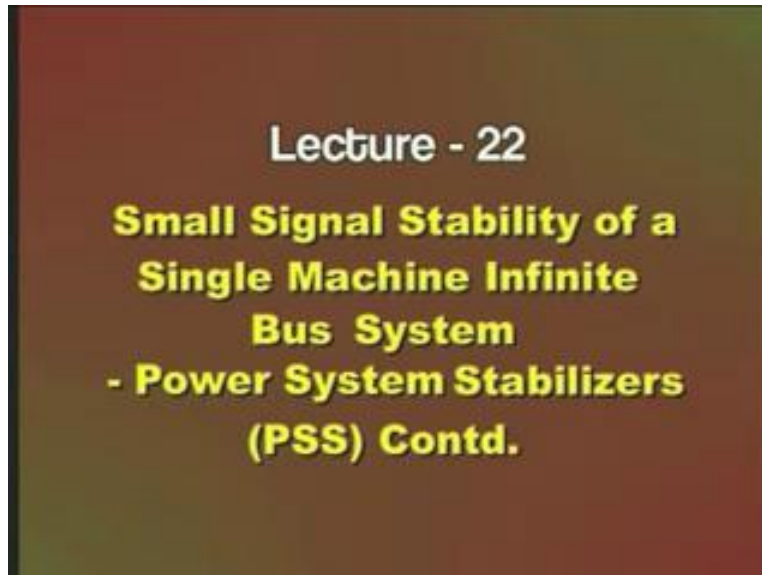


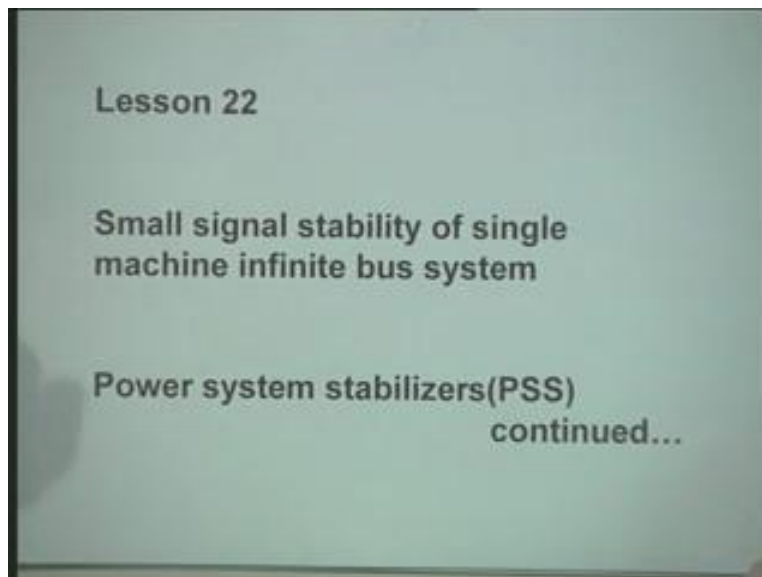
**Power System Dynamics**  
**Prof. M. L. Kothari**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Delhi**  
**Lecture - 22**  
**Small Signal Stability of a Single Machine Infinite Bus System-Power System Stabilizers (PSS) (Contd...)**

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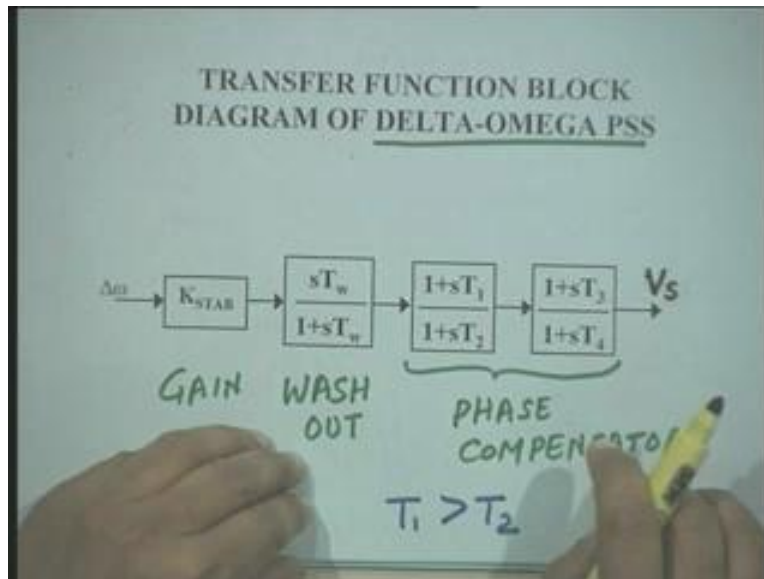
Okay gentleman, we start today on the the study of power system stabilizers.

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Last time I have introduced the function of power system stabilizer and we have seen actually that the power system stabilizer is provided to damp electromechanical modes of oscillations. Okay and in order to provide the required damping to the electromechanical mode of oscillations, we provide power system stabilizer.

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The structure of the power system stabilizer is shown here. This power system stabilizer is given the name delta omega power system stabilizer. The reason for giving this name as delta power system stabilizer is that the input signal is derived from the deviation of the rotor speed right. Now it has primarily 3 blocks, the first block we denote as K stabilizer it is the gain block, the second block is denoted as s times  $T_w$  or 1 plus s times  $T_w$  this is called wash out block. The third block in this particular case we have shown 2 transfer functions right this 2 put together will form the phase compensator because the the power system stabilizer is to compensate for the phase lag between between the between the exciter input to the air gap torque okay and to produce a torque in phase with the speed deviation right.

This power system stabilizer is to be design, so that we compensate for the phase lag of the system that the phase lag of the system is the phase lag of the exciter and the synchronous generator. Now, I will now discuss in detail the role of different blocks of the PSS. Let us first start with the wash out block this wash out block is provided to allow to allow the high frequency signal of interest to pass and end not to allow the steady state deviation to pass and end change the terminal voltage because if this block is not provided then if there is a steady state change in the rotor speed then this steady state change will be reflected in the terminal voltage change.

Now if you look here at the transfer function of this block under steady state condition, s is 0, under steady state condition s is 0 therefore when I substitute s equal to 0 in this transfer function right then the this the gain of this block becomes 0 right and therefore

any signal which comes in to the power system stabilizer there will be no output coming from this block. However, when the frequency  $s$ , frequency of oscillate rotary such that if this term  $1 + s T_w$  is right.

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The image shows a whiteboard with handwritten mathematical expressions. At the top, it says  $s = j\omega$ . Below that, the transfer function  $\frac{j\omega T_w}{1 + j\omega T_w} \approx 1$  is written. Underneath, the conditions for this approximation are given:  $\text{if } |j\omega T_w| \gg 1$  and  $\omega T_w \gg 1$ .

Now this term will become or you can say that it becomes for  $s$  equal to  $j\omega$  right. The transfer function of the wash out block can be return as  $j\omega T_w$  divided by  $1 + j\omega T_w$  for any frequency of oscillation of the rotor.

Now if magnitude of this term is  $j\omega T_w$  is very large as compared to 1 or I can say if  $\omega T_w$  is very large as compared to 1 right. In that case this transfer function a gain equal to 1 because this 1 can be 1 is 1 plus  $j$  times  $\omega T_w$  can be written as  $j$  times  $\omega T_w$  and therefore this transfer function will behave as if its gain is equal to 1 right and therefore the speed deviation signal right will be allowed to pass right and therefore this wash out block is also known as high pass filter. Okay and generally the characteristic of this wash out block will depend upon of the value of time constant  $T_w$  right. In the literature lot of studies have been done on the choice of  $T_w$ .

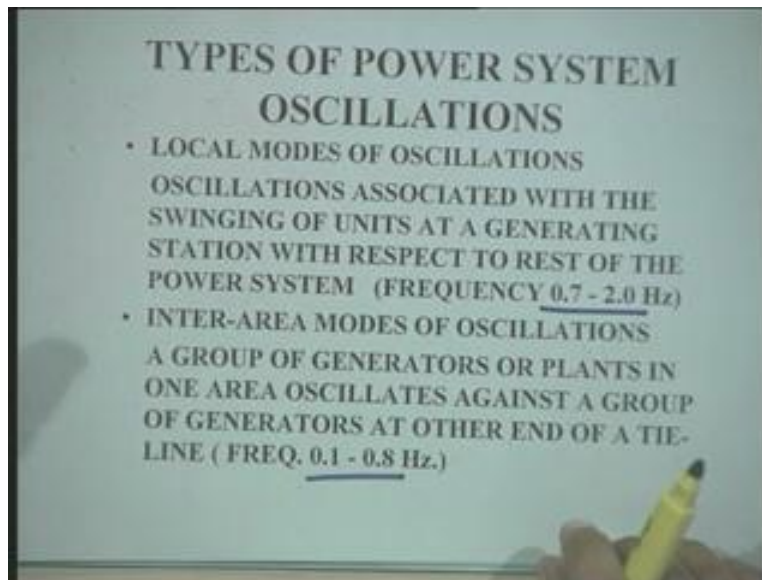
We have also studied the effect of variation of  $T_w$  and we found actually that this  $T_w$  is not very critical and I can choose the value of  $T_w$  in the range of 1 to 20 in the wide range that is  $T_w$  equal to 1 second right up to 20 seconds they have been chosen. Now generally the a time constant  $T_w$  equal to 10 second is consider to be quite suitable. Further actually some times what happens is that the system gets islanded because of suppose some the major for in the system and view to resort islanding, in that case what will across the frequency of this islanded system may go up or go down and at that time at that time this frequency is the steady state deviation right at that time also we do not want actually this phases phases to create to any problem in terms of variation of terminal voltage right.

Therefore when you  $T_w$  equal to 10 or around that right then the desired characteristic of the wash out block is okay the next transfer function block which we have to consider is the phase compensatory. Now here I have shown 2 stage phase compensator that 2 stages which you have shown here is normally call 2 stage phase compensator okay.

Now I will discuss, how to design 1 stage phase compensator and in case you require to provide 2 stages then the second stage can also be design accordingly the basic function of this phase compensator is to provide phase lead right and therefore we normally call this as a phase lead controller or phase lead compensator. Now in order to provide the phase lead this time constant  $T_1$  should be greater than  $T_2$  okay.

Now I will further discuss, how the phase lead controller is designed and how this can be physically realized using operational amplifiers because when you want to realize this phase lead compensator right for implementation, okay then we have to see that how this transfer function is physically realized using operational amplifiers although there are other ways to realize this transfer function right but the modern practice is to make use of operational amplifiers.

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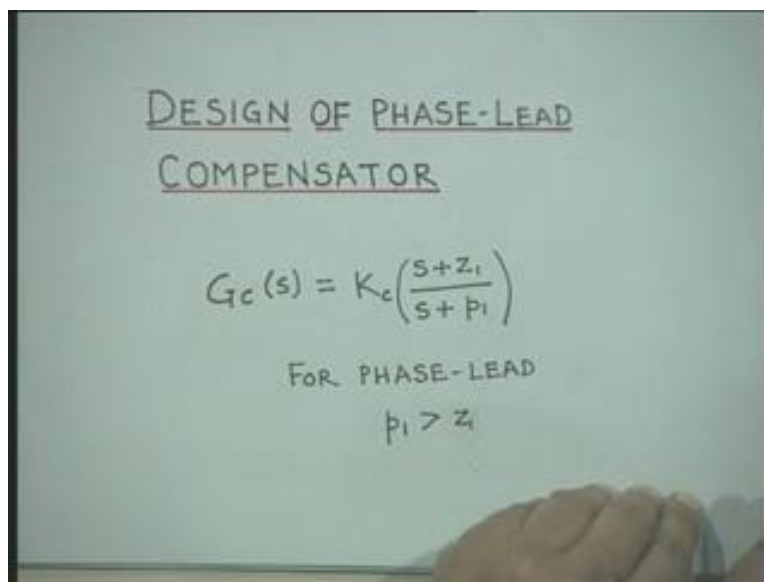


Now when we design the phase lead compensator, we have to very careful and design in such phase so that it provides the damping to the oscillations right which are which are local modes of oscillations and interior modes of oscillation because we we have seen earlier that local modes of oscillations have a frequency range of around .72 hertz, okay while interior modes of oscillations have frequency of the order of point 1 to .8 hertz and therefore if you look the total spectrum right the frequency range over which we want to produce damping is varying from .1 to 2 hertz and the and the phase shift or phase lead controller is to be designed.

So that we get the required phase compensation right but as we know as we have seen actually that you can design this compensator to compensate exactly the phase lag of the system at 1 particular frequency okay.

Now further we have to understand very carefully that the phase lag of the system is not constant it depends upon the system operating condition and system parameters. Okay and therefore as the system operating condition changes the phase lag characteristic of the system also changes and therefore actually the design of the phase compensator, so that it meets the requirement over the over the wide range of frequency is a real challenge, okay.

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DESIGN OF PHASE-LEAD  
COMPENSATOR

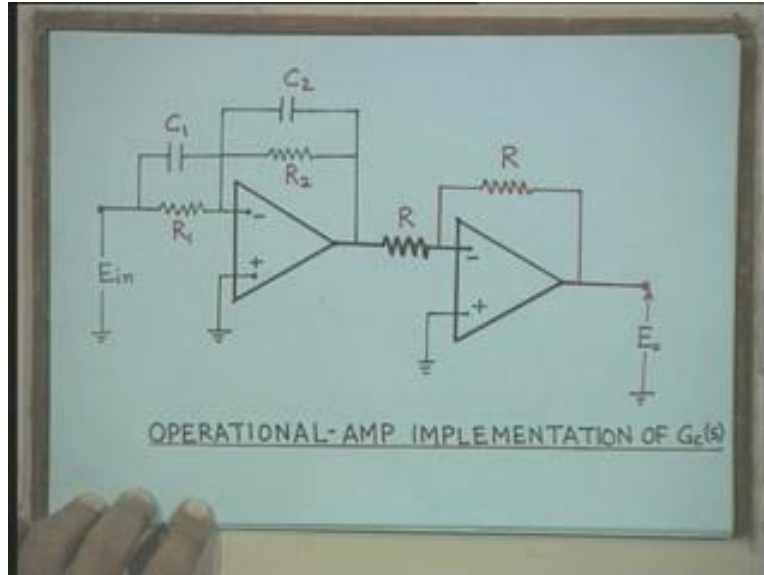
$$G_c(s) = K_c \left( \frac{s + z_1}{s + p_1} \right)$$

FOR PHASE-LEAD  
 $p_1 > z_1$

Now this this phase lead phase lead compensator the transfer function of phase lead compensator can be written in this form. In general I am just putting a general any phase lead compensator can be written as gain  $K_c$  in to 1 gain  $K_c$  multiplied with this transfer function that is  $s$  plus  $z_1$  divided by  $s$  plus  $p_1$ , this  $z_1$  is the 0 of this transfer function and  $p_1$  is the pole of this transfer function and for this transfer function to provide the phase lead  $p_1$  should be greater than  $z_1$ , okay.

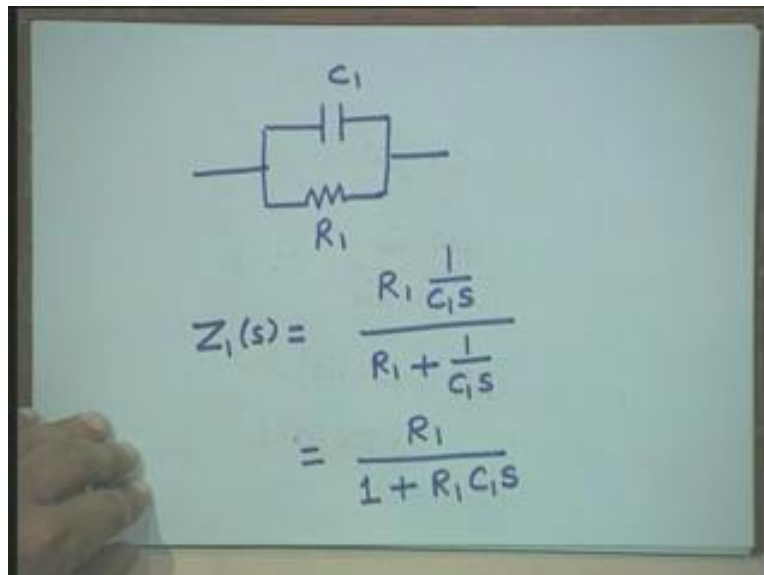
Now here to realize this phase lead controller or phase lead compensator, the simplest arrangement for its implementation using operational amplifier see shown here that is in this amplifier, in this portion of the operational amplifier circuit this is the feedback impedance pass 1 and this is the input impedance right. Now in the feedback portion we have taken a circuit comprising of resistance  $R_2$  in parallel with capacitor  $C_2$ . Similarly, in the input a circuit is comprising of a resistance  $R_1$  in parallel with  $C$  okay.

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Now you all know actually that when use 1 operational amplifier right there is a phase inversion. Okay therefore to correct for this phase inversion the make use 1 more stage of operational amplifier. So that we correct this phase inversion because this will in this particular case the gain is 1 or the the feedback resistance is taken same as the input resistance.

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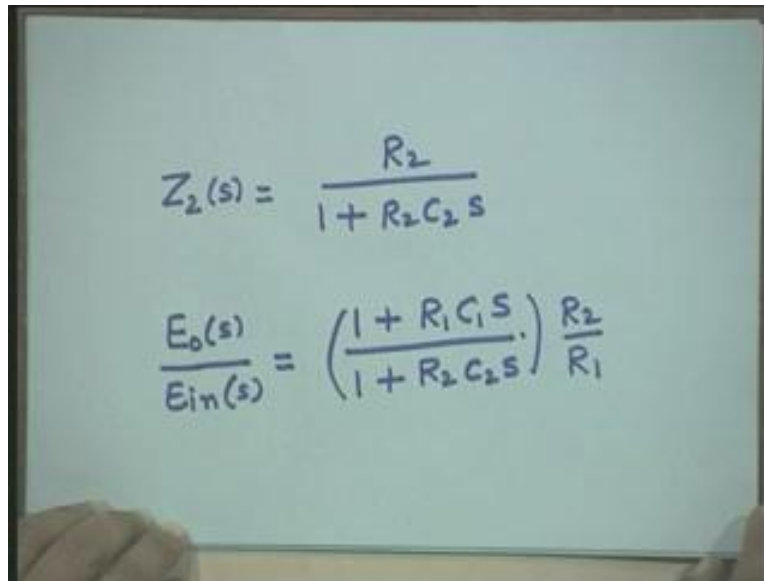


So that the gain of this operational amplifier is 1 and the phase shift is 180 degree therefore the phase shift created by this operational amplifier is corrected by second operational amplifier. The transfer function relating the output voltage  $E_o$  to the input  $E_{in}$

can be simply obtained by writing the writing the operational impedance of this  $R_2$   $C_2$  in parallel and  $R_1$   $C_1$  in parallel.

Suppose, you have circuit in comprising of resistance a capacitance  $C_1$  and the resistance  $R_1$  right the we I can call the operational impedance of this a  $Z_1$ , this will be equal to parallel combination of these 2 impedance that is  $R_1$  in parallel if 1 upon  $C_1$  s divided by  $R_1$  plus 1 upon  $C_1$  right which can be simplified and put in the form  $R_1$  divided by 1 plus  $R_1 C_1 s$ , okay.

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$$Z_2(s) = \frac{R_2}{1 + R_2 C_2 s}$$

$$\frac{E_o(s)}{E_{in}(s)} = \left( \frac{1 + R_1 C_1 s}{1 + R_2 C_2 s} \right) \frac{R_2}{R_1}$$

Similarly, you can write down for the of feedback impedance that is I call this a feedback as  $Z_2$  as the operational impedance of the feedback impedance that can be written as  $R_2$  divided 1 plus  $R_2 C_2 s$  and therefore we can write down the transfer function  $E_o$  divided by  $E_{in}$  that is  $E_{os}$  divided by  $E_{ins}$  as it is  $Z_f$  divided by  $Z_1$   $Z_f$  is here. This quantity and  $Z_1$  we have already therefore when you divide it will come out be 1 plus  $R_1 C_1 s$ , 1 plus  $R_2 C_2 s$  this multiplied by  $R_2$  by  $R_1$ . Okay therefore you can see here actually the transfer function is similar to what we are looking for but you are interested in having a transfer function of the form 1 plus  $T_1 s$  divided by 1 plus  $T_2 s$ .

Therefore, this the simple operational amplifier with the with the of feedback comprising of a resistance in parallel with capacitance, similarly input also a resistance in parallel with capacitance. We can get a simple transfer function which and by choosing the value of this  $R_1 C_1$ ,  $R_2 C_2$  right. We will in a position to obtain the require phase characteristic of the transfer function, okay.



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$$\begin{aligned}
 G_c(s) &= \frac{E_o(s)}{E_{in}(s)} \\
 &= \left( \frac{C_1}{C_2} \right) \left( \frac{s + 1/R_1 C_1}{s + 1/R_2 C_2} \right) \\
 &= K_c \frac{s + z_1}{s + p_1} \\
 C_1 &= C_2 = C \\
 G_c(s) &= \frac{R_2}{R_1} \left( \frac{1 + R_1 C s}{1 + R_2 C s} \right) = \frac{1}{a} \left( \frac{1 + a T s}{1 + T s} \right) \\
 a &= \frac{R_1}{R_2} \quad T = R_2 C
 \end{aligned}$$

Now here I have written this expression, we just now we derived you call this as a  $G_c$  that is the transfer function of the compensator  $G_c$ , c stand for compensator as  $E_{os}$  by  $E_{ins}$ . Now this can be put in the form the 1 which we have just now derived right it can be now put in a different form right that is you can put this in a form  $C_1$  by  $C_2$  s plus 1 by  $R_1 C_1$  s plus 1 by  $R_2 C_2$  and therefore we can say that this is this transfer function is of the form gain  $K_c$  s plus  $Z_1$  s plus  $p_1$  where we can identify the  $Z_1$  as 1 by  $R_1 C_1$  and  $p_1$  as 1 by  $R_2 C_2$ , okay.

Now in order to in order to reduce the number of independent parameters of the circuit, we can choose  $C_1$  equal to  $C_2$  equal to  $C$  that is this these 2 capacitors can be chosen to be equal, once you choose the  $C_1$  equal to  $C_2$  equal to  $C$  this transfer function can be written here in this form it is put  $C_1$  equal to  $C$ ,  $C_2$  was also equal to  $C$  and you divide this whole expression by  $R_1$  and denoted by  $R_2$ .

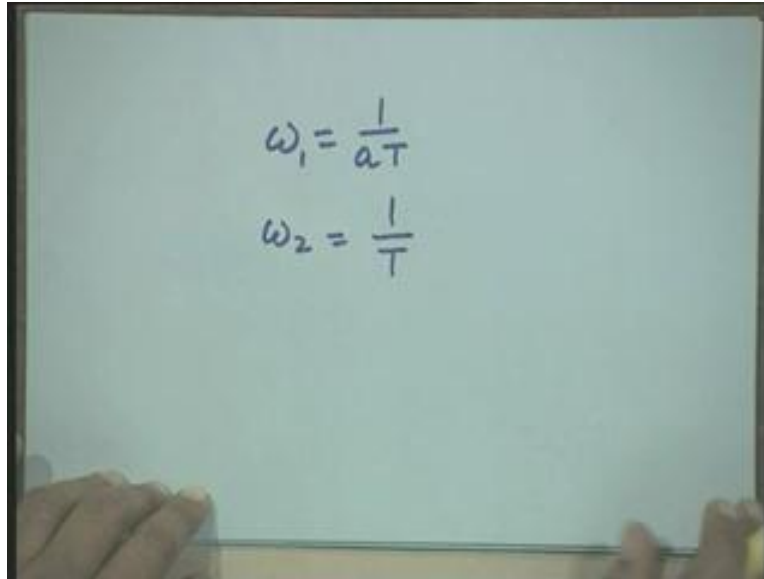
Okay then finally it is written in this form at  $R_2$  by  $R_1$ , 1 plus  $R_1 C s$ , 1  $R_2 C s$  which can be  $R_2$  by  $R_1$  can be written as 1 by  $a$  where  $a$  is equal to  $R_1$  by  $R_2$  and  $T$  can be written  $s R_2$  into  $C$ , okay therefore now I get the transfer function of the compensator in the form 1 by  $a$  multiplied by 1 plus  $a$  times  $T s$ , 1 plus  $T s$  right therefore what we have done is that we have now 2 parameters to be computed 1 is the  $a$ , another is the time constant  $T$  and I as have told actually that that  $T_1$  is greater than  $T_2$  and therefore  $a$  is greater than 1,  $a$  is always greater than 1 to realize  $R_2$  realize a phase lead controller right.

Now the problem arises is that how to choose the values of these 2 parameters  $a$  and  $T$ . Now this this quantity 1 upon  $a$  will be consider as part the stabilizer gain, you have this is a multiplying factor a constant therefore this can be club with the stabilizer gain. Okay now we will now a study how to design the this phase lead compensator to to achieve the required phase compensation. Now if you look here in this transfer function is, excuse me sir, yes.



No no here here this, if you look at this transfer function this portion of transfer function right that this represents  $T_1$ , this represents  $T_2$  and we want phase lead compensator and for this phase lead compensator this  $a$  in to  $T$  should be greater than  $T$ , therefore  $a$  has to be greater than 1 as if you want to get the phase lag then  $a$  will be less than 1 because sometimes we do required phase lag compensators in the design of controllers right at that time  $a$  will be less than 1 and this  $a$  equal to  $R_1$  by  $R_2$  the values of  $R_1$  and  $R_2$  can be so chosen, so that we get the required ratio.

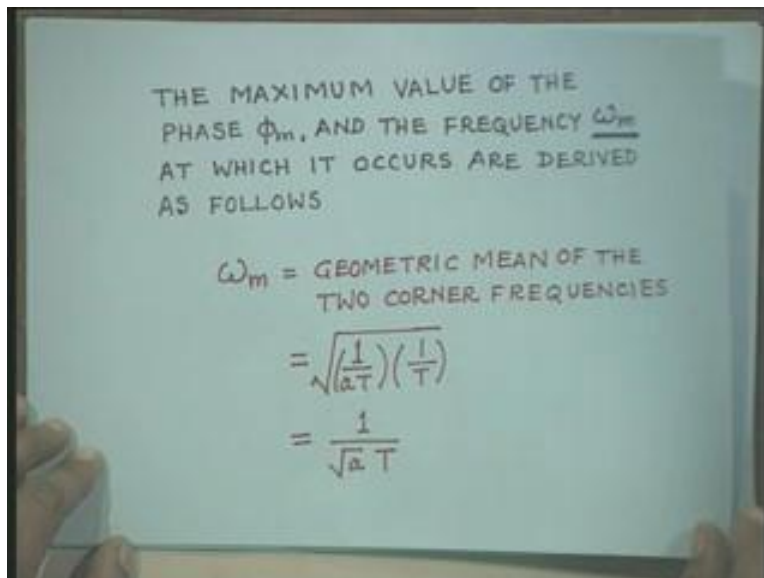
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A photograph of a whiteboard with two handwritten equations. The first equation is  $\omega_1 = \frac{1}{aT}$  and the second equation is  $\omega_2 = \frac{1}{T}$ . The equations are written in blue ink.

$$\omega_1 = \frac{1}{aT}$$
$$\omega_2 = \frac{1}{T}$$

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A photograph of a whiteboard with handwritten text and a formula. The text reads: "THE MAXIMUM VALUE OF THE PHASE  $\phi_m$ , AND THE FREQUENCY  $\omega_m$  AT WHICH IT OCCURS ARE DERIVED AS FOLLOWS". Below the text, the formula for  $\omega_m$  is derived as the geometric mean of the two corner frequencies:  $\omega_m = \text{GEOMETRIC MEAN OF THE TWO CORNER FREQUENCIES}$ ,  $= \sqrt{\left(\frac{1}{aT}\right)\left(\frac{1}{T}\right)}$ , and  $= \frac{1}{\sqrt{a} T}$ . The text and formula are written in blue ink.

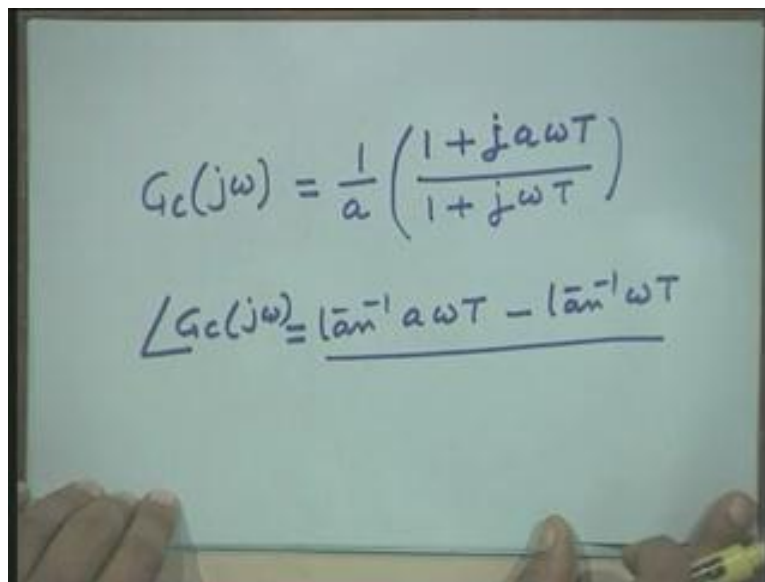
THE MAXIMUM VALUE OF THE PHASE  $\phi_m$ , AND THE FREQUENCY  $\omega_m$  AT WHICH IT OCCURS ARE DERIVED AS FOLLOWS

$$\omega_m = \text{GEOMETRIC MEAN OF THE TWO CORNER FREQUENCIES}$$
$$= \sqrt{\left(\frac{1}{aT}\right)\left(\frac{1}{T}\right)}$$
$$= \frac{1}{\sqrt{a} T}$$

Now the simplest approach to design this compensator is that we identified 2 corner frequencies of this transfer functions, this transfer function as 2 corner frequencies 1 is omega equal to 1 upon aT quite omega 1 another corner frequency we can call it omega 2 equal to 1 upon T and the this particular transfer function will provide you the maximum phase lead at a frequency which will be the geometric mean of these 2 corner frequency right.

Therefore here we consider this omega m as a geometric mean of the 2 corner frequencies which is obtained as square root of 1 upon 80 in to 1 upon T that is these 2 corner frequencies we identify we find out the geometric mean. Okay and the geometric mean of these 2 corner frequencies is equal to 1 divided by square root a in to T, the idea here is that when you plot plot for this transfer function right the phase angle versus frequency right then the maximum phase angle of this transfer function will occur at a frequency equal to omega m and this frequency is the geometric mean of these 2 corner frequencies, okay.

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$$G_c(j\omega) = \frac{1}{a} \left( \frac{1 + ja\omega T}{1 + j\omega T} \right)$$

$$\angle G_c(j\omega) = \tan^{-1} a\omega T - \tan^{-1} \omega T$$

For this transfer function for this transfer function  $G_{cs}$  for s equal to G omega. Let us write down what is the phase angle of this transfer function right to obtain the phase angle of this transfer function what we do is that we write down  $G_c j \omega$  equal to 1 upon a remains 1 plus j times a omega T that is for s equal to j omega 1 plus j omega okay, what is the phase angle of this transfer function. Obviously, the phase angle is that phase angle of the numerator minus the phase angle of the denominator and the phase angle of the numerator is tan inverse a omega T and of the denominator is tan inverse omega T and this is actually the phase angle of  $G_c j \omega$  that is for this function for this transfer function phase angle is given by this expression.

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$$\begin{aligned}\angle G_c(j\omega) &= \tan^{-1} \omega a T - \tan^{-1} \omega T \\ \tan \phi &= \frac{\omega a T - \omega T}{1 + (\omega a T)(\omega T)} \\ \text{For } \phi &= \phi_m \\ \omega &= \omega_m = \frac{1}{\sqrt{a}T} \\ \tan \phi &= \frac{a-1}{2\sqrt{a}} \\ \sin \phi &= \frac{a-1}{a+1} \quad \text{or} \quad a = \frac{1+\sin \phi_m}{1-\sin \phi_m}\end{aligned}$$

A very simple formula to compute the now my interest is my interest is to express the this phase difference right phase difference in a simple form that is in terms of  $a$  and  $T$ . Okay now what we do here is that we make use of this identity that is phase angle of this function is let us say  $\phi$  then  $\phi$  is equal to  $\omega a T$  minus  $\omega T$  divided by  $1$  plus  $\omega a T$  in to  $\omega T$  that is, we know that its suppose your this function transient  $\theta_1$  minus  $\theta_2$  right this is equal to  $\tan \theta_1$  minus  $\tan \theta_2$  divided by  $1$  plus  $\tan \theta_1$ ,  $\tan \theta_2$ .

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$$\tan(\theta_1 - \theta_2) = \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$$

Okay, now here  $\tan \theta_1$  is equal to  $\omega aT$   $\tan \theta_2$  is equal to  $\omega a$   $\omega T$  not  $a$  and the phase angle of this transfer function is  $\theta_1$  minus  $\theta_2$ , you will call this quantity as  $\phi$ . So that now I can write down  $\tan \phi$  equal to  $\omega aT$  minus  $\omega a$  divided by  $1 + \omega aT$  in to  $\omega a$ , okay a very simple expression of the phase angle of this transfer function.

Now we know that for  $\omega$  equal to  $\omega_m$  equal to  $1/\sqrt{aT}$  the phase shift is maximum. Okay and therefore what we do here is that we write down the expression for  $\tan \phi_m$  in this form that is you substitute in this expression  $\omega$  equal to  $\omega_m$ , okay and when you substitute  $\omega$  equal to  $\omega_m$  in this expression and simplify you will find that you get an expression for  $\tan \phi_m$  equal to  $-1$  divided by  $2$  times square root of  $a$ , this step you can just substitute and verify, okay.

Now this expression is further written in the form  $\sin \phi_m$  equal to  $-1$  or  $+1$  or  $a$  can be expressed as  $1 + \sin \phi_m$  divided by  $1 - \sin \phi_m$  then we have obtained very simple expression for  $a$  in terms of the required phase angle of the transfer function or the phase compensator that is if I want actually that this phase compensator should provide a phase shift equal to  $\phi_m$  right then what should the value of  $a$  and the value of  $a$  is obtain by the simple formula  $a$  equal to  $1 + \sin \phi_m$  divided by  $1 - \sin \phi_m$ .

This intermediate steps can be easily verified right then we have also now an expression  $\omega_m$  equal to  $1/\sqrt{aT}$  that is if I know that this is the frequency at which I want to get the required phase shift there is  $\omega_m$  known to me I start with this  $\omega_m$  the frequency at which I want to compensate for the phase lag of the system.

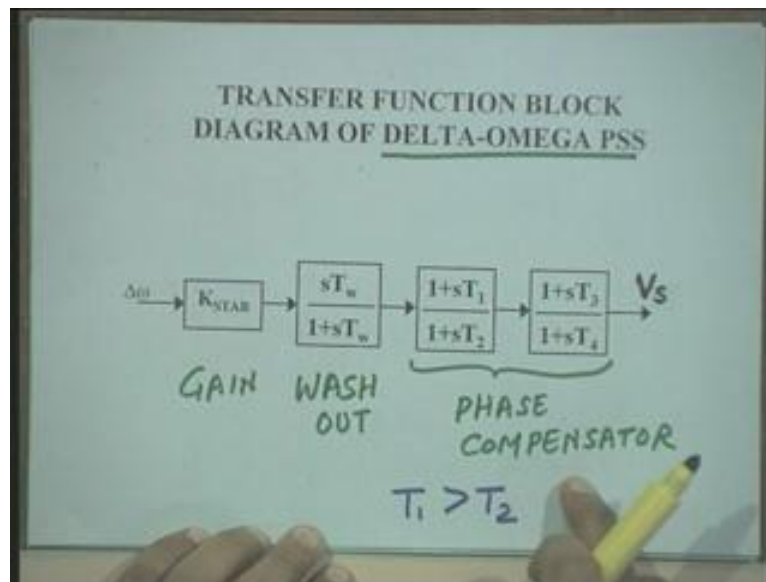
Okay, therefore  $\omega_m$  is known to me, so that using this expression and using this equation to I know both this things, I know what is the what is the phase shift of achieve that is the how much should be the phase shift or phase lead this phase compensator is to provide. I also know at what frequency these 2 provide okay. Therefore knowing these 2 information I have a very simple formula we are first I compute the expression for  $a$  because  $\phi_m$  is known to me and then use this expression use the value of  $a$  to obtain the expression or the value of  $T$  right. So that I can now I have now both the parameters  $a$  and  $T$  both.

Now the question arises that what is the frequency at which I should design my power system stabilizer this frequency is the rotor frequency, rotor oscillation frequency. However this rotor oscillation frequency we have seen varies over wide range, over a wide right and therefore this power system stabilizer should be design in such a fashion.

So that you design for 1 frequency but when the frequency varies from that value on or increases or decreases it still it should be in the position to give you reasonable phase compensation but I just now mens I told you very clearly that we would like to design power system stabilizer in a such a fashion.

So that so that it is it may have some under compensation but it should not have over compensation. The moment you have as a some degree of under compensation, we get the advantage of additional synchronizing torque coefficient, positive synchronizing torque coefficient and is helpful right but we should we should not target to over compensate because the moment you try to over compensate it will produce a negative synchronizing torque coefficient.

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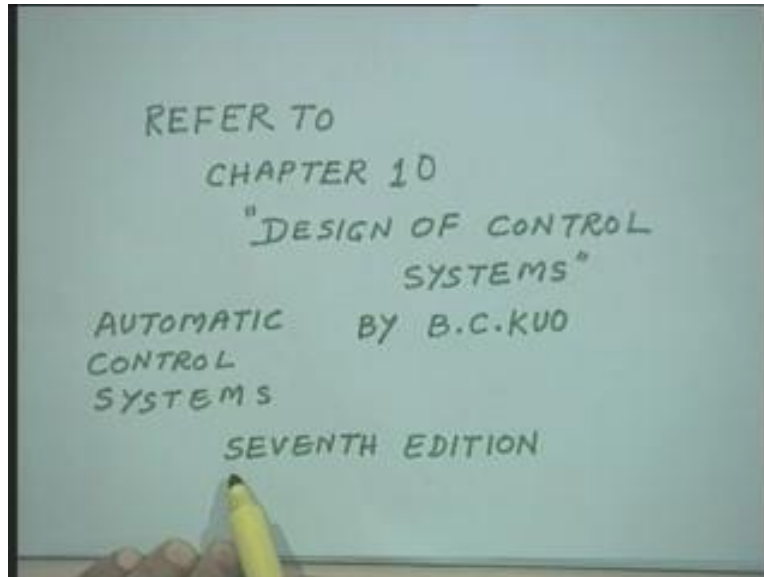
Okay a many times the required phase shift cannot be obtained using 1 stage of the phase lead compensator and in that case we go for 2 stages that is called 2 stage phase lead compensator, many times 1 can have 2 identical blocks that is here I shown this phase compensator right.

I have discussed how to design 1 stage phase lead compensator and we we can have 2 stages identical in that case suppose I want to compensate for say 85 degree let us say right therefore, instead of designing 1 phase compensator to compensate for this total 85 degrees I can design the so that 1 is for 85 by 2 another is for 85 by 2 generally the thumb rule is that 1 phase lead compensator block should be design to compensate around 50 degrees, the to understand the basic concepts of designing the phase lead compensator or for that purpose any linear compensator, I will advise you to read this chapter 10 which is design of control systems that is a title of the book is design of control systems written B. C. Kuo.

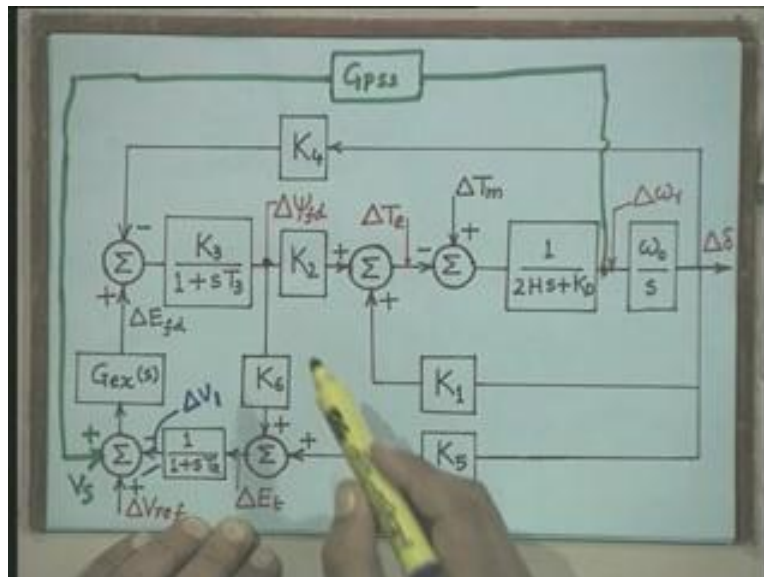
I am just sorry, let me correct it title of this chapter 10 is design of control systems the title of the book is automatic control systems the 7th edition by B. C. Kuo, B. C. Kuo is a very standard text book on automatic control systems and I am sure that you will be in a position to very clearly understand how this to be design. Now I will just quickly tell you how to obtain the state space model of the complete system that is when you have incorporated in a power system stabilizer right automatic voltage regulator in the system

therefore, we can develop a complete state space model of the single machine infinite bus system including AVR and PSS.

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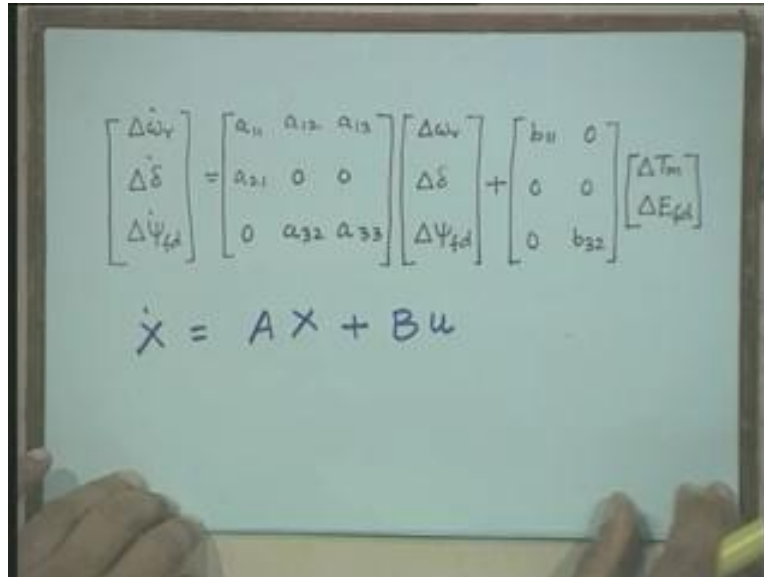
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We had developed earlier the state space model model considering considering the input signal from the exciter as  $\Delta E_{fd}$  that is this transfer function model actually if you just look here. This transfer function model I am sorry, the from this transfer function model that is if you see this is this portion actually that is we are not considering PSS we are not considering the excitation system and AVR therefore the state variables which are required to model the system in the state space form  $\Delta \omega_r$ ,  $\Delta \delta$  and  $\Delta E_f$ .

$\psi_{fd}$ . With this 3 state variables right 1 can write down the state space model of the system and input to this model or input to this state space model will be  $\Delta T_m$  and  $\Delta E_{fd}$  right.

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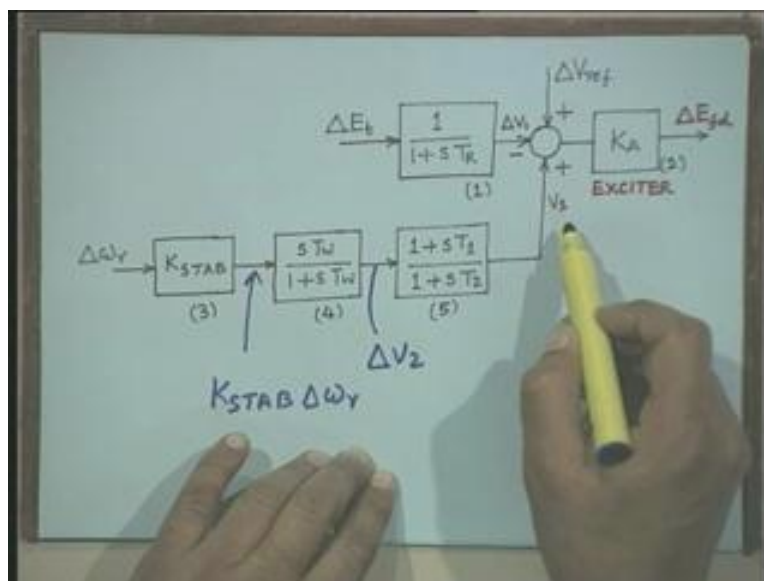


$$\begin{bmatrix} \Delta \dot{\omega}_r \\ \Delta \dot{\delta} \\ \Delta \dot{\psi}_{fd} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \Delta \omega_r \\ \Delta \delta \\ \Delta \psi_{fd} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & 0 \\ 0 & b_{32} \end{bmatrix} \begin{bmatrix} \Delta T_m \\ \Delta E_{fd} \end{bmatrix}$$

$$\dot{X} = AX + Bu$$

The the model is written in the form  $\Delta \omega_r$  dot  $\Delta \delta$  dot  $\Delta \psi_{fd}$  dot at this is my  $X$  dot and this is the  $A$  matrix  $A$ , this is the state vector this becomes the input matrix  $B$  and this is your input vector  $B_u$  right therefore in this case the input is comprising of  $\Delta T_m$  and  $\Delta E_{fd}$ .

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Now you can obtain more detailed model including including the AVR and PSS. Now to develop this model I will just tell you few salient steps, the additional portion of the transfer function to be incorporated in the model is shown here is the additional portion because we already developed actually the transfer function model of the third order system. Now the exciter is represented by a gain  $K$  only you can represent the exciter depending upon the what type of exciter you are using okay.

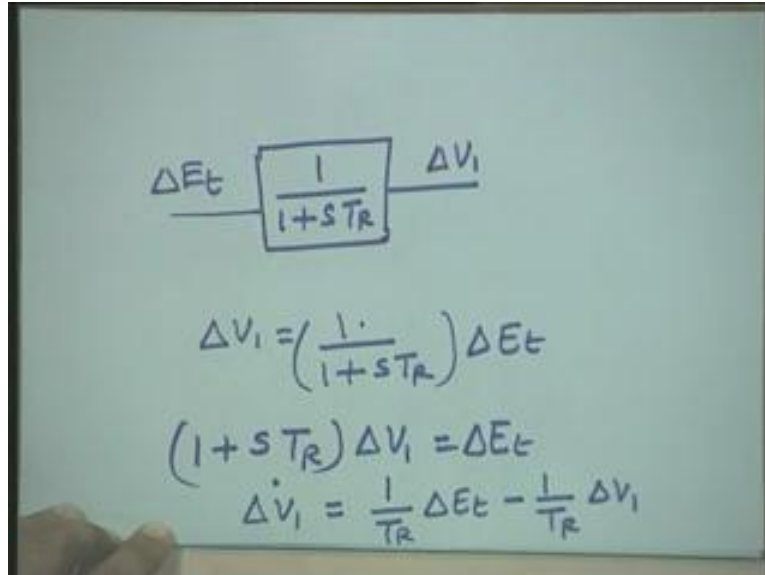
We know the detail models of the different type of excitation system therefore here for the purpose of simplicity we are representing this by a constant  $K_a$  only and this representing static excitation system IEEE  $sT_1$  a model. Okay this is the change in the terminal voltage  $\Delta E_t$  this is the transfer function of voltage transducer terminal voltage transducer that is we get a  $\Delta V_1$  this is the transfer function model of the PSS, this portion you can just see here I have shown in this model a single stage phase lead compensator right. If if we necessary 1 can put 1 more stage 2 stage phase lead compensator right and accordingly you can modify your study space model and the question is that how do we develop the differential equations, linear differential equations for this system.

So that they can be augmented with the existing or the existing system model is augmented by adding these equations additional equations. Now the state variables which we will have to define new state variables 1 is because earlier these are the only in put signal here right use state variables which your define will be  $\Delta v_1$  which is the output of this transfer functional block,  $V_s$  the output of power system stabilizer then here then we call this another state variable you call this is a  $\Delta V_2$ .

Okay and here there is no state variable required because this is only gain therefore at this point it is the signal is  $K_{stab}$  into  $\Delta \omega$ , okay therefore so far this this block is concern in this block the input is this quantity output is  $\Delta V_2$ . So far this second block or phase compensator is concern input is  $\Delta V_2$  output is  $V_s$ , this block 1 is concerned input is  $\Delta E_t$  output is  $\Delta V_1$ . Therefore, these 3 state variables are required to to develop the complete model of the machine infinite bus system including PSS and AVR right.

Now the equations can be written very easily let us **us** write the equation for first a block that is here transfer function is  $1 / (1 + s T_R)$  output of this is  $\Delta V_1$  input of this is  $\Delta E_t$ , okay. Now this model which we are talking about is small perturbation model and in small perturbation model all these state variables initially are 0. There is the perturbed values like see  $\Delta V_1$   $\Delta E_t$  right a tan equal to 0,  $R_0$  and therefore we can replace  $s$  by  $p$  okay and you can write down the equation in this form  $\Delta V_1$  equal to  $1 / (1 + S \text{ times } T_R)$  into  $\Delta E_t$  or you can write down here now,  $1 / (1 + S \text{ times } T_R)$  into  $\Delta V_1$  equal to  $\Delta E_t$  a very simple right.

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The image shows a handwritten block diagram and three equations on a whiteboard. The block diagram consists of a box with the transfer function  $\frac{1}{1+sT_R}$ . An input signal  $\Delta E_t$  enters the box from the left, and an output signal  $\Delta V_1$  exits the box to the right. Below the diagram, the following equations are written:

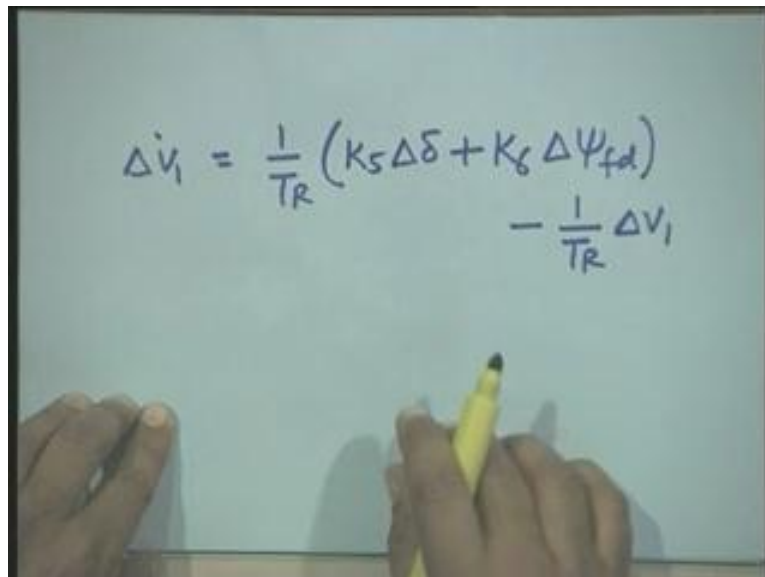
$$\Delta V_1 = \left( \frac{1}{1+sT_R} \right) \Delta E_t$$

$$(1+sT_R) \Delta V_1 = \Delta E_t$$

$$\Delta \dot{V}_1 = \frac{1}{T_R} \Delta E_t - \frac{1}{T_R} \Delta V_1$$

Now here if a replace  $S$  by  $d$  by  $dt$ ,  $S$  is replace by  $d$  by  $dt$ , then you will and divided by  $T_R$  all through you can write the equation in the form that  $\Delta \dot{V}_1$  will be equal to  $1$  upon  $T_R$   $\Delta E_t$  minus  $1$  upon  $T_R$   $\Delta V_1$ . Okay, now to make this equation complete equation because  $\Delta E_t$  is not a state variable  $\Delta E_t$  is not a state variable but  $\Delta E_t$  can be expressed in terms of the state variables.

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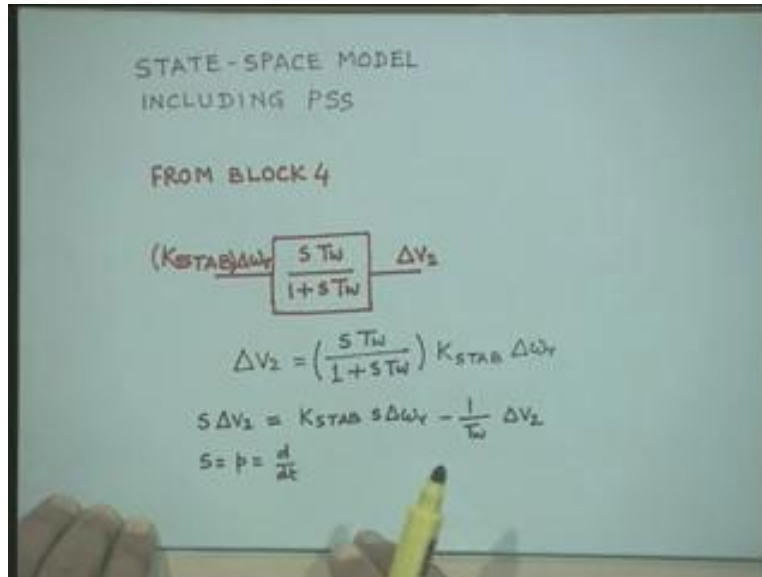
The image shows a handwritten differential equation on a whiteboard. The equation is:

$$\Delta \dot{V}_1 = \frac{1}{T_R} (K_5 \Delta \delta + K_6 \Delta \psi_{fd}) - \frac{1}{T_R} \Delta V_1$$

Therefore  $\Delta \dot{V}_1$  is written as  $1$  upon  $T_R$   $\Delta E_t$  is equal to  $K_5$   $\Delta \delta$  plus  $K_6$   $\Delta \psi_{fd}$ , okay minus  $1$  upon  $T_R$   $\Delta V_1$  okay therefore now I have written 1 additional equation which is a differential equation it is expressed in terms of the these 3

state variables  $\Delta \delta$ ,  $\Delta \psi$ ,  $\Delta f_d$  and  $\Delta V_1$  okay. Similar approach is to be used for deriving additional state variable equations right I will just illustrate for  $\Delta V_2$  also.

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Now to understand how to develop the equation corresponding to this wash out block, we can write down  $\Delta V_2$  equal to  $s T_w$  or  $1 + s$  times  $T_w$  input is  $K_{STAB}$  into  $\Delta \omega_r$ .  $\Delta \omega_r$  is a state variable. Now you perform the similar operation you will find that  $s$  times  $\Delta V_2$  equal to  $K_{STAB} s \Delta \omega_r$  minus  $1$  upon  $T_w$   $\Delta V_2$ . Okay that is you cross multiply and then collect the terms that you put  $s \Delta V_2$  on left hand side and all other terms on right.

Now in this equation you find here  $s \Delta \omega_r$ ,  $s$  is  $d$  by  $dt$ . So that this is basically your  $\Delta \omega_r$  dot term  $\Delta \omega_r$  dot term because whenever you write differential equation on right hand side we will have only the expression which is the linear function of state variables right therefore what we do is that this  $\Delta \omega_r$  dot is replaced by this equation, we have the equation here  $\Delta \omega_r$  dot is equal to  $a_{11} \Delta \omega_r$  plus  $a_{12} \Delta \delta$  plus  $a_{13} \Delta \psi$  plus  $b_{11} \Delta T_m$  we have it right.

So that when you now substitute the expression for  $d$  by  $dt$  of  $\Delta \omega_r$  in this expression, you will get a equation for  $\Delta V_2$  dot and this  $\Delta V_2$  dot is now written in terms of all the state variables like  $\Delta \omega_r$ ,  $\Delta \delta$ ,  $\Delta \psi$ ,  $\Delta f_d$ ,  $\Delta V_2$  and  $\Delta T_m$  because when you write this right this differential equation it will be function of state variables plus input variables.

Now here input is  $\Delta T_m$  right you follow the similar approach for writing the transfer function of phase lead compensator that is in the phase lead compensator the input is  $\Delta V_2$ , output is  $V_s$  the transfer function is  $1 + s$  time  $T_1$ ,  $1 + s$  time  $T_2$  right now  $V_s$  is one of the state variables. So that we can write down the equation in this form you

have to simplify the expressions following the same approach, ultimately our requirement is that the left right hand side of the expression should be function of all state variables and input way the last equation is for this block a summation block here.

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$$\begin{aligned}\dot{\Delta V}_2 &= K_{STAB} \left( a_{11} \Delta \omega_r + a_{12} \Delta \delta + a_{13} \Delta \psi_{fd} \right. \\ &\quad \left. + \frac{1}{2H} \Delta T_m \right) - \frac{1}{T_w} \Delta V_2 \\ &= a_{51} \Delta \omega_r + a_{52} \Delta \delta + a_{53} \Delta \psi_{fd} \\ &\quad + a_{55} \Delta V_2 + \frac{K_{STAB}}{2H} \Delta T_m\end{aligned}$$

where,

$$\begin{aligned}a_{51} &= K_{STAB} a_{11} \\ a_{52} &= K_{STAB} a_{12} & a_{54} &= a_{56} = 0 \\ a_{53} &= K_{STAB} a_{13} \\ a_{55} &= -\frac{1}{T_w}\end{aligned}$$

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FROM BLOCK-5

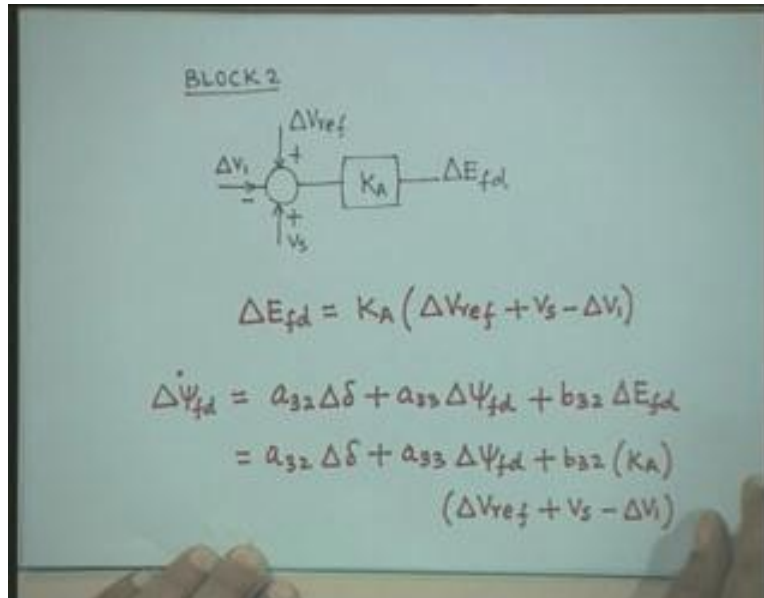
$$V_3 = \left( \frac{1 + sT_1}{1 + sT_2} \right) \Delta V_2$$

$$V_3 + T_2 s V_3 = \Delta V_2 + T_1 s \Delta V_2$$

$$s V_3 = \frac{T_1}{T_2} s \Delta V_2 + \frac{1}{T_2} \Delta V_2 - \frac{1}{T_2} V_3$$

$$\begin{aligned}&= \frac{T_1}{T_2} \left( a_{51} \Delta \omega_r + a_{52} \Delta \delta + a_{53} \Delta \psi_{fd} \right. \\ &\quad \left. + a_{55} \Delta V_2 + \frac{K_{STAB}}{2H} \Delta T_m \right) \\ &\quad - \frac{1}{T_2} \Delta V_2 - \frac{1}{T_2} V_3\end{aligned}$$

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Now what we do is that delta  $E_{fd}$  is written in terms of  $K$  that is delta  $E_{fd}$  is equal to  $K$  into delta  $V$  reference plus  $V_s$  minus delta  $V_i$ . Okay, now this delta  $E_{fd}$  appeared in our earlier equation for delta  $\psi_{fd}$  that is if you see this equation for delta  $\psi_{fd}$  this delta  $\psi_{fd}$  is a substitute actually the express what delta  $efd$  because when delta  $\psi_{fd}$  was written delta  $\psi_{fd}$  dot equal to right function of delta delta delta omega  $R$  delta  $\psi_{fd}$  plus function of delta  $efd$  therefore this  $efd$  is now not state variable is to be eliminated.

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$$\dot{\Delta \Psi}_{fd} = a_{32} \Delta \delta + a_{33} \Delta \Psi_{fd} + a_{34} \Delta V_i + a_{35} V_s + b_{32} K_A \Delta V_{ref}$$

Okay, when all these things are done there is you can eliminate this delta  $efd$  and you will get the expression in the form delta  $\psi_{fd}$  dot equal to  $a_{32}$  delta delta  $a_{33}$  delta  $\psi_{fd}$   $a_{34}$

for  $\Delta V_{1a36}$   $V_s$   $a_{b32}$   $K_a$   $\Delta V$  reference that is what we have done here is that we have written 3 additional equations, okay and modified the **the** third equation the reverse corresponding to  $\Delta \psi f_d$  dot.

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$$\begin{bmatrix} \dot{\Delta \omega_r} \\ \dot{\Delta \delta} \\ \dot{\Delta \psi_{fd}} \\ \dot{\Delta V_1} \\ \dot{\Delta V_2} \\ \dot{V_s} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & a_{36} \\ 0 & a_{42} & a_{43} & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & a_{55} & 0 \\ a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} \Delta \omega_r \\ \Delta \delta \\ \Delta \psi_{fd} \\ \Delta V_1 \\ \Delta V_2 \\ V_s \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & 0 \\ 0 & b_{32} \\ 0 & 0 \\ b_{51} & 0 \\ b_{61} & 0 \end{bmatrix} \begin{bmatrix} \Delta T_m \\ \Delta V_{ref} \end{bmatrix}$$

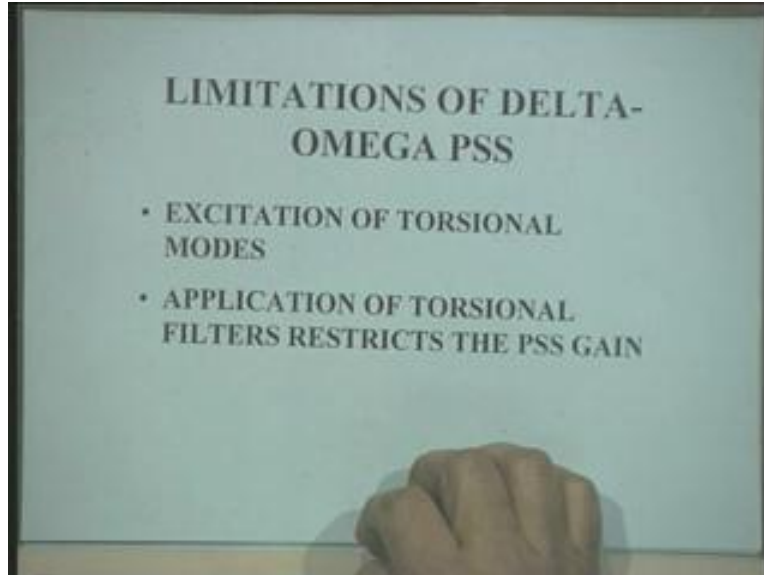
$$\dot{X} = AX + BU$$

Okay and the the total model of the system can be written in this form it comes out to be a 6th order model there is there are 6 difference state variables  $\Delta \omega_r$   $\Delta \delta$   $\Delta \psi_{fd}$   $\Delta V_1$   $\Delta V_2$   $V_s$ . Okay and we can write down this model in the form again  $\dot{X} = AX + BU$ , now this  $u$  is  $\Delta T_m$  and  $\Delta V$  reference that is input is now the mechanical talk that is the the incremental change in mechanical dot  $\Delta T_m$  and the change in reference voltage,  $\Delta V$  reference and  $\Delta e_{fd}$  has now been eliminated from the model.

Now in this model once the model is developed we know actually that all these parameters that is coefficient the the elements of this a matrix, they all function of loading condition and system parameters right, one can find out the Eigen values of this a metrics to understand the stability of the system. Okay, now I will just quickly tell you 1 or 2 more points before I close the discussion.

The basic limitations of  $\Delta \omega_r$  PSS basic limitation the the limitations are 1 is that this  $\Delta \omega_r$  PSS can excite the torsional modes, can has the has the potential of exciting the torsional modes. In case you have large gain of the PSS, if the PSS gain is kept large then it can excite the torsional modes. Many times to overcome the problem of excitation of torsional modes, we provide filter in the PSS path the moment you provide filter right then this is again restricts the PSS gain that is we cannot have the gain as much as we want because if you if you want to can say have the desired dumping right.

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Then the gain setting of the PSS will give the required dumping ratio right but if you try to design the PSS so that you achieve the required dumping for electromechanical mode right then there are other problems the other problems are the possibility of excitation of torsional modes and the limitation on the PSS gain. Okay, let me some of here that we have discussed today the details of the power system stabilizer to be applied we have analyzed the analyze the system with PSS. Okay and we have also seen how this PSS can be realize using operational amplifiers and at the end we have derived the complete state space model of the machine infinite bus system including AVR and PSS.

I will suggest that all of you should derive this model yourself right and that you can study the behavior of the system with PSS, you can make use of the mat lab software package to find out the Eigen values of the system matrix and to simulate the dynamic performance your system by giving some perturbations may be perturbations may be in terms of perturbation in mechanical torque you can just make  $\Delta T_m$  equal to some .01 or change in the reference voltage and see the behavior of the system. Thank you!