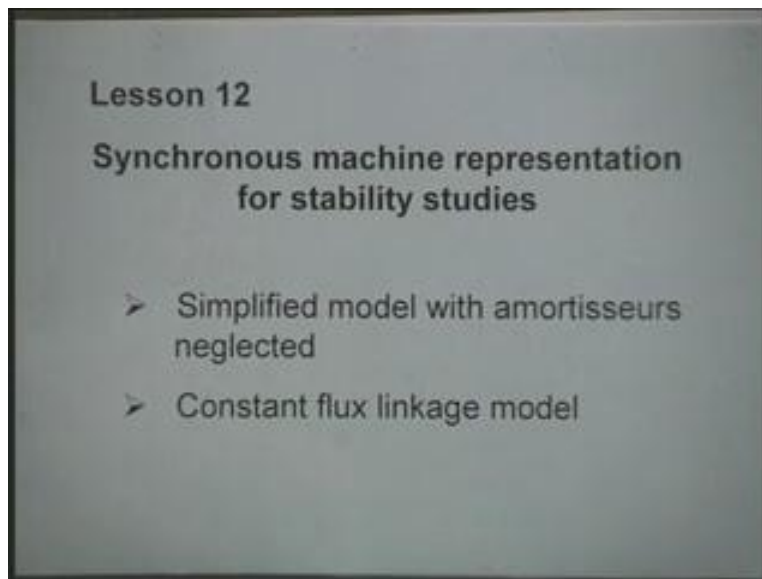


Power System Dynamics
Prof. M. L. Kothari
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Lecture - 12
Synchronous Machine Representation for Stability Studies

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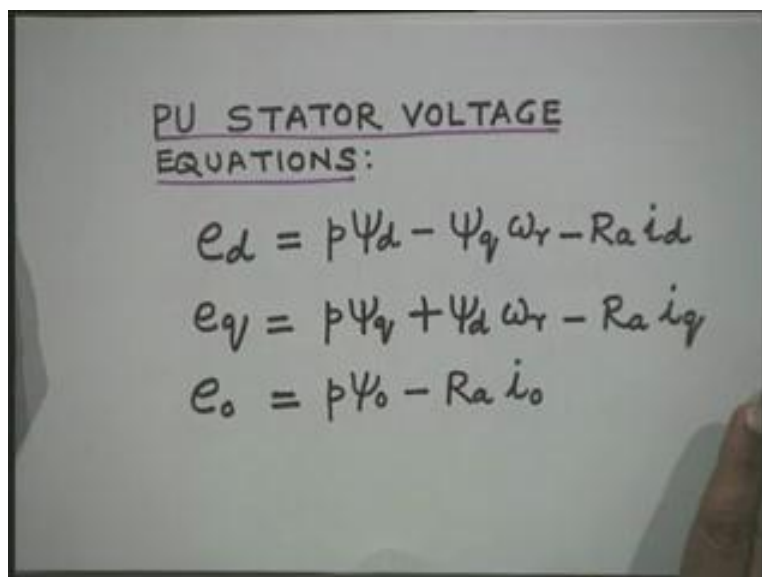


Lesson 12

Synchronous machine representation for stability studies

- Simplified model with amortisseurs neglected
- Constant flux linkage model

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PU STATOR VOLTAGE
EQUATIONS:

$$e_d = p\psi_d - \psi_q \omega_r - R_a i_d$$
$$e_q = p\psi_q + \psi_d \omega_r - R_a i_q$$
$$e_o = p\psi_o - R_a i_o$$

Friends, today we shall study about the synchronous machine representation for stability studies. In this we will study the simplified model with amortisseurs neglected and a constant flux linkage model. Now here, before I discuss these simplified models let us see the per unit system of equations which we have developed so far. I will just summarize the per unit system of equations which we have developed for the synchronous machine.

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PU ROTOR VOLTAGE EQUATIONS

$$E_{fd} = p\psi_{fd} + R_{fd} i_{fd}$$

$$0 = p\psi_{1d} + R_{1d} i_{1d}$$

$$0 = p\psi_{1q} + R_{1q} i_{1q}$$

$$0 = p\psi_{2q} + R_{2q} i_{2q}$$

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PU STATOR FLUX LINKAGE EQUATIONS

$$\psi_d = -(L_{ad} + L_l) i_d + \underline{L_{ad}} i_{fd} + \underline{L_{ad}} i_{1d}$$

$$\psi_q = -(\underline{L_{aq}} + L_l) i_q + \underline{L_{aq}} i_{1q} + \underline{L_{aq}} i_{2q}$$

$$\psi_o = -L_o i_o$$

These 3 equations represent the stator circuit equations in the dqo frame and all these terms are expressed in per unit. All the currents, flux linkages, the voltages are expressed in per unit, the time is also expressed in per unit. The second set of equations are the per unit rotor voltage equations. Now here while writing the per unit rotor voltage equations I have considered one amortisseur on the d axis and 2 amortisseurs on the q axis. The field winding circuit voltage equation is $e_{fd} = p \psi_{fd} + R_{fd} i_{fd}$. Similarly, the equations are written for amortisseurs. For the first amortisseur on the d axis, $0 = p \psi_{1d} + R_{1d} i_{1d}$. Now here I put this substitute one to indicate that there is 1, 1 amortisseur on the d axis or amortisseur number 1 on the d axis. If there is a 2 that will become the second winding also on the d axis, okay $0 = p \psi_{2d} + R_{2d} i_{2d}$ and similarly the last equations.

Then in these equations which we have written, we need the expressions for stator flux linkages. Therefore another set of equations are the stator per unit stator flux linkage equations. These are written as $\psi_d = -L_{ad} i_{fd} + L_{ld} i_d + L_{ad} i_{fd}$. Now here if you see here this L_{ad} is the mutual inductance between the stator and field winding, okay. Now in the per unit system of representation, we have made all mutual inductances on the d axis equal so that you will find that this L_{ad} appears here.

Similarly, this term in this term represents the mutual inductance between the direct axis amortisseur and d axis of the stator winding. Right therefore, again this comes out to be the same. Similarly, similarly for quadrature axis windings, we have flux linkage $\psi_q = -L_{aq} i_{1q} + L_{lq} i_q + L_{aq} i_{1q}$ plus $L_{aq} i_{2q}$ okay. If again these are all mutual inductances are made equal the last equation is the flux linkage due to the zero sequence currents.

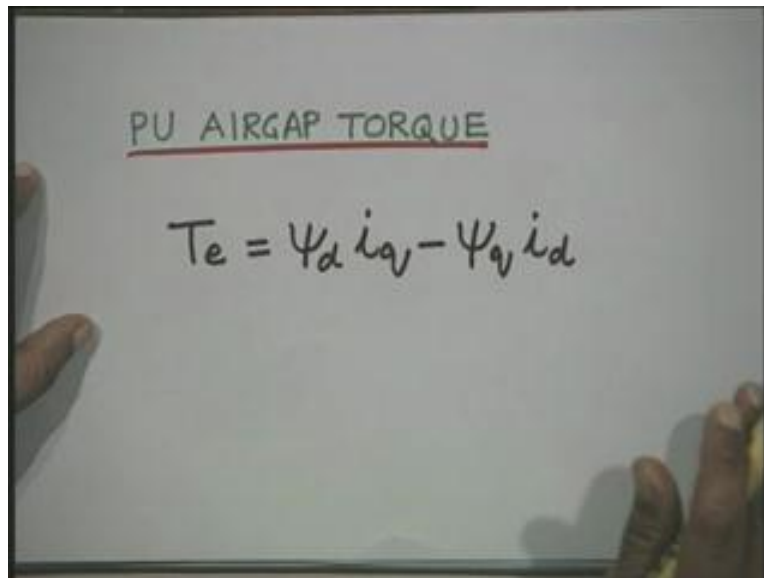
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PU ROTOR FLUX LINKAGE EQNS

$$\begin{aligned}\psi_{fd} &= L_{ffd} i_{fd} + L_{fd1} i_{1d} - \underline{L_{ad} i_d} \\ \psi_{1d} &= L_{fd1} i_{fd} + L_{l1d} i_{1d} - \underline{L_{ad} i_d} \\ \psi_{1q} &= L_{l1q} i_{1q} + L_{aq} i_{2q} - \underline{L_{aq} i_q} \\ \psi_{2q} &= L_{aq} i_{1q} + L_{22q} i_{2q} - \underline{L_{aq} i_q}\end{aligned}$$

Then we have equations for rotor flux linkage equations which are expressed in per unit rotor flux linkage equations okay now again if you see here again we find actually that in the rotor circuit also we get the these terms L_{ad} L_{ad} okay.

(Refer Slide Time: 05:47)



PU AIRGAP TORQUE

$$T_e = \psi_d i_q - \psi_q i_d$$

Now we have to see the last equation is this per unit air gap torque T_e is equal to $\psi_d i_q$ minus $\psi_q i_d$ this is the per unit torque in the synchronous generator. Now this is the complete set of equations for representing a synchronous machine okay and if you want to study the dynamics of the synchronous machine then this set of equations plus the swing equation have to be solved and you have seen that these equations are non-linear differential equations, okay and these equations can be solved by applying numerical technique. Now in case actually the system is small right then we can easily solve these equations and obtain the complete solution but when we talk about the power system stability problem where we may come across hundreds of generators thousands of buses a realistic power system which we have today even in our country right has large number of generators and large number of buses.

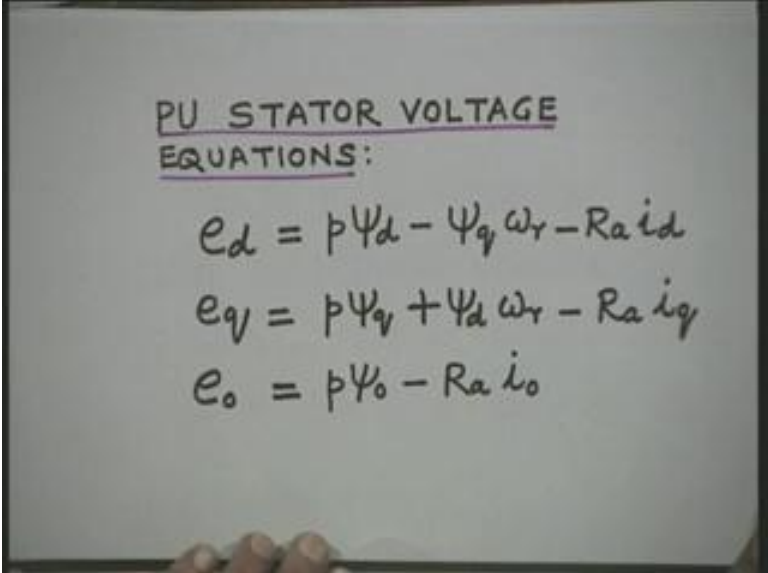
Therefore the dimension of the system is very large and for analyzing the stability of the system right if we use this detailed model which we have developed while developing this model we have not done any approximations, no simplifications were made further actually I would just emphasize that in these equations which we have written right the all inductances are the saturated values of these inductances that is you account for the magnetic saturation also while computing the inductances okay.

Now if you want to use this equations for stability studies it becomes very very difficult because the system becomes very large and for large system the number of equations become very large therefore, when we look in terms of simplifying the system model we always look from the point of view of reducing the dimension of the model that is we want to reduce the number of

the differential equations, second is that we would like to have numerical technique which can be used to solve our these equation quickly that is the fast numerical techniques and low order model of the system. Okay these are the main thrust looking from that point of view we shall be studying actually today that how we can make some approximations, simplifications.

Now what we will do that we will consider some various degrees of approximations okay that can be made to simplify the model minimizing the data requirement and the computational effort many times we have a problem also that ah certain data's which are required to use the model are not available, one example is that normally the synchronous machine data related to the amortisseurs are very are not easily available right and therefore we have to see that what approximation can be made, so that computations can be fast or computational efforts are reduced as well as the data's which are not available right can be or we can do the studies without using those data's or getting some approximate data's.

(Refer Slide Time: 10:23)



PU STATOR VOLTAGE
EQUATIONS:

$$e_d = p\psi_d - \psi_q \omega_r - R_a i_d$$

$$e_q = p\psi_q + \psi_d \omega_r - R_a i_q$$

$$e_o = p\psi_o - R_a i_o$$

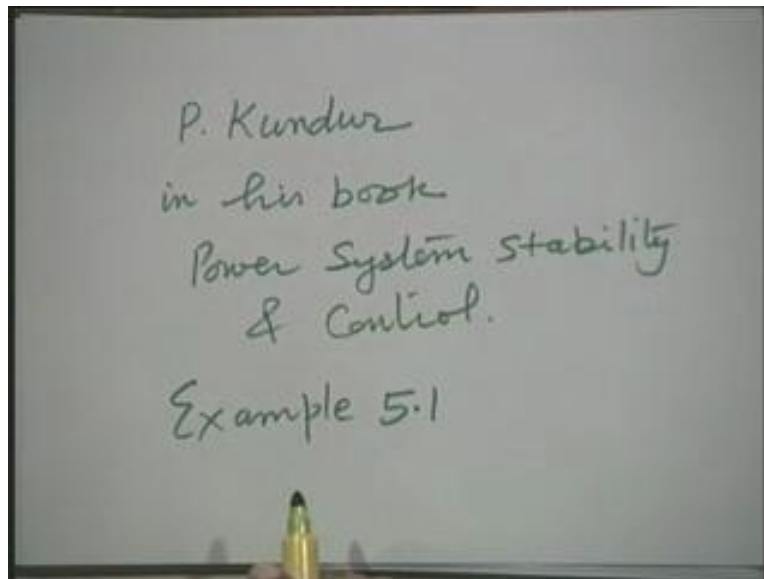
Now while looking at these simplifications, the first simplification which has been looked into is that we neglect this $p\psi_d$ and $p\psi_q$ terms in the stator circuit equations this is the first simplification. Now let us see what is the justification for making this simplification or neglecting the $p\psi_d$ and $p\psi_q$ terms, these is terms are the transformer voltages. Okay earlier I talked about these equations and mentioned that these transformer voltages terms are smaller as compared to the speed voltage terms right but if you see this equation this stator circuit equation only because of this transformer voltage terms these equations are differential equations right and if we neglect this transformer voltage term or the derivative terms right then the stator voltage equations become algebraic equations in fact these terms represent the stator circuit transients, okay.

Now I will just discuss what is the justification for neglecting these terms? Now the stator circuit is connected to the transmission network right. Now for so far the transmission side or you say the transmission network transients are concerned. Okay there is a transmission network transients the time constants of this transient is generally very low and whenever whenever some transient phenomena occurs in transmission system it quickly dies time constant is very small right and therefore for system stability studies, we ignore the transmission system transients. We assume that the transmission system is in steady state.

We consider only the sinusoidally varying voltages and currents in the transmission system although whenever disturbances occur in the system like say fault we will find actually that when the fault occurs it will have decaying DC components of the fault current it will have a high frequency currents which are also adjusting in the during the fault system okay and but because of the the low time constants, we always ignore the transmission network transients. Okay now if we ignore the transmission network transients can we consider the stator transients because they are part of the same circuit.

Therefore, if you consider the stator transients it becomes inconsistent and therefore therefore for all system stability studies this assumption is made then the question arises whether this assumption is a realistic assumption whether the results which we will get by ignoring these terms will be acceptable, people have done lot of research on this and after performing a detail studies the conclusions drawn are that yes we can ignore these terms that is the the stator circuit transients can be neglected.

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I will suggest you to look into the discussion given by P. Kundur in his book on power system stability and control, power system stability and control. In this he has discussed the results reported in some research papers and compared the the dynamic performance of the system

including including the stator circuit transients and excluding the stator circuit transients and compared, okay and once you read this you will, you will come to a very important influence that yes for stability studies one can, one can comfortably neglect the stator circuit transients right and therefore the moment you neglect the stator circuit transient we get very big simplification that the stator circuit voltage equations become algebraic equations right that is e_d equal to minus ψ_q ω_r minus $R_a i_d$ e_q equal to ψ_d ω_r minus $R_a i_q$ and e_o equal to $p \psi_o$ minus $R_a i_o$ okay they are becoming the algebraic equations, okay the another important assumption which is made or approximation which is made is in this equation this rotor speed ω_r is assumed to be equal to be synchronous speed that is in per unit system, we make ω_r equal to one right this is another assumption.

Now in practice by making this ω_r equal to 1 there is not much simplification in the system model however, the very interesting thing which is there is that if you assume ω_r equal to one then this assumption to some extent counter balances this assumption that is where you neglect the stator circuit transients that is if you analyze the system performance neglecting only $p \psi_d$ term $p \psi_q$ term and $p \psi_o$ term. Okay second time you do the studies neglecting both these terms right then not neglecting both the terms sorry by making the second assumption correction that is we are neglecting this transformer voltage terms or stator circuit transients and assuming this rotor speed to be equal to the synchronous speed right, if you make both these assumptions right then the results which we get are more closer to the to the actual results right and therefore this assumption is also a very desired assumption.

Now in the same book, Kundur in his example 5.1 in example, 5.1 he has established established considering small perturbations that effect of neglecting the speed variation, counter balances the effect of neglecting the stator circuit transients I will suggest that you should carefully study this example 5.1, okay now these are the two major assumptions which have been made.

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With per unit $\omega_r = 1.0$, the stator equations reduce to,

$$e_d = -\psi_q - i_d R_a \quad (12.3)$$

$$e_q = \psi_d - i_q R_a \quad (12.4)$$

Now with these assumptions, let us see develop the model the simplified model which we will get after okay that is if you make this assumption then the equations will becomes like this for a balanced three phase system e_d will become equal to minus ψ_q , where ω_r is 1 minus $i_d R_a$ e_q equal to ψ_d minus $i_q R_a$ that is these are the two stator circuit equations which we shall be using for further system studies and no more simplifications are done so far the stator circuit equations are concerned.

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Relationship between $\underline{P_e}$ and $\underline{T_e}$

$$\underline{P_t} = \underline{e_d i_d} + \underline{e_q i_q}$$


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Substituting $\underline{e_d}$ and $\underline{e_q}$ from above equations

$$\underline{P_t} = \underline{(\psi_d i_q - \psi_q i_d)} - \underline{R_a(i_d^2 + i_q^2)} \quad (12.5)$$

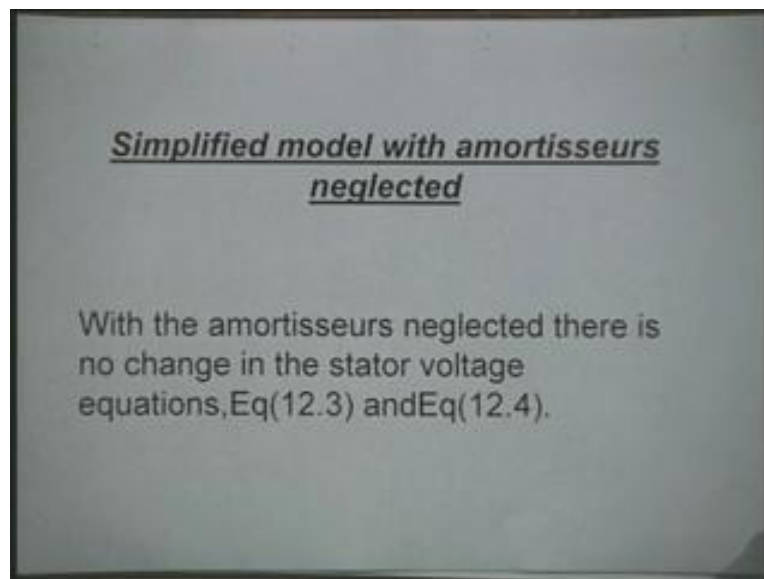
$$= \underline{T_e} - \underline{R_a I_t^2}$$

Now with these simplifications, we can derive the expression for the electrical power output and electromagnetic torque or air gap torque. We will see that the power output P_t is $e_d i_d$ plus $e_q i_q$, now you substitute the expressions for E_d and E_q from these equations, okay these algebraic equations you substitute this.

With this substitution, we will find actually that the terminal power P_t will come out to be equal to $\psi_d i_q$ minus $\psi_q i_d$. Okay $\psi_d i_q$ minus $\psi_q i_d$ this is the expression minus R_a times i_d^2 square plus i_q^2 square, this term can be identified as stator circuit loss or stator circuit resistance loss. Therefore, we say that terminal power is equal to the air gap torque minus armature loss of the stator that is why I said that when you have to calculate actually the torque T_e right or air gap power we first calculate the terminal power we know it to that we add the armature resistance drop I am I am sorry armature resistance loss not drop and this is very important equation which we shall be using.

Okay, now after making these 2 major simplifications we can make some further simplifications that is if you look into the model in the stator circuit equations become the algebraic equations, okay but when you look at the rotor circuit equations then the rotor circuit equations are differential equations. Okay now next next set of simplifications which can be made are that we neglect the, neglect the amortisseurs.

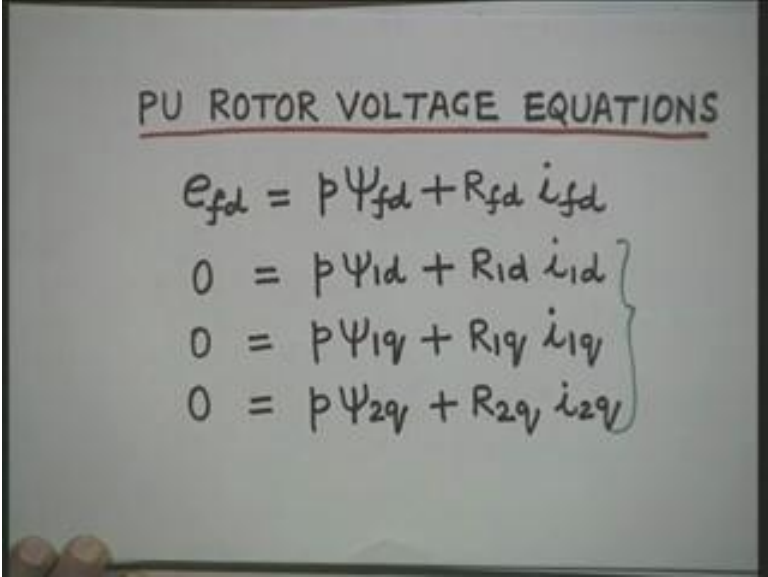
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We have amortisseurs located on the rotor, okay now here I would like to just question you that many times many times people have considered the amortisseurs in the mathematical model right but we can make some simplification here by neglecting the amortisseurs. Now the justification for neglecting amortisseurs is like this one is that the synchronous machine parameters related to amortisseurs or synchronous machine parameters related to amortisseurs are generally not available or not easily available okay this is one and therefore if the data is not

available we have to take some approximate data and use it, this is one second is that this transients associated with the amortisseurs are also fast transients their time constants are also low and generally they decay fast.

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PU ROTOR VOLTAGE EQUATIONS

$$\begin{aligned}
 e_{fd} &= p\psi_{fd} + R_{fd} i_{fd} \\
 0 &= p\psi_{1d} + R_{1d} i_{1d} \\
 0 &= p\psi_{1q} + R_{1q} i_{1q} \\
 0 &= p\psi_{2q} + R_{2q} i_{2q}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

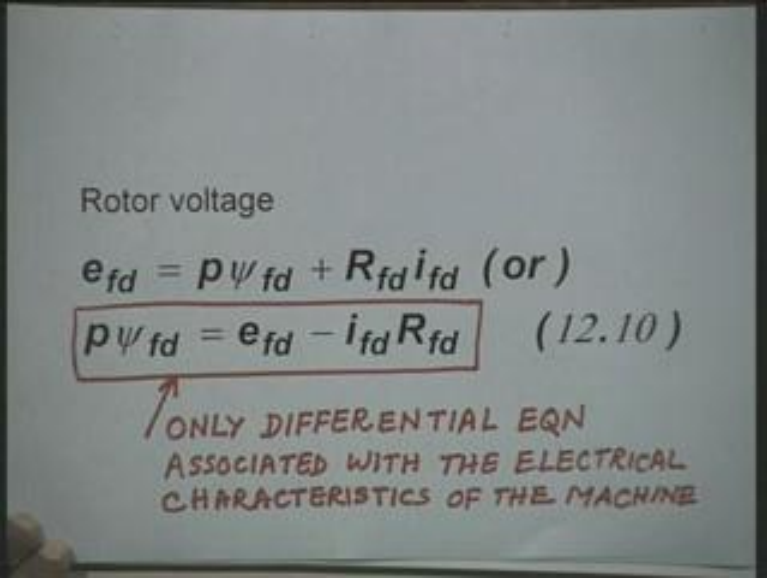
Therefore, the moment you neglect the amortisseurs in the system or amortisseurs on the rotor circuit what we gain, what do we gain just can you tell me just tell me, what will you what do you gain actually by this because, I had mentioned that these are the 4 equations related to the voltage, related to the rotor circuits. Okay this is the equation related to the field voltage and the field circuit sorry this is related to the field circuit, this is related to the amortisseur on the d axis and these are the two related to the amortisseurs on the q axis.

Now if I neglect this amortisseurs then these 3 equations disappear and these are the 3 differential equations okay and therefore, the immediate advantage which I get is the ah the model becomes of lower order number of differential equations which I have to write down for the system will reduce this is one advantage which I get immediately that is the model order gets reduced and you can easily understand that if the number of differential equations are less right then the solution becomes faster.

The second advantage which we will get here is that while performing the simulation studies or dynamic simulation studies, we can use higher or the larger time step for integration because the time step which we use for integration of the differential equations it depends upon, it depends upon the time constants of the system right and in case you want to capture the complete dynamics of the system right, our time step should be small enough to take care of all the time constants

In case the moment we neglect these 3 differential equations, we get very important advantage and that advantage is that the remaining equations have larger time constant and we can use larger time step for simulation and once you use larger time step then our computational efforts are reduced therefore we get two major advantages both one in terms of reduction in the order of the system, second is reduction in the computational time or simulation time by using higher or larger value of time step for integration. Okay now with this 2 assumptions which we have made here. The stator circuit you know equations are now algebraic equations.

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Rotor voltage

$$e_{fd} = p\psi_{fd} + R_{fd}i_{fd} \text{ (or)}$$

$$p\psi_{fd} = e_{fd} - i_{fd}R_{fd} \quad (12.10)$$

ONLY DIFFERENTIAL EQN
ASSOCIATED WITH THE ELECTRICAL
CHARACTERISTICS OF THE MACHINE

Okay the rotor circuit the the equations related to amortisseurs we have neglected then we are left only with one differential equation that is e_{fd} equal to $p\psi_{fd}$ plus $R_{fd}i_{fd}$. This equation is corresponding to the field circuit of the synchronous generator. Now this can be written in this form $p\psi_{fd}$ equal to e_{fd} minus $i_{fd}R_{fd}$ this is a differential equation, we write down the derivative term on the side and therefore one has to note down very carefully that this is the only differential equation associated with the electrical characteristic of the machine right.

The the swing equation are always there right therefore whenever you study the dynamics of the system then we have to consider the differential equation associated with the synchronous machine the stator and rotor circuits but after the simplifications we find that we have only one differential equations which is associated with the electrical characteristic of the system and therefore, my model becomes very very much simplified. Now here I will just present how the alternatives form of machine equations.

So far the equations which we have written right these equations were written in terms of the field current okay and flux linkages. Now another form of equations this there is only the change in the form actually there is nothing new is that we define the terms like this. Here, we define the term E_f this is slight a mistake here E_f a voltage proportional to i_{fd} that is E_f equal to $L_{ad}i_{fd}$ a

voltage proportional to i_{fd} that is you know field current right, field current is i_{fd} and a term when you multiply this by L_{ad} mutual inductance right then all these things are all expressed in per unit quantity right. Therefore this E_t term is proportional to i_{fd} because this L_{ad} becomes a proportionality constant therefore this is one term which we define here, second term which we define here is a voltage E_q prime this is proportional to the field flux linkage that is ψ_{fd} that is instead of talking about the field flux linkages.

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Alternative form of machine equations

$$E_t = L_{ad} i_{fd}$$

= voltage proportional to i_{fd}

$$E'_q = \frac{L_{ad}}{L_{ffd}} \psi_{fd}$$

= voltage proportional to ψ_{fd}

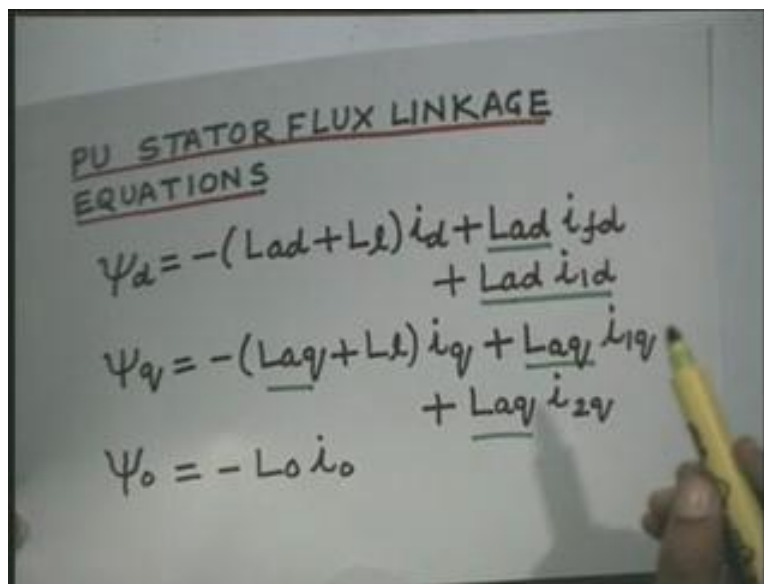
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$$\underline{E_{fd}} = \frac{L_{ad}}{R_{fd}} \underline{e_{fd}}$$

= voltage proportional to e_{fd}

We talk about a voltage term and this voltage term is proportional to its flux linkages and this is the proportionality constant and the third term which we define is E_{fd} . Now this E_{fd} is the voltage applied to the field circuit right we transform this by multiplying with a multiplying factor and again these are per unit quantities and therefore this is a constant and E_{fd} a voltage term proportional to the field voltage okay therefore instead of calling this term E_{fd} , I will now express in terms of this term E_{fd} which is related to this voltage by this multiplying term that is the coefficient.

(Refer Slide Time: 31:46)



PU STATOR FLUX LINKAGE EQUATIONS

$$\psi_d = -(L_{ad} + L_l) i_d + \underline{L_{ad}} i_{fd} + \underline{L_{ad}} i_{1d}$$

$$\psi_q = -(\underline{L_{aq}} + L_l) i_q + \underline{L_{aq}} i_{1q} + \underline{L_{aq}} i_{2q}$$

$$\psi_o = -L_o i_o$$

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In terms of these new variables

$$\psi_d = -L_d i_d + E_I \quad (12.11)$$

Multiplying both sides by L_{ad} / L_{ffd}
Eq(12.9) becomes

$$E'_q = -\frac{L_{ad}^2}{L_{ffd}} i_d + E_I \quad (12.12)$$

Okay now when I use this when I use these terms now or the newly defined terms and use our equations. Okay the equations which we have got are these are the basic equations therefore, let us write down the equation for ψ_d , okay the equation for ψ_d can be written now after making all the simplifications here ψ_d equal to minus $L_d i_d$ plus E_f .

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$$\underline{L_d - L'_d} = \frac{L_{ad}^2}{L_{ffd}}$$

Substituting in Eq(12.12)

$$\underline{E'_q = E_f - (L_d - L'_d) i_d} \quad (12.13)$$

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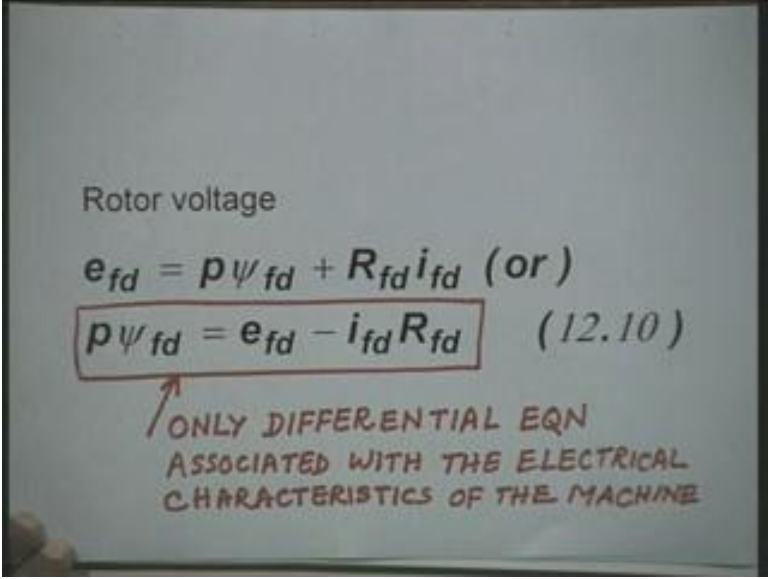
Multiplying Eq(12.10) by $\underline{L_{ad}/L_{ffd}}$, we get

$$p \left[\frac{L_{ad}}{L_{ffd}} \psi_{fd} \right] = \frac{L_{ad}}{R_{fd}} \left(\frac{R_{fd}}{L_{ffd}} \right) e_{fd} - \frac{R_{fd}}{L_{ffd}} L_{ad} i_{fd}$$

$$\boxed{pE'_q = \frac{1}{T_{do}} (E_{fd} - E_f)} \quad (12.14)$$

You can just look into the simplified equations which we have developed I will just show you again anyway okay. Similarly, you can write down another equation for E_q prime using the previous equations that is equation number 12.9. Okay you multiply these both sides by L_{ad} and divided by L_{ffd} , you will get the equation in the form E_q prime equal to minus L_{ad} square divided by L_{ffd} i_d into E_i . This term which is the coefficient of i_d has been identified as difference of difference of the direct axis inductance minus direct axis transient inductance. In fact, this I shall establish in my next lecture that is L_d minus L_d prime comes out to be equal to L_{ad} square upon L_{ffd} . This derivation is to be done separately therefore, if you make this substitution in the equation 12.1 2, 12.12, we will get the result in the form that is E_q prime is equal to E_i minus L_d minus L_d prime i_d right. Now here this i_d is normally this is small i_d not capital i_d , we use this direct axis component of the stator current okay.

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Rotor voltage

$$e_{fd} = p\psi_{fd} + R_{fd}i_{fd} \text{ (or)}$$

$$p\psi_{fd} = e_{fd} - i_{fd}R_{fd} \quad (12.10)$$

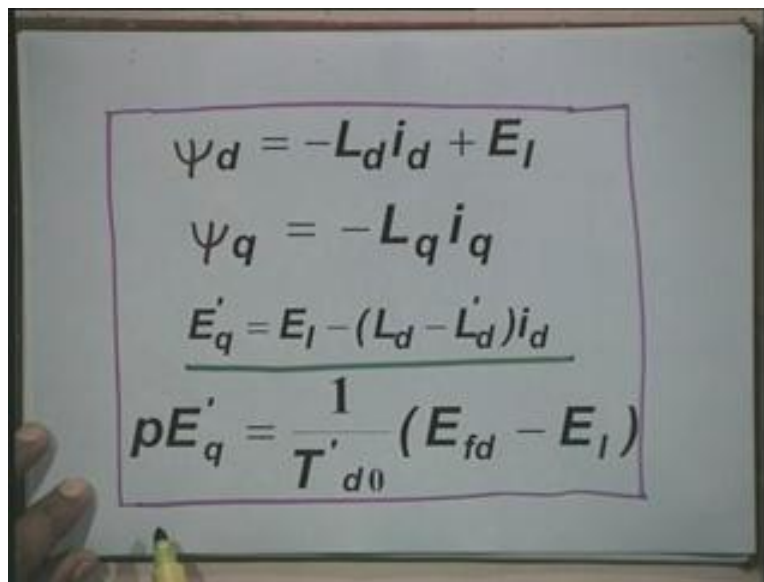
ONLY DIFFERENTIAL EQN
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CHARACTERISTICS OF THE MACHINE

Similarly, similarly we can use this equation 12.10 to obtain an equation of the form. This pE_q prime equal to 1 upon T_{do} prime E_{fd} minus E_i . Now here I think I will just show that equation, if you take this equation this is the equation where $p\psi_{fd}$ equal to e_{fd} minus $i_{fd}R_{fd}$. Okay you multiply both sides of this equation by L_{ad} over L_{ffd} , okay when you multiply both sides we will have the term here p multiplied by L_{ad} divided by L_{ffd} into ψ_{fd} and this term is identified as E_q prime as per our new definition. Okay similarly, this e_{fd} when it is multiplied by L_{ad} upon L_{ffd} you multiply both side by R_{fd} and divide by R_{fd} that is you multiply and divide this term by this term R_{fd} .

So that what happens is that we can identify this term as one upon T_{do} prime this is called the open circuit field time constant, open circuit field time constant because what it this L_{ffd} it is the this is the inductance of the field circuit and R_{fd} is the resistance therefore, the L by R is the time constant okay and this time constant term is a very important term for our stability studies this time constant term is generally of the order of 5 to 10 seconds. Okay, therefore this is the

equation now which is put in the form $pE_q' = \frac{1}{T_{d0}'} (E_{fd} - E_I)$ that is the equation which we have earlier which was in this form the differential equation $p\psi_d = E_{fd} - i_{fd}R_{fd}$ this has been transformed in terms of the newly defined variables that is E_{fd} is the voltage proportional to field voltage, E_I is the voltage proportional to field current and E_q' is the voltage proportional to flux linkages, field flux linkages. Okay therefore now you have in this equation you find that we have only voltage terms not the current or flux linkage term this is the only difference.

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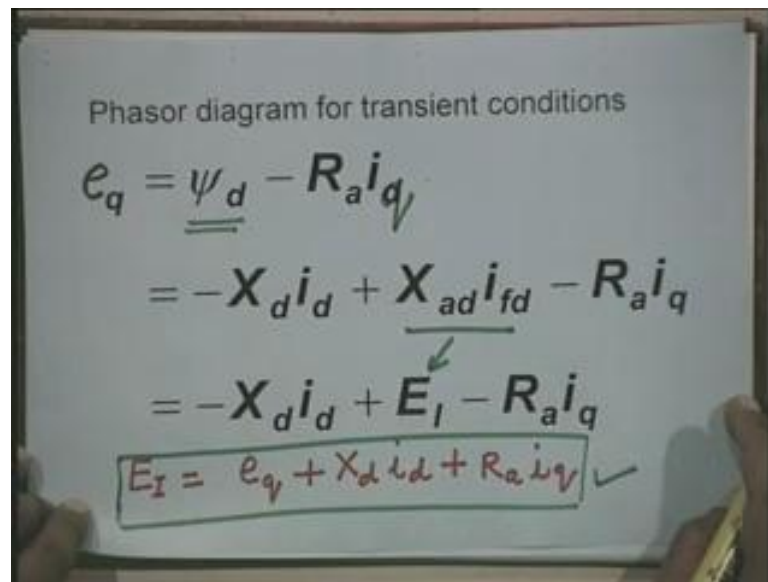
$$\begin{aligned}\psi_d &= -L_d i_d + E_I \\ \psi_q &= -L_q i_q \\ E_q' &= E_I - (L_d - L_d') i_d \\ pE_q' &= \frac{1}{T_{d0}'} (E_{fd} - E_I)\end{aligned}$$

Now I am here summarizing the model of the synchronous machine which is most commonly used for system stability studies that is we write down ψ_d the flux linkage of the d axis equal to minus $L_d i_d$ plus E_I okay, ψ_q is equal to minus $L_q i_q$ E_q' prime expression for E_q' is E_I minus L_d minus L_d' i_d and pE_q' that is the d by d_T of E_q' is equal to one upon T_{d0}' E_{fd} minus E_I . Therefore in these four equations what we see here is that these three are the algebraic equations and one is the differential equation therefore, the synchronous machine is very commonly represented by these equations only one differential equation and what this differential equation is if you look it very carefully this differential equation is related to the field circuit of the synchronous generator the field circuit.

You know we started with the this was the equation you know E_{fd} equal to $p\psi_d$ plus $R_{fd} i_{fd}$ this is the equation for the field rotor field circuit okay and this has been transformed in this particular form and the terms like E_I is the related to $i_d L_d$ like this similarly E_q' which is in this equation is related to other terms. Okay therefore these are the inter related terms this is the differential equation but these algebraic equations are also required to be solved along with the differential equations.

Okay now next very important thing is that we can we can draw a phasor diagram, phasor diagram under transient conditions. Sometimes you even feels like actually that how can we draw a phasor under transient conditions but since we have neglected the stator circuit transients and therefore, so far the stator circuits are concerned it carries the balanced 3 phase sinusoidally varying currents right and therefore you can represent or you can draw a phasor diagram under transient conditions.

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Phasor diagram for transient conditions

$$e_q = \underline{\psi_d} - R_a i_q$$

$$= -X_d i_d + \underbrace{X_{ad} i_{fd}}_{\downarrow} - R_a i_q$$

$$= -X_d i_d + E_I - R_a i_q$$

$$E_I = e_q + X_d i_d + R_a i_q \checkmark$$

Now to draw the phasor diagram, we again start with our basic model there is original equations we start with original equations note there is a lot of papers here. We start with these equations because we have developed these equation ,okay e_d equal to minus ψ_q minus R_a into i_d or i_d into R_a . Okay, similarly e_q equal to ψ_d minus i_a i_q into R_a okay therefore we will make use of these equations to develop a phasor diagram .

Now let us take this term e_q equal to ψ_d minus $R_a i_q$, $R_a i_q$ there is a mistake here okay now this ψ_d term this is the flux linkage term can be replaced by ψ_d can be replaced by minus $L_d i_d$ plus E_I and in per unit system the inductance is equal to the reactance that is L_d can be written as X_d in per unit system right because X_d is nothing but ω into L_d and ω is one therefore I can write down this equation now e_q is equal to minus $X_d i_d$ plus $X_{ad} i_{fd}$ minus $R_a i_d$ that is this term is identified as E_I and therefore now i can write an equation very interesting equation for E_I .

You can see this started with e_q this is the fundamental stator circuit voltage equation okay or stator circuit equation. Now in this equation we are substituting the expression for ψ_d which we have already developed and we are also, we are also replacing this L_d terms or L_{ed} terms by X_d or X_{ad} terms because in per unit system they are same the reactance is the same as the inductance per unit. You get the equation in the form E_I equal to e_q plus $X_d i_d$ plus $R_a i_q$ this is

one very one important equation what we do is that this is actually the algebraic equation that these are all magnitudes of these quantities we have got.

This can be transformed into phasor equation what we do is we multiply both the sides by j , you multiply both the sides by j then you will get j times E_I prime E_I equal to j times e_q j time $X_d i_d$ plus j times $R_a i_q$, you note that anything accept that this equation is multiplied by j all through once you multiply this by j then this term $j E_I$ can be identified as a phasor right therefore I can represent this as $E_I \Delta$ what will be the location of this phasor it is going to be along be quadrature axis because there is a term j associated with it j times e_q again it is located along the quadrature axis then here in this term j times $X_d i_d$, I put $j X_d i_d$ tilde.

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Handwritten equations on a whiteboard:

$$j E_I = j e_q + j X_d i_d + j R_a i_q$$

IN TERMS OF PHASOR NOTATIONS

$$\tilde{E}_I = \tilde{e}_q + j X_d \tilde{i}_d + R_a \tilde{i}_q$$

$$E_q' = E_I - (L_d - L_d') i_d$$

$$= e_q + X_d i_d + R_a i_q - X_d i_d$$

$$= e_q + X_d' i_d + R_a i_q$$

$$\tilde{E}_q' = \tilde{e}_q + j X_d' \tilde{i}_d + R_a \tilde{i}_q$$

Now this j is not replaced because i_d is a reference therefore i_d when it is a reference I can call this as a i_d phasor ah straight away okay since it is a reference there is no but in the case of e_q $j e_q$ is the phasor because it lies along the q axis. Similarly, like this right therefore I get a equation in this form now when you see this equation here. You will find actually that this this is e_q which is along the q axis this term is also along the q axis that is i_d when it is multiplied by j times X_d becomes comes across the q axis this is anyway along the q axis therefore, the this term E_I which we defined as as voltage proportional to field current is is lying along the q axis. Okay then we make use of this equation E_q prime okay.

Now in this equation, we have E_q prime equal to E_I minus L_d minus X_d prime into i_d okay here we replace this E_I by this term e_q plus $X_d i_d$ plus $R_a i_q$ after this replacement and replacing X_d L_d and L_d prime by X_d and X_d prime we will get the equation E_q prime in the phasor form E_q prime in phasor form as E_q prime equal to e_q plus j times X_d prime i_d plus $R_a i_q$ that is this again these 3 terms you can see here they are this this sum is lying along the q axis that is this E_q prime also lies along the q axis E_I also lies along the q axis okay.

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$$\begin{aligned}\psi_d &= -L_d i_d + E_f \\ \psi_q &= -L_q i_q \\ E'_q &= E_f - (L_d - L'_d) i_d \\ pE'_q &= \frac{1}{T'_{d0}} (E_{fd} - E_f)\end{aligned}$$

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$$\begin{aligned}\tilde{E}_q &= \tilde{E}_t + (R_a + jX_q)\tilde{I}_t \\ &= j[X_{ad} i_{fd} - (X_d - X_q) i_d] \\ &= jX_{ad} i_{fd} - j(X_d - X_q) i_d \\ \tilde{E}_q &= \tilde{E}_I - j(X_d - X_q) \tilde{I}_d \quad \checkmark \\ \text{OR } \tilde{E}_I &= \tilde{E}_q + j(X_d - X_q) \tilde{I}_d \quad \checkmark\end{aligned}$$

Then earlier we had defined one term E_q that is the voltage behind this impedance R_a plus j times X_q that is terminal voltage to this terminal voltage we added this term that is R_a plus j times X_q into I_t this term also came out to be along the q axis. Okay therefore, this E_q in the phasor form is represented as E_f minus j times X_d minus $X_q i_d$ that is we can express this E_q which was defined earlier as a voltage behind this impedance okay this is also along the q axis and we have seen actually that it is this voltage phasor which define the location of q axis with respect to the terminal voltage therefore whenever we start drawing the phasor diagram, the first requirement is

that we have to establish the location of d and q axis with respect to the terminal voltage and that now we have established the relationship between this E_q and E_t okay. Now this can be written in this final form.

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Handwritten equations on a whiteboard:

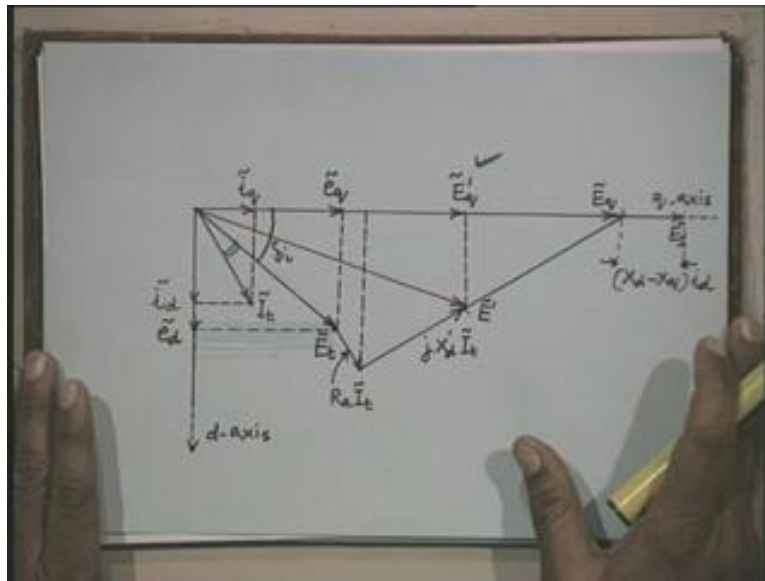
$$\begin{aligned} \text{DEFINE} \\ \checkmark \tilde{E}' &= \tilde{E}_t + (R_a + jX_d') \tilde{I}_t \\ \tilde{E}_t &= e_d + j e_q \\ \tilde{I}_t &= i_d + j i_q \\ \tilde{E}' &= \tilde{E}_d + R_a \tilde{i}_d + j X_d' \tilde{i}_q \\ &\quad + \tilde{E}_q + j X_d' \tilde{i}_d + R_a \tilde{i}_q \\ \checkmark E_q' &= q\text{-axis component of } E' \end{aligned}$$

This term define not define, define we can define one more term here. Now let us see what this um can you just interpret this equation, for example I had the equation in this form E_q equal to E_t plus R_a plus j times X_q into I_t okay therefore we have another term if we divide E prime, E prime equal to E_t plus j times E_t plus this impedance into I_t , this impedance is R_a plus j times X_d prime a sub transient direct axis reactance.

Okay it means this is the voltage behind this impedance terminal voltage plus voltage drop in this impedance therefore, these voltages and these voltages can be simplified and expressed in the form you just make the substitutions right E_t can be written as e_d plus j times e_q I_t can be written as i_d plus j times i_q and therefore this E prime can be written as as e_d plus $R_a i_d$ plus j times X_d prime i_q plus e_q plus j times X_d prime i_d plus $R_a i_q$, this this whole thing will come that is substitute the value of e_d e_q from the expressions for e_d and e_q which we have already developed that is in the previous expressions I think okay.

Now here one thing which we observe is that if you take this voltage term, okay then this has two components one will be along the d axis another will be along the q axis. The q axis component of this quantity comes out to be E_q prime the E_q prime which we define as a voltage proportional to field flux linkages right and the q axis component of E prime comes out to be quantity E_q prime right and therefore now I have drawn a simple phasor diagram to represent all these quantities.

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This is not a very unknown phasor diagram to you right but to make the things complete what we do is that you start with terminal voltage with E_t . Okay let us say this is the terminal current now this and the phase difference between the terminal voltage and terminal current is always known to you then to this E_t we add this voltage drop $R_a I_t$ plus plus j times $X_d I_t$ $R_a I_t$ plus j times $X_q I_t$ you will get this term E_q and this E_q will define the position of q axis.

Once this q axis is defined you resolve the terminal voltage into two components E_t as E_q and e_d . Similarly, the terminal current can be resolved into two components I_q and i_d right then in this phasor diagram we can show this term E prime a voltage behind direct axis transient reactance that is what you do is that you take E_t to this you add $R_a I_t$ plus j times X_d prime I_t therefore, this is the location of E prime and and the when you resolve this E prime into 2 components the q axis component of E prime is E_q prime right therefore, most of the ah relations which we have derived can be easily remembered with the help of this phasor diagram.

Okay, now any doubt I will suggest that you start with our basic equations and make the simplifications step by step, okay and establish establish the new equations or equations in the in terms of new variables. Once they are established in terms of new variables right you can express these variables as phasors that is E_t is a phasor E_q prime is a phasor and E_q is a phasor right and then also the term which you defined is E prime this as a phasor then the whole everything is expressed in the form of a simple phasor diagram okay.

Now let me summarize ah what we have studied today. We started with the complete model of the synchronous machine then we have made the simplifications, we have discussed the simplifications, simplifications discussed are that the stator circuit transients are ignored another simplification we have considered is the the ωr is assumed to be constant then third simplification which we have incorporated is we have neglected the amortisseur circuits.

With this the resulting resulting model of the synchronous machine to describe the electrical characteristic has one differential equation and that differential equation is ultimately written in terms of in terms of the the variables which we have defined in terms of E_q prime in terms of E_f and e_{fd} , okay and this model is very commonly used although we do make some further simplification to obtained a classical model right but this is the model which is very commonly used and any simplification beyond this will not give you the desired results although it will give you the information right but sometimes you can get approximate information about the stability and then go for detail information. Okay with this I conclude my today's presentation. Thank you very much.