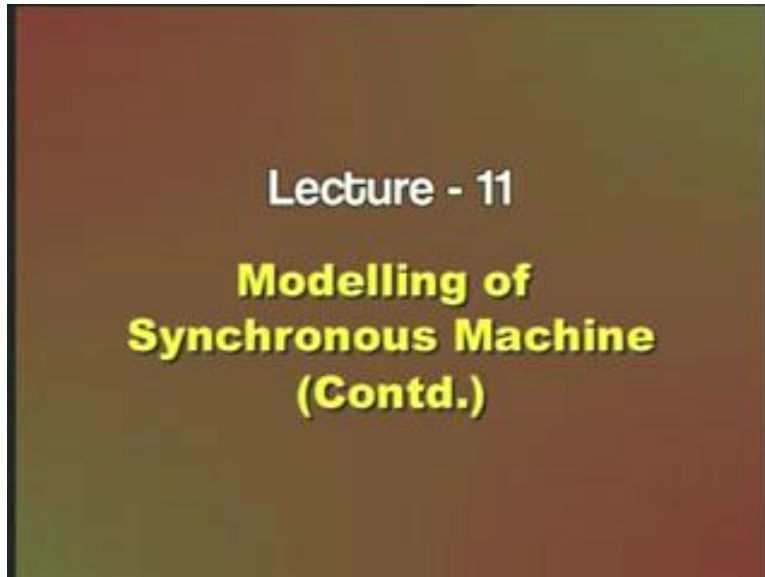
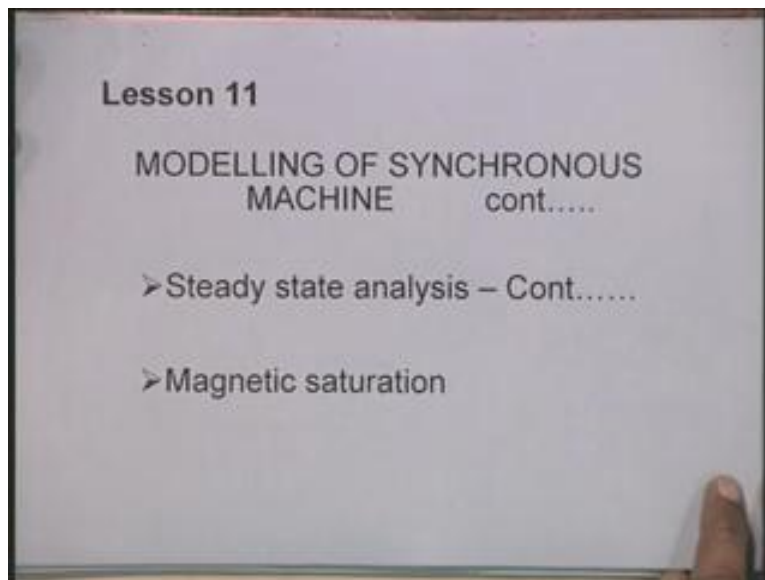


Power System Dynamics
Prof. M. L. Kothari
Department of Electrical Engineering
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Lecture - 11
Modeling of Synchronous Machine (Contd....)

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Friends, today we will continue our discussion on modeling of synchronous machine, okay. We will continue on the steady state analysis and then we will talk about the magnetic saturation. Now while discussing the steady state analysis we had discussed earlier that a voltage, a voltage behind voltage behind the impedance R_a plus j times X_q lies along the q axis and that gives us the location of q axis with respect to terminal voltage.

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In order to define the d and q axes positions relative to $\vec{E}_t$, let us define

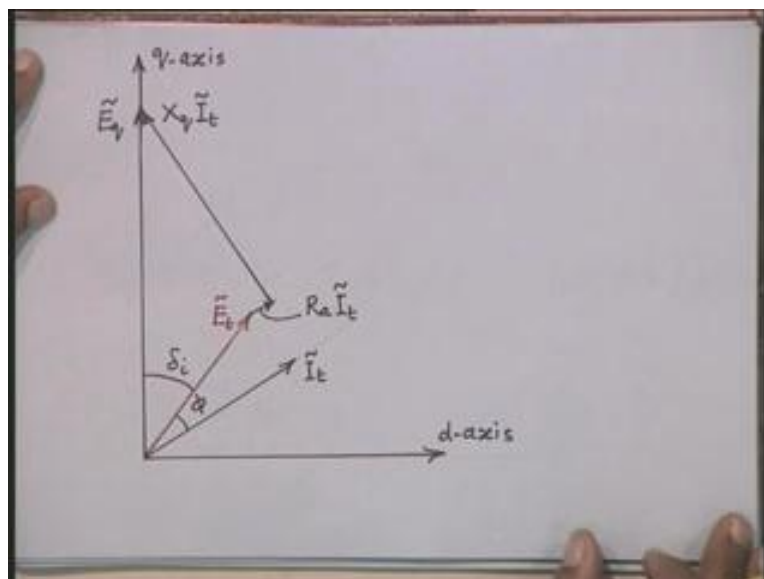
$$\underline{\tilde{E}_q} = \underline{\tilde{E}_t} + (R_a + jX_q)\underline{\tilde{I}_t} \quad (8.124)$$

$$= (\vec{e}_d + j\vec{e}_q) + (R_a + jX_q)(i_d + ji_q)$$

$$\underline{\tilde{E}_q} = j[X_{ad}i_d - (X_d - X_q)i_d] \quad (8.125)$$

Once we establish the location of q axis we are in a position to obtain the d and q components of voltage and currents which are required, okay.

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Now here this is the phasor diagram which shows the that voltage E_q bar or E_q is along the q axis what we have done is that this is the terminal voltage, this is the load current or terminal current, the phase difference between the terminal voltage and the current is ϕ to this we are adding this voltage drop $R_a I_t$ and adding the voltage drop $X_q I_t$ this gives you E_q .

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Rotor angle

Under no load conditions

$$\psi_d = L_{ad} i_{fd}$$

$$\psi_q = 0$$

$$e_d = 0$$

$$e_q = X_{ad} i_{fd} \quad \text{and}$$

$$\tilde{E}_t = e_d + j e_q = j X_{ad} i_{fd} \quad (8.126)$$

Now this voltage E_q is along the q axis and the angle between E_t terminal voltage and this E_q that is the q axis right this is denoted by the angle δ . Now we will establish a term which is a rotor angle or the power angle, although we have mentioned right now that the power angle is the angle between the q axis and the terminal voltage.

Now we will establish actually that in case there is no load on the machine that is machine is under no load condition right then the terminal voltage, terminal voltage E_t is along the q axis and therefore when the machine is loaded right E_t falls back with respect to the q axis right now to establish this we start with our original equations they are the stator voltage equations that is e_d equal to minus $\omega r \psi_q$ minus $R_a i_d$ e_q equal to $\omega r \psi_d$ minus $R_a i_q$ and the field voltage e_{fd} is equal to $R_{fd} i_{fd}$ while the flux linkages are minus $L_d i_d$ plus $L_{ad} i_{fd}$ ψ_q equal to minus $L_q i_q$ and the flux linkages in the field winding is ψ_d is equal to $L_{ffd} i_{fd}$ minus $L_{ad} i_d$.

In the amortisseur on the d axis ψ_{1d} equal to minus L_{f1d} , now we are putting amortisseurs is identified as 1, K equal to 1 i_{fd} minus $L_{ad} i_d$ ψ_{1q} equal to ψ_{2q} . I have we have assumed here that they are 2 amortisseurs on the q axis minus $L_{aq} i_q$. Now when you consider the system under no load condition right under no load condition there is the i_d and i_q will be 0. Okay now once i_d and i_q are 0, you can see here ψ_q will be 0. Okay and once ψ_q is 0 and i_d is 0 therefore what you see here is that e_d become 0.

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$$e_d = -\omega_r \psi_q - R_a i_d \quad (8.97)$$
$$e_q = \omega_r \psi_d - R_a i_q \quad (8.98)$$
$$e_{fd} = R_{fd} i_{fd} \quad (8.99)$$

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$$\psi_d = -L_d i_d + L_{ad} i_{fd} \quad (8.100)$$
$$\psi_q = -L_q i_q \quad (8.101)$$

Okay now with this substitution we write here the under no load condition the d axis flux linkage is equal to $L_{ad} i_{fd}$ that is in these equations you substitute i_d , i_q and e_d equal to 0 and ψ_q is also 0 therefore, we when we make the substitution we find here that ψ_d is equal to $L_{ad} i_{fd}$ that is the direct axis flux linkage is directly proportional to field current this is very important relationship under no load conditions ψ_q is 0 e_d is 0 e_q is equal to $X_{ad} i_{fd}$ that is the q axis component of the terminal voltage is directly proportional to the field current. Okay now we know the terminal voltage E_t is equal to e_d plus j times e_q right that is what we established earlier. Now since e_d is 0 e_d is 0 and e_q equal to $X_{ad} i_{fd}$.

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$$\psi_{fd} = L_{ffd} i_{fd} - L_{ad} i_d \quad (8.102)$$
$$\psi_{1d} = -L_{f1d} i_{fd} - L_{ad} i_d \quad (8.103)$$
$$\psi_{1q} = \psi_{2q} = -L_{aq} i_q \quad (8.104)$$

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Rotor angle

Under no load conditions

$$\psi_d = L_{ad} i_{fd}$$
$$\psi_q = 0$$
$$e_d = 0$$
$$e_q = X_{ad} i_{fd} \quad \text{and}$$
$$\tilde{E}_t = e_d + j e_q = j X_{ad} i_{fd} \quad (8.126)$$

So that E_t comes out to be equal to j times $X_{ad} i_{fd}$ that is in the no load condition the terminal voltage is in quadrature with the direct axis that is j is here and is proportional to the field current i_{fd} right.

Now this is all known to all of you because under no load condition the terminal voltage is directly proportional to the field current excitation and this voltage is called excitation voltage under no load condition the terminal voltage is the excitation voltage. Now we can establish a very simple equivalent circuit under steady state operating conditions for a

round rotor synchronous generator that if you consider the round rotor synchronous generator then X_d and X_q are equal and this we will denote as X_s synchronous reactance, X_d equal to X_q equal to X_s for a round rotor right.

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Steady state equivalent circuit

If saliency is neglected

$$\underline{X_d} = \underline{X_q} = \underline{X_s}$$

Where X_s is the synchronous reactance

$$\tilde{E}_q = \tilde{E}_t + (R_a + jX_s)\tilde{I}_t \quad (8.126)$$

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In order define the d and q axes positions relative to E_t , let us define

$$\tilde{E}_q = \tilde{E}_t + (R_a + jX_q)\tilde{I}_t \quad (8.124)$$

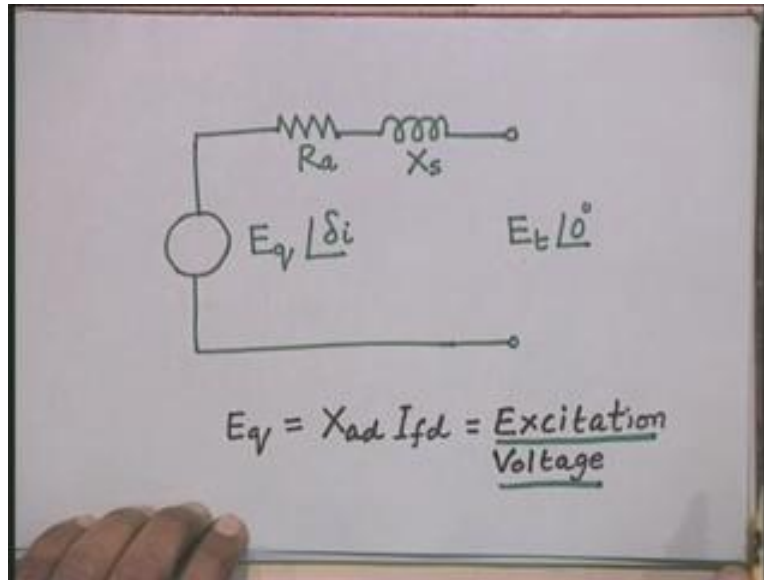
$$= (e_d + je_q) + (R_a + jX_q)(i_d + ji_q)$$

$$\tilde{E}_q = j[X_{ad}i_{fd} - (X_d - X_q)i_d] \quad (8.125)$$

Now when you substitute the value of substitute the value of X_d equal to X_q equal to X_s in the equation for E_q because E_q is now becomes E_t plus R_a plus j times $X_s I_t$, we have seen earlier that E_q is equal to j times $X_{ad} i_{fd}$ minus X_d minus $X_q i_d$ now the moment the moment the i_d is 0 i_d is 0 E_q is equal to j times $X_{ad} i_{fd}$ right. Now this is what we are trying to establish here that if you do not neglect the armature resistance R_a will come

right and X_d X_q neglecting the leakage reactance right X_{ad} becomes X_s right. Otherwise, X_d is equal to X_q equal to X_s while X_{ad} is the mutual reactance okay.

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Active and reactive power

$$\begin{aligned}
 S &= \tilde{E}_t \tilde{I}_t^* \\
 &= (e_d + je_q)(i_d - ji_q) \\
 &= \underline{(e_d i_d + e_q i_q)} + \underline{j(e_q i_d - e_d i_q)}
 \end{aligned}$$

Therefore, we can write draw a simple a simple equivalent circuit of the synchronous generator where terminal voltage is denoted as E_t angle 0 and the excitation voltage as E_q angle delta i and we put an impedance R_a plus R_a and X_s that is impedance is R_a plus j times X_s that is a very simple equivalent circuit which is obtained and where this we have establish that this E_q the magnitude of the E_q is equal to $X_{ad} I_{fd}$ that is called excitation voltage, a very simple expression. Now we can derive the expression for the complex

power in terms of d and q axis components okay, complex power S is equal to E_t into I_t^* that is I_t^* stands for conjugate now when we substitute the expression for E_t and I_t right in this expression, we get S equal to $e_d i_d$ plus j times $e_q i_q$ plus j times $e_q i_d$ minus $e_d i_q$. Therefore, this term is identified as the real power and this term is the reactive power.

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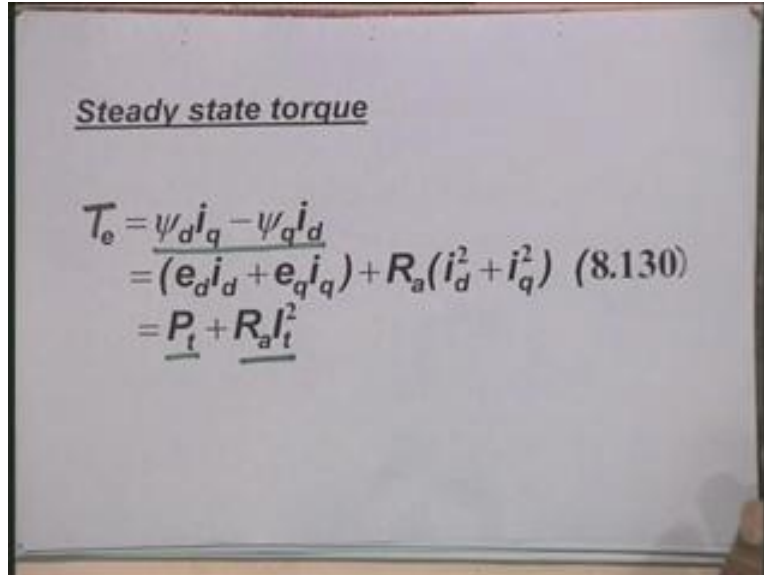
$$P_t = \underline{e_d i_d + e_q i_q} \quad (8.128)$$

$$Q_t = \underline{e_q i_d - e_d i_q} \quad (8.129)$$

So that we can write down here that the real power at the terminal of the machine is $e_d i_d$ plus $e_q i_q$ that is you know the d is component of voltage the d is component of the current q is component of the voltage q is component of the current right you multiply the corresponding voltage and currents and add them you get P_t . Similarly, you obtain the expression for Q_t as $e_q i_d$ minus $e_d i_q$. Okay now we will establish one very interesting expression for torque because we have seen that the expression for torque is $\psi_d i_q$ minus $\psi_q i_d$, this expression is the very general expression which we have established.

Now in these expressions we substitute the expression for ψ_d and ψ_q from the previous relationships established relationships and then simplifies this equation. You will find that this torque T_e that is a air gap torque will come out to be equal to the terminal power plus R_a into I_t^2 or we can say that $I^2 R$ loss what we what do we understand here.

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Steady state torque

$$\begin{aligned} T_e &= \frac{\psi_d i_q - \psi_q i_d}{\omega} \\ &= (e_d i_d + e_q i_q) + R_a (i_d^2 + i_q^2) \quad (8.130) \\ &= \underline{P_t} + \underline{R_a I_t^2} \end{aligned}$$

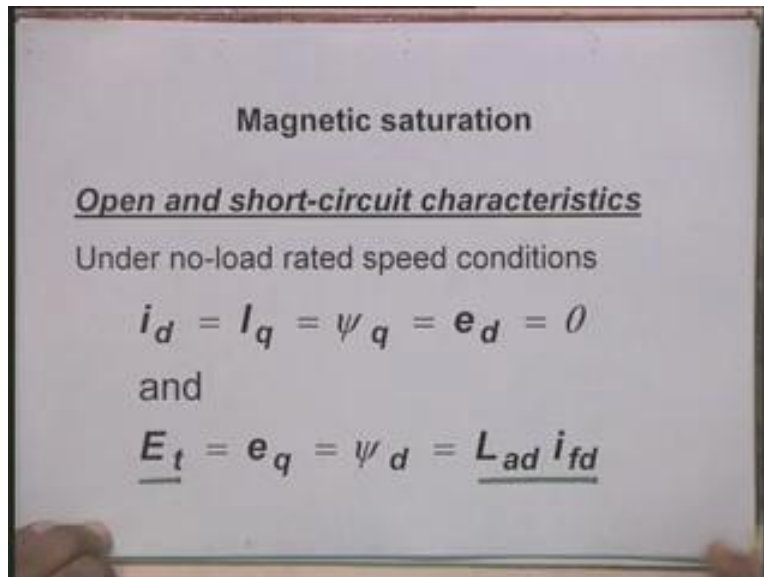
Now since in per unit quantities power and torque are equal when we use the per unit quantities then the torque is same as the power. Now this torque is the air gap torque or we can and this air gap torque can be obtained as the terminal power plus the armature resistance loss right and therefore whenever you performed stability steady analysis this is one computation which is required to be performed that first we find out what is the terminal power then to that terminal power we add the armature resistance loss okay and that gives us the electro magnetic torque and that is what we will be substituting in the expression for swing equations that is in the swing equation.

We have to substitute T_e the T_e is computed using these expressions and under steady state conditions right the mechanical torque is equal to the electrical torque therefore mechanical torque is computed by this expression right while electrical torque will be will continue to be computed by this expression because under dynamic conditions this is the expression for electromagnetic torque under dynamic conditions for under steady state condition we can say that the steady state torque is equal to terminal power plus losses. This is a very important step in case you neglect the armature resistance then terminal power is same as the electromagnetic torque but today with the availability of computing facility right and the capability of the digital computers there is no need to neglect the armature resistance because armature resistance losses are substantial okay and they have bearing on the stability results.

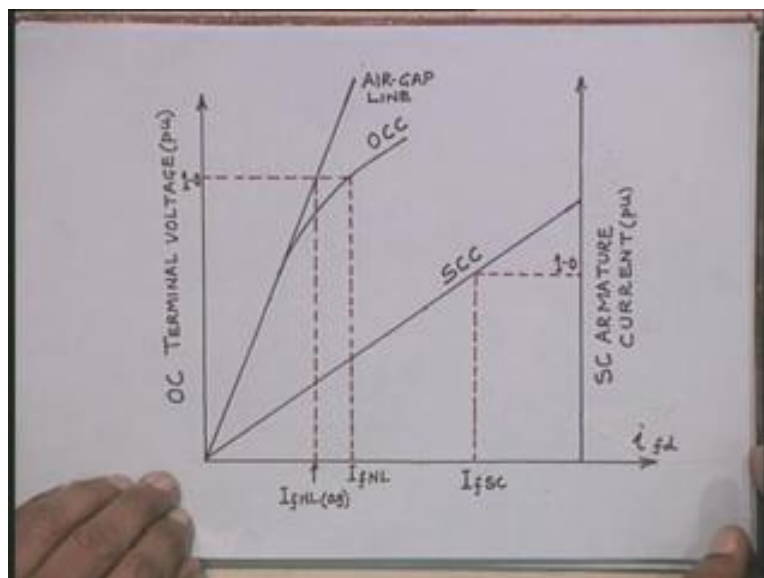
Now, we come to the very important aspect of modeling of the system. So for, so for we have neglected the saturation in the magnetic circuit. This was neglected to simplify the mathematical model. However, however the magnetic saturation need to be accounted and if we neglect it then the errors in the results are sometimes substantial and therefore there is a necessity to understand how do we account for the saturation in the magnetic circuit. Now to understand how do we account for a saturation in the magnetic circuit, we will look into the open and short circuit characteristic of the synchronous generator all of

you are aware of open circuit characteristic of the synchronous generator short circuit characteristic of the synchronous generator and we will see that the open circuit characteristic is very useful in determining the saturation characteristic of the machine okay.

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Now we write under no load conditions we established earlier also i_d i_q ψ_q and e_d these four terms where 0 when the machine is not loaded right the i_d and i_q components of the current are 0 ψ_q is 0 and e_d is also 0 this is what we have established in our previous discussion. Then the terminal voltage E_t came out to be equal to $L_{ad} i_{fd}$ that is this

terminal voltage is directly proportional to the field current. Okay and this terminal voltage is also called excitation voltage under no load conditions under no load condition this is also the excitation voltage now let us look at the at this characteristic.

On this axis I am I have shown the field current i_{fd} and on this axis we are showing the open circuit terminal voltage in per unit. Okay when you perform the open circuit characteristic or perform the experiment to obtain the open circuit characteristic what we do is we run the machine at rated speed okay and note down the terminal voltage for different values of the field current.

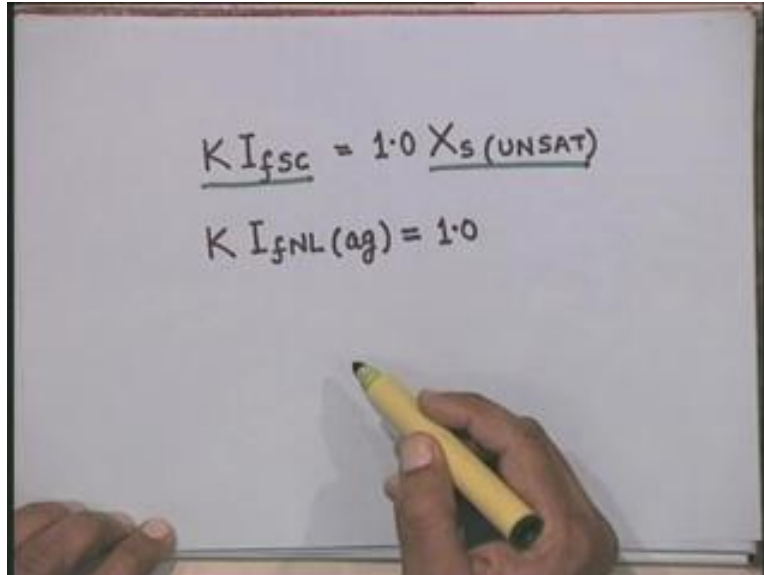
Now here while plotting this characteristic I have neglected the residual flux, okay otherwise there is some residual flux and you will get that even if the field current is 0 there will be some that is that has been neglected here. Now the the open circuit characteristic will look like this. Okay that is it remains straight line and then starts getting saturation, now if you draw a tangent to the open circuit characteristic in the initial portion right then we get a straight line, we can draw a straight line which is tangent to the initial portion of the OCC, we call this air gap line.

Now here **here** we will define two field currents that is if the terminal voltage is one per unit, the field current required when you consider the saturation is I_{fNL} . In case we ignore the saturation then the field current required is $I_{fNL} (ag)$, ag stands for that or the air gap line considering the air gap line. Okay now if we plot short circuit characteristic that is on this axis I am putting short circuit armature current in per unit there is again the same field current. This short circuit characteristic comes out to be a straight line while you perform the short circuit characteristic the machine is run at rated speed and the terminals of the machine are put short shorted okay.

Now there is no saturation when we are performing short circuit characteristic that is you consider actually the short circuit current right for different values of the field current even beyond the rated current, you will find there is hardly any saturation. The basic reason for **for** the short circuit characteristic comes coming out to be straight line can be attributed that under short circuit condition right the armature current is having the demagnetizing affect okay and because for the demagnetizing affect the resultant magnetic field which is produced in the air gap or in the iron core is very low and therefore saturation does not take place and this is very important point.

Now here we can now define unsaturated reactance of the machine and then we define saturated reactance of the machine. Now to define this that is what we do here is that when the synchronous machine, when the synchronous machine terminals are shorted and let us say it carries one per unit current right at that time whatsoever the induced emf due to this field current is used in overcoming the drop in the short circuited path.

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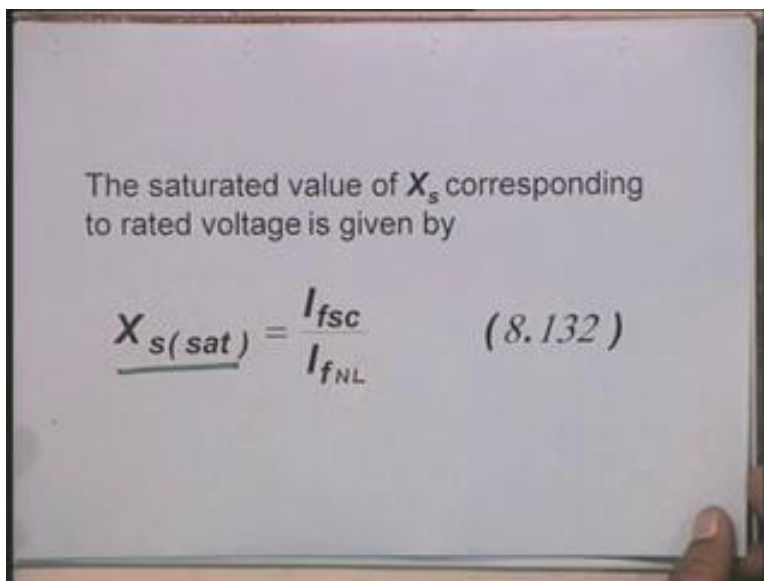


A photograph of a whiteboard with two handwritten equations. The first equation is $K I_{fsc} = 1.0 X_s (UNSAT)$ and the second is $K I_{fNL} (ag) = 1.0$. A hand holding a yellow marker is visible at the bottom of the frame.

$$K I_{fsc} = 1.0 X_s (UNSAT)$$
$$K I_{fNL} (ag) = 1.0$$

If we neglect the armature resistance then we can say that the induced emf is a some constant times I_{fsc} equal to 1 into X_s unsaturated okay that is there under short circuit condition there is no saturation in the system right therefore the reactance which we talk about the unsaturated reactance and the voltage induced is some constant time in the field current okay, then if we see here the open circuit characteristic then one per unit voltage is produced when the current is equal to I_{fNL} okay.

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A photograph of a whiteboard. The top part contains the text 'The saturated value of X_s corresponding to rated voltage is given by'. Below this is the equation $X_{s(sat)} = \frac{I_{fsc}}{I_{fNL}} (8.132)$. A hand is visible at the bottom right corner.

The saturated value of X_s corresponding to rated voltage is given by

$$X_{s(sat)} = \frac{I_{fsc}}{I_{fNL}} \quad (8.132)$$

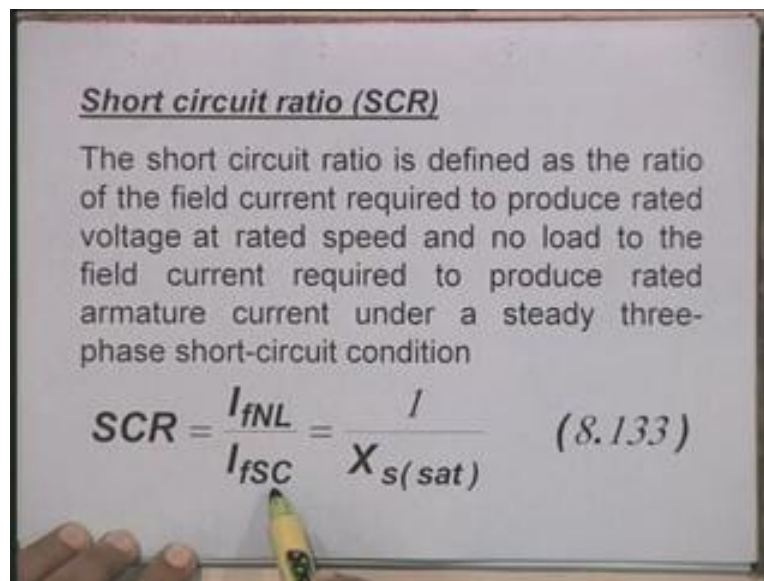
Now being the proportionality constant same that is $K I_{fNL} ag$ equal to one right therefore I using these 2 equations we can establish that the unsaturated value of

synchronous reactance is equal to I_{fSC} divided by I_{fNL} (ag), this is a very important term that is what is the value of unsaturated synchronous reactance or unsaturated value of synchronous reactance that is X_s unsaturated is obtained using the open circuit characteristic open circuit characteristic and short circuit characteristic that is on this short circuit characteristic we draw we find out what is the field current required to produced one per unit current in the armature under short circuit condition.

Similarly, we find out how much is the how much is the current which is required to produce one per unit voltage under on air gap line again this is I am putting a air gap line therefore this ratio of I_{fSC} upon I_{fNL} the ratio is of I_{fSC} and I_{fNL} (ag) gives you a saturated unsaturated value of synchronous reactance. Okay now when we look at the saturated value of the synchronous reactance.

Now we can find out the value of saturated synchronous reactance that to produce one per unit voltage under open circuit condition the field current required is I_{fNL} , okay this I_{fNL} ag was on air gap line while I_{fNL} is on OCC right therefore, since open circuit characteristic there is some saturation taking place therefore the field current required is I_{fNL} and following the same approach as we have done earlier now we can write down that saturated value of the synchronous reactance is I_{fSC} divided by I_{fNL} . Normally, we perform the open circuit and short circuit test to find out the synchronous reactance of the machine the synchronous reactance which we compute by doing open circuit and short circuit test are the will give you the saturated value of the synchronous reactance okay.

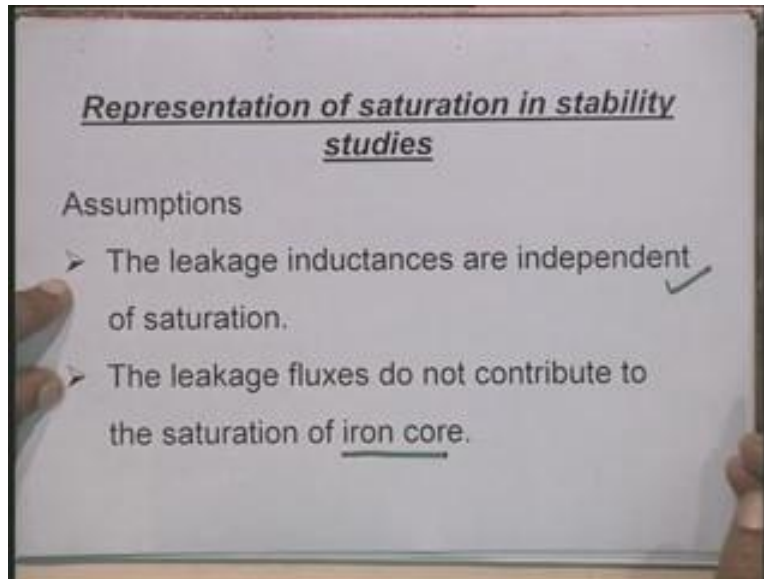
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Now our task is that to relate to relate the unsaturated value of synchronous reactance to the saturated value of the synchronous reactance. Okay now to relate this before I relate this one more term which is very commonly used I am just introducing that is the short circuit ratio, the short circuit ratio is defined as I_{fNL} divided by I_{fSC} and it comes out to be the reciprocal of reciprocal of saturated synchronous reactance right the if the if the

since short circuit ratio is low that is what that synchronous reactance is high and in fact in industry we very very frequently used this term short circuit ratio and it also gives the information about the quality of the machine.

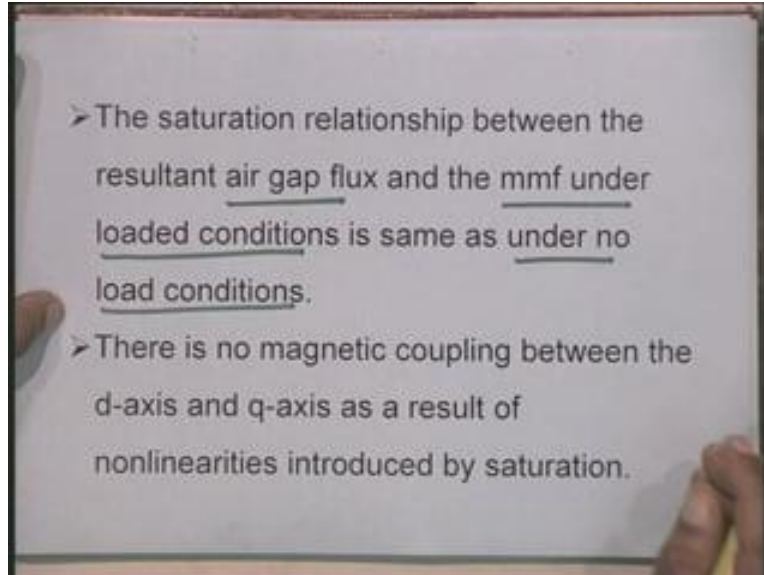
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Now for representing the magnetic saturation for stability studies we make some realistic assumptions the assumptions made are that the leakage inductances are independent of saturation when we talked about a various inductances, we found mutual inductances as well as the leakage inductances right. For example, L_d direct axis inductance L_d is equal to L_{ad} plus LL , LL is the leakage component okay and L_{ad} is L_a e_d is due to the flux which **which** crosses air gap goes through the field winding and again return backs to the armature okay.

Now since this leakage flux is mostly takes path through the air right and it crosses the iron but **but** it is but it is it does not contribute toward the saturation of the iron right because the reluctance to the leakage flux path is mostly due to the air and therefore we make this realistic assumption that is the leakage inductances are independent of saturation, second is the leakage fluxes do not contribute to saturation of iron cores that is the same thing because leakage fluxes do pass through the iron core but we assume here that do not contribute to the saturation. This is an assumption the realistic assumption but not very very strictly they given but acceptable for stability studies.

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This is another very important is that saturation relationship between the resultant air gap flux and the mmf under loaded conditions the same as under no load conditions. Now this is very important assumption here that we will establish the saturation relationship using the open circuit characteristic right while while we assume here that that this relationship is also applicable under loaded conditions.

Okay that is the saturation relationship between the resultant air gap flux and mmf under loaded conditions is same as under no load conditions, okay that is why the open circuit characteristic is very useful and used for determining the saturation characteristic okay then another is the there is no magnetic coupling between d-axis and q-axis as a result of nonlinearities between the nonlinearities introduced by saturation because here here under linear conditions we assume that the d axis and q axis are magnetic circuit do not have any coupling okay and even if there is nonlinearities now in the magnetic circuit, we assume that that coupling does not exist even under nonlinearities this is what is the meaning of this. Now with this assumption we will now concentrate upon the these terms mutual inductance that is L_{ad} and L_{aq} , this now this L_{ad} is the mutual inductance considering saturation and we express L_{ad} their mutual inductance with saturation is equal to K_{sd} times mutual inductance unsaturated, u stands for unsaturated L_{ad} u and this term K_{ad} is called direct axis saturation coefficient K_{sd} .

Okay similarly for quadrature axis we can write down L_{aq} is equal to K_{sq} L_{aq} L_{aqu} that u again stands for unsaturated value of mutual inductance and this K_{sd} and K_{aq} are the saturation coefficients but our basic task is to obtain this coefficients. Okay now here again we look at the open circuit characteristic.

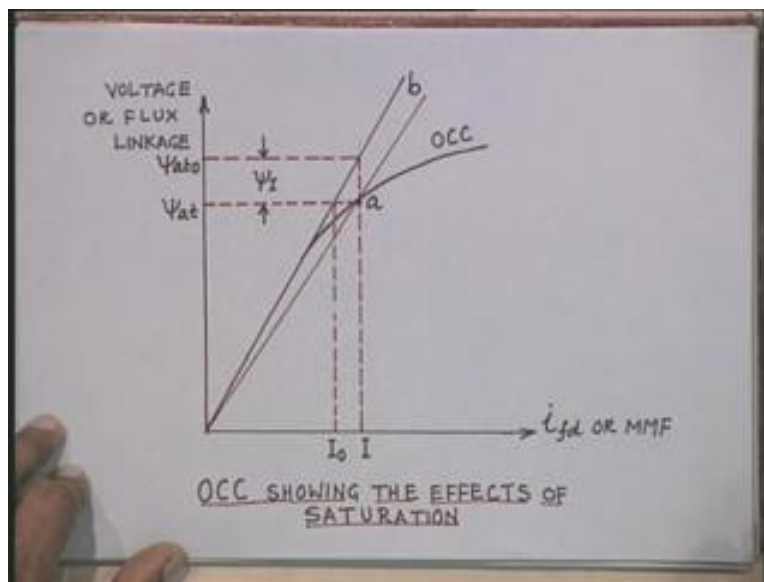
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From above assumptions

$$\underline{L_{ad}} = K_{sd} \underline{L_{adu}} \quad (8.134)$$

$$\underline{L_{aq}} = K_{sq} \underline{L_{aqu}} \quad (8.135)$$

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Now in this open circuit characteristic on this axis that is on y axis, we are putting the voltage open circuit voltage or flux linkages because we have established that the open circuit voltage is proportional to the flux linkage okay and on this axis you can put field current or the mmf. Okay, now we see two important things here that for a given field current I right, if I consider the saturated characteristic that is open circuit characteristic I find that the total air gap flux produced is ψ_{at} , okay if you neglect the saturation then the for the same field current the flux produced is ψ_{ato} is not because this is a line this is air gap line okay and we are operating at this point the field current is say I , okay for

the same for this field current the flux produce is ψ_{at} when you consider the OCC and the flux produced is equal to ψ_{at0} if I consider the air gap line right.

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The saturation factor K_{sd} is given by,

$$K_{sd} = \frac{\psi_{at}}{\psi_{at0}} \quad (8.136)$$

or

$$K_{sd} = \frac{I_0}{I} \quad (8.137)$$

The saturation coefficient K_{sd} is defined as ψ_{at} divided by ψ_{at0} this is the saturation coefficient on direct axis or direct axis saturation coefficient K_{sd} is equal to ψ_{at} that is the flux linkage corresponding to current I on open circuit characteristic and ψ_{at0} is the flux linkage on air gap line for the same current and you can see here very carefully that this ratio is same as ψ_{at0} , I_0 by I and it is going to be always less than 1.

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defining

$$\psi_I = \psi_{at0} - \psi_{at} \quad (8.138)$$

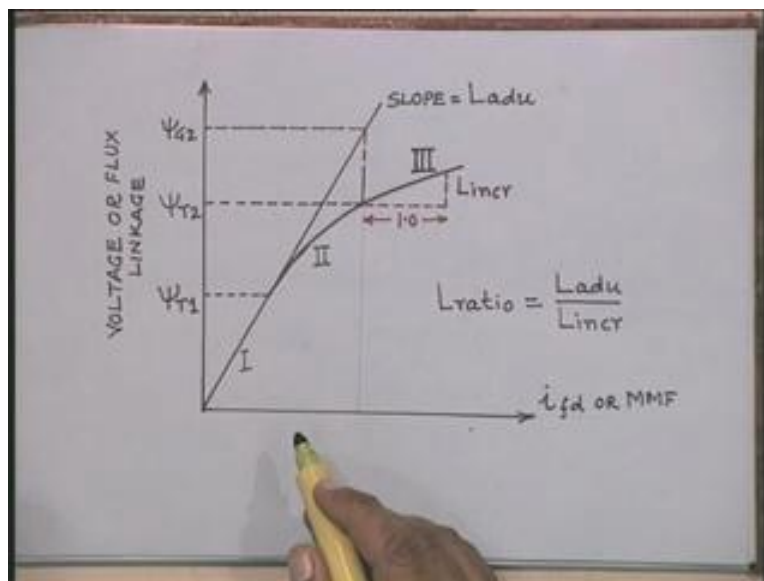
The expression for the saturation factor becomes

$$\underline{K_{sd}} = \frac{\psi_{at}}{\psi_{at} + \underline{\psi_I}} \quad (8.139)$$

The saturated value of the inductance is going to be less than the unsaturated value of the inductance therefore we can see here actually that whether I take the ratio of ψ_{atO} to ψ_{at} or ψ_{at} to ψ_{atO} or I take ratio of I_0 to I they will give the same result okay. Now we define here difference that is ψ_{atO} minus ψ_{at} this difference flux linkage is designed by ψ_I and this ψ_I is very important to determine the saturation characteristic of the machine or we define ψ_I as ψ_{atO} minus ψ_{at} and K_{sd} can now be written as ψ_{at} divided by ψ_{at} plus ψ_I that is I replace this ψ_{atO} by ψ_I plus ψ_{at} right therefore the major exercise will be that for a given value of ψ_{at} , we have to find out what is the value of this ψ_I .

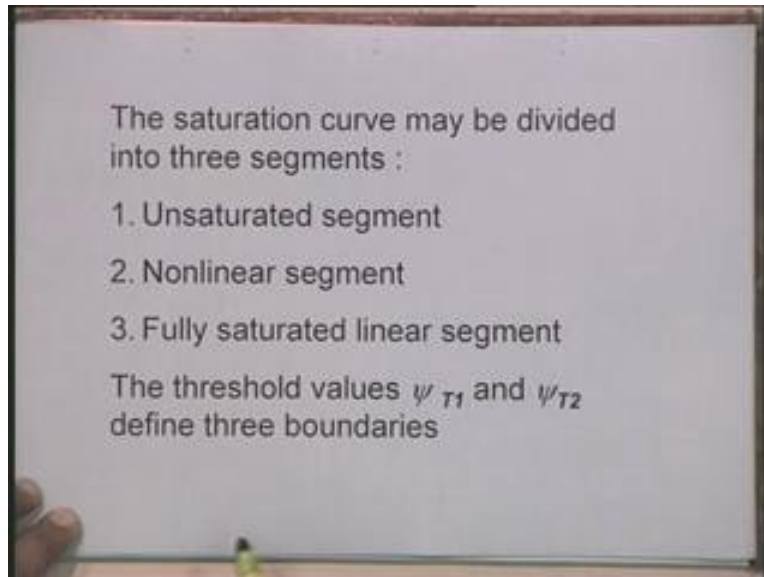
So that we can compute the saturation coefficient K_{sd} now here, now we have to basically model this OCC open circuit characteristics to be modeled that is we have to develop a model for the open circuit characteristic once we know this this is the model of the open circuit characteristic I can always find out find out this ψ_I for a given value of ψ_{at} okay.

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Now for modeling if you look at this characteristic the initially this characteristic is a straight line. Okay then we will find actually that from then this characteristic is practically following exponential shape and then once it gets fully saturated again it will become a straight line, if you carefully examine the open circuit characteristic right then you can divide this open circuit characteristic into 3 segments that is the first segment is one where the flux linkage is proportional to the field current that is in the unsaturated condition, second segment yes the iron core starts getting saturated and the characteristic is a non-linear one.

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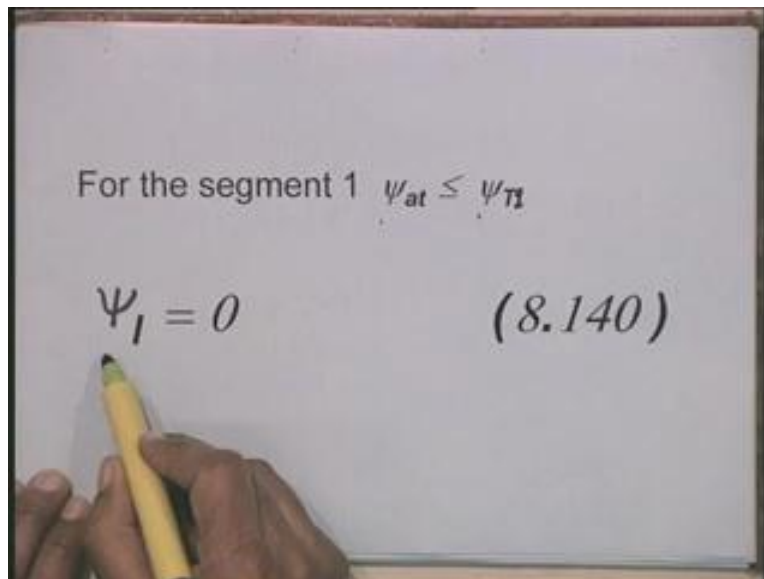


Okay, then in the third segment this characteristic becomes a straight line under saturated condition what happens is that when you increase the mmf the air gap flux is going to increase right but this increase is going to be linear relationship again therefore, for modeling the open circuit characteristic or magnetization characteristic of the machine we divide this characteristic into 3 segments and these 3 segments are unsaturated segment, non-linear segment and third is fully saturated linear segment.

This is the new thing which you have to very carefully understand okay now these 3 segments are shown here and ψ_{T1} is the boundary value of flux linkage for the segment one ψ_{T2} is the boundary value of the flux linkage for second segment ψ_{T2} and beyond this it is the linear linear characteristic okay.

Therefore now what we do is that whenever you to try model right we this characteristic is available to us experimentally you obtain the characteristic whichever way you obtain it, this characteristic is known to you. Okay, now I can define here in this linear portion if I take one per unit value on the on the x axis that is I take this as a x axis and I call this vertical height as L_{incr} incremental L incremental right then I can write down the slope equal to the slope of this characteristic can be written as i_{Lncr} that is this is the straight line characteristic with slope you have to obtain. Okay our the these 3 segments can be model very easily.

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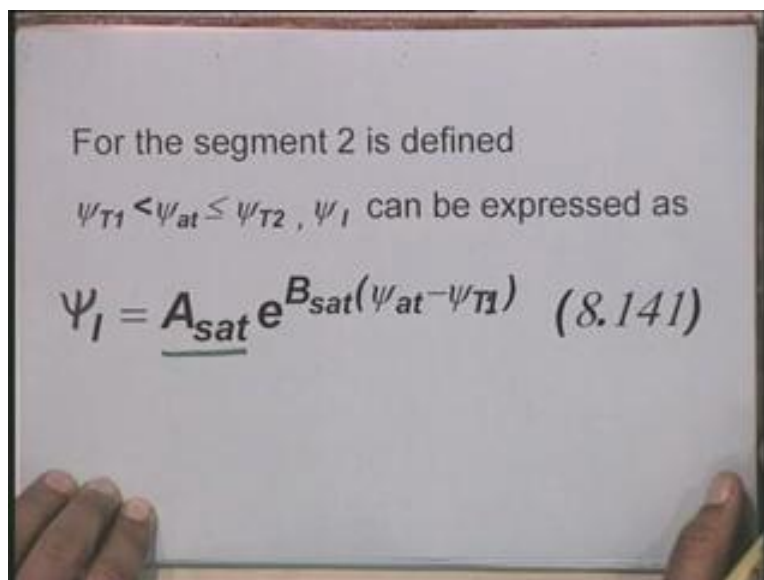
For the segment 1 $\psi_{at} \leq \psi_{T1}$

$$\psi_I = 0 \quad (8.140)$$

A hand holding a yellow marker is visible at the bottom left, pointing towards the equation.

So for actually the segment one is concerned segment one is concerned the ψ_{at} is less than equal to ψ_{T1} which is the boundary for segment one right and therefore the this ψ_I term is 0, now this the actual OCC and the air gap line they are coinciding right and therefore the ψ_I which is difference between ψ_{at} and ψ_{T1} is 0. The second segment is a non-linear segment okay.

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For the segment 2 is defined

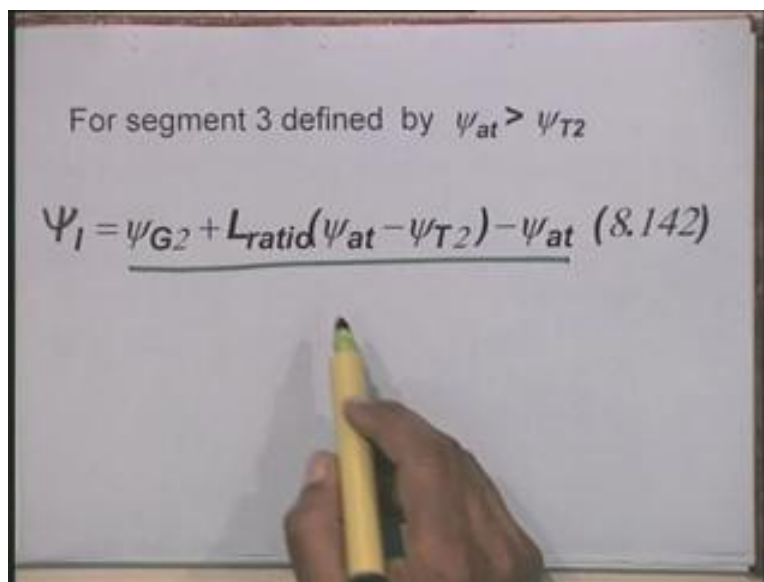
$$\psi_{T1} < \psi_{at} \leq \psi_{T2}, \psi_I \text{ can be expressed as}$$
$$\psi_I = \underline{A_{sat}} e^{B_{sat}(\psi_{at} - \psi_{T1})} \quad (8.141)$$

Hands are visible at the bottom corners of the whiteboard.

The boundary values are ψ_{at} should be greater than ψ_{T1} and less than equal to ψ_{T2} , okay this is the range we have already established for the second segment that is the second segment is the ψ_{at} anywhere on this line right this is going to be in this range ψ_{T1} and ψ_{T2} . Now this can be modeled by the equation of this form the exponential form of equation the ψ_I can be written as ψ_I can be written as A_{sat} that is A saturation this is a coefficient this the subscript sat is used to indicate that we are trying to model the saturation that is ψ_I equal to A saturation e to the power B saturation multiplied by ψ_{at} minus ψ_{T1} what is this ψ_I , this is the difference between the ψ_{at} and ψ_{T1} and this ψ_I can be model by this expression where you for any value of ψ_{at} you find out this difference ψ_{at} minus ψ_{T1} is known to you and these coefficients have to be obtained, these coefficients have to be obtained actually for a given machine okay.

Once these coefficients are known ψ_I can be computed. Now here if you see that if I put ψ_{at} equal to ψ_{T1} right that is actually at the beginning of the segment ψ_{at} is equal to ψ_{T1} , you can again look here at this point right at this point ψ_I is 0 right but when I use this equation ψ_I will come out to be equal to A_{sat} , this is a slight discrepancy at this boundary. However, in practice this term ψ_{at} is small right and therefore this discrepancy is not very significant because when I use this equation and try to put actually the initial value that is ψ_{at} equal to ψ_{T1} then ψ_I should come out to be 0 but it is not coming out to be 0 by this equation right.

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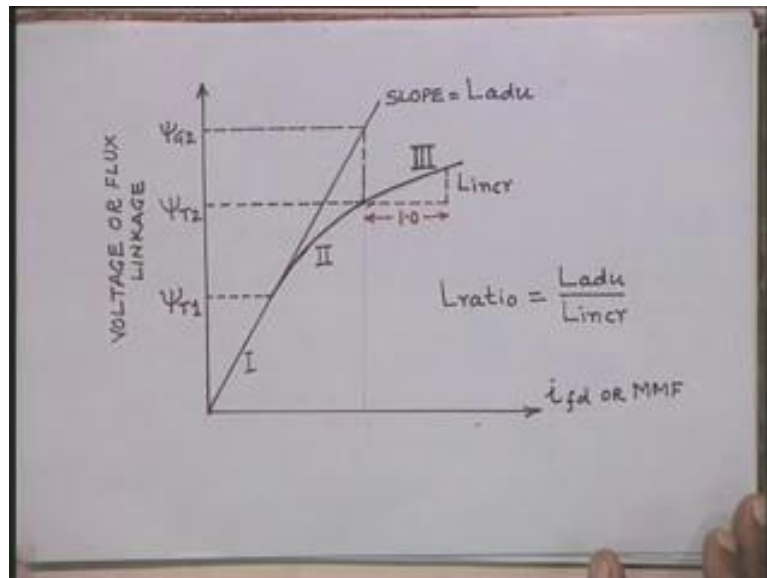
For segment 3 defined by $\psi_{at} > \psi_{T2}$

$$\psi_I = \psi_{G2} + \frac{L_{ratio}(\psi_{at} - \psi_{T2})}{A_{sat}} - \psi_{at} \quad (8.142)$$

However since A_{sat} this coefficient is generally very small and therefore this discrepancy is a negligible or can be ignored because again let me emphasize here that we trying to develop the model okay and for this developing the model we have to very carefully obtain these coefficients because when I said is exponential I have to do curve

fitting okay and once these coefficients are obtained we obtain actually the model for ψ I. Now the last segment which is again a straight line fully saturated segment third segment the fully saturated one okay.

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Now here we have to find out what is the value of ψ I when ψ at is beyond ψ T2 right now to obtain this what we really do here is we may have to write down the equation for this segment is a straight line, you can write down the equation of the segment in the form of y equal to mx plus c a straight line okay and then you know this equation for this line what is the equation for this air gap line, you simply write down ψ at equal to L_{adu} into i_{fd} because this is a path line passing through origin while this line is not going to pass through origin therefore, you have to make use of the initial conditions that is actually that our initial conditions are that when the flux is equal to ψ T2 right ψ T2 the field current is i_{fd} something, you can note it down that is because here this is the segment which we are trying to model right.

Now you do this exercise look this exercise you can do and obtain the expression for the ψ I which is which is the difference between actual ψ at and ψ at O right. Now ψ I has been obtained in this case as ψ_{g2} plus L ratio into ψ at minus ψ T2 minus ψ at. Now this I will suggest all of you to derive the expressions ψ I equal to ψ_{g2} plus L ratio ψ at minus ψ T2 minus ψ at, I have already checked it, it comes out to be okay there is no discrepancy.

Now here what is done here is that when the flux linkage is ψ T2 right we find out actually the value of flux linkage on air gap that is for the ψ T2, you note down this current and for the same current you can find out a flux linkage which is on air gap line corresponding to the air gap line that is called ψ_{g2} and using this information that for

this particular current this point is lying on the third segment characteristic right and using this value that the flux linkage is ψ_{g2} therefore basically at this point at this point ψ_I is equal to ψ_{g2} right.

Therefore if I substitute here in this equation in this equation right if ψ_I if I take this ψ_I at if I take this ψ_I at equal to ψ_{T2} and this term become 0 right and ψ_I becomes ψ_{g2} minus ψ_{at} , no no ψ_{T2} is nothing but ψ_{at} right therefore this equation is satisfied. Okay and this can be derived now once you are in a position to obtain this quantity ψ_I the coefficient K_{sd} is known okay now next point is how to obtain the saturation coefficient for q axis okay.

Now, so far actually d axis is concerned we have OCC for q axis we do not have similar open circuit characteristic we cannot plot it. Now here generally we made one assumption that in the q axis the saturation is 0, a very realistic assumption that you can make this assumption that on q axis that is the saturation coefficient K_{sq} is equal to 1 because the magnetic the flux takes its path, so larger air gap when you talk about the q axis of the machine, okay.

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For a operating condition K_{sd} is computed as a function of the corresponding air gap flux linkage given by,

$$\underline{\Psi_{at}} = \sqrt{(\psi_{ad}^2 + \psi_{aq}^2)} \quad (8.143)$$

Now when we are talking about this the flux ψ_{at} is equal to ψ_{ad} square plus ψ_{aq} square that is the flux which we have been talking about right is the total flux in the air gap ψ_{at} equal to ψ_{ad} square plus ψ_{aq} square.

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The d- and q-axis air gap flux linkages are given by ,

$$\begin{aligned}\Psi_{ad} &= \psi_d + L_l i_d \\ &= (e_q + R_a i_q) + L_l i_d \quad (8.144)\end{aligned}$$
$$\begin{aligned}\Psi_{aq} &= \psi_q + L_l i_q \\ &= (-e_d - R_a i_d) + L_l i_q \quad (8.145)\end{aligned}$$

Now here we will just establish one very simple relationship using our expressions for ψ_{ad} and ψ_{aq} , ψ_{ad} and ψ_{aq} these are the expressions which were known to you the relationships are that is you write down the expressions for ψ_{ad} that is you write down the expressions for ψ_{ad} which is then ψ_d plus L_{li} and ψ_{aq} as ψ_q plus L_{li} .

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Therefore , ψ_{at} in per unit is equal to the air-gap voltage

$$\underline{\tilde{E}_a} = \underline{\tilde{E}_t} + \underline{(R_a + jX_l)\tilde{I}_t} \quad (8.146)$$

Okay and ψ_d can be written by this expression ψ_d ψ_{ad} is equal to e_q plus $R_a i_q$ ψ_q is written as minus e_d minus $R_a i_d$ these are the equations which we have already

developed earlier and now if you substitute this in the equation for ψ at right, you will find actually that ψ at when you express in per unit because we are all doing per unit calculations this ψ at will come out to be simply as a voltage E_a E_t plus R_a plus j times X_l into I_t that is this ψ at is a voltage behind ψ at is a voltage behind or is equal to the voltage behind behind this impedance that is R_a plus j times X_l that is E_q when we are talking about that was the voltage behind R_a plus j times X_q . Okay now here it is only talking about the leakage reactance this is this is established by substituting these expressions. Now with this we have established how do we obtain the saturated value of the reactance is required for calculations.

Now if you see it very carefully here then for each operating condition you know for each operating condition, you have to find out what is the saturated value it is not same for all operating condition this coefficient K_{sd} is different for different operating condition you can easily see here that ψ at when I say ψ at is obtained corresponding to one particular operating condition. Okay and for that we have to find out ψ atO, okay and this ratio ψ at upon ψ atO gives you saturation coefficient and this ψ at is different and therefore we get the different value of saturation coefficient and that is why, when you perform the stability studies stability studies right the saturation has to be computed as the loading condition keeps on changes for different loading condition different value of saturation coefficient is to be used. With this I am concluding here the modeling of synchronous machines for stability studies. Today, we have talked about the method of modeling, modeling the saturation right.

How how it has been done basically we started with the basic definition of what we mean by saturation coefficient and then the OCC open circuit characteristic is divided into 3 segments and for each segment we have written the expression for the flux linkage ψ I which is the difference between ψ atO and ψ at that is ψ atO is the flux linkage for a given field current on air gap line right and then we compute the value of saturation coefficient. Further, we have assumed here the saturation coefficient on q axis as 1 that is saturation is neglected and this can be considered with this.

Let me say that we have completed the synchronous machine modeling this modeling is complete in the next chapter we will study the synchronous machines model for stability studies. Okay when we talk about the actual model of the synchronous machine we are considering here now the stator circuit, rotor circuit transient while in the stator circuit we have we have made the assumption that the derivative terms are negligible as compared to the speed voltage terms therefore, this assumption will be carried over for a stability studies also that is when we have written the stator circuit equations we have ignored the transformer voltages and therefore this assumption will be carried over beyond for stability studies also.

However, there will be some more assumptions that can be made and simplified to obtain are made to obtain the simple synchronous machine model for stability studies. Now when we talk about the modeling the next step will be to model the excitation system and the models for turbines and governors that makes the complete model for the energy system that is turbine, governor, excitation system and synchronous generator that is the

complete thing, we will devote few more terms to develop the models for the excitation system and this okay. Thank you very much.