Power System Dynamics

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Lecture -18

Small Signal Stability of a Single Machine Infinite Bus System

Friends, today we start with the study of small signal stability. We shall devote our discussion towards the small signal stability of a single machine infinite bus system that is for understanding the concepts related to the small signal stability, we will consider a simple system and that is a single machine connected to an infinite bus. Okay the small signal stability is the ability of the system to maintain synchronism under small perturbations. The small perturbations continuously occur in any power system due to changes in loads and generations.

Now for analyzing the small signal stability of any system, the system model can be linearized around an operating point that is the disturbances are considered to be so small or incremental in nature. So that we can develop a linear model of the system around the operating point, once we develop the linear model of the system we can understand the behaviour of the system under small perturbations, various parameters of the system which affect the stability of the system further the moment we have a linear model we can apply the linear control system theory for designing the controllers.

Now here when I talk about the controller particularly we are interested in designing the excitation system control that is the voltage regulator and the power system stabilizers as we will see that in any power system right, the actual system is some what complex it is not as simple as a machine connected to infinite bus is always a multi machine system right. Now in any multi machine system as you know that the system will have different modes of oscillations, the modes of oscillations are classified as as local modes of oscillations, inter area modes of oscillations and the control modes okay.

Now, primary requirement of the system is that this system should have for stability this will have positive synchronizing torque coefficient and positive damping torque coefficient. The stability of the system will be affected if any of these two torques or any of these two torque coefficients become negative. Okay now to start with we will study first a simple model considering constant flux linkages in the field winding that is we will first start with the constant flux linkage model, next we will include the the field winding dynamics that is we will consider next step the constant field voltage and must we assume these voltages applied to the field winding as a constant okay we the model is developed so that we take care of the the dynamics of the field winding that is changes in the field flux linkages.

This particular stage when we say that the applied voltage is fixed means it is a practically a manual control after studying this model with constant field voltage, we will extend it to the model which will include the automatic voltage regulator and we will study the affect of the parameters of the excitation system and gain setting of the automatic voltage regulator on the stability of the system. Then the next step will be we include auxiliary controllers that is the power system stabilizer that is this the stage by step development will give us the complete insight into the small signal stability problem.

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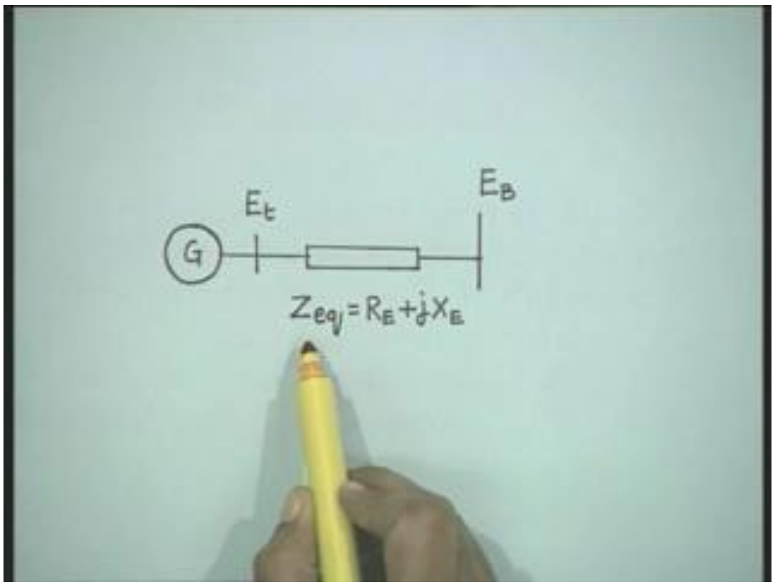

The system which we consider for small signal stability to start with is a machine infinite bus system where a synchronous generator is connected to a large system which can be characterized by a infinite bus, the equivalent circuit can be represented as generator the we represent the terminal voltage of the generator, the line is represented by an equivalent impedance Z equivalent equal to R_E plus j times X_E and connected to an infinite bus right.

Now here here even in a multi machine system, multi machine system right. Now if you if you take out one generator and the connected line right then just will be system if you represent by a infinite bus then it becomes a machine infinite bus system. Now as we continue our studies to simplify our model and understanding the resistance of the line or the equivalent resistance R_E we may may be ignored.

Now when we consider a classical model that is we assume the field flux linkage is to be constant. Okay now when we consider the classical model of the synchronous generator then the classic the synchronous generator is represented by a constant voltage behind direct axis transient reactance right. Now here we will represent the voltage behind transient reactance by the symbol E prime and the voltage of the infinite bus as E_B, okay and the phase angle, phase angle between E prime and E_B is denoted by delta right and that is the power angle of the system.

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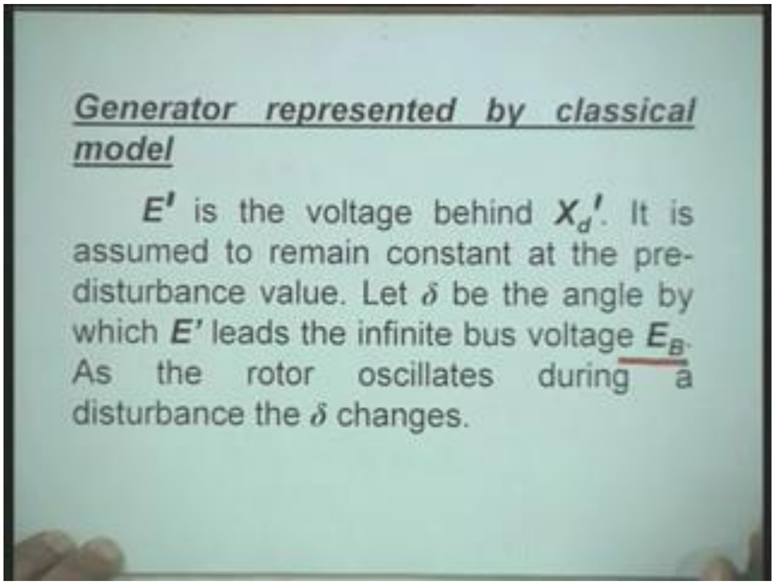

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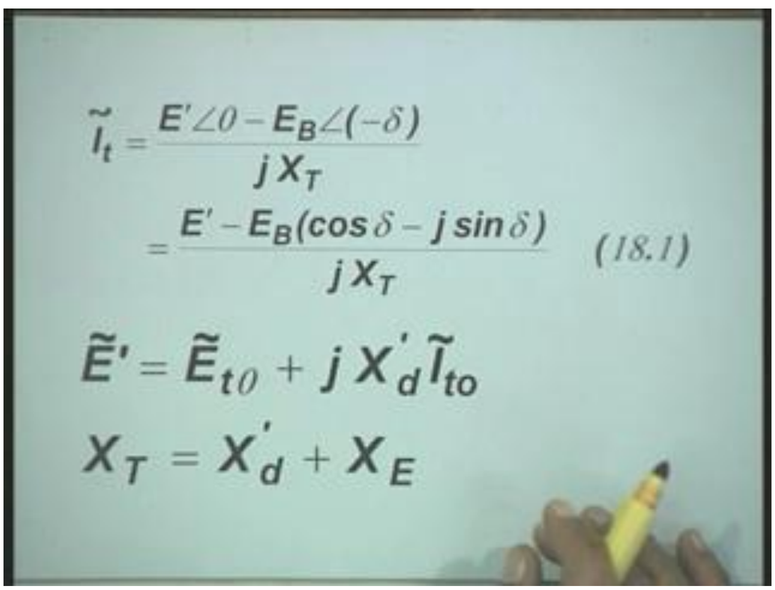

Now for this machine infinite bus system we have derived earlier the the power angle characteristic okay the power angle characteristic to derive we can start with the the terminal current of the synchronous generator as the E prime minus E_B. Now here we are representing the ah voltage behind transient reactance if it is represented as the as E prime angle 0 that is you you consider this as a reference voltage generally we do other way around we consider the infinite bus voltage as reference and the internal voltage leads the reference voltage by some angle delta okay but it does not matter very much.

We can write down the expression for current I_t this is the first step for solving this problem by this what we know will be the the you know whenever we start solving the problem the terminal voltage of the machine will be known to us right and therefore, once the terminal voltage is known we can find out the internal voltage by this formula E prime equal to E_{to} plus j times X_d prime into I_{to}, where here now we denote this total reactance X_d, X_T as X_d prime plus X_E earlier the power angle characteristic which we derived was relating the terminal voltage with the internal voltage but now here we will be deriving the power angle characteristic relating the infinite bus voltage with respect to the internal voltage. Okay therefore the total reactance which comes will be X_d prime plus X_E where X_E is the reactance of the transmission line okay.

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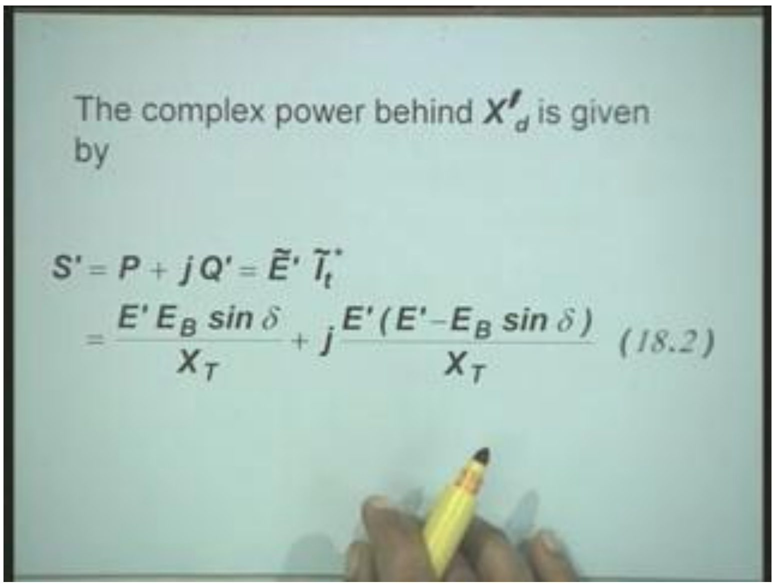

Then the complex power behind X_d prime is given by S pi, S prime is equal to P plus j times Q prime you can call it P prime. Okay this is E prime into I_t complex I am sorry I_t conjugate, I_t conjugate that is the complex power is P prime plus j times Q prime and this is obtained by multiplying this voltage E prime by conjugate of I_t, okay when you make the simplifications.

We can write down the expression for air gap torque. Now we all know actually that in case we neglect losses right then the air gap torque is equal to the the power output impairing your system that when you consider the per unit system then the power output at the terminal of the machine is same as the air gap torque. Okay and this is our standard power angle characteristic, now here at this stage I would like to mention that in case instead of having a non-salient pole machine, if we have a salient pole machine right then the power angle characteristic will be different from what is given here, it will have a another term which is known as the reluctance torque. Okay and one can write down the expression for the power angle but this is the primary requirement to start with. The next step is you will linearize you linearize the expression for air gap torque around an operating point.

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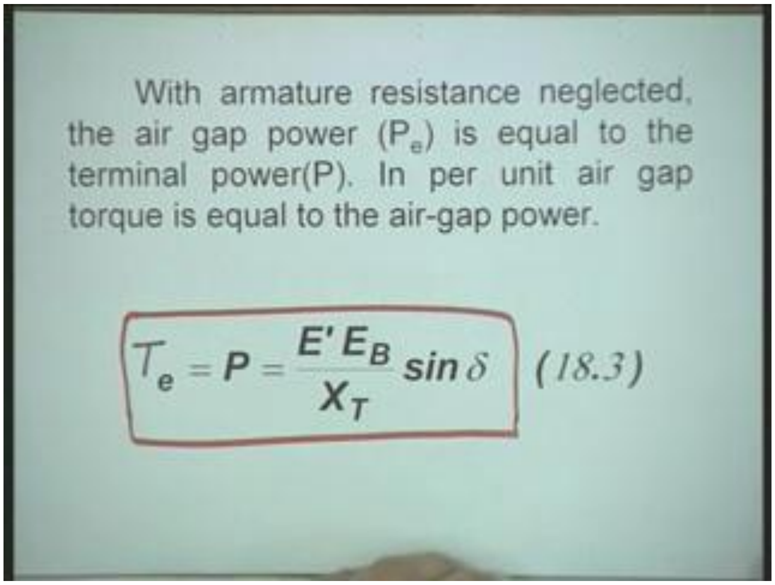

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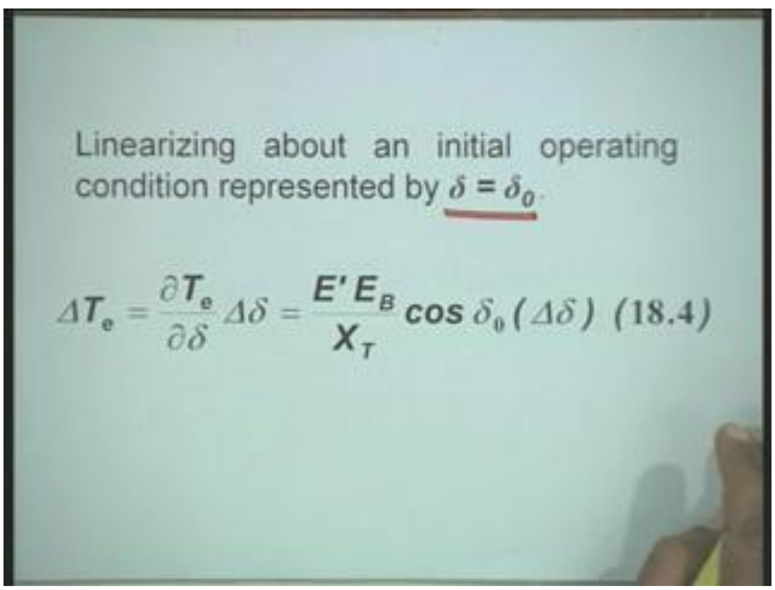

Our initial operating condition is characterized by the angle delta equal to delta naught that when we are operating and transferring certain amount of power right the initial value of delta is delta equal to delta naught, okay. The linearized equation is written in the form delta T_e is equal to partial derivative of T with respect to delta into delta delta where this expression is very simple there is only a T_e is function of function of one variable only here. We are assuming this E prime and E_B constant, okay. The infinite bus voltage assumed to be constant the voltage behind transient reactance is assumed to be constant the total line reactance is constant and therefore the torque is function of delta only one variable and therefore when you obtain the small change in the air gap torque delta T is equal to partial derivative of delta T divided delta delta into delta delta.

Therefore, here basically this partial derivative becomes derivative term itself right there is no other you can say parameter on which the T_e depends okay but in case actually these are also variables right then when I talk about this term it is the partial derivative with respect to delta not delta when you take other terms also then you have to write the partial derivative of this quantity with respect to not delta say when you take this partial derivatives there there so many variables will be there.

You have to keep on taking all the variables then you will have delta delta delta E prime delta E prime like this it is basically it is actually the Taylor series exponents time to expand the equation around the operating condition right. Now we come to the basic equations of motion, we have developed earlier the swing equation of the synchronous generator and the swing equation is a second order non-linear differential equation. This equation can be represented by two first order linear I am sorry 2 first order non linear differential equations it { it will continue to be non linear.

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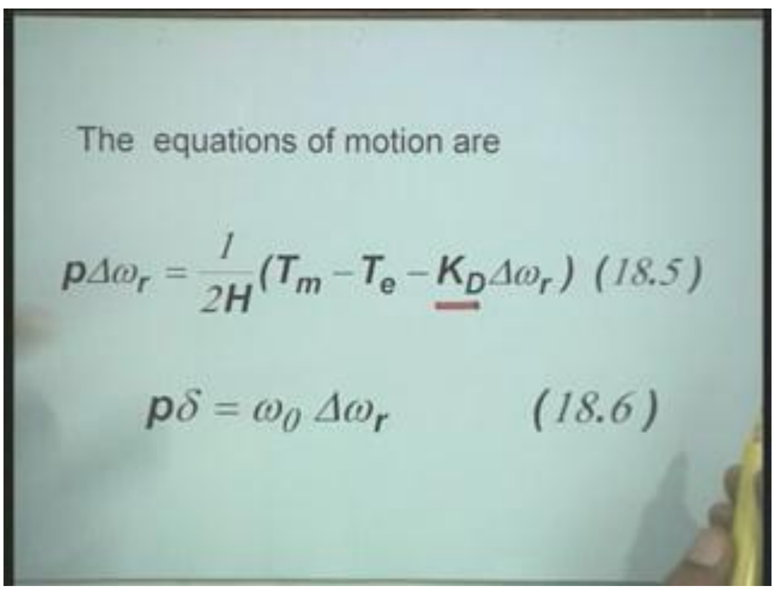

These two equations are written here as p delta omega r equal to 1 upon 2H T_m minus T_e minus K_D delta omega r and second equation is p delta equal to omega naught delta omega r, where delta omega r is the deviation of the rotor speed with respect to synchronous speed right. The K_D is the damping torque coefficient to for the sake of completeness right, we include damping term also well earlier when we discussed actually the K_D was not explicitly included in the equation but the complete equation is in this form.

Now when the equation is written in this form the speed deviation the delta omega r x is is expressed in per unit delta omega r is expressed in per unit the mechanical torque, electrical torque are also expressed in per unit. We know how to define the initial constant H the time in this equation is the time instead of putting in per unit we always prefer to express time in second is right and therefore, the second equation has p delta equal omega naught into delta omega.

Now these are the 2 basic equations around which the the we will be you can say developing the small signal model using these equations, small signal model and studying the basic concepts of small signal stability. Now you what we do is that this this these two equations that is 18.5 and 18.6, we will linearize around the operating condition okay.

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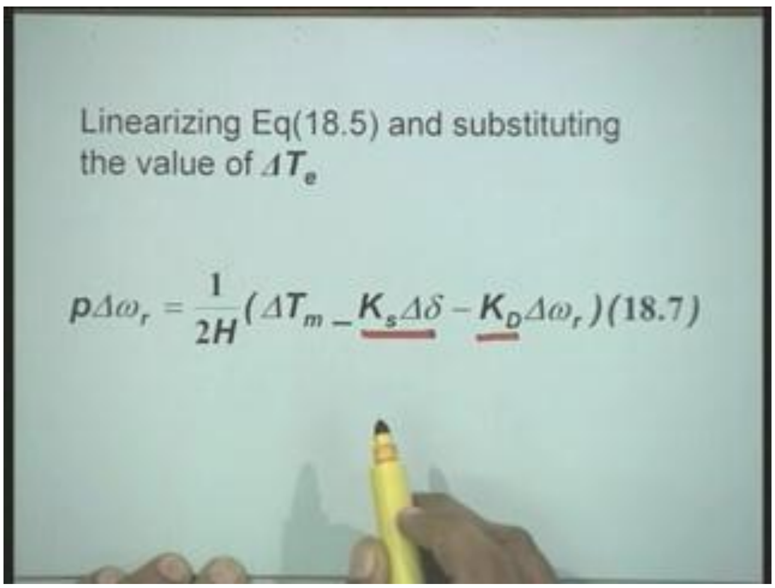

Now to linearize this around the operating condition one assumption here which we are making is that there is a mechanical power input remains constant that is T_m is constant right and T_m is equal to the initial power output that is T_{eo}. Okay and therefore when we talk about the small perturbations our equation will become p delta omega r equal to 1 upon 2 H delta T_m minus K_s delta delta minus K_D delta omega r this delta T_m delta T_m is considered to be a change in mechanical torque.

Okay but generally this change is included in the model included in the model to obtain the dynamic performance by giving a small change in the mechanical torque right. Otherwise the mechanical torque remains constant this delta T term has been replaced by K_s delta delta and this term K_s is synchronizing torque coefficient right and what will the unit of this coefficient unit per unit torque. Now in this equation actually be the delta delta is expressed in radians electrical radians. Okay similarly, the unit for K_D the torque in per unit per, sir in previous equation T delta equals to omega r naught into delta omega r, so all this equation comes because T delta equal to I think omega naught plus delta omega delta r. Now if you see the basic equation it is like this, d delta by dt is equals to omega minus omega naught that omega naught is the synchronous speed okay. Now what we do here is that this difference we do not by in fact actually this speed actual speed is called omega r is the speed of the rotor. Okay this will do not this difference is noted by delta omega r. Now what we do is that divide this by omega naught, so that this quantity becomes in per unit and multiplied by omega naught.

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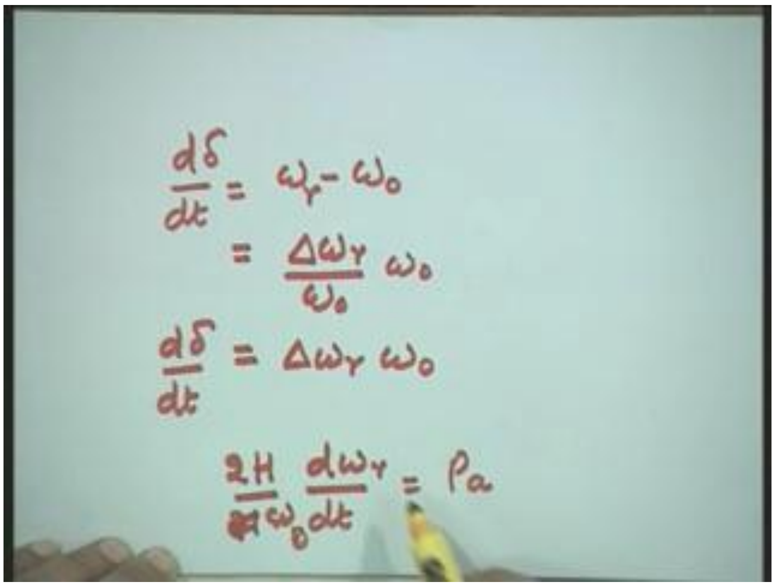

So that I can represent d delta by dt as as delta omega r into omega naught while this delta omega r is expressed in per unit. Okay while in this equation in this whole equation delta is expressed in electrical radians because when you solve these problems, you have to be very clear about the units. Okay see originally when you see the our swing equation right, it is of the form d omega divided dt d omega r divided by dt 1 upon 2, I am sorry 2H divided by s 2H divided by omega r omega r, okay equal to accelerating power okay.

Now here when I express this omega r in per unit this is this is omega naught not omega r this is omega naught this is omega r right then this term is combined here. Okay so that I have this coefficient equal to 2H only that is that is when you write the equation in this form, 2H d omega r by dt equal to P_a. Okay here the this omega r is a per unit further omega r can be represented as as omega naught plus delta omega r that the omega naught is in this particular case when it is per unit omega naught will become 1 that is you can put there is a 1plus delta omega r therefore, when you substitute this expression here I can write down the equation as 2 times H d by dt of delta omega r equal to P_a, is it okay that is that is since we know actually the derivative of this 1 is 0 therefore d by dt of delta omega r is same as d omega r by dt and that is why when you look at the swing equation which we have written here.

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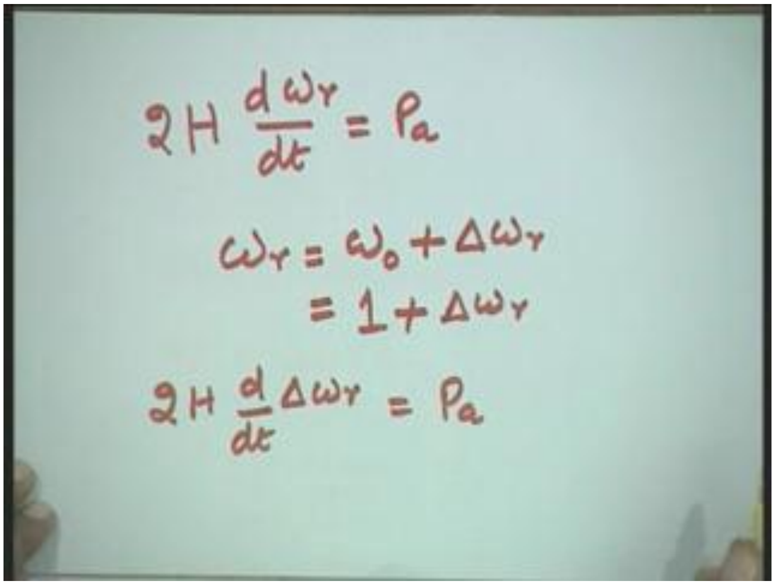

Okay original swing equation this is not a linearized equation we can write down this delta omega r. Okay is it clear actually ah these 2 equations are very important actually because when you solve a given problem right the it is very important to understand that how the quantities are expressed what quantities are in per unit? what the unit of time? what is the unit of delta? what is the unit of speed deviation? all these terms are very important. Okay now when you look at this equation.

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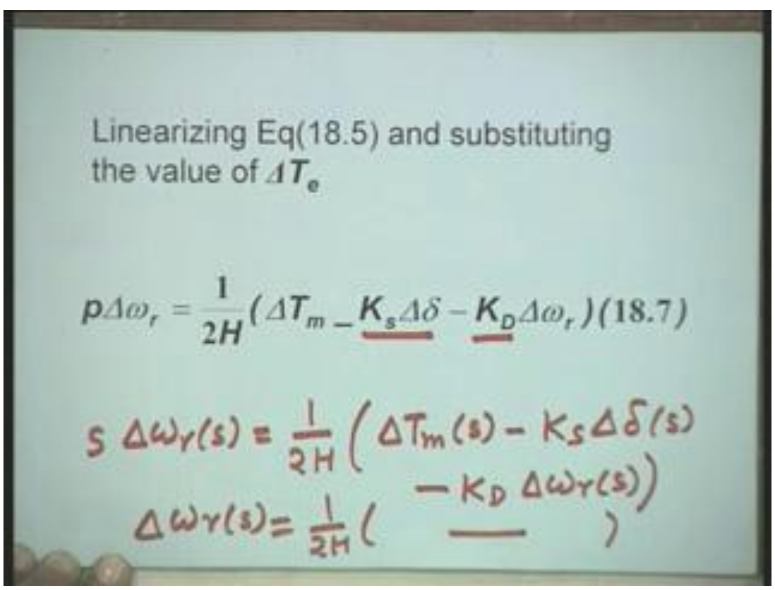

This is a equation in terms of small variations of speed angle and mechanical torque, okay and therefore we can take the Laplace transform of this equation right. When you take the Laplace transform it will become something like this s times delta omega r, s equal to 1 upon 2 H delta T_m (s), okay minus K_s times delta delta s minus K_D times delta omega r s that is in this equation all these variables are the Laplace transform. Okay and therefore now what we can do is that we can write down we can write down delta omega rs as 1 upon 2H times this whole expression okay.

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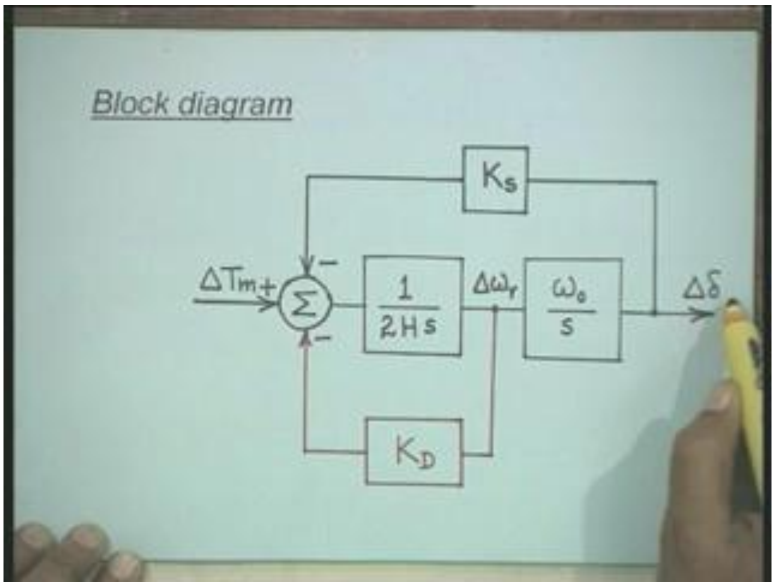

Now this equation is represented in the block diagram form here at this summasing at this summation point we have delta T_m K_s times delta delta that is your delta T_e that is delta delta is here delta T and K_D times delta omega r. Now when I represent this in the block diagram form all these variables are the Laplace transform but for the sake of convenience, we may not write ah delta delta as delta delta with s. Okay but we understand that in the in the block diagram or the transfer function model all the variables which we represent they are the Laplace transform of the original variables. Okay delta T_m also delta T_m (s) therefore this is the summation point where we derive a quantity which is given in this bracket delta T_m minus K_s delta delta minus K_D delta omega.

Now this quantity when it is multiplied, now here it is 2 times H_s when you multiply this by this block is one upon 2 times Hs right, we get delta omega r here okay. Now the next step here is to linearize the the second equation. Our second equation was p delta equal to omega naught into delta omega r but this delta delta is our our actually the delta can be written as delta o plus delta delta and therefore p delta is same as p delta delta because delta o is constant that is if you take the derivative of this term that is d delta by dt right then this comes out to be same as d delta delta by dt.

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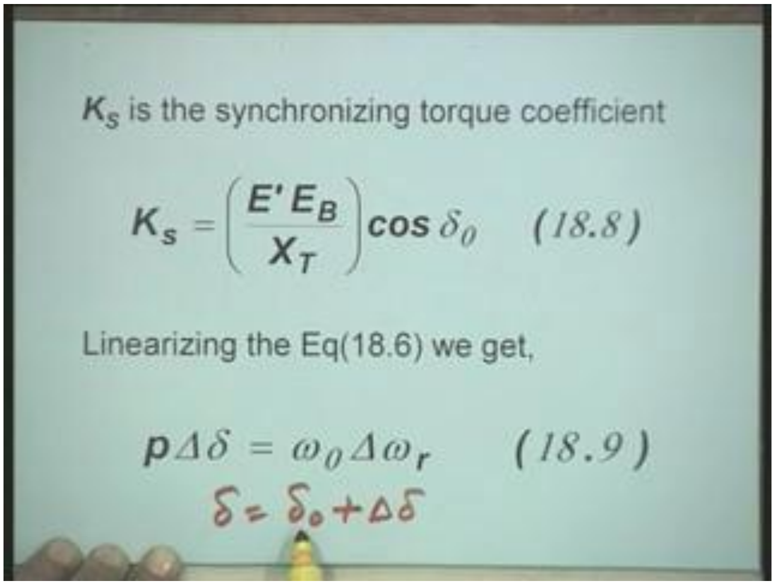

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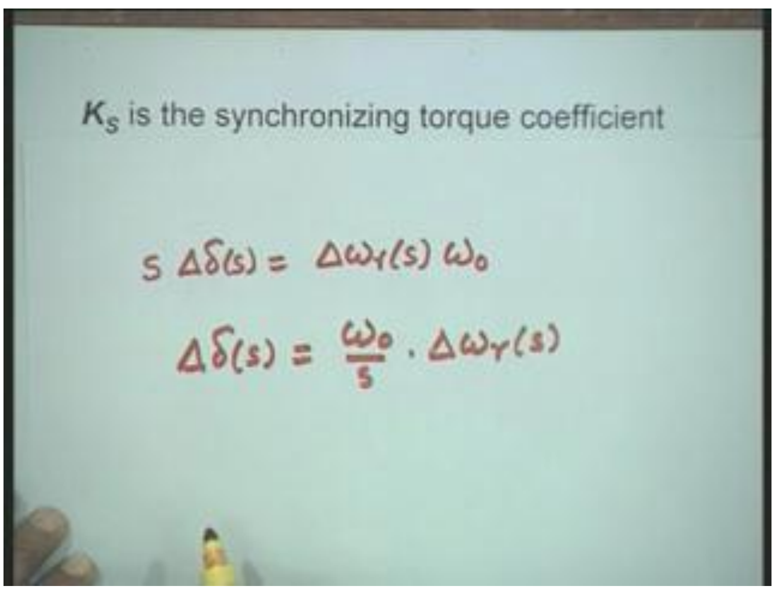

Okay now if you take the Laplace transform of this equation then it will be s times, s times delta delta s equal to delta omega r s into omega naught therefore, I can now write down delta delta s equal to omega naught by s into delta omega r s. Okay therefore the the speed deviation right is related to the angle deviation by this transfer function omega naught by s. Okay and therefore in this block diagram the this transfer function omega naught by s relates the angle deviation that is delta delta to delta omega r. Okay and therefore this block diagram is extremely important to understand the small signal stability of a power system because we will be developing the complete model including including additional blocks to this basic block diagram right. Now the next step to understand the whole thing is that we develop a characteristic equation characteristic equation of this system right.

Now to develop the characteristic equation, we can make use of this block diagram and relate this output delta delta this output delta delta is to be related to change in mechanical this is the input right therefore, if you look at this block diagram input is changed in mechanical torque output is delta delta therefore this is a single input, single output system. Okay they normally they call it SISO single input single output system and you can simplify this model and obtain a simple transfer function relating the delta delta to delta T_m that is first we write down what is delta delta as omega naught by s 1 upon 2 time Hs into this whole quantity which is delta Tm minus K_s delta delta minus K_D delta omega, okay.

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$$\Delta\delta = \frac{\omega_0}{s}\left[\frac{1}{2Hs}\left(-K_s\Delta\delta - K_D\Delta\omega_r + \Delta T_m\right)\right]$$

$$= \frac{\omega_0}{s}\left[\frac{1}{2Hs}\left(-K_s\Delta\delta - K_D s\frac{\Delta\delta}{\omega_0} + \Delta T_m\right)\right] \quad (18.11)$$

Now in this equation what we do is that we replace this delta omega r, delta omega r by delta delta upon omega naught into s right because this relationship is there in the block diagram. You can see this block diagram relationship delta delta is equal to omega naught by s into delta omega r right. So that when you look at this equation we have now delta T_m delta delta and other constant terms.

Okay and therefore when you rearrange the whole thing, you can write the equation in this form s square delta delta K_D by 2H s delta delta plus K_S by 2H omega naught delta delta equal to omega naught by 2H delta T_m right. Now when you equate this expression to 0, we get the characteristic equation for the system that this is this is the input input term right.

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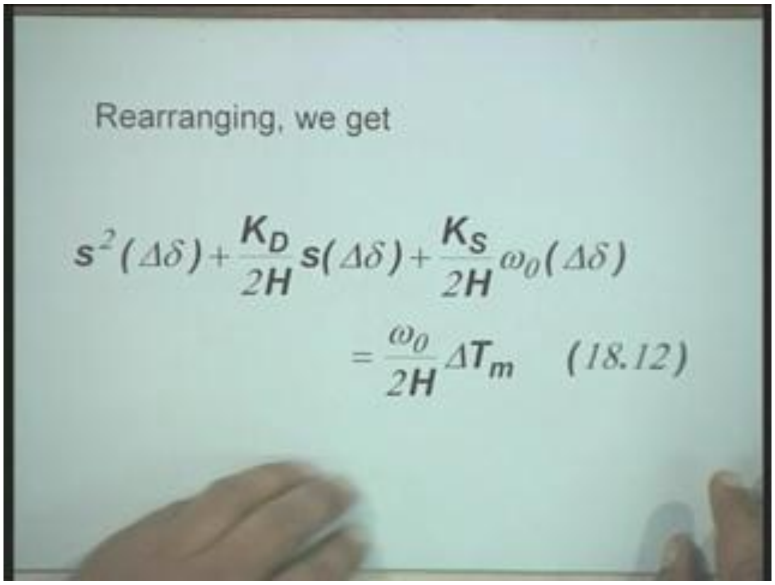

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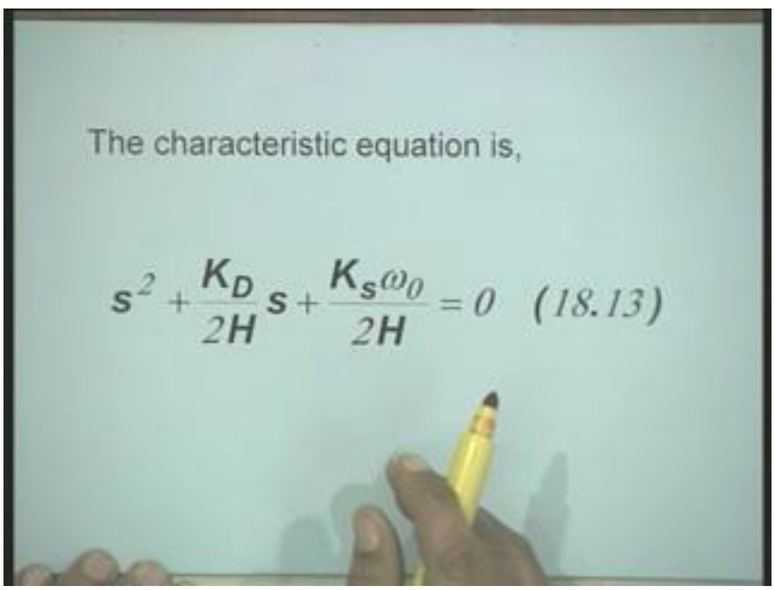

Therefore to to obtain the characteristic equation we can use this expression therefore the characteristic equation is in the form s square plus K_D by 2 Hs plus Ks omega naught by 2H equal to 0. Since the system which we have considered is a second order system therefore characteristic equation is also a second order characteristic equation. Now you can see here the the coefficients are depending upon the damping term K_D the system inertia constant H the synchronizing torque coefficient K_s. Now these are the system parameters further you can easily see that K_s depends upon the operating condition. Okay it also depends upon the line reactance whether the expression for K_s is for this single machine infinite bus system the expression for synchronizing torque coefficient is E prime into E_B divided by X_t into cos delta o right and therefore we can easily see here actually that in this characteristic equation the the coefficients are function of loading that is system operating condition and system parameters also the X_t depends upon the line reactance.

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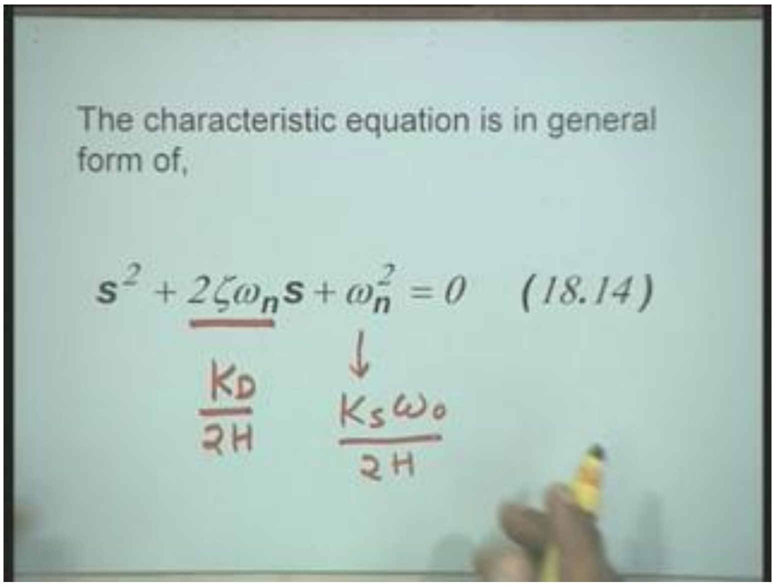

Similarly, it depends upon direct access transient reactance right, now when this equation is similar to our standard equation in this form s square plus to times zeta omega n into s plus omega n square and you can identify identify that this coefficient 2 times zeta omega n, 2 times zeta omega n is equal to K_D by 2H. Similarly, this omega n square term that is this term is equal to K_D by 2H while this omega n square term is equal to K_s omega naught divided 2H.

Okay now when you when you obtain the roots of this characteristic equation right the the roots can be written as s_1, s_2 the small s s_2 equal to minus zeta omega n plus minus j times j times omega n square root of 1 minus zeta square that is in this case the roots will depend upon whether they are complex conjugate or real on the value of zeta, if zeta is if zeta is greater than 0 and less than 1 right. We will get these 2 roots as complex conjugate roots right.

Now a system whose zeta is greater than 0 and less than equal to one right we say that the system is under damped under damped. A system whose zeta is equal to 1 is critically damped is critically damped and a system whose zeta is greater than 1 is over damped right and and system whose zeta is less than 0 is unstable it is in negative damping system is unstable right in any system, we have to ensure that the damping is adequate.

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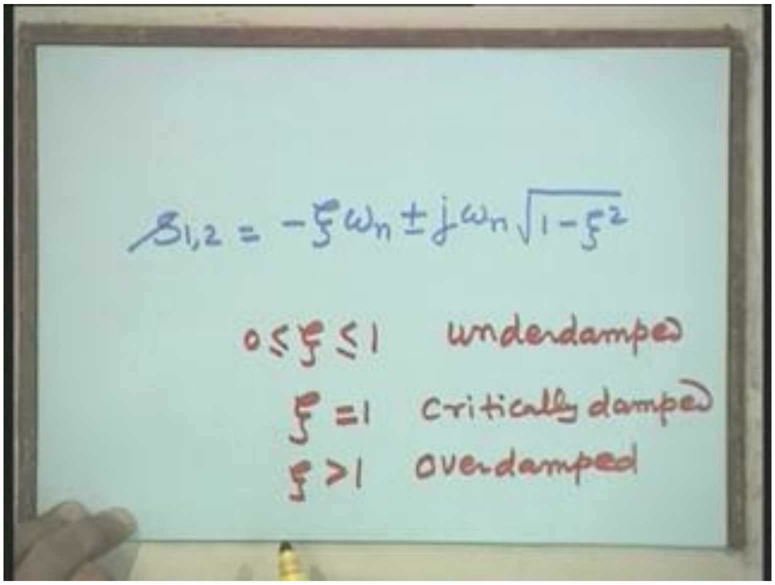

So that the oscillations which are generated are damped, whenever we design the controller for the system we have to achieve certain minimum damping for all the modes which are present in the system right. Now here we can also give some more interpretation to the damping term zeta that is if you plot these roots in the s plane then assuming that damping zeta is positive the roots will appear in the s plane because here we, so the real part we call it sigma.

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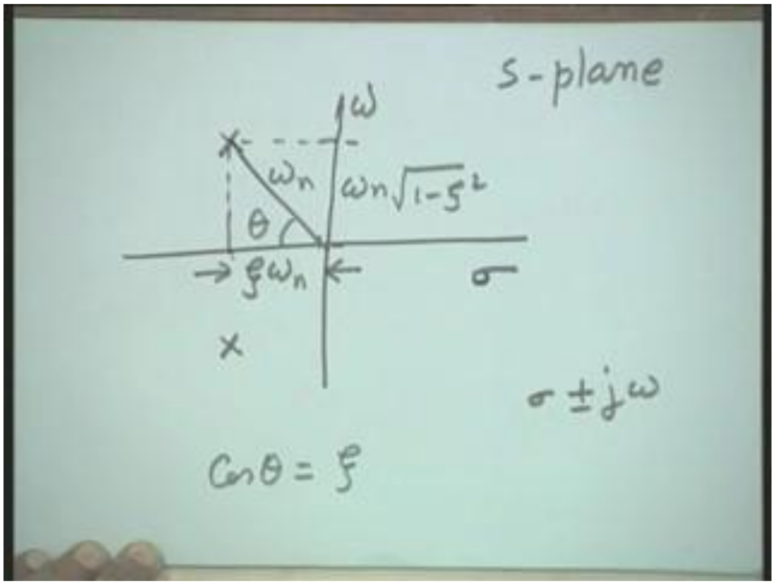

We call this as omega and this axis and any any root is written as sigma plus minus j omega right. Now if the system is a stable system like having its complex conjugate or the roots lying in the left half of the s plane then this term that is this can be this can be is written equal to how much zeta omega n while this term will be equal to how much omega n square root of 1 minus zeta square what will be this quantity omega n therefore, this omega n is the distance of the root from the origin okay.

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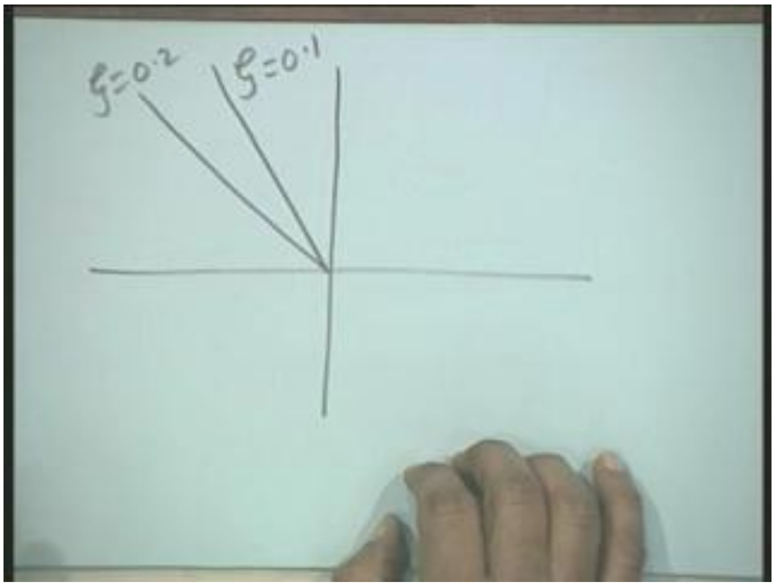

Now if you find out this if you look at this angle theta right then cosine theta is equal to zeta right therefore whenever we perform this Eigen value analysis of the system. We we can draw in the s plane a line which will represent certain value of zeta that is you can that is suppose this is your s plane right and if I say that this line represents zeta equal to say .1, this line will represent zeta equal to say something more than .1, .2 like this.

Now in case all the roots lie on this side of this line it means each mode or there is no mode whose damping is less than .2 right therefore whenever we design the control for this for improving the system stability particularly the AVR tuning or power system stabilizer tuning right, we perform the root locus analysis okay and vary the parameters like gain setting and see actually that you get the optimum performance but at the same time you ensure actually that no root is having its damping less than the desired value that therefore this this is a very important way to to ensure that we do not go to a stage where any do any of the roots is less than or any of the roots is having its damping less than desired value okay.

Now for this system which we are considering here the omega n is the natural frequency of oscillation it is given by this formula omega n is equal to square root of K_s omega naught by 2H and damping ratio zeta is one upon 2 times K_D divided by square root of K_s 2H into omega naught. Okay these 2o terms you can derive and see that you get the value of omega n and zeta like this. Now the next point which we will study is, see this the two differential equations which we have derived the linearized differential equations one differential equation was written in this form, okay the.

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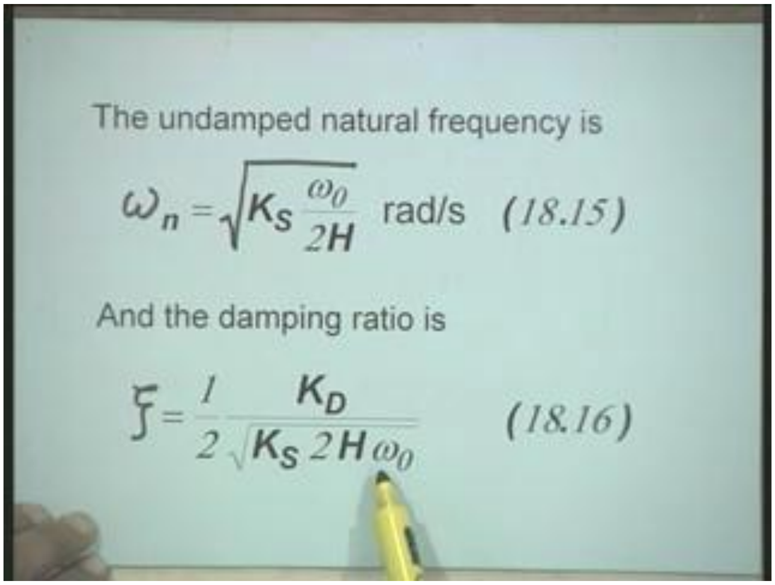

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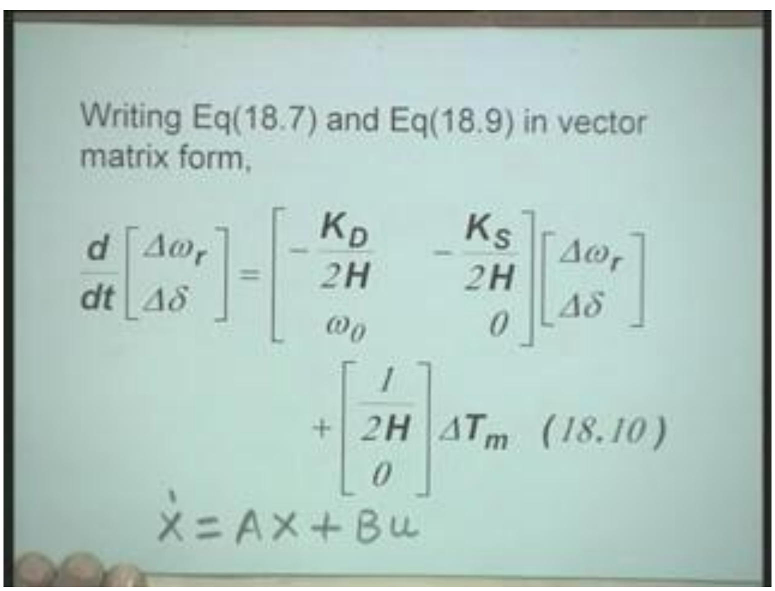

Similarly, we are written data written the second differential equation in the linearized form, okay. Now you can arrange these two differential equations right and write the model in the vector matrix form that is finalizing the stability of the system right the most convenient form of representing the model is vector matrix form that is we write the model in the form of X dot equal to A_x plus B_u right.

Now these two equations which we have derived can be arranged in this form that is d by dt of delta omega r equal to minus K_D by 2H delta omega r minus K_s by 2H delta delta plus1by 2H delta T_m second equation is d by dt of delta delta equal to omega naught into delta omega r that is all. Okay therefore, these two equations which we have derived can be written in the vector matrix form and when we make use of the the say mat lab for analyzing the system performance right then we can find out the Eigen value of the system matrix A right for studying the stability characteristic of the system right.

Now you have to understand very clearly here that this matrix A is function of function of operating condition and system parameters this I will be you know emphasizing again and again here that whenever you write down the model in the form X dot equal to Ax plus Bu right then the basic stability characteristics are determined from the system A matrix. Okay therefore live let me summarize here that what we have done till now is we have developed the small signal model for the machine infinite bus system and we have represented this model in the form that is vector matrix form X dot equal to A_x plus B_u.

We can write down the A matrix we can write down the B matrix and the state variables in this model are delta omega r and delta delta, these are the state variables while delta e delta T_m in this case is a input, okay. Now I will just give one more important information that the importance of this term 2H okay, now to understand this importance of this term 2H, let us look at our swing equation again.

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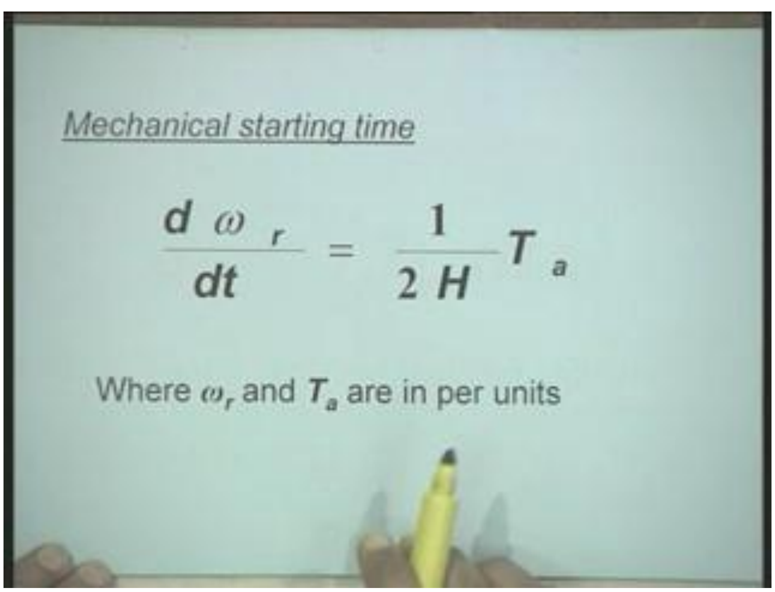

Our swing equation is d by dt omega r equal to 1 upon 2H T_a okay, where omega r is expressed in per unit, okay. Now now suppose you have a synchronous generator and which is at stand state 0 speed okay and if you apply the rated torque rated torque then in how much time the synchronous generator will attain its the rated speed right. Now to obtain the time in which this synchronous generator will attain its rated speed can be obtained by integrating this equation that is you integrate this equation assume T_a equal to 1 per unit okay, T_a equal to 1 per unit and just tell me in uh quickly in how much time the synchronous generator will attain its rated speed.

Okay the synchronous generator will attain its rated speed starting from 0 towards rated value when it is applied with a rated rated accelerating torque that is T_a is equal to 1 per unit and therefore this time 2H is also denoted as the mechanical starting time of the synchronous generator that is T_M is equal to 2H therefore, if you see in the model we have a term 1upon 2H therefore this 2H is similar to a time constant and is the mechanical starting time of the synchronous generator in literature you will find actually that many times we represent this term this T_M by a symbol capital M that is you will find actually when you read the literature that this block 1 upon 2 times H_s is also represented as one upon M_s.

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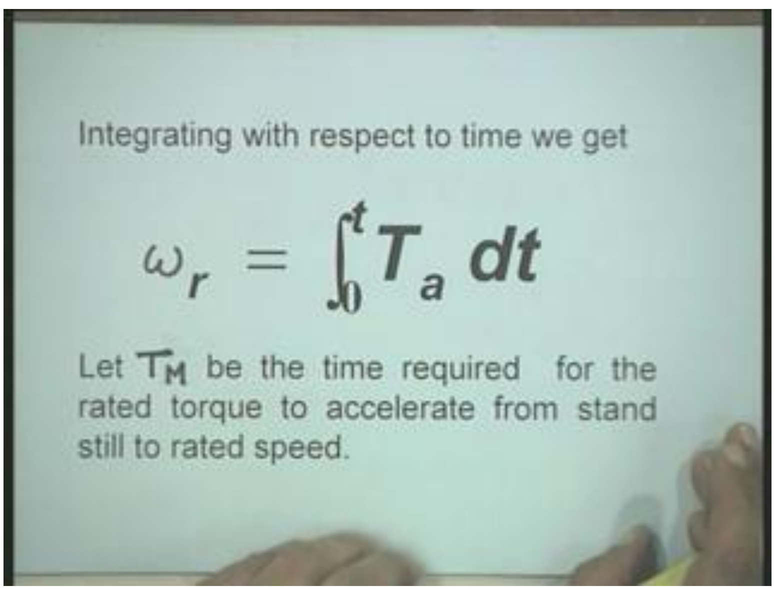

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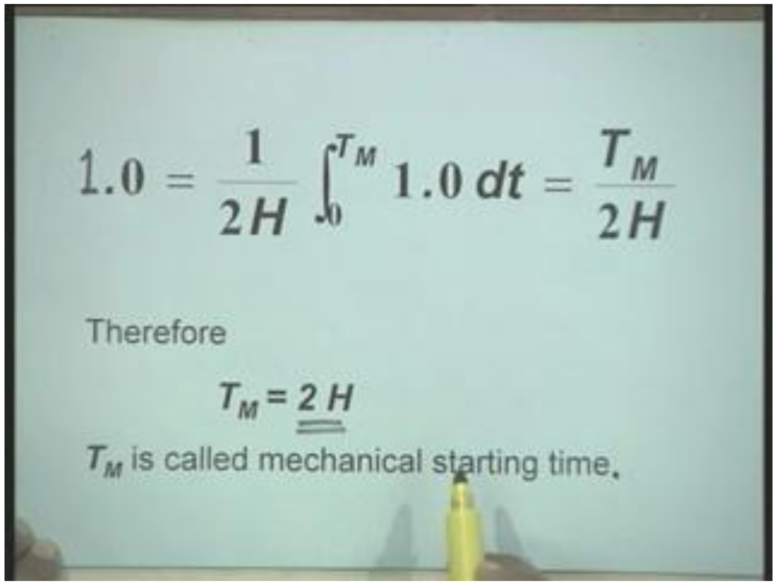

Now here you should not get confused with the term M, the angular momentum of the rotor right the M here is actually the in the small del will be equal to 2H and which is the mechanical starting time of the synchronous generator rotor when it is subjected to a rated torque. Similarly, if the machine is running at constant speed a rated speed and if you apply a retarding torque equal to the rated torque then its speed will come down to 0 in a time equal to 2H right.

Therefore, that also gives the information that how much time the machine will take to retard from its lateral speed to 0speed therefore with this today I will conclude my presentation by saying that we have developed a linear model of machine infinite bus system considering considering constant field flux linkages. Okay we have also studied the ah the the the importance of damping ratio zeta right and whenever we design the system right the zeta is going to be one of our design parameters, okay. Thank you!