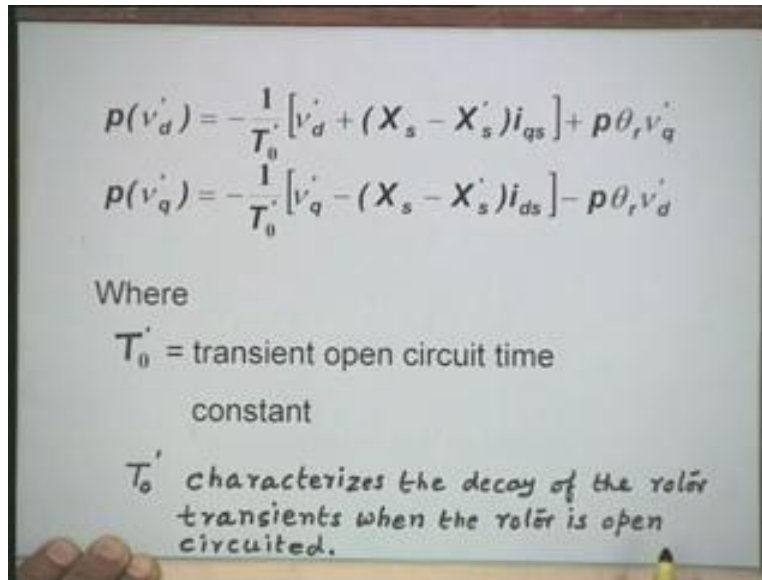
$$\tilde{V}_s = v_{ds} + jv_{qs}$$

$$\tilde{V}' = v'_d + jv'_q$$



We can represent this voltage  $\bar{V}_s$  as  $R_s$  plus  $j$  times  $\bar{X}_s$  into  $\bar{I}_s$  plus  $V$  prime this is an important voltage component okay when you represent these voltages in terms of D and Q axis components we can write down  $V_s$  equal to  $V_{ds}$  plus  $j$  times  $V_{dqs}$   $V$  prime equal to  $V_d$  prime plus  $j$  times  $V_q$  prime.

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$$p(\dot{v}_d) = -\frac{1}{T_0'} [\dot{v}_d + (X_s - X_s') i_{qs}] + p\theta_r \dot{v}_q$$

$$p(\dot{v}_q) = -\frac{1}{T_0'} [\dot{v}_q - (X_s - X_s') i_{ds}] - p\theta_r \dot{v}_d$$

Where

$T_0' =$  transient open circuit time constant

$T_0'$  characterizes the decay of the rotor transients when the rotor is open circuited.

Then here the 2 components of this voltage  $V$  prime  $V$  prime at these 2 components  $V_d$  prime and  $V_q$  prime right these these voltages are obtained from these 2 differential equations that is  $p v_d$  prime is given this equation and  $p v_q$  prime is given by this equation that is they are the 2 differential equations in fact basically these these equations represent the rotor dynamics, rotor dynamics as I have told you that we do not neglect the rotor dynamics here if you neglect {lo} rotor dynamics then this model also becomes a static model okay.

Now here there is a term very important term  $T_0$  prime this is the transient open circuit time constant, transient open circuit time constant and this time constant this  $T_0$  prime characterizes the decay of the rotor transients when the rotor is open circuited, I am sorry there is mistake here this when the stator is open circuited when the stator is open circuited. The differential equation for the induction motor just like actually the we have a swing equation for synchronous jetter.

Similarly, we have a swing equation for induction motor is  $d$  by  $dt$  of  $p$  omega  $r$  bar equal to  $1$  upon  $2H$   $T_e$  minus  $T_m$ . Now here here electrical power right is the input and mechanical power is the output therefore  $T_e$  minus  $T_m$  is the accelerating torque right therefore this is the swing equation for the induction motor and this electrical torque is obtained by this equation  $T_e$  equal to  $v_d$  prime  $i_{ds}$  plus  $v_q$  prime  $i_{qs}$ . Further the load torque load torque to this  $T_m$  is the load torque this  $T_m$  is the load torque right this load torque also depends upon the speed and therefore the equation which is used to represent the load torque  $T_m$  is  $T_o$  omega  $r$  to the power  $m$  omega  $r$  bar here omega  $r$  bar means it is actually the speed of the rotor expressed in terms of or we can say

synchronous speed of the machine. Okay that is you can say it is a per unit speed  $\omega_r$  bar is a per unit speed another can express in for torque may be  $T_m$  equal to  $T_o$  into A times  $\omega_r$  square plus B times  $\omega_r$  plus C.

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$$p\theta_r = \frac{\omega_s - \omega_r}{\omega_s}$$

$$p(\bar{\omega}_r) = \frac{1}{2H} (\bar{T}_e - \bar{T}_m)$$

$$\bar{T}_e = \bar{v}_d \bar{i}_{ds} + \bar{v}_q \bar{i}_{qs}$$

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$$\bar{T}_m = T_o (\bar{\omega}_r)^m$$

$$\bar{T}_m = T_o \left[ A(\bar{\omega}_r)^2 + B\bar{\omega}_r + C \right]$$

Now let me conclude the today's presentation, today we have discussed the static and dynamic models for the loads which are required to be modelled in the stability studies of the power system we have modelled the various types of loads and the loads which we consider are actually

the static loads and the dynamic loads, in dynamic loads particularly particularly we have determined the dynamic model for the induction model. Thank you!