

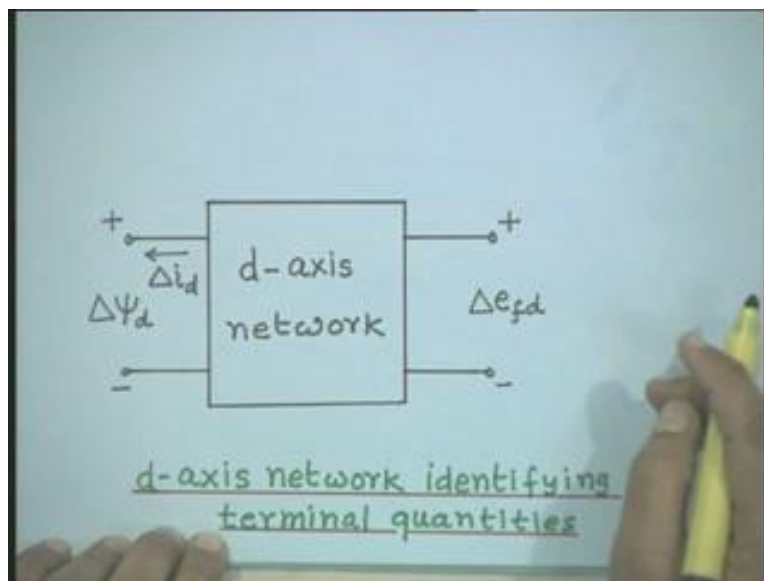
Power System Dynamics
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Lecture - 13
Synchronous Machine Representation (Contd....)

Friends, today we will discuss about the synchronous machine representation. Now, so far we have developed the synchronous machine model, considering the rotor circuit and stator circuit resistances and inductances. Now with this stator circuit and resistor circuit, rotor circuit inductances the complete model of the synchronous machine is developed.

Now however there is 1 practical problem that the **the the** various inductances which are used actually in modeling the synchronous machine are in general not available or cannot be measured directly from the certain standard tests and the parameters that is the **the** inductances and resistances of the stator and rotor circuits are which are used in modeling right have been called fundamental parameters or basic parameters.

Now the standard ah the the practical practice is now to use derived parameters which can be measured by performing test at the terminals of the synchronous machine. Now these derived parameters are very important and the machine data are given in terms of the derived parameters. Okay therefore, today we will try to relate the the derived parameters with the fundamental parameters of the synchronous machine.

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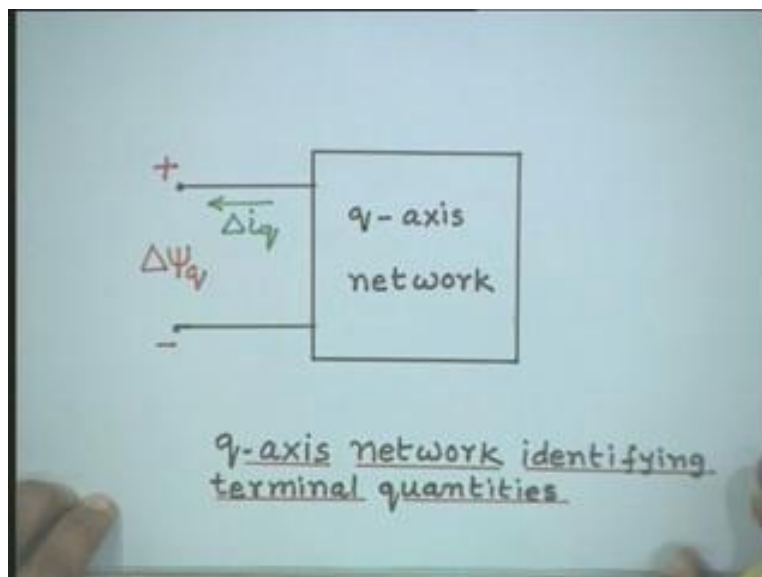


Now to understand this concept of derived parameters, let us first look at the direct axis network of the synchronous generator. Let us say that this black box represents the direct axis network and we have 2 pairs of terminals shown here. This pair of terminals represents the stator side of the synchronous generator, this pair of terminals represent the rotor side of the synchronous generator that is at these terminals we **we we** inject the field voltage. Okay and at these terminals the the stator circuit current is coming out okay.

Now we are we will be representing this machine by 2, 2 networks 1 direct axis another quadrature axis right. Now in the direct axis network the the direct axis current is the output from this network and we are looking for what is the flux linkage in the direct axis due to this particular excitation at the field winding and the load which is supplied by the synchronous generator terminals.

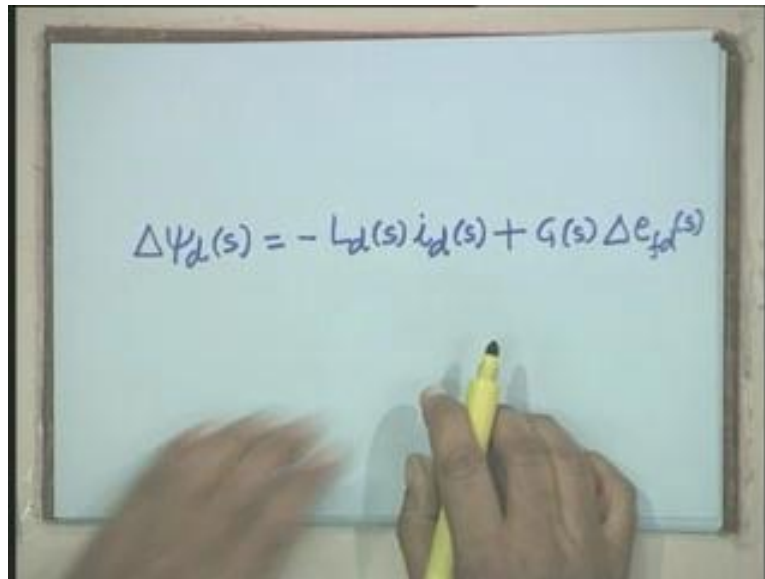
Now here the convention is that we have used the synchronous generator convention where the terminal the the currents are leaving the stator terminals that is the generator convention. Now with the generator convention the direct axis current is leaving the terminal of the d axis network okay.

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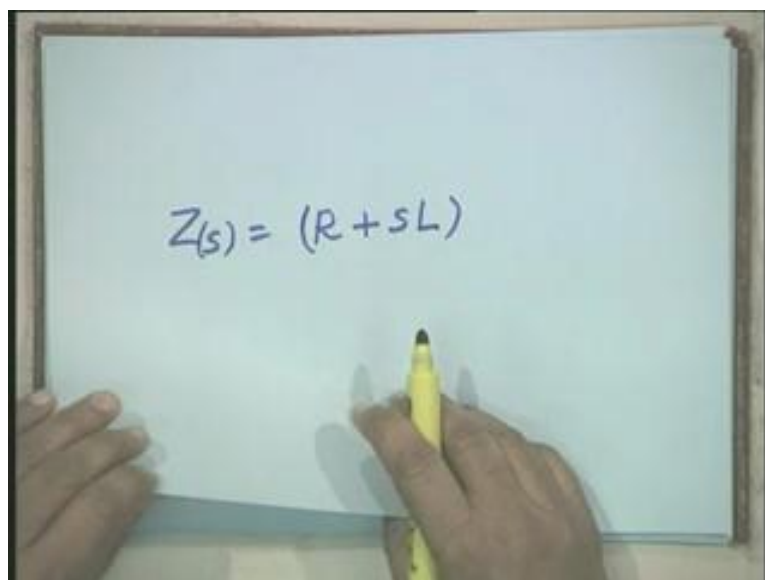
This network, this diagram represents the q axis network in this q axis network we represents the current leaving the network as i_q and the at the terminals we look for the quadrature axis flux linkage ψ_q . Now here in this 2 diagrams which I have shown here instead of showing the current i_d flux linkage ψ_d and applied voltage e_{fd} , we are we will be representing this in terms of small perturbation. So that input quantity shown are Δe_{fd} and the output quantity shown here are Δi_d and $\Delta \psi_d$, similar is the situation in the q axis network.

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$$\Delta \Psi_d(s) = -L_d(s)i_d(s) + G(s)\Delta e_{fd}(s)$$

Now here we will be representing the mathematical model relating the flux linkage delta psi d, now when we develop this model we will be developing first the model in terms of what we call as a operational parameters, operational parameters that is we will represent delta psi d as equal to minus L_{ds} into i_{ds} plus a transfer function we will call as G_s delta e_{fds} .

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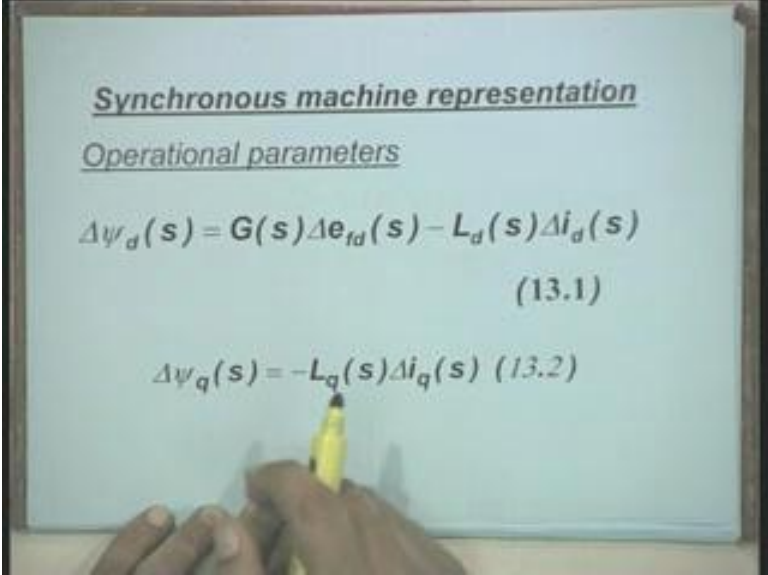

$$Z(s) = (R + sL)$$

This is a very standard way to start with that is we are trying to relate the the current flux linkage and the voltage applied to the terminals of the synchronous at the field winding of the

synchronous generator in terms of what we call as the operational parameters that is this L_{ds} is the operational direct axis inductance, operational we understand what is the meaning of operational here right but this parameter s which I have shown here the s is the familiar Laplace transform operator. Now if we recollect actually, your circuit theory then in circuit theory. We represent the operational impedance of a RL circuit as R plus sL is it not, this is the operational impedance we can call it Zs that the same approach is used here.

Now the our our problem basically will be to obtain the expression for L_{ds} and G_s in terms of fundamental fundamental parameters or we call it the basic parameters of the synchronous generator.

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Synchronous machine representation

Operational parameters

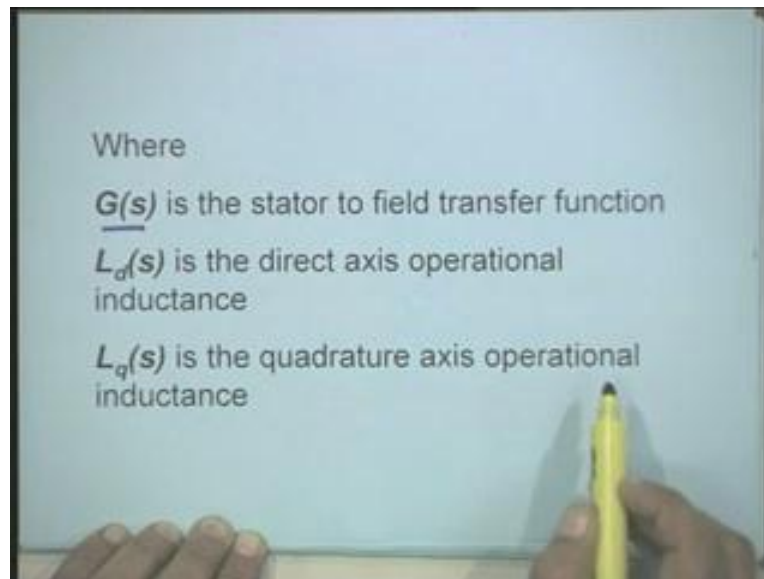
$$\Delta\psi_d(s) = G(s)\Delta e_{fd}(s) - L_d(s)\Delta i_d(s) \quad (13.1)$$

$$\Delta\psi_q(s) = -L_q(s)\Delta i_q(s) \quad (13.2)$$

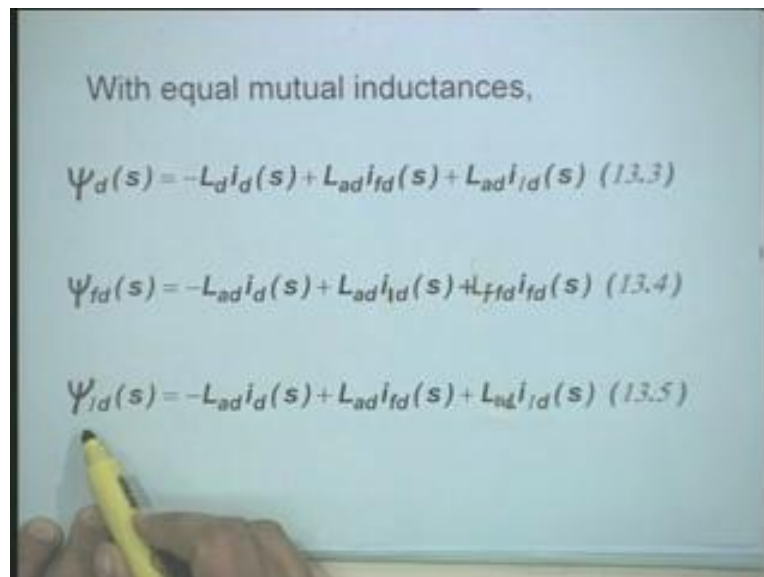
Now these this equation I have written here because when I consider the small perturbations the same equation is written here okay, the minus L_{ds} into Δi_{ds} plus G_s into Δe_{fds} okay equals $\Delta\psi_{ds}$. Now similarly we can write an expression for quadrature axis network the $\Delta\psi_{qs}$ equal to minus L_{qs} Δi_{qs} because in the case of quadrature circuit there is no applied voltage and there is no field circuit on the q axis okay, on the q axis we have always the damper circuits.

Now in this expression this transfer function G_s is the stator to field transfer function, stator to field transfer function you can easily see here that G_s suppose Δi_d is 0 that is this machine is under no load condition, okay then you can easily see that $\Delta\psi_{ds}$ will be equal to G_s times Δe_{fds} therefore, this transfer function relates the stator to field transfer function L_{ds} is the direct axis operational inductance. Okay L_{qs} quadrature axis operational inductance okay.

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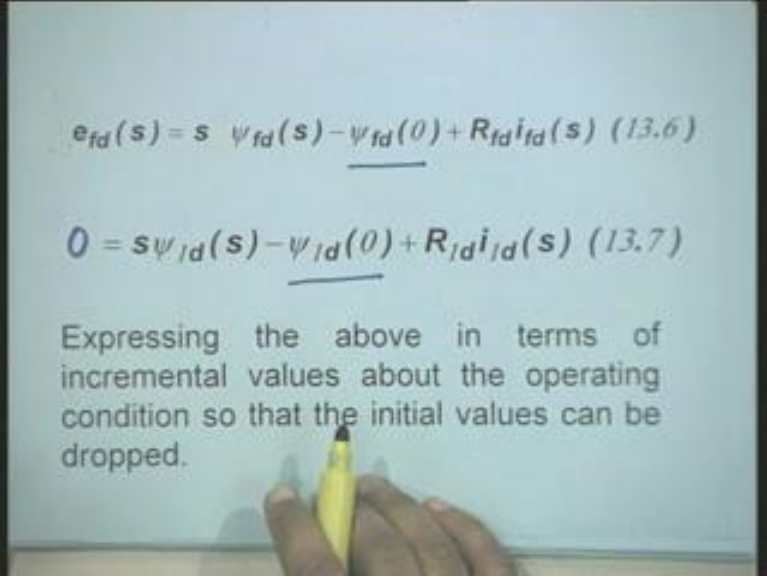


Now our interest here is now to develop a relationship between the operational inductances and this transfer function G_s with the fundamental circuit parameters or fundamental parameters of the synchronous generator. Now for doing this, we again start with our basic equations of the synchronous generator okay. Now in this case of synchronous generator, we had developed the flux linkage equations these flux linkage equations now are written in the operational form that is ψ_d can be written as minus $L_d i_{ds}$ the current is now operational current $L_{ad} i_{fds}$ plus $L_{ad} i_{lds}$.

Now here while writing the equations I am considering 1 damper on the direct axis right therefore, we were denoting the damper winding by the symbol k_d , k used to stand for the number associated with the damper winding therefore, since k is 1 in this case it becomes L_{1d} further we are assuming that the all mutual inductances are same, so that the this L_{ad} appears in both the terms okay.

similarly we write down the flux linkage for equation ψ_{fd} as minus $L_{ad} i_{ds}$ plus $L_{ad} i_{1ds}$ plus $L_{ffd} i_{fds}$ right and the flux linkage of the damper winding in the per operational form is written here. Now these 3 equations are were derived earlier and what we have simply d_1 is that we have now put in the operational form okay.

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$$e_{fd}(s) = s \psi_{fd}(s) - \psi_{fd}(0) + R_{fd} i_{fd}(s) \quad (13.6)$$

$$0 = s \psi_{1d}(s) - \psi_{1d}(0) + R_{1d} i_{1d}(s) \quad (13.7)$$

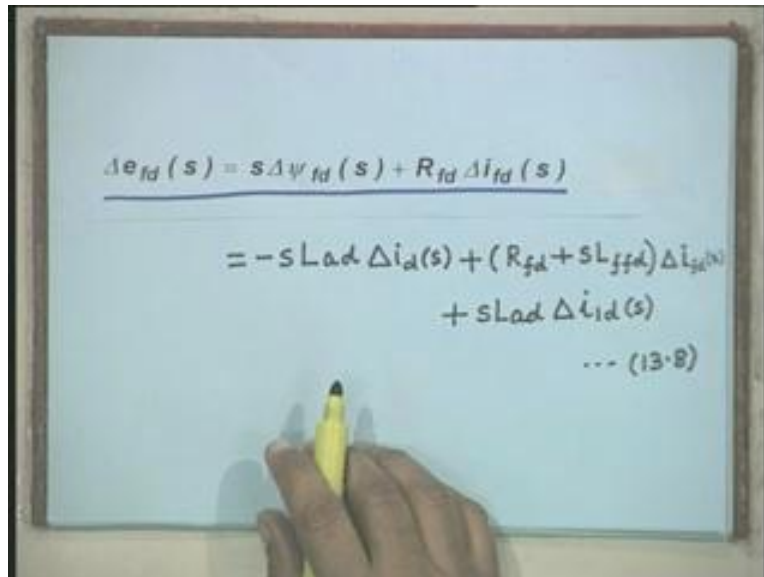
Expressing the above in terms of incremental values about the operating condition so that the initial values can be dropped.

Similarly, the e_{fds} when it is written that is the field winding voltage right, we are writing now expression for e_{fds} in operational form again. Now when you write this expression for the operational form e_{fds} the we will write the equation right then it comes out to be s times ψ_{fds} minus ψ_{fd0} plus $R_{fd} i_{fds}$ that is you take again the standard equation for the field circuit right because in the case of field circuit, we have the term d by dt of ψ_{fd} e_{fd} has a term d by dt of ψ_{fd} and therefore when you take the derivative and then when you take the Laplace transfer of the derivative, you will have the initial condition that will appear here because in the previous equation these were the algebraic equations.

Now equation for e_{fd} it has a derivative term and therefore this appears similarly, actually we write down an equation for an equation for the damper winding there is no applied voltage right but still we have d by dt of ψ_{1d} therefore, again you will find that initial term come therefore

now what we do is that in order to avoid the initial values we express these equations in terms of incremental values, okay so that under steady state conditions the incremental values are 0.

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$$\begin{aligned} \Delta e_{fd}(s) &= s \Delta \psi_{fd}(s) + R_{fd} \Delta i_{fd}(s) \\ &= -s L_{ad} \Delta i_d(s) + (R_{fd} + s L_{ffd}) \Delta i_{fd}(s) \\ &\quad + s L_{ad} \Delta i_{1d}(s) \end{aligned} \quad \dots (13.8)$$

Okay therefore, when you use this incremental values the equation will have straight away the form Δe_{fd} equal to s times $\Delta \psi_{fd}$ plus $R_{fd} \Delta i_{fd}$ this is very important equation is very straight forward actually if you look in terms of our basic equation right then we have applied voltage is equal to R_i plus L_{di} by dt . Okay therefore you can write that R_i plus $d \psi$ by dt and this is what we have already derived.

Now in this expression you substitute the expression for $\Delta \psi_{fd}$ okay $\Delta \psi_{fd}$, we have already written earlier that is use this expression for this $\Delta \psi_{fd}$ and here these are all constants you can convert this Δi_d Δi_{1d} Δi_{fd} . Okay and you substitute the expression for $\Delta \psi_{fd}$ here in this expression and then simplify when you simplify this Δe_{fd} will appear as minus s times $L_{ad} \Delta i_{ds}$ plus R_{fd} plus s times $L_{ffd} \Delta i_{fds}$ plus s times $L_{ad} \Delta i_{1ds}$ that is we have written now incremental field voltage right in terms of the 3 currents. Okay similarly you write down this expression for damper winding also because in case of damper winding the applied voltage is 0 right therefore, you have this is minus $s L_{ad} \Delta i_{ds}$ plus $s L_{ad} \Delta i_{1ds}$ plus R_1 plus $s L_{11d} \Delta i_{1ds}$.

Now at this stage, at this stage again me let me reiterate actually that we want to relate the stator circuit quantities to the rotor circuit quantities right in terms of the stator variables and rotor applied field voltage that is that is the expression which I have if you look at this expression $\Delta \psi_{ds}$ is equal to $G_s \Delta e_{fds}$ minus $L_{ds} \Delta i_{ds}$ in this expression or in the second expression the rotor currents do not appear rotor circuit currents that is i_{1d} or i_{fd} these 2 currents do not appear therefore, what we do is that we eliminate this currents that is from this using these equations what we do is that you eliminate this currents.

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$$0 = -sL_{ad} \Delta i_d(s) + sL_{ad} \Delta i_{fd}(s) + (R_{fd} + sL_{ffd}) \Delta i_{fd}(s) \quad (13.9)$$

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Synchronous machine representation
Operational parameters

$$\Delta \psi_d(s) = G(s) \Delta e_{fd}(s) - L_d(s) \Delta i_d(s) \quad (13.1)$$
$$\Delta \psi_q(s) = -L_q(s) \Delta i_q(s) \quad (13.2)$$

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$$\Delta i_{fd}(s) = \frac{1}{D(s)} \left[(R_{fd} + sL_{lfd}) \Delta e_{fd}(s) + sL_{ad}(R_{fd} + sL_{lfd}) \Delta i_d(s) \right] \quad (13.10)$$

$$\Delta i_d(s) = \frac{1}{D(s)} \left[-sL_{ad} \Delta e_{fd}(s) + sL_{ad}(R_{fd} + sL_{lfd}) \Delta i_d(s) \right] \quad (13.11)$$

Now to eliminate this what you do is that that you use this equations and write the expression for Δi_{fd} and Δi_d in terms of Δe_{fd} and Δi_d that is these 2 equations associated with the other equations which are already derived earlier right what we do is that we write the expression for Δi_{fd} and Δi_d in terms of this quantity and this known quantities that is we are trying to eliminate the field current and the damper circuit current.

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With equal mutual inductances,

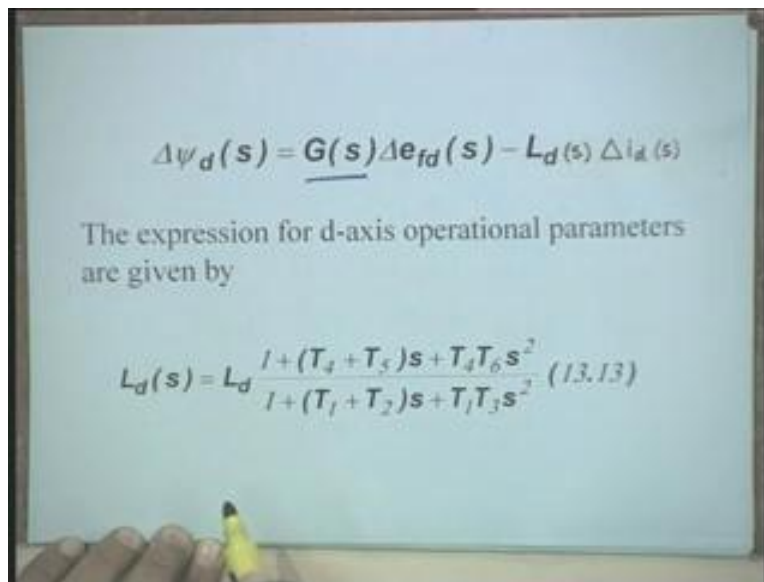
$$\Psi_d(s) = -L_d i_d(s) + L_{ad} i_{fd}(s) + L_{ad} i_{ld}(s) \quad (13.3)$$

$$\Psi_{fd}(s) = -L_{ad} i_d(s) + L_{ad} i_{ld}(s) + L_{ffd} i_{fd}(s) \quad (13.4)$$

$$\Psi_{ld}(s) = -L_{ad} i_d(s) + L_{ad} i_{fd}(s) + L_{lld} i_{ld}(s) \quad (13.5)$$

Now this step is little what involved step but you can obtain it. Once you obtain this expression next step will be to substitute this in the expression for delta psi ds because if you look actually the expression for delta psi ds, delta psi d this is that equation for delta psi d or psi d equation equation for psi d is here. In this equation we have i_d , i_{fd} and i_{1d} , okay what we have now d_1 is that i_{fds} the this will be expressed in incremental form then once you express in incremental form in this expression you replace this delta i_{fds} and delta i_{1ds} using those expressions.

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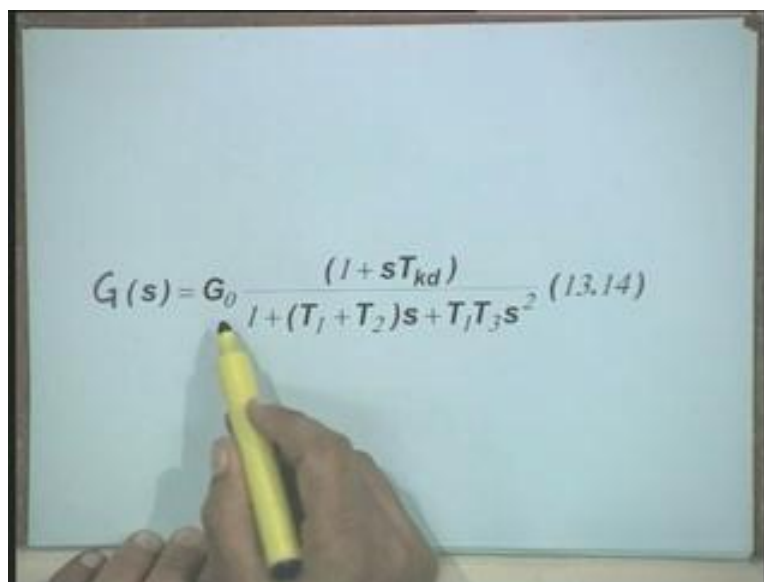
The image shows a whiteboard with a handwritten equation and a line of text. The equation is $\Delta\psi_d(s) = \underline{G(s)} \Delta e_{fd}(s) - L_d(s) \Delta i_d(s)$. Below it, the text reads "The expression for d-axis operational parameters are given by". At the bottom, the equation $L_d(s) = L_d \frac{1 + (T_4 + T_5)s + T_4 T_6 s^2}{1 + (T_1 + T_2)s + T_1 T_3 s^2} \quad (13.13)$ is written. A hand holding a yellow marker is visible at the bottom left of the whiteboard.

$$\Delta\psi_d(s) = \underline{G(s)} \Delta e_{fd}(s) - L_d(s) \Delta i_d(s)$$

The expression for d-axis operational parameters are given by

$$L_d(s) = L_d \frac{1 + (T_4 + T_5)s + T_4 T_6 s^2}{1 + (T_1 + T_2)s + T_1 T_3 s^2} \quad (13.13)$$

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The image shows a whiteboard with a handwritten equation. The equation is $G(s) = G_0 \frac{(1 + sT_{kd})}{1 + (T_1 + T_2)s + T_1 T_3 s^2} \quad (13.14)$. A hand holding a yellow marker is visible at the bottom left of the whiteboard, pointing towards the equation.

$$G(s) = G_0 \frac{(1 + sT_{kd})}{1 + (T_1 + T_2)s + T_1 T_3 s^2} \quad (13.14)$$

Once you express you will find that you can write the expression for delta psi d_s as G_s delta efd_s minus L_d i_d so you appreciate these steps. Now once you do this thing what will happen is that this expression for G_s which you get will be written as that is 2 expressions L_{ds} and G_s, the L_{ds} will be written as L_d multiplied by an expression of this form 1 plus T₄ plus T₅ into S plus T₄, T₆ S square divided by 1 plus T₁ plus T₂ S plus T₁, T₃ S square that is when you do this simplification and express delta psi d_s in terms of delta efd_s and delta i_d right.

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Where

$$G_0 = \frac{L_{ad}}{R_{fd}} \quad T_{kd} = \frac{L_{ld}}{R_{ld}}$$

$$T_1 = \frac{L_{ffd}}{R_{fd}} \quad T_2 = \frac{L_{lld}}{R_{ld}}$$

$$T_3 = \frac{1}{R_{ld}} \left(L_{ld} + \frac{L_{ad} L_{fd}}{L_{ad} + L_{fd}} \right)$$

$$T_4 = \frac{1}{R_{fd}} \left(L_{fd} + \frac{L_{ad} L_l}{L_{ad} + L_l} \right)$$

Then the coefficient of delta efd term will come out to be, coefficient of delta efd term will come out to be in this form, that is G₀ equal to ah G_s equal to G₀ into 1 plus divided by 1 plus T₁ plus T₂ S, T₁ 3S square while the expression for L_d will come out to be equal to this the the yes, then these constants which are shown in the this equation are related to the fundamental parameters of the synchronous generator by this expression that is G₀ equal to L_{ad} by R_{fd}, T_{kd} is L_{ld} by R_{ld}, T₁ equal to L_{ffd} by R_{fd}, T₂ equal to L_{lld} by R_{ld}, T₃ 1 upon R_{ld} multiplied by L_{ld} plus L_{ad} L_{fd} divided by L_{ad} plus L_{fd} T₄ 1 upon R_{fd} L_{fd} plus L_{ad} into L_l divided by L_{ad} plus L_l and T₅ as 1 upon R_{ld} multiplied by L_{ld} plus L_{ad} L_l, L_l divided by L_{ad} plus L_l and similarly, T₆.

You can easily see that when you perform this simplification right right, the you will be in a position to express the L_{ds} that express this G_s as a ratio of 2 polynomials right. The denominator here is of the second order and numerator is 1 order less than the denominator similarly, L_{ds} is expressed as ratio of 2 polynomials where the order is same further the denominator of L_{ds} and G_s are same that is this denominator for L_{ds} is same as that of the okay, this is these derivations can be carried out.

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$$T_s = \frac{1}{R_{fd}} \left(L_{fd} + \frac{L_{ad}L_f}{L_{ad} + L_f} \right)$$

$$T_6 = \frac{1}{R_{fd}} \left(L_{fd} + \frac{L_{ad}L_{fd}L_f}{L_{ad}L_{fd} + L_{fd}L_f + L_{ad}L_f} \right)$$

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$$G_1(s) = G_0 \frac{(1 + sT_{kd})}{1 + (T_1 + T_2)s + T_1T_3s^2} \quad (13.14)$$

Now at this stage what is d_1 is we write the expression for L_{ds} that is the operational direct axis inductance as L_d multiplied by by numerator that is by this expression which is the numerator as 1 plus s time T_d prime into 1 plus s time T double prime 1 plus s times T_{d0} prime plus into 1 plus s time T_{d0} double prime that is what we do is now this this operational inductance will express in terms of time constants and these time constants as we will see are are the the standard parameters of the synchronous generator because these are the parameters that is going to determine the the the transient performance of the system okay.

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From Eq(13.13) and (13.14)

$$L_d(s) = L_d \frac{(1 + sT_d')(1 + sT_d'')}{(1 + sT_{d0}')(1 + sT_{d0}'')} \quad (13.16)$$

$$G(s) = G_0 \frac{(1 + sT_{kd})}{(1 + sT_{d0}')(1 + sT_{d0}'')} \quad (13.17)$$

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Expression for q-axis operational inductance is

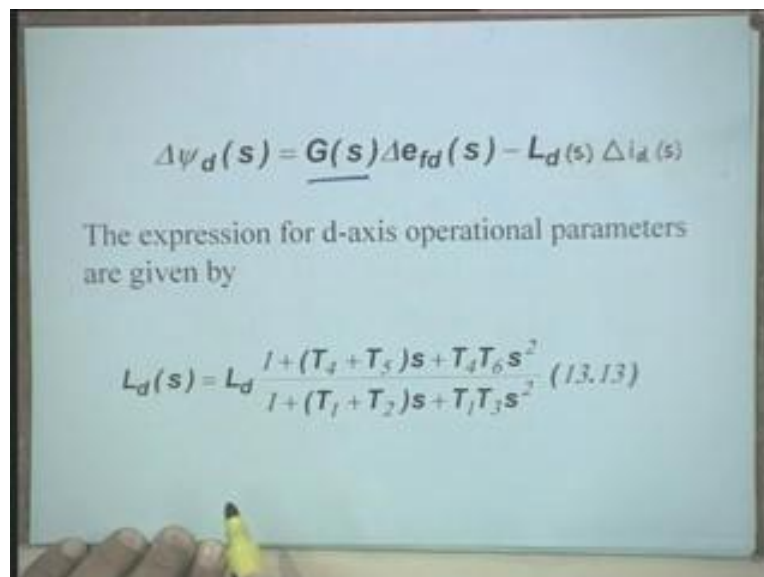
$$L_q(s) = L_q \frac{(1 + sT_q')(1 + sT_q'')}{(1 + sT_{q0}')(1 + sT_{q0}'')} \quad (13.18)$$

Similarly since G_s has the same denominator right therefore, we when you put in this formula where G_s can be written as $1 + s$ time T_{d0} prime and T double prime okay now our interest is that we know we know the expression for this time constant T_1 to T_6 in terms of basic circuit parameters is it not these are all known. We want to relate this these standard parameters that is this time constant T_{d0} prime T_{d0} double prime T_d prime T_d double prime to our basic parameters or fundamental parameters. Now to correlate this before I correlate I will just write 1 more expression that if we follow the same approach as we have d_1 for d axis we can write the

expression for L_{qs} as that is operational quadrature axis inductance as L_q multiplied by $1 + s$ times T_q prime into $1 + s$ times T_q double prime divided by $1 + s$ times T_{qo} prime $1 + s$ times T_{qo} double prime. Now again these constants T_q prime T_q double prime T_{qo} prime T_{qo} double prime these 4 time constants right are again the standard parameters of the synchronous generator right.

Now to obtain an expression for these parameters in terms of T_1 to T_6 what we have to do is that we compare this denominator and numerator terms after all actually we had written earlier, let us say L_{ds} is written in this form see this L_{ds} is written as L_d multiplied by this expression. Okay and L_{ds} we are now writing in the form of in this form in terms of this time constants, okay while these are our standard parameters and as we will see that they can be measured from the certain tests which can be conducted at the tests on the synchronous generator right. Now at this stage we make to we see here actually this expression look at this expression.

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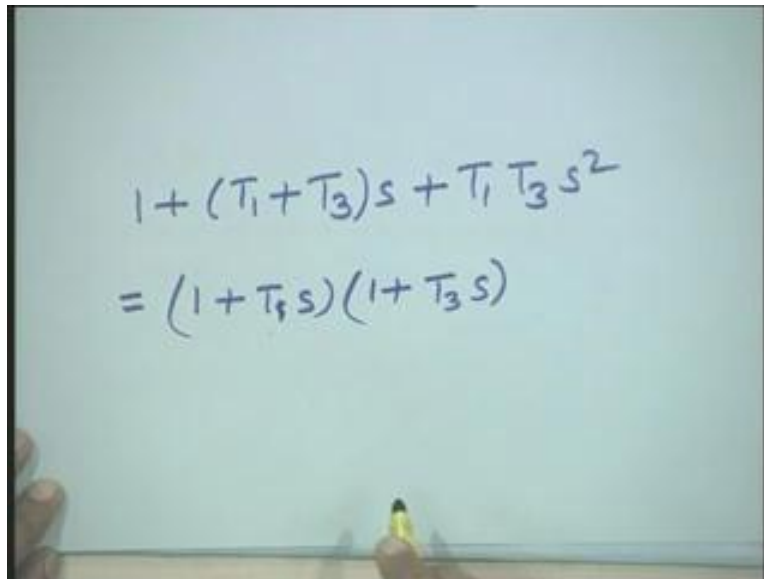
$$\Delta\psi_d(s) = \underline{G(s)}\Delta e_{fd}(s) - L_d(s)\Delta i_d(s)$$

The expression for d-axis operational parameters are given by

$$L_d(s) = L_d \frac{1 + (T_4 + T_5)s + T_4T_6s^2}{1 + (T_1 + T_2)s + T_1T_3s^2} \quad (13.13)$$

First I will just look at the denominator $1 + T_1 + T_2 s + T_1 T_3 s^2$. Now in case suppose you have instead of T_2 here as T_3 then this can be broken into fractions, you know suppose actually you have the expression of this form $1 + T_1 + T_3$ into $s + T_1 T_3 s^2$ square the the partial, you can find out the factors of this expression the factors will be $1 + T_1 s$ into $1 + T_3 s$. Okay now if it is in this form then directly you can identify that T_1 is equal to T_{do} prime and T_3 equal to T_{do} double prime but incidentally the expression is here $1 + T_1 + T_2 s + T_1 T_3 s^2$.

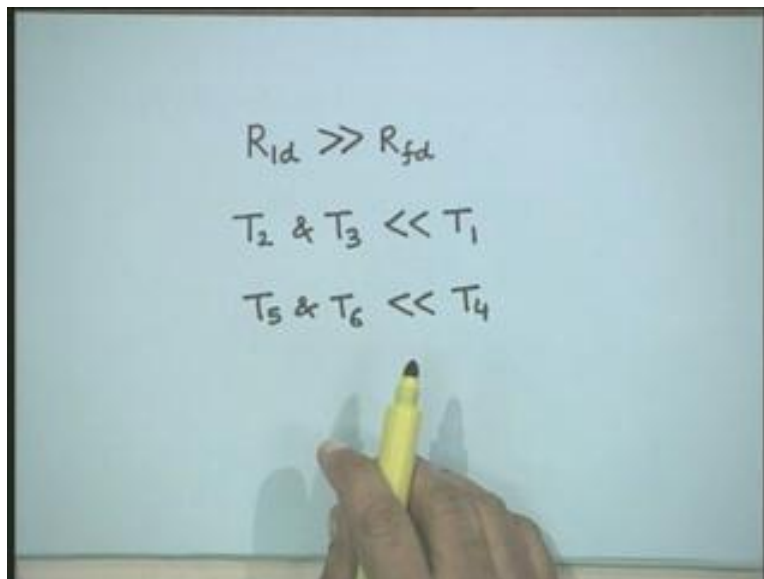
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A hand-drawn equation on a whiteboard. The equation is written in two lines. The first line is $1 + (T_1 + T_3)s + T_1 T_3 s^2$. The second line is $= (1 + T_1 s)(1 + T_3 s)$. A hand holding a yellow marker is visible at the bottom of the frame.

$$1 + (T_1 + T_3)s + T_1 T_3 s^2$$
$$= (1 + T_1 s)(1 + T_3 s)$$

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Three lines of handwritten text on a whiteboard. The first line is $R_{1d} \gg R_{fd}$. The second line is $T_2 \text{ \& } T_3 \ll T_1$. The third line is $T_5 \text{ \& } T_6 \ll T_4$. A hand holding a yellow marker is visible at the bottom of the frame.

$$R_{1d} \gg R_{fd}$$
$$T_2 \text{ \& } T_3 \ll T_1$$
$$T_5 \text{ \& } T_6 \ll T_4$$

Now here we make use of 1 simplification or approximation we can say, the approximation is that in practice in practice R_{1d} is very very large as compared to R_{fd} , R_{1d} which is the resistance of the damper winding on the d axis or amortisseur on the d axis is very very large as compared to the field winding resistance. When you make this assumption T_3 and T_2 , T_2 and T_3 come out to be very small as compared to T_1 and T_5 and T_6 will come out to be very small as compared to T_4 right therefore in the expression for the the expression which we have is in the denominator we have 1 plus T_1 plus T_2 therefore this T_2 is negligible as compared to T_1 okay.

Now T_3 is also very small as compared to T_1 therefore first step suppose I neglect T_2 I will not create much error second actually step is that suppose I add T_3 to this term then also I am not going to create big error therefore what we do is that this denominator this is actually 1 plus T_1 plus T_2 into s plus $T_1 T_3$ s square right will be replaced by this term 1 plus T_1 plus T_3 s plus $T_1 T_3$ s square, the justification I have just now told you that T_2 was is very small as compared to T_1 similarly, T_3 is also very small as compared to T_1 right therefore what we do is that we neglect T_2 but we add T_3 . We do not create much error but we simplify our results and expression.

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Standard parameters

$$(1 + sT'_{do})(1 + sT''_{do}) = 1 + s(T_1 + T_2) + s^2 T_1 T_3 \quad (13.19)$$

$$(1 + sT'_d)(1 + sT''_d) = 1 + s(T_4 + T_5) + s^2 T_4 T_6 \quad (13.20)$$

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Parameters based on classical definitions

$$(1 + sT'_{do})(1 + sT''_{do}) = (1 + sT_1)(1 + sT_3) \quad (13.21)$$

$$(1 + sT'_d)(1 + sT''_d) = (1 + sT_4)(1 + sT_6) \quad (13.22)$$

With this you can see here $1 + s \text{ times } T_{do} \text{ prime}$ becomes $1 + s \text{ times } T_{do} \text{ double prime}$ which is of course is standard. Now when you make this assumptions I just told you the approximations not assumption the approximations then you can write down this this. You can equate the 2 denominators and you get this type of expressions, in the case of numerator the approximation is that T_5 and T_6 are very small as compared to T_4 right therefore, we are now getting a simple expression of this form. Remember that, remember that when I am making this assumption that T_3 is very small as compared to T_1 right in the product term $T_1 T_3 s^2$ I am not making any approximation $T_1 T_3$ remain there.

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$$T'_{do} = T_1 = \frac{L_{ffd}}{R_{fd}}$$

$$T''_{do} = T_3 = \frac{1}{R_{1d}} \left(L_{1d} + \frac{L_{ad} L_{fd}}{L_{ad} + L_{fd}} \right)$$

$$T'_d = T_4$$

$$T''_d = T_6$$

(13-23)

Now when you do this you will find actually that $T_{do} \text{ prime}$ can be identified as T_1 $T_{do} \text{ double prime}$ can be identified as T_3 , $T_d \text{ prime}$ can be identified as T_4 and $T_d \text{ double prime}$ can be identified as T_6 while the while we know that T_1 is equal to L_{ffd} divided by R_{fd} . Similarly, $T_{do} \text{ double prime}$ is equal to 1 upon R_{1d} multiplied by this expression and expression for T_d , T_4 and T_6 were also derived therefore they are all time constants.

You can easily see that the this this term T_1 is equal to ratio of inductance to resistance that means it has the characteristic of a time constant. Okay, here also in this bracket you have inductance term divided by resistance therefore, this also is a time constant right and now we are relating actually these 2 time constants that is the we call actually the standard time constants right are related to the the basic parameters of the synchronous generator we will see actually that this time constant can be measured by performing tests okay

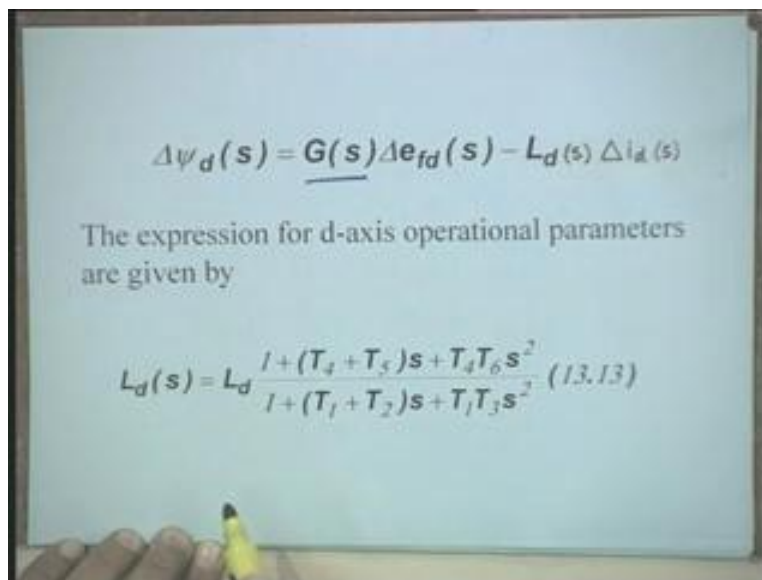
Now at this stage let me just sum up what we have d_1 till now is that we have started with with the ah operational model for d axis and q axis winding for the synchronous generator okay and then using the the standard equations. Okay for the synchronous generator stator winding and field winding right we have established the transfer functions for G_s and L_d and similarly, for L_q

then assuming that assuming that this can be expressed in terms of some time constants or the standard parameters. We have established actually that this time constant can be related to the basic parameters okay.

Now till now actually in my all discussion I have not talked about what is the transient reactance? what is the transient inductance? what is the sub transient reactance, sub transient inductance but we all know actually that whenever there are sudden changes in the synchronous machine operation, let us say a fault okay then the currents are induced in the rotor circuits and this currents will decay at certain rate and the rate of decay of these currents are determined by the time constants. A certain time constant which determine the rate of decay of the currents I will just take the case that whenever there is a short circuit currents will be induced in the damper windings they will induced in the field winding right similarly, in the damper winding of the q axis and so on and they decay with certain time constants okay.

Now first we will try to establish that what will be the inductances offered offered by the synchronous machine when sudden changes take place or the transient take place. Now to understand this what we do is we start with, let us look at this expression for L_{ds} this is the expression for L_{ds} okay.

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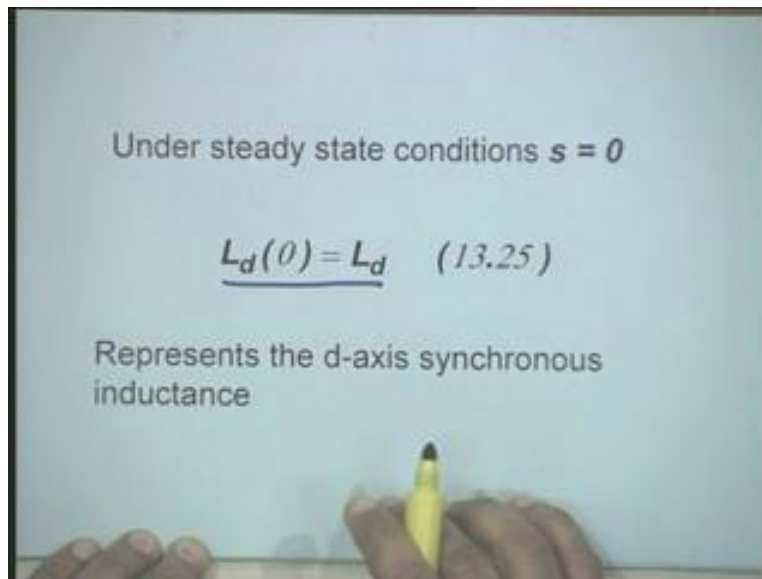
$$\Delta\psi_d(s) = \underline{G(s)}\Delta e_{fd}(s) - L_d(s)\Delta i_d(s)$$

The expression for d-axis operational parameters are given by

$$L_d(s) = L_d \frac{1 + (T_4 + T_5)s + T_4T_6s^2}{1 + (T_1 + T_2)s + T_1T_3s^2} \quad (13.13)$$

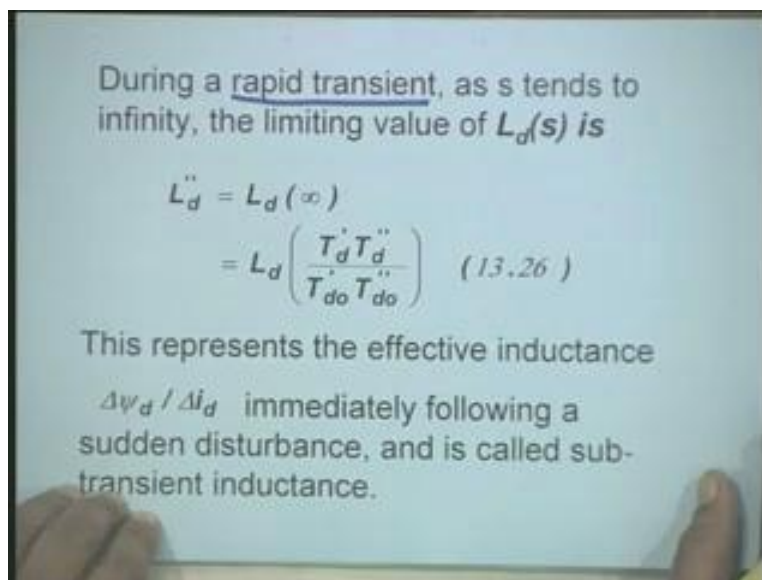
Now suppose I want to know the behaviour of the machine under sustained disturbance condition that is the when the machine is carrying the steady state currents at that time I can substitute s equal to 0, s equal to 0. When I substitute s equal to 0 then this L_{ds} will become equal to L_d that is this operational inductance for s equal to 0 will become L_d that is L_d is your synchronous direct axis synchronous inductance because at that time the currents are under steady state conditions in sustained conditions.

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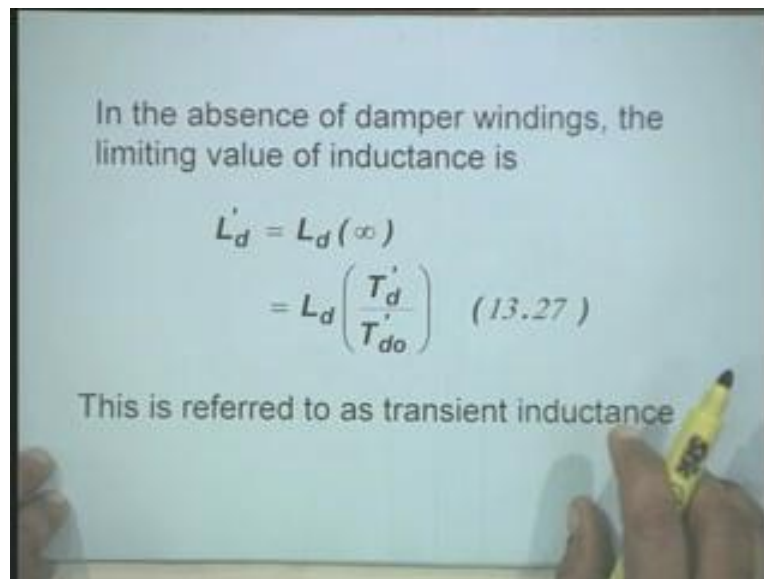
Therefore, now I can say that under steady state condition s is 0 and therefore, L_{do} that is the operational impedance for s equal to 0 is same as the synchronous inductance or d axis synchronous inductance. Now you consider another case let us say s is approaching infinity, a very high frequency transient when s approaches infinity it means actually the transient is of short duration right then the frequency associated will be very large.

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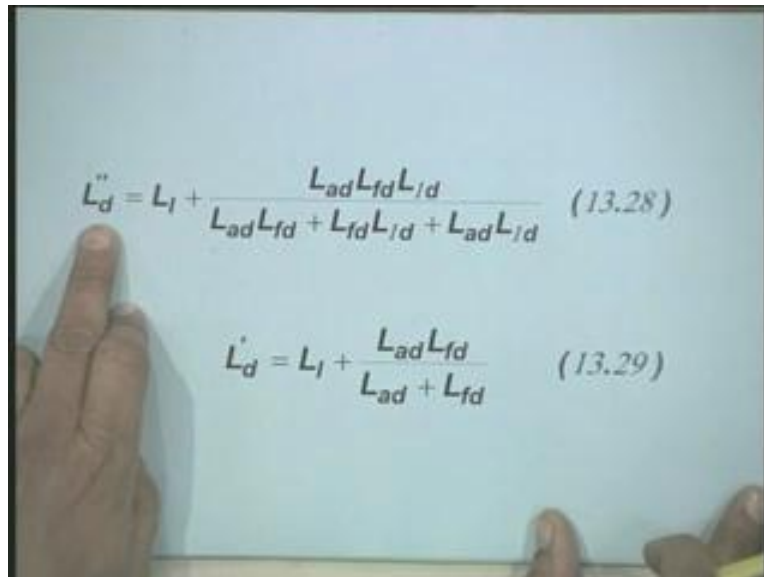
If you make this situation that is the rapid transient condition then this L_d double prime, we will represent this operational impedance for s equal to infinity as L_d double prime right and this will come out to be equal to L_d multiplied by this expression. Now what we are trying to say is that the inductance offered by the direct axis under rapid transients is denoted as sub transient inductance and this sub transient inductance is related to the synchronous inductance by this expression these time constants further further if you assume actually that there is no damper winding in the circuit, let us say that there is no damper winding in the circuit direct axis only there is a field winding right.

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Now at that time, at that time when there is a rapid transient right the inductance is denoted as L_d prime and again s is approaching infinity right, this will come out to be equal to L_d into T_d prime upon T_{do} , T_{do} prime therefore now I can say here that we have identified the 3 different type of reactances offered by the synchronous machine that is synchronous inductance, sub transient inductance L_d double prime and transient inductance L_d starting from the operational inductance point and the expressions for this if you just look into the complete expression because when you substitute here the expression for T_d prime, T_{do} prime, okay in terms of the fundamental parameters you will get the full expression for L_d double prime okay and this these are expressed in terms of the basic parameters the expressions are quite lengthy you can write it down.

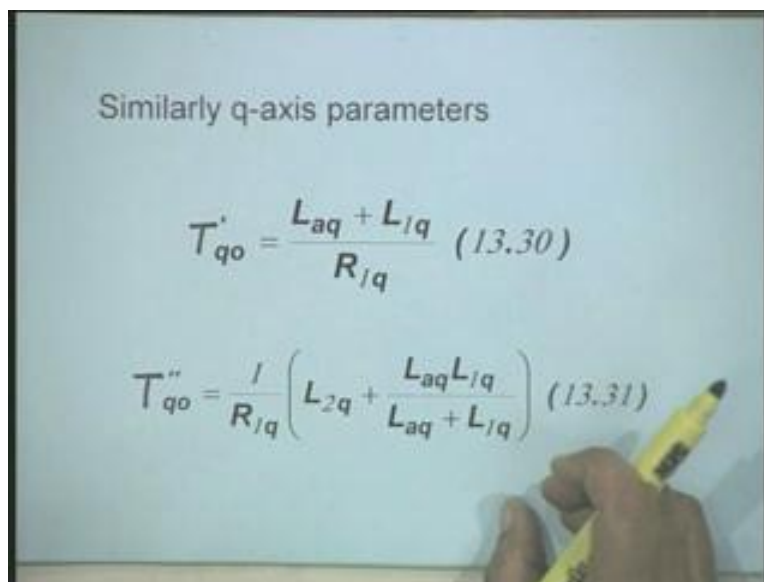
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$$\ddot{L}_d = L_l + \frac{L_{ad}L_{fd}L_{ld}}{L_{ad}L_{fd} + L_{fd}L_{ld} + L_{ad}L_{ld}} \quad (13.28)$$

$$\dot{L}_d = L_l + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}} \quad (13.29)$$

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Similarly q-axis parameters

$$T'_{qo} = \frac{L_{aq} + L_{lq}}{R_{lq}} \quad (13.30)$$

$$T''_{qo} = \frac{1}{R_{lq}} \left(L_{2q} + \frac{L_{aq}L_{lq}}{L_{aq} + L_{lq}} \right) \quad (13.31)$$

Similar exercise can be performed for q axis that is you can perform the same exercise for q axis and you will find actually that the time constants are T'_{qo} prime equal to this and T''_{qo} double prime equal to this again expressed in terms of fundamental circuit parameters right and this transient and sub transient inductances of the quadrature axis winding damper, winding and their parameters are also given. Now we have to just look slightly in more detail about these time constants.

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And the transient and subtransient inductances

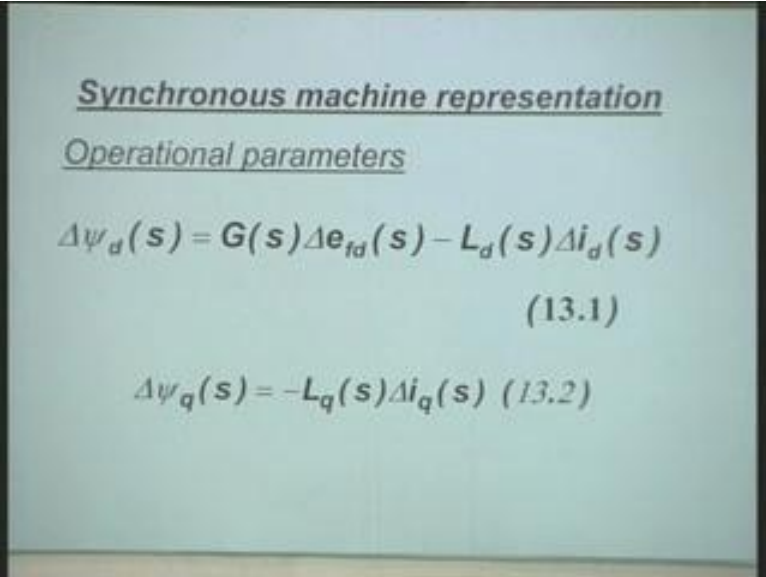
$$L_q'' = L_l + \frac{L_{aq}L_{lq}L_{2q}}{L_{aq}L_{lq} + L_{lq}L_{2q} + L_{aq}L_{2q}} \quad (13.32)$$
$$L_q' = L_l + \frac{L_{aq}L_{lq}}{L_{aq} + L_{lq}} \quad (13.33)$$

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Parameters for salient pole machines

$$L_q = L_l + L_{aq}$$
$$L_q'' = L_l + \frac{L_{aq}L_{lq}}{L_{aq} + L_{lq}} \quad (13.42)$$
$$T_{q0}'' = \frac{1}{R_{lq}} \left(L_l + \frac{L_{aq}L_{lq}}{L_{aq} + L_{lq}} \right)$$

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Synchronous machine representation
Operational parameters

$$\Delta\psi_d(s) = G(s)\Delta e_{fd}(s) - L_d(s)\Delta i_d(s) \quad (13.1)$$
$$\Delta\psi_q(s) = -L_q(s)\Delta i_q(s) \quad (13.2)$$

Now here if suppose I perform a test on the synchronous generator with with stator terminals open that is under no load condition. When we perform a test right, where we we we consider the synchronous terminals open then this term will absent, this terminal will be absent under this case actually we have relation with the field direct axis field flux linkages, direct axis linkages with respect to Δe_{fd} therefore this transfer function G_s affects the affects the change in d axis flux linkages as the change in the applied voltage of the in the field circuit. Okay and this change is determined by the 2 time constants, 1 time constant is T_{do} prime another is T_{do} double prime.

Now since this test is performed under open circuit condition, okay therefore this time constant T_{do} prime is called direct axis open circuit time constant or we always call the open circuit direct axis time constant T_{do} prime. Okay and the T_{do} double prime is called called the open circuit sub transient direct axis time constant here actually the T_{do} prime is called basically full full name is it is open circuit direct axis transient time constant, the transient word is also there T_{do} prime. Okay and this time constant is very important actually when you model our system in fact 1 can perform 1 test very simple test actually on the synchronous generator and that is normally called the steady state frequency response no not steady state h what you can call another term is stand still frequency response test the stand stand still frequency response test what is d_1 is that you keep the machine stationary not rotating, okay you apply field voltage and vary the frequency of the applied voltage.

Okay and measure the output voltage of the machine and plot plot actually the phase plot the gain versus that is the applied voltage at the field winding and the output voltage at the terminal of the synchronous generator for for a range of frequency right once you make this plot and then this plot can be that is this can be this curve can be fitted with the characteristic of this G_s . You can do the curve fitting right and by doing the curve fitting you can identify the time constant T_{do} prime and T_{do} double prime and T_{kd} also right.

Now over the years in the literature actually there is lot of lot of emphasis has been laid down on measuring the synchronous machine standard parameters therefore, when I talk about standard parameters the standard parameters are the synchronous inductance sub transient inductance transient inductance then these time constants right.

Now these are parameters called as the standard parameters and these parameters can be measured by by performing tests at the synchronous machine terminals and lot of work has been going on actually to measure accurately these standard parameters of the synchronous generator particularly those who are actually working in the field actually right. Sometimes you will come across where you have to measure these parameters by performing the standard tests and a lot of literature is available on performing this test even IEEE has given some standards, standard test to be conducted for measuring these standard parameters.

Now in the case of salient pole machine, salient pole machine there is 1 small additional point to be learnt and it is something like this in the case of salient pole machine we provide the damper winding in the phase of the pole there is a the pole is a laminated pole, you know while actually if you talk the round rotor machine the rotor is as solid it is not laminated 1 right and therefore you can always identify or you can represent actually quadrature axis by 2 windings, 2 damper windings right while in the case of salient pole machine there is 1 damper winding provided there is only 1 damper winding.

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Parameters for salient pole machines

$$L_q = L_l + L_{aq}$$

$$L_q'' = L_l + \frac{L_{aq}L_{lq}}{L_{aq} + L_{lq}} \quad (13.42)$$

$$T_{qo}'' = \frac{1}{R_{lq}} \left(L_l + \frac{L_{aq}L_{lq}}{L_{aq} + L_{lq}} \right)$$

Now once there is only 1 damper winding, you cannot have any distinction between the transient and sub transient inductances L_d prime and L_q prime and L_q double prime this distinction is not possible, they are same okay and therefore this for salient pole machine the sub transient and

transient they are same that is the transient there is no mention of transient we always call of sub transient okay because there is they are considering the damper winding 1 damper winding only these are the expressions for salient pole machine.

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Typical values of standard parameters

$$X_d \geq X_q > X'_q > X'_d > X''_q > X''_d \quad (13.34)$$

$$T'_{do} > T'_d > T''_{do} > T''_d > T_{kd} \quad (13.35)$$

$$T'_{qo} > T'_q > T''_{qo} > T''_q \quad (13.36)$$

Further you will see actually in the literature actually after this discussion will show you that these are the relationship between the different types of reactance because when we talk in per unit system the inductance is same as the reactance okay these are the standard relations which are given.

Now, I will just quickly tell you 1 more point actually before I close my discussion here that when we started the study of transient stability I talked about the classical model and in the classical model I said that the synchronous generator can be represented by a constant voltage behind transient reactance and this representation is valid, when we assume the field flux linkage ψ_{fd} constant that is if you assume this ψ_{fd} constant right then the voltage behind transient reactance or direct axis transient reactance becomes constant.

Okay now this we will quickly derive actually and show you that yes how it happens now the steps involved are like this, you write down the per unit flux linkage in the d axis ψ_{ad} , you can just write down ψ_{ad} is equal to minus $L_{ad} i_d$ plus $L_{ad} i_{fd}$. Here, we are assuming the mutual inductance is same, okay ψ_d is equal to ψ_{ad} minus $L_1 i_d$. Okay this is the difference between ψ_{ad} and ψ_d ψ_{fd} is ψ_{ad} plus $L_{fd} i_{fd}$ these equations are standard. We all know it about now using this last expression what we do is that is we find out the expression for i_{fd} .

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Constant flux linkage model

Classical model

The per unit flux linkages identified in d-axis are given by

$$\Psi_{ad} = -L_{ad}i_d + L_{ad}i_{fd}$$
$$\Psi_d = \Psi_{ad} - L_l i_d$$
$$\Psi_{fd} = \Psi_{ad} + L_{fd}i_{fd}$$

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$$i_{fd} = \frac{\Psi_{fd} - \Psi_{ad}}{L_{fd}}$$

Substituting i_{fd} in expression for Ψ_{ad} ,

$$\Psi_{ad} = -L_{ad}i_d + \frac{L_{ad}}{L_{fd}}(\Psi_{fd} - \Psi_{ad})$$

We find the expression for i_{fd} , the i_{fd} comes out to be equal to Ψ_{fd} minus Ψ_{ad} upon L_{fd} . You eliminate we eliminate the term i_{fd} from the first equation that is Ψ_{ad} which is equal to minus $L_{ad} i_d$ plus $L_{ad} i_{fd}$, you substitute the expression for i_{fd} then Ψ_{ad} will be written in the form minus $L_{ad} i_d$ plus L_{ad} upon i_{fd} Ψ_{fd} minus Ψ_{ad} . Okay when you rearrange the whole thing you rearrange the term that is Ψ_{ad} you take it out on this side and express Ψ_{ad} in terms of other parameters it will come out to be equal to Ψ_{ad} is equal to L_{ad}' into this term where L_{ad}' is defined like this okay. Similarly, for Ψ_{aq} 1 can write down the Ψ_{aq} as equal to $L_{q'}$

prime minus L_1 . Now here we will make 1 more small assumption here that the transient saliency, saliency we will neglect we assume that X_d prime is same as X_q prime right.

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Rearranging the terms we get,

$$\Psi_{ad} = \dot{L}_{ad} \left(-i_d + \frac{\psi_{fd}}{L_{fd}} \right)$$

where

$$\dot{L}_{ad} = \frac{\frac{1}{L_{ad}}}{\frac{1}{L_{ad}} + \frac{1}{L_{fd}}} = \dot{L}_d - L_1$$


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Similarly, for the q-axis

$$\Psi_{aq} = \dot{L}_{aq} \left(-i_q + \frac{\psi_{lq}}{L_{lq}} \right)$$

where

$$L_{aq} = \dot{L}_{q'} - L_1$$


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The d-axis stator voltage is given by

$$e_d = R_a i_d - \omega \psi_q$$

$$= -R_a i_d + \omega (L_l i_q - \psi_{aq})$$

With this assumption, now we can write down our d axis voltage e_d equal to $R_a i_d$ minus omega psi q which is written in terms of psi aq i_d and i_q this is e_d that is in this expression for e_d , you replace this psi aq now once you replace the psi aq e_d can be written in terms of i_d i_q and e_d prime that is this expression for e_d is written as minus $R_a i_d$ plus X_d prime into i_q plus E_d prime this E_d prime is a important term here which is expressed here as E_d prime equal to minus omega L_{aq} prime into this. If we assume this psi 1q constant that is the flux linkage in the q circuit also constant right then this term is a constant term.

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Where $\omega = \omega_r = \omega_0 = 1pu$. Substituting ψ_{aq}

$$e_d = -R_a i_d + \omega L_l i_q - \omega \dot{L}_{aq} \left(-i_q + \frac{\psi_{lq}}{L_{lq}} \right)$$

$$= -R_a i_d + \omega (L_l + \dot{L}_{aq}) i_q - \omega \dot{L}_{aq} \left(\frac{\psi_{lq}}{L_{lq}} \right)$$

$$= -R_a i_d + X_d' i_q + E_d'$$

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Where

$$E'_d = -\omega L'_{aq} \left(\frac{\psi_{lq}}{L_{lq}} \right)$$

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Similarly q-axis stator voltage is given by

$$e_q = -R_a i_q - X'_d i_d + E'_q$$

where

$$E'_q = \omega L'_{ad} \left(\frac{\psi_{fd}}{L_{fd}} \right)$$

Similarly when you can write down for e_q where you it will be written in this form minus $R_a i_d i_q$ minus $X'_d i_d$ plus E'_q , where this E'_q prime is written here as $\omega L'_{ad}$ prime into ψ_{fd} upon L_{fd} . Okay therefore this E'_q prime is written in terms of ψ_{fd} and E'_d prime is written in terms of ψ_{ad} therefore, this E'_q prime and E'_d prime if you assume these 2 fluxes to be constant they will remain constant right.

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Using phasor notation, we have

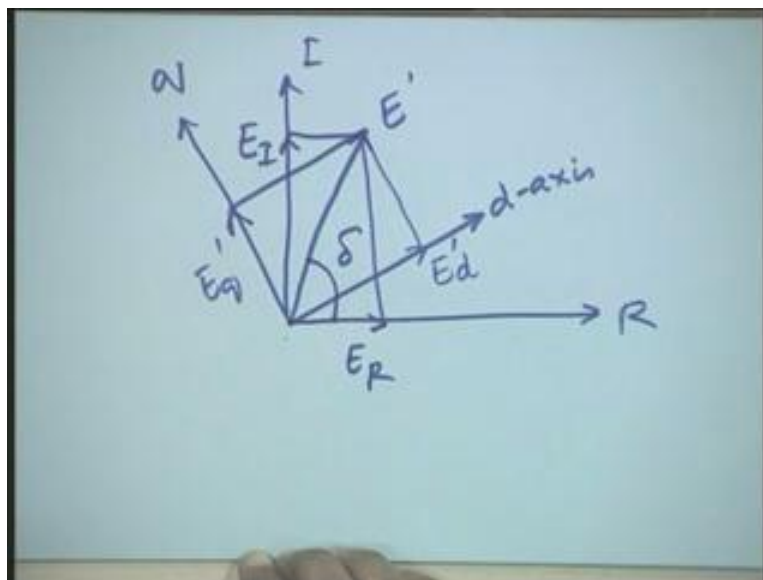
$$\tilde{E}_t = \tilde{E}' - (R_a + jX'_d)\tilde{I}_t \quad (12.29)$$

where

$$\begin{aligned} \tilde{E}' &= E'_d + jE'_q \\ &= L'_{ad} \left(-\frac{\psi'_{lq}}{L_{lq}} + j\frac{\psi'_{fd}}{L_{fd}} \right) \end{aligned}$$

Once these 2 voltages are constant I have established here I have establish here that this voltage E prime that is the the voltage E prime that is the voltage behind transient reactance can be written as E'_d prime plus j times E'_q prime and when these 2 voltages are constant this E prime is also constant.

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Now here I you you have to look like this that if suppose consider that this is my q axis, okay this will become d axis let us say this is our E prime. This quantity is going to be E'_q prime this is

going to be your E_d prime. Now if E_q prime is constant E_d prime is constant E prime is constant if suppose these are our q axis let us say that the reference axis is shown somewhere here this is R and I these are our reference axis's and we measure the angle with respect to the reference axis that is E prime this angle is measured as δ .

Now you can easily see here actually is that so far this E prime is concerned right the magnitude of E prime is constant and its position with respect to q axis is fixed right and therefore for studying the stability I can measure the angle of E prime with respect to some arbitrary reference it may be synchronous rotating reference prime or so on therefore here you will find actually that the component of E prime call it say E_R and E_I this is our actually I and R they will be varying as the δ varies therefore there are 2 important points to sum up here 1 is that this E prime remains constant when we make these assumption second point is that position of this E prime with respect to q axis is fixed and therefore, I can measure the angle with respect to angle between E prime and any reference frame for performing my stability studies right. Let me sum up here that today, we have developed the relations between the standard synchronous machine parameters with respect to the basic parameters and also we have established that that the classical model in a classical model the the if you assume this flux linkage is constant then the voltage behind transient reactance is constant right. Thank you!