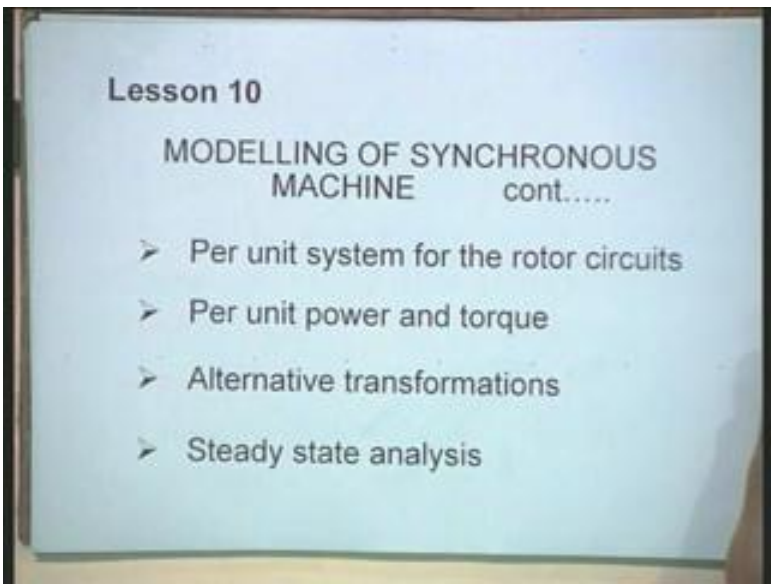
**Power System Dynamics**
**Prof. M. L. Kothari**
**Department of Electrical Engineering**
**Indian Institute of Technology, Delhi**
**Lecture -10**
**Modeling of Synchronous Machine (Contd….)**

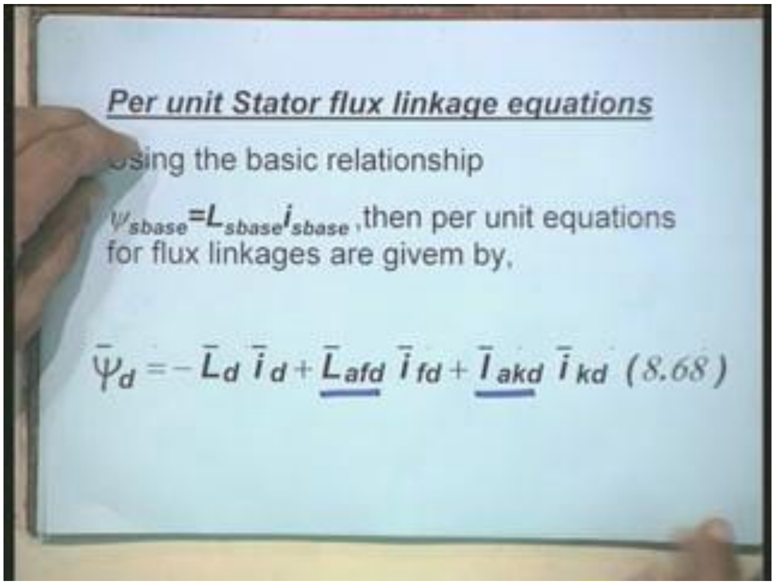
Okay, friends today we will continue with the study of modeling of synchronous machine.

(Refer Slide Time: 01:07)

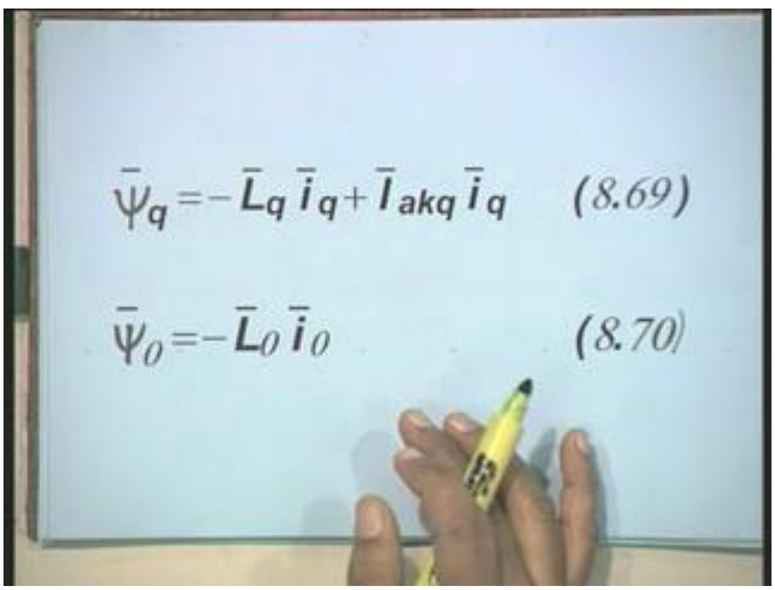



Today we will address some of these issues per unit system for rotor circuits, per unit power and torque and alternative transformations. We have discussed dqO transformation and we will also talk about the alternative transformation which has also been proposed in the literature and then we then, we will talk about the steady state analysis of the synchronous machine. Now we have developed the per unit equations for the stator circuits. Okay we have also developed that to the per unit equations for rotor circuits.

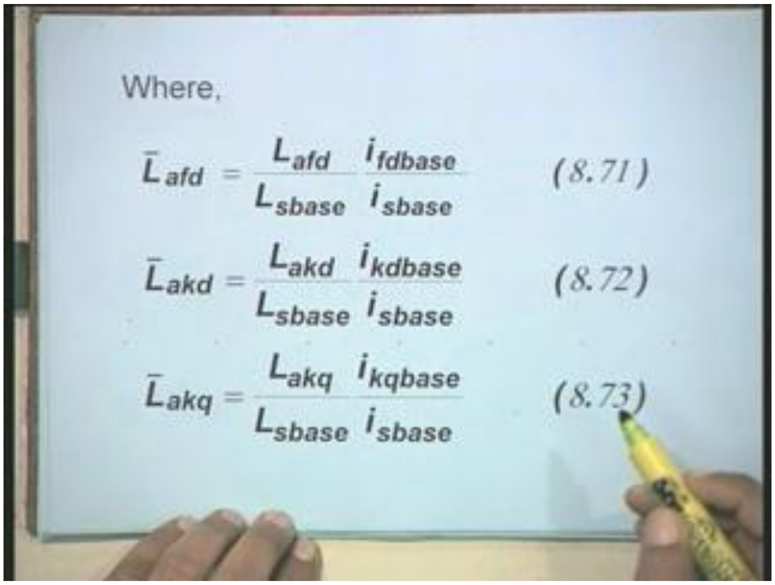
(Refer Slide Time: 02:20)




(Refer Slide Time: 02:29)

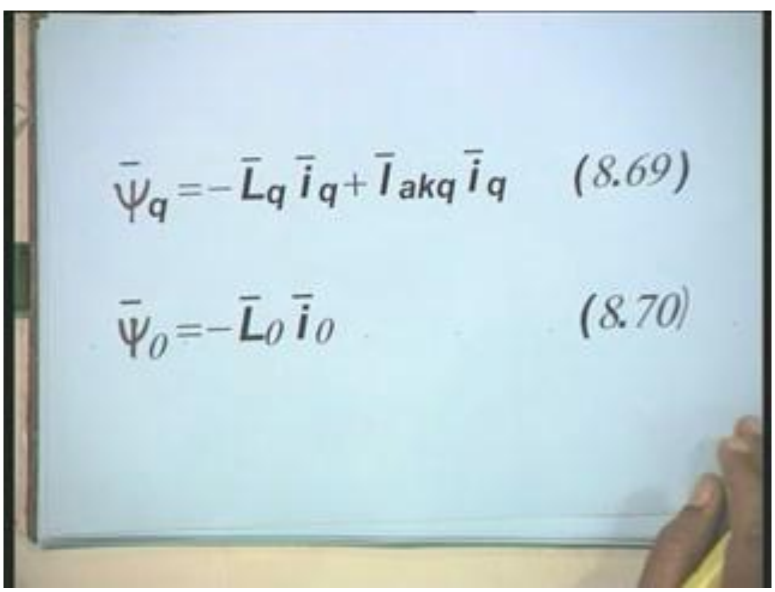


(Refer Slide Time: 02:35)

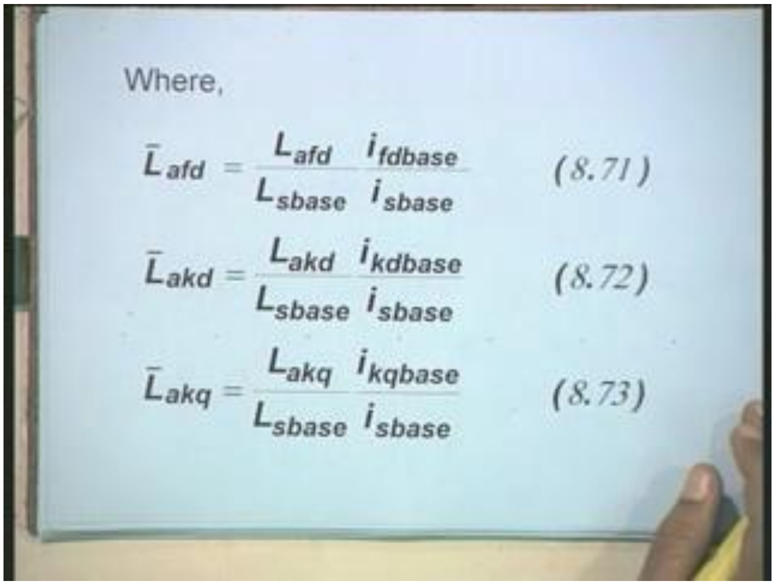



Now these terms were defined and the definitions were given like this $L_{afd}$ bar equal to $L_{afd}$ divided by $L_{sbase}$, $i_{fdbase}$ divided by s base that is these three terms were defined that they are basically in the stator circuit equations you see this that the stator circuit flux linkage equations psi d, psi q and psi o.

(Refer Slide Time: 02:57)

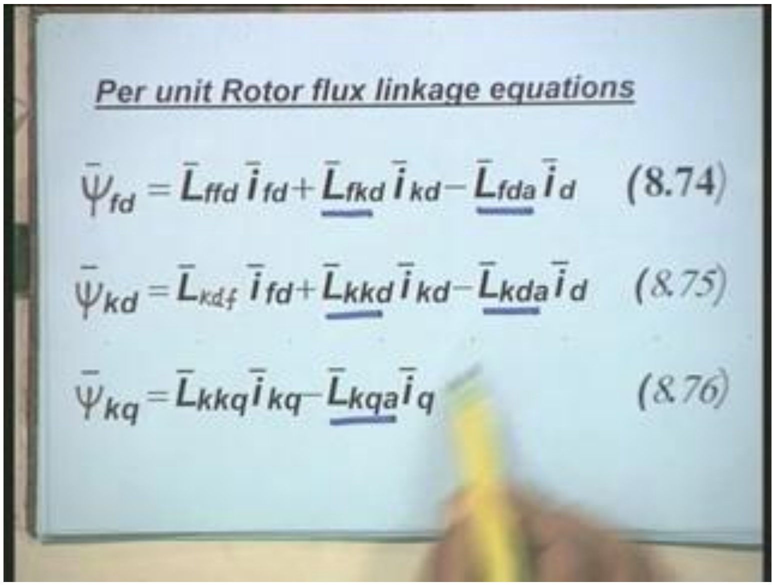


(Refer Slide Time: 03:01)



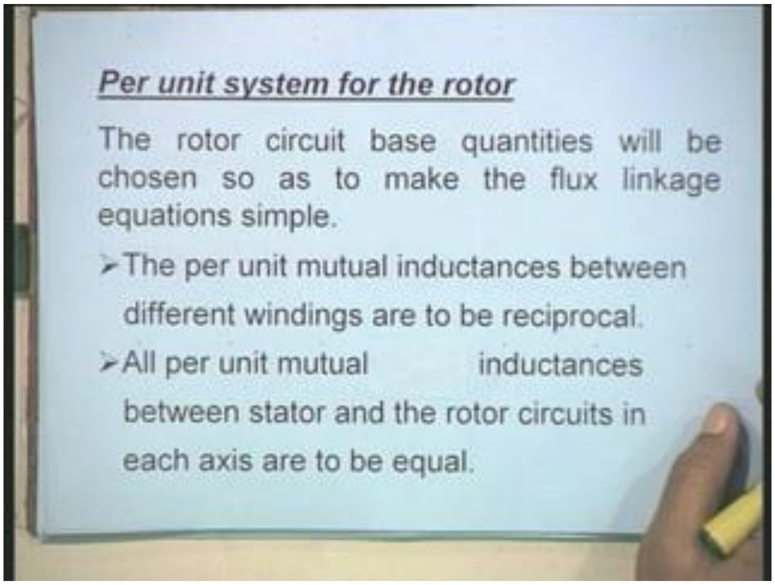

Therefore in these three equations we had defined these per unit inductances now these per unit inductances are the mutual inductances okay.

(Refer Slide Time: 03:12)




Similarly, when we talked about the per unit rotor flux linkage equations, we had define these terms these mutual terms $L_{fkd}$, $L_{fda}$ these are all per unit quantities which were defined.

(Refer Slide Time: 03:38)




Now, now we want to establish the per unit system for the rotor. Now when I am trying to say per unit system for the rotor in the sense that we have chosen the base quantities in the stator circuits. Now how should we choose the base quantities for the rotor circuits, the rotor circuits are rotor field winding and rotor amortisseurs. Okay now the primary consideration for choosing the base quantities in the rotor are to simplify the the flux linkage equations to make this flux linkage equation as simple as possible.

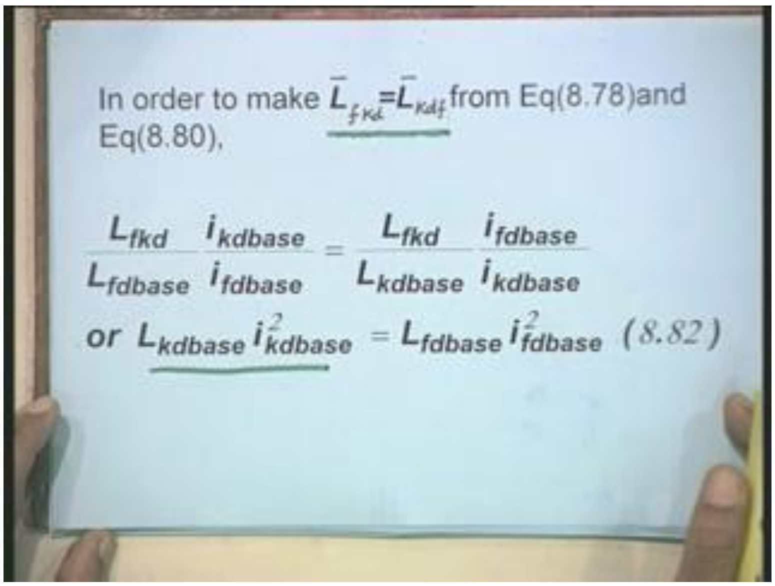
Now one requirement which we have put is or one way of simplifying is we make mutual inductances between different windings reciprocal. We make mutual inductances between different windings reciprocal. Now when I say that we make these mutual inductances reciprocal in the sense that you there is a mutual inductance between the stator winding and the field winding okay.

Now the if I write down the mutual inductance between the stator winding and field winding I can write down $L_{afd}$. When I write down between field winding and stator winding it can written as $L_{fda}$ we want to make these two inductances equal. So that that is what is the meaning of reciprocal okay another thing which we would do to simplify the equations will be that the mutual inductances between stator and rotor circuits on each axis are to be equal that is when I say each axis means, we have two axis d axis and q axis. The on the d axis we have the stator direct axis winding, field winding and amortisseurs on the d axis therefore, there exists mutual inductances between these windings we would like to make them equal.

Similarly, there exists mutual inductance between the q axis winding on the stator and q axis amortisseur we want to make these mutual inductance also equal therefore we are trying to do two things, one is that we will try to make or we will make the mutual inductances reciprocal and the mutual inductances on each axis equal okay.

Now this will simplify the equations to a great extent. Now how do how do we make the mutual inductances reciprocal, now to make the mutual inductance reciprocal you just look at the suppose you look actually this equation I just put this, this is slightly I have to make that I think $L_{akd}$. Okay, now this $L_{akd}$ and another is $L_{kda}$ we want to make them equal okay. Now these quantities were defined now when to make them equal reciprocal, okay so that what is to be done is that we equate the expression for this with the expression for $L_{kda}$, okay.

(Refer Slide Time: 07:31)

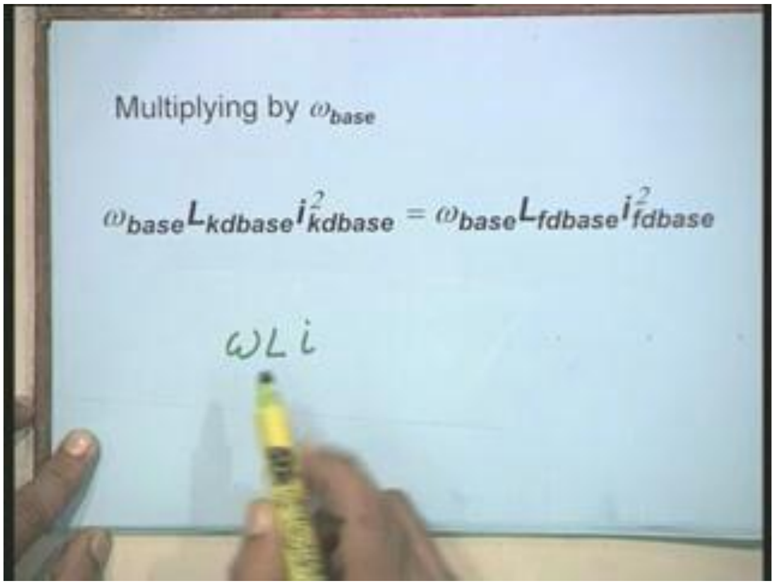



Now we will start first that is let us say that we want to make the per unit mutual inductance $L_{fkd}$ equal to per unit mutual inductance $L_{kdf}$ right that is the amortisseur and field winding, okay that is these are the two, you know mutual inductances one between between field winding and amortisseur another is between amortisseur and field winding and want to make these reciprocal.

Now these quantities were defined now as per the definition this quantity L bar fkd that is the per unit mutual inductance between field and amortisseur windings were defined like this that is $L_{fkd}$ upon $L_{fdbase}$, $i_{kdbase}$ upon ifdbase that is in the denominator here is $L_{fdbase}$, $i_{fdbase}$ here you have $L_{kdbase}$, $i_{kdbase}$. Now this is by definition which we have defined actually when we wrote the equations for flux linkage equations in the rotor circuit. Now when we equate these two, okay we will get this expression that is you just cross multiply

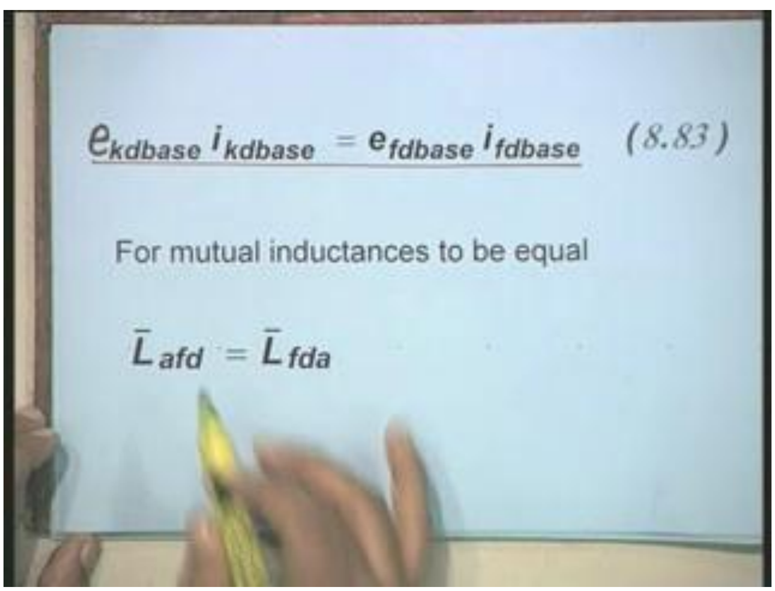
You will find that it becomes $L_{kdbase}$ into $i_{kdbase}$ square equal to $L_{fdbase}$, $i_{fdbase}$ square you can just check it it comes out to be like this that is $i_{kdbase}$ comes from this side, so okay and this these two cancel out they are equal okay and we get this equation. Now here what we do is we multiply both sides of this equation by omega base, once you multiply by omega base it becomes L omega base into Lkdbase, ikdbase square similarly, omega base equal to Lfdbase into ifdbase square.

(Refer Slide Time: 09:20)




Now you can identify this quantity omega L into i this quantity this is actually the voltage this is a voltage therefore I take this base frequency in radiance per second the base inductance and multiply this by current, base current this will come out to be base voltage okay and now we are taking these quantities for amortisseur the quantities are for amortisseur $L_{kd}$ $i_{kd}$ therefore the base voltage will become the amortisseur circuit base voltage this d axis amortisseur circuit base voltage similarly, here.

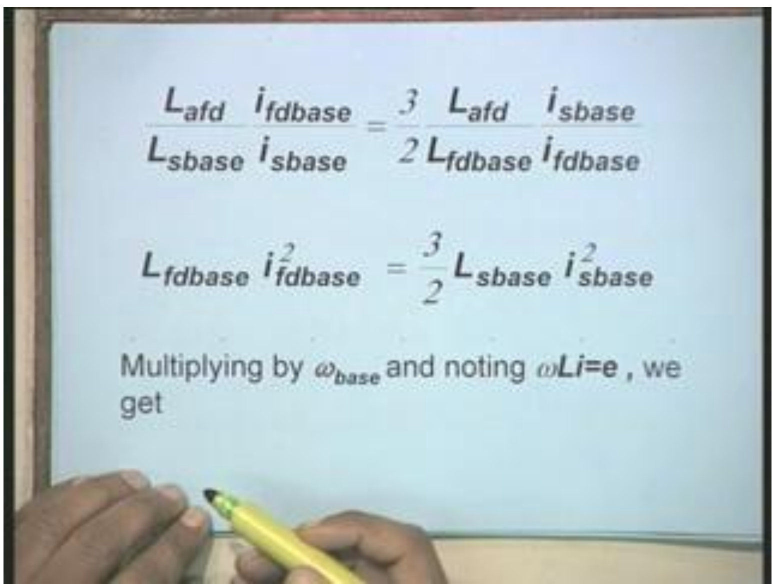
(Refer Slide Time: 10:23)



Now when you do this you will find this is very important relationship that is the base voltage in the direct axis amortisseur circuit multiplied by the base current in the direct axis amortisseur circuit this product must be equal to the the base voltage in the field circuit into base current in the field circuit, it is very simple that is the the base $v_i$ in the amortisseur circuit should be equal to base $v_i$ in the field circuit

Now if you do this you will find actually that the mutual mutual inductances right between the field winding and the amortisseur winding will become reciprocal. Okay therefore this is one important requirement that base VA in the field circuit should be same as the base VA in the amortisseur circuit now we do one similar exercise for mutual inductance between the stator winding that is the stator direct axis winding and the field winding that is stator and field that is $L_{afd}$, we want to make this as $L_{fda}$ that we want to make these two per unit mutual inductances equal right now these quantities were again defined when we wrote the equations for flux linkages okay.

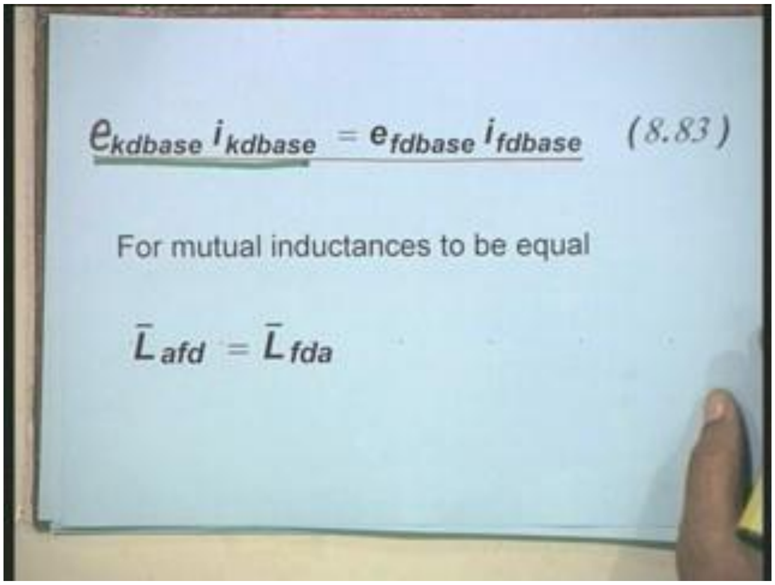
(Refer Slide Time: 11:59)




Now when we equate this two terms and follow the same procedure as we have done for the for the mutual inductance between field and amortisseur. Okay therefore here again we come to very important result and that result is that vibase, v is that is the efd base into ifd base that is the vibase in the field circuit comes out to be equal to 3 phase VAbase for stator, 3 phase VAbase for the stator that is in the stator circuit when you assume that the the base voltage as peak value of the phase voltage, the the base current in the stator circuit is the peak value of the phase current and we have established that this quantity is equal to 3 times VA, 3 times VAbase for the stator that is 3 phase VAbase for the stator the total volt ampere rating right.

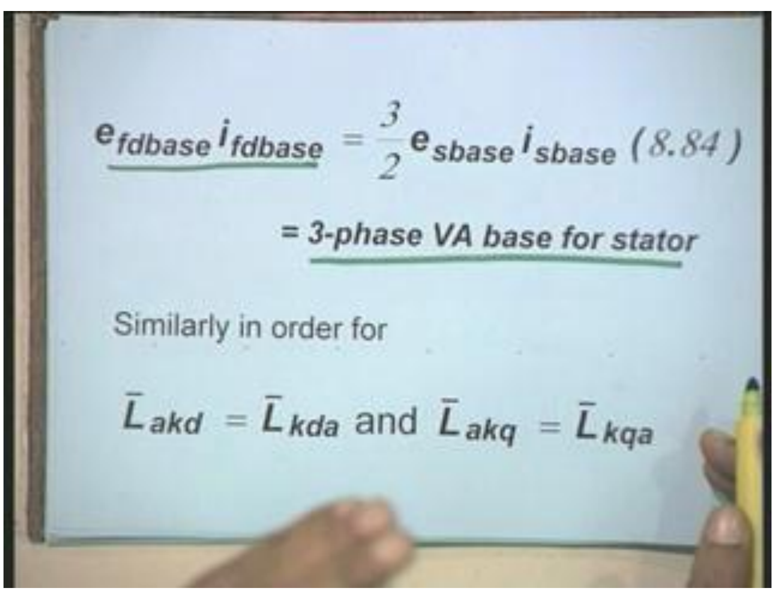
Therefore, this is a very important relationship because in the we have also established the relationship that the base VA in the kd circuit direct axis amortisseur should be same as that in the field circuit it means now we can conclude actually that these 3 windings one in the field winding, another stator winding, third amortisseur winding, direct axis amortisseur winding.

(Refer Slide Time: 12:17)




Now if you want to make this mutual inductances reciprocal then the volt ampere rating or volt ampere base in the stator circuit must be equal to the volt ampere base in the field circuit and volt ampere base in the amortisseur circuit because in the stator with the the volt ampere base is equal to the 3 phase $VA_{base}$ 3 phase VA total MVA rating.

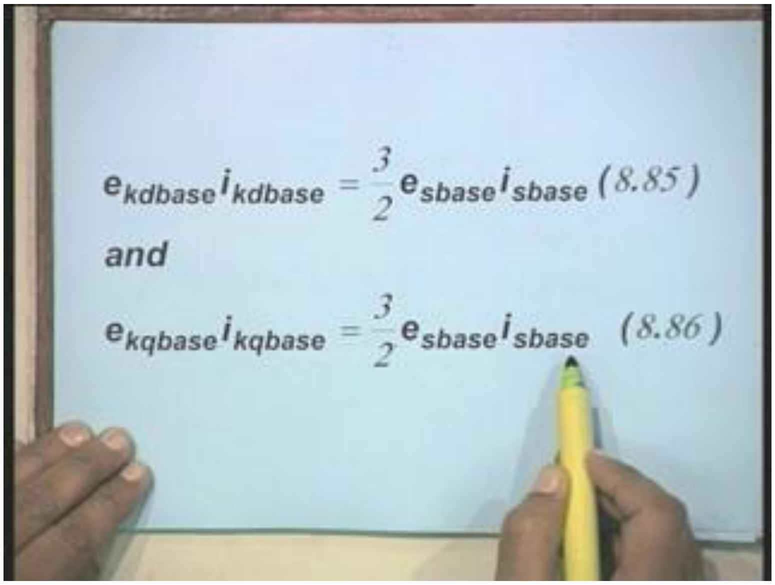
(Refer Slide Time: 15:09)



Okay this is exactly similar to what you normally do in any system we take total 3 phase MVA as the base quantity and then we talk about the base for the rotor circuits we find actually that but rotor circuit now if the field circuit if you consider it it has a applied voltage Vefd and the current flowing is ifd therefore efdbase into base ifdbase becomes the this product becomes the VAbase for the field circuit therefore this one condition will be established that if we want to make, we want to make the mutual inductances between the circuits reciprocal then this is the criteria required for choosing the VAbase for these circuits, okay.

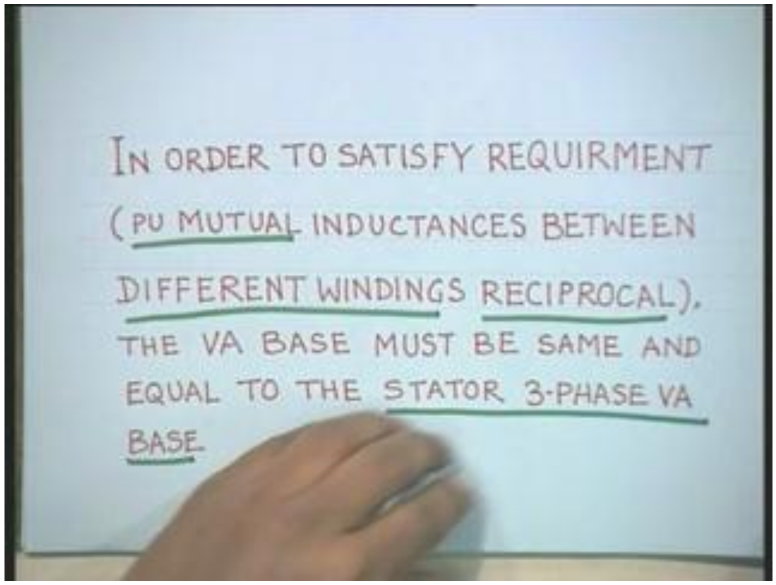
Now this the similar exercises have to be done for for quadrature axis, stator winding and the quadrature axis amortisseur because so for we have talked about the d axis and here also we established the relationship and the relationship is that ekqbase into ikqbase is equal to 3 by 2 esbase into isbase not now this quantity is nothing but the 3 phase VAbase for the stator therefore ultimately what is our conclusion that all the circuits state all the all the circuits that is all which are in the rotor field circuit, amortisseur circuits right these would have their VAbase equal to the 3 times or 3 phase VAbase of the stator circuit this is the and this will make all the mutual inductances reciprocal.

(Refer Slide Time: 15:30)



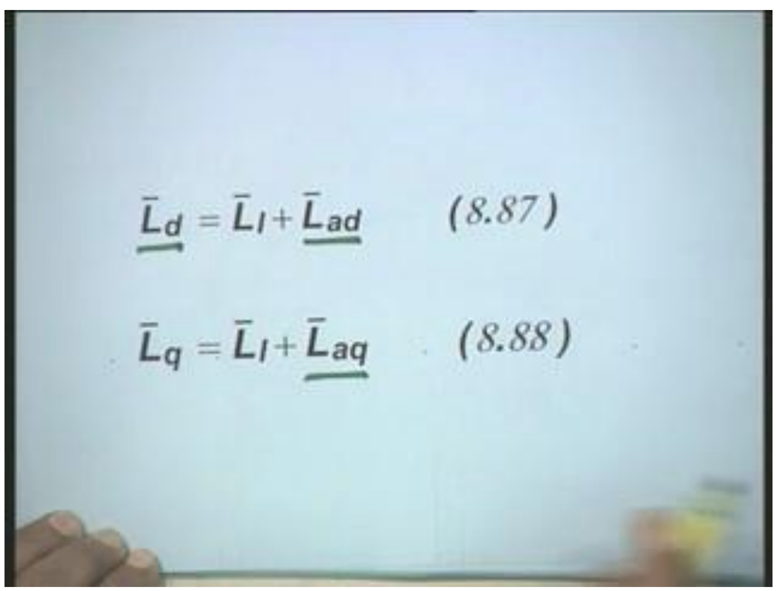

Therefore, now I conclude here that in in order to satisfy the requirement that per unit mutual inductances between again the emphasis is that we are trying to make the per until mutual inductances reciprocal actual mutual inductances which exist, we have seen actually that they are not reciprocal when you see the actual equations which we have written right they are they appear in the equations in a different fashion between different windings the reciprocal okay and the requirement is that VAbase must be same and equal to the stator 3 phase VAbase, this is the main conclusion okay.

(Refer Slide Time: 16:14)




Now the next requirement which we have posed here is posed here is that the mutual inductances on the same axis we want to make them equal first we make them reciprocal and then requirement was there want to make them equal.

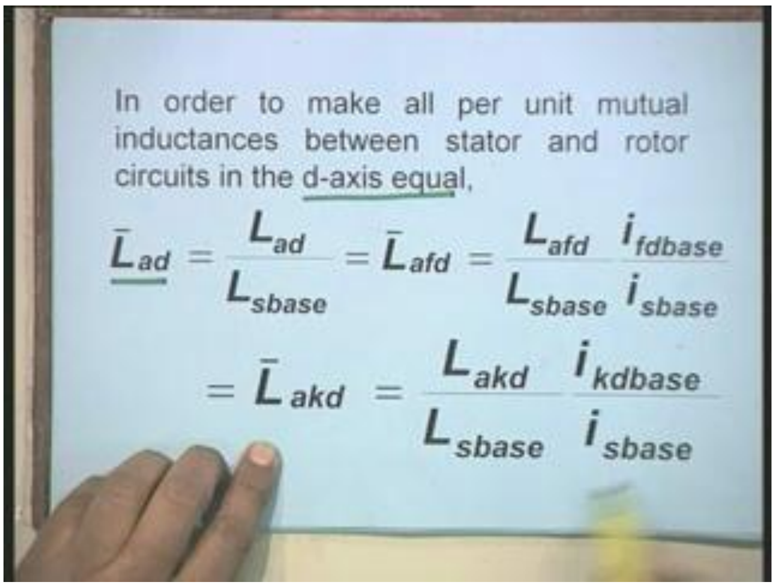
(Refer Slide Time: 17:22)




Now to make them equal we we define the per unit self inductance of the d axis winding, d axis stator winding as leakage inductance per unit leakage inductance plus per unit mutual inductance that is when you look at the d axis winding on the stator. Okay the total per unit inductance is defined as $L_d$ bar you know this bar stands for rotor super bar stands for per unit.

Now the total flux which is produced right which links the d axis winding does not pass through the field circuit or the amortisseur circuits d axis amortisseur circuit there is some flux which is the local leakage flux therefore this $L_l$ is accounts for the inductance of the d axis stator winding due to leakage. Okay therefore this is this becomes our mutual inductance $L_{ad}$ bar similarly, for q axis.

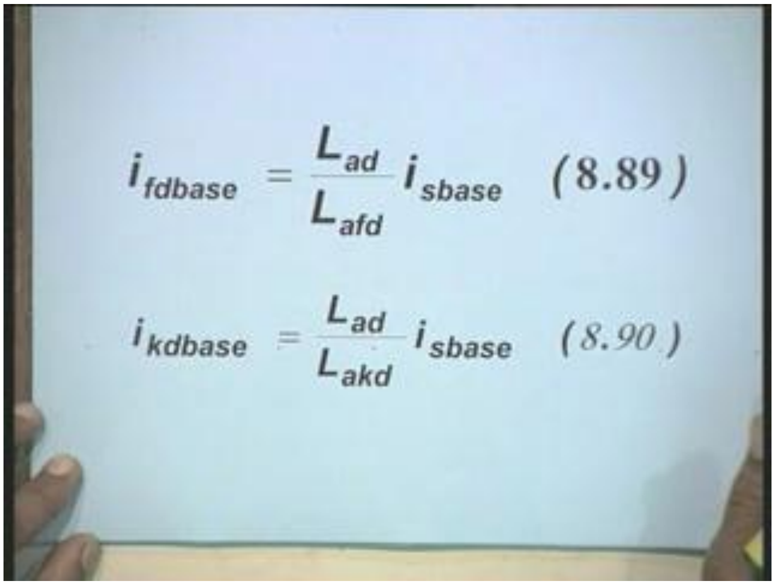
(Refer Slide Time: 18:44)



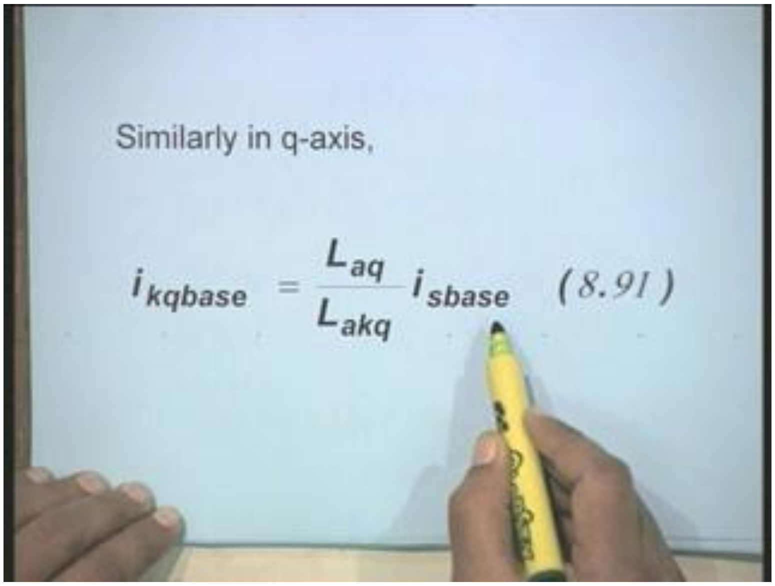

Now here our requirement is that in order to make all per unit mutual inductances between stator and rotor circuits in the d axis equal that is we have in the stator circuit a fictitious d axis winding okay and the rotor circuit we have on the d axis two windings okay and we want to make them equal for making them equal or this by definition this is actual value of mutual inductance $L_{ad}$ divided by the $L_{sbase}$ okay and this this is equal to $L_{afd}$ which has which was earlier defined by this expression when we wrote the wrote the mutual or we can say when we write the expressions for the psi d psi q and psi naught right, the $L_{afd}$ was defined like this per unit value of $L_{afd}$ was defined by this expression that you can refer to the pervious equations okay.

Similarly, now we want to make $L_{ad}$ to equal to the per until $L_{akd}$ okay while $L_{akd}$ was defined by this expression. Okay now what we want that we equate these 2 as we did in the previous case and when we equates these 2 we come to one very important relationship that the $i_{fdbase}$ should be equal to $L_{ad}$ upon $L_{afd}$, $i_{sbase}$. Similarly, $i_{kdbase}$ comes out to be $L_{ad}$ upon $L_{akd}$ divided by $L_{sbase}$ while these quantities are the actual inductances, actual mutual inductances right while what we trying to make them equal are the per unit mutual inductances. Okay, now this this gives a relationship that suppose I have chosen the stator base current then the base current in the field circuit should be given by this equation.

(Refer Slide Time: 20:16)



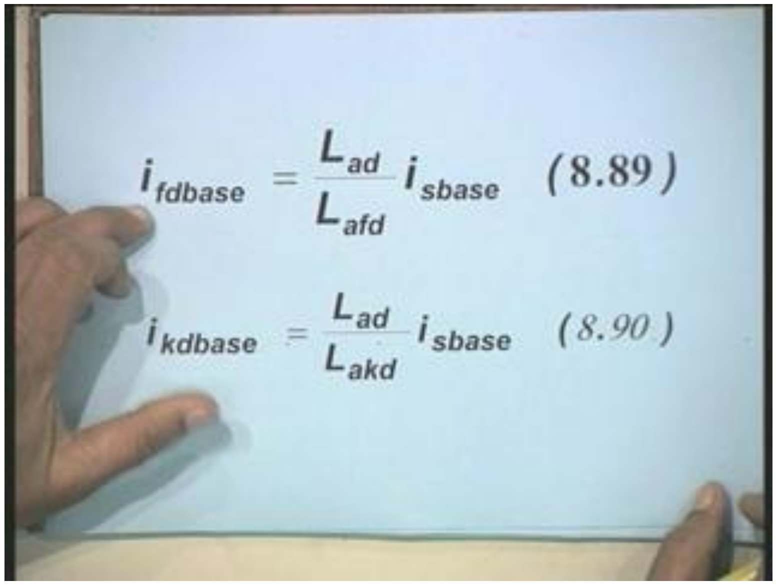

(Refer Slide Time: 21:25)



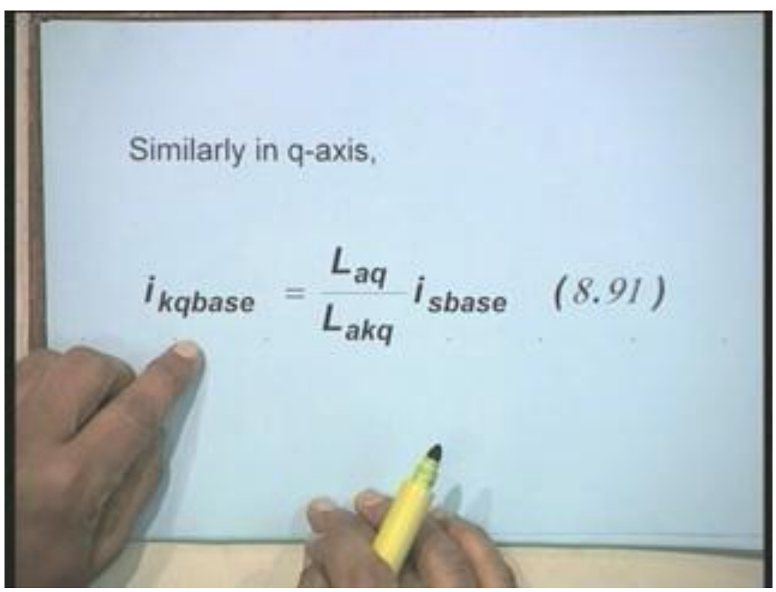

Similarly, if I choose in the base current in the stator as is base then the base current in the amortisseur circuit, direct axis amortisseur circuits will be given by this expression. Okay similarly you can do for q axis also that is q axis will give you that $i_{kq}$ base should be equal to $L_{aq}$ divided by $L_{akq}$, $i_{sbase}$ I hope you understand this one is this is this is a straight forward okay, what we are trying to do is that what we want to make them equal. We want to make the per unit per unit the mutual inductances on the same axis d axis equal that is right, we want to make $L_{ad}$ per unit equal to $L_{afd}$ which is the definition of $L_{ad}$ same as $L_{fd}$ we want to make this $L_{akd}$ and $L_{afd}$

equal to make them equal you equate these quantities which was the basic definitions for this per unit $L_{afd}$ and per unit $L_{akd}$.

(Refer Slide Time: 22:24)




(Refer Slide Time: 22:32)

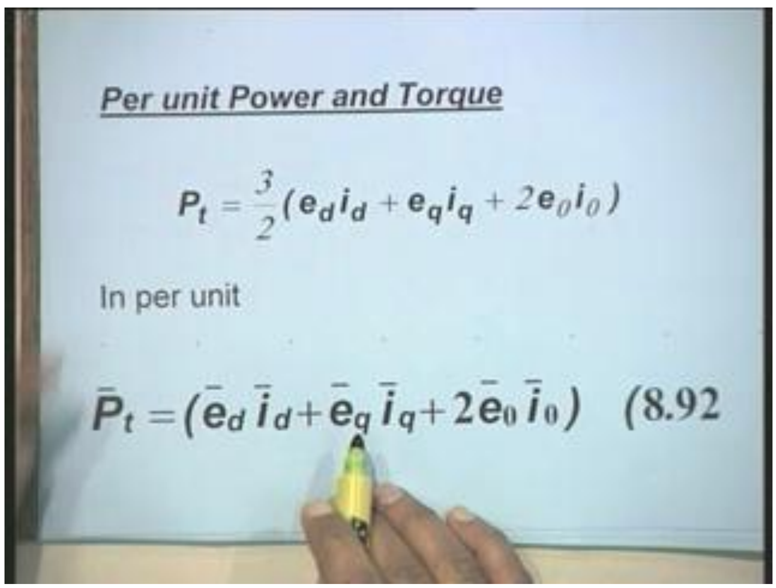



Okay, and we finally get the relationship that the field currents base value of field current base value of the amortisseur circuit current and the base value of the quadrature axis amortisseur therefore current are expressed in terms of the stator, the stator circuit base current okay Similarly, we had also established that VA base for all the rotor circuits in terms of the 3phase

VA base of the stator and therefore since actually when I talk about the VA that the field voltage base into ah field voltage current base okay.

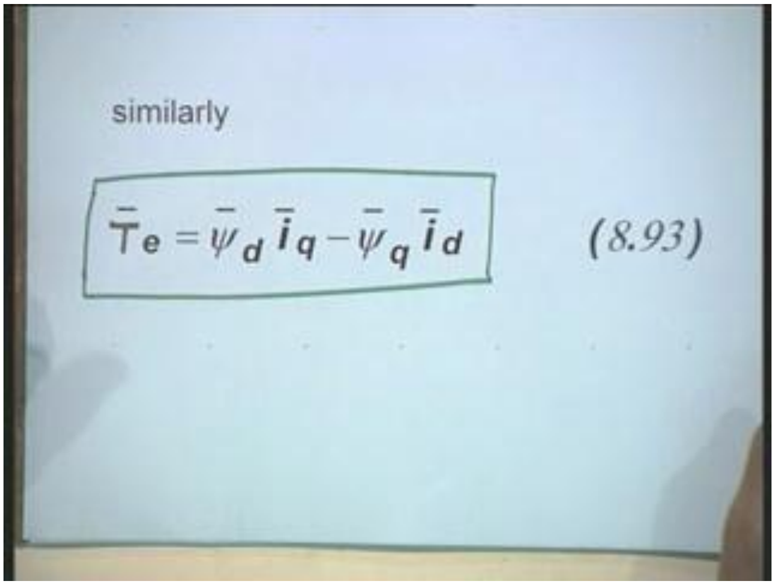
Now this product is equal to the three phase VA of the stator current right but the individually currents will be defined by this expression now once the current is current base is given for these circuits the , we compute the voltage base for the rotor circuits using the previous relationship I hope it is clear to all of you now once we have done this and define the the model for the synchronous machine is complete, the synchronous machine model is complete we for all for all further discussions what we will do is that this super bar which we had put to identify these quantities as the per unit quantities we will drop it but we will always keep in our mind that these are per unit quantities once these are per unit quantity because every time writing super bar is not very convenient okay.

(Refer Slide Time: 24:17)




Now further we can express the per unit power and torque per unit power and torque the in our previous derivations the terminal power, total terminal real power was expressed as 3 by 2 $e_d$, $i_d$, $e_q$, $i_q$ plus 2 times $e_o$, $i_o$. Okay that you can recollect now here what you do is that you replace all these quantities by that is you divide by base 3 phase base VA of the stator here that is you divide this by base volt ampere, you divide all these quantities by the base voltage and currents when you do this exercise the power in per unit can be written as ed bar id bar plus eq bar iq bar plus 2 times eo bar io bar.

(Refer Slide Time: 25:56)

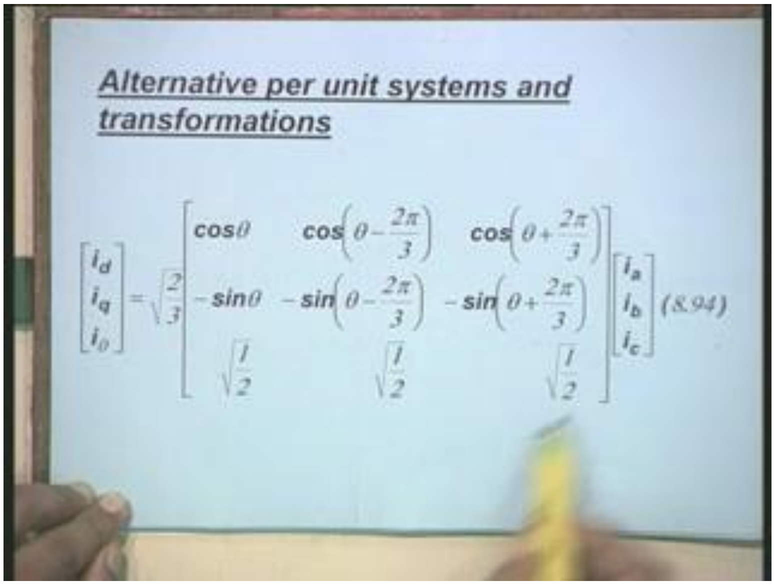



Okay that is only thing which happens is that 3 by 2 term which appears in this expression disappears in the expression for per unit generator power, okay other expressions are same except that the per unit quantities this exercise I will just suggest you to verify, you substitute the values and then you verify that you get these 3 by 2 is then we obtained the per unit electrical torque or air gap torque which will come out to be equal to per unit direct axis flux linkage psi d into $i_q$ minus psi q into $i_d$. In fact this this is the most important expression when we talk about the synchronous machine model why is this expression is, so important can you tell me.

Other, other stator circuit rotor circuit equations are also ah equally important right because that gives the model complete model we have to write down the stator circuit equations, rotor circuit equations, stator circuit flux linkages, rotor circuit flux linkages. Okay and all these things have to be written in per unit by establishing proper per unit base but when I say that the per unit air gap torque is given by this expression and this is quite important, can you tell me why? yes instead of calculating the torque in terms of three phase we are getting the torque just by 2 values. No, because the torque is computed in per unit using the per unit flux linkages and per unit currents this expression is going to be used in our swing equation.

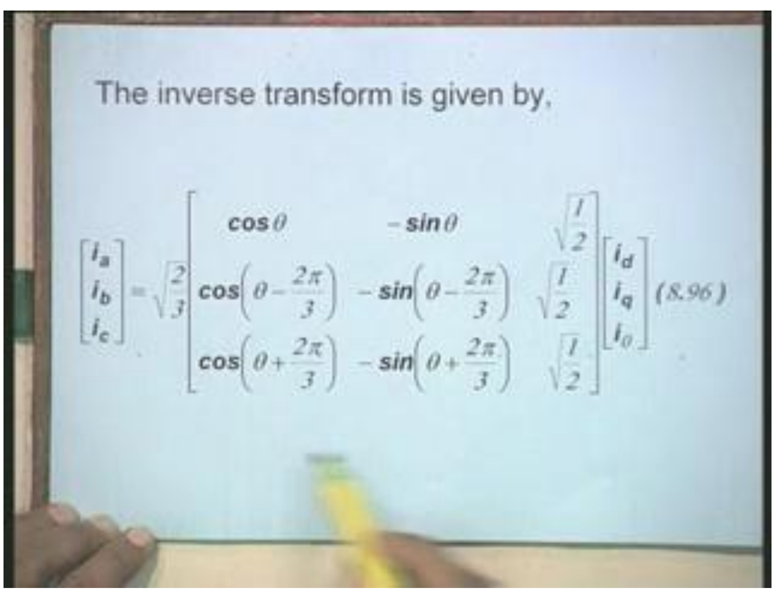
When you have written the swing equation in the swing equation, we have to write down mechanical torque minus electrical torque and this this is the electrical torque that will come in the swing equation okay.

(Refer Slide Time: 27:56)




I will just mention briefly about alternative per unit systems and transformation that is the dqO transformation which we discussed till now and another form of dqo transformation which has been discussed in the literature and also used by some people. Here, in this the the dqo quantities are expressed in terms of this transformation matrix into the phase currents that is the phase variables $i_a$, $i_b$ and $i_c$.

(Refer Slide Time: 29:22)

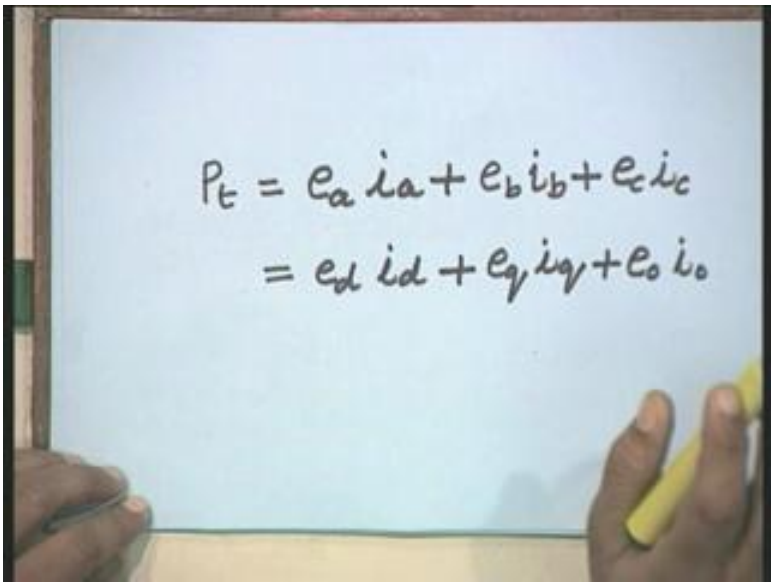


Now if you see this transformation matrix the main difference here is that the dqO transformation which we used right this quantity or this term kd this is a kd was a kd or kq they have both taken equal to 2 by 3. Here they are taken as square root of 2 by 3 and correspondingly the the last row that is the third row the terms become instead of 1 by 2, it becomes square root of 1 by 2 and so on.

Now with this transformation if you write the inverse transformation that is you write down the phase currents $i_a$, $i_b$ and $i_c$ in terms of dqo quantities right. Then this this this becomes the inverse of our transformation matrix, this is the transformation matrix we use therefore if you invert this matrix right then we can write down using this inverted matrix the relationship between the phase currents $i_a$, $i_b$ and $i_c$ in terms of dqO components. This inverse if you just see here then this inverse is nothing but the transpose transpose of the transformation matrix.

You just see here actually that this row is the column here right while this row appears as column in the inverse okay and therefore this is this inverse this is the inverse of the transformation matrix but is also the the transpose of the transformation matrix. Now whenever you have this type of situation we call this transformation as orthogonal transformation now when you resort to this transformation one very important result which we get is like this.

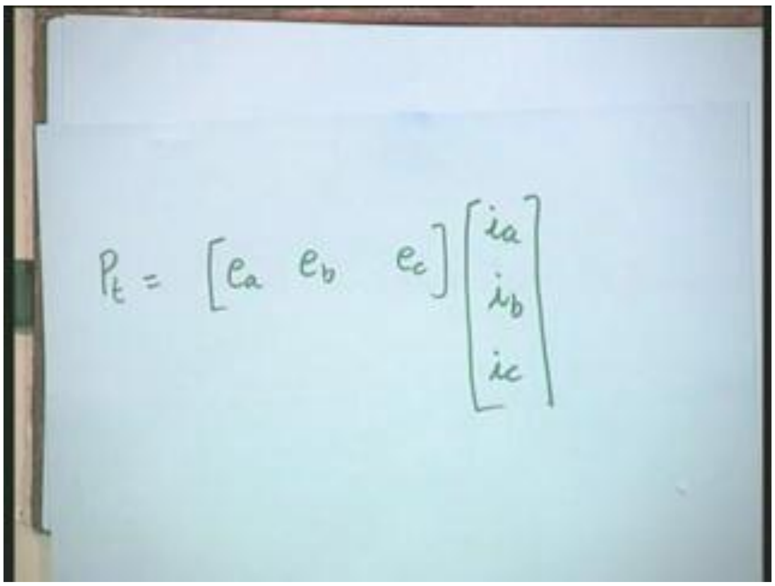
(Refer Slide Time: 30:45)




The terminal power $P_t$ right which when we write down as $e_a$ into $i_a$ plus $e_b$ into $i_b$ plus $e_c$ into $i_c$ this is a terminal power okay. This $P_t$ is average or instantaneous, instantaneous but what is average?

The average and instantaneous become equal in a 3 phase system, 3 phase system the total power is total or the average power is constant, instant we say instant is power we calculate this sum will come out to be equal to average power right therefore now if you substitute here that is you write this equation, that is this equation can be written as $P_t$ can be written as $e_a$, $e_b$, $e_c$ multiplied by $i_a$, $i_b$, $i_c$ is it not this expression is written you can put in the matrix form this is row vector multiplied by this column vector this will give you $e_a$, $i_a$ plus $e_b$, $i_b$ plus $e_c$, $i_c$. Now you replace this row vector by dqO components you replace this also by dqO components and multiply these 2 matrices when you do this multiplication the end result comes out to be of this form that is the $P_t$ will come out to be equal to $e_d$, $i_d$ plus $e_q$, $i_q$ plus $e_o$, $i_o$ it is something like this that when I had 3 voltages and 3 currents okay I got the terminal power by multiplying this quantity.

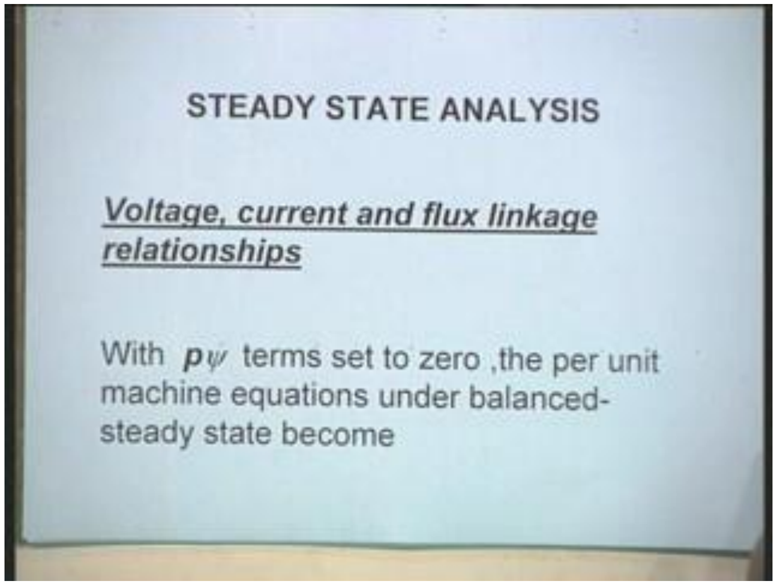
(Refer Slide Time: 31:45)



Similarly, I have now the direct axis voltage quadrature axis voltage and zero sequence voltage similarly the direct axis current quadrature axis current and zero sequence current I multiply and add this comes out to be equal to right now for this this type of therefore this transformation is called power invariant transformation, power invariant transformation and it has some merits which has been discussed in literature but it has some demerits also and that is why in most of the models which have been developed the dqo transformation which was discussed earlier is used where we use $k_d$ and $k_q$ equal to 2 by 3. Okay having developed the synchronous machine model you have developed is synchronous machine model completely in using per unit quantities and the model is developed in dqo frame of reference okay.

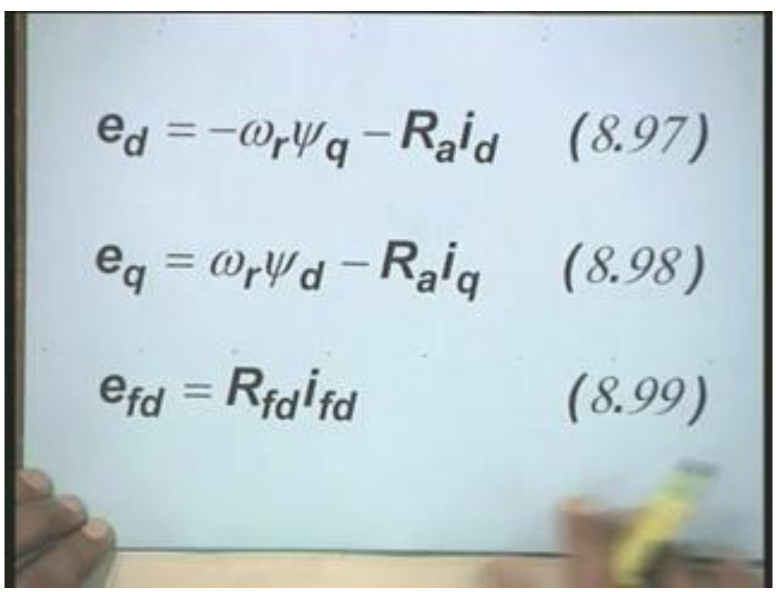
Now this model is the complete model and now what we will study from this point onwards will be first we will study the steady state aspect okay and then we will go how this model can be simplified for our stability studies. Okay that is this is the complete model actually right but sometimes we can make some assumptions and simplify the model okay and how do we simplify this model which will be suitable for stability studies that is what we will study in the subsequent lectures.

(Refer Slide Time: 34:58)




Now let us quickly do the steady state analysis because steady state analysis is one which is known to everybody all of you know this analysis but now we will develop the steady state equations for the machine starting from our dynamic model of the synchronous machine. Okay now when we talk about steady state conditions we are presuming that the synchronous machine is operating under steady state condition and the stator currents and voltages are balanced 3phase currents and voltages that is this is the assumption that these steady state operating condition and the stator and stator quantity stator currents as well as the stator voltages are balanced.

(Refer Slide Time: 36:21)

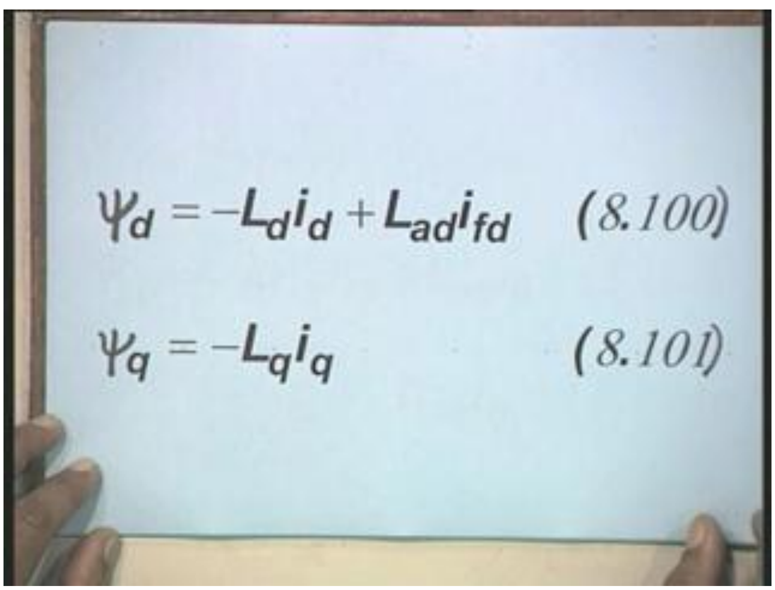


With this assumption once it is steady state this derivative terms will be absent, in the steady state condition we do not have anything like d by dt of psi suppose, I take this psi will be constant therefore this derivative term derivative term will be absent, second is that all all zero sequence terms will also be absent because it is a balanced 3 phase system okay.

With this assumption the stator circuit equations can be written now in per unit again remember that these are all per unit stator circuit equations the bar is not written here while the stator circuit equations are written in the form ed equal to minus omega r psi q minus $R_a$ $i_d$ $e_q$ equal to omega r psi d minus Raid and efd equal to $R_{fd}$, $i_{fd}$ it is very simple actually, you can just in all the 3 equations the p psi d terms have been p psi terms that is the derivative terms have been set equal to 0 okay.

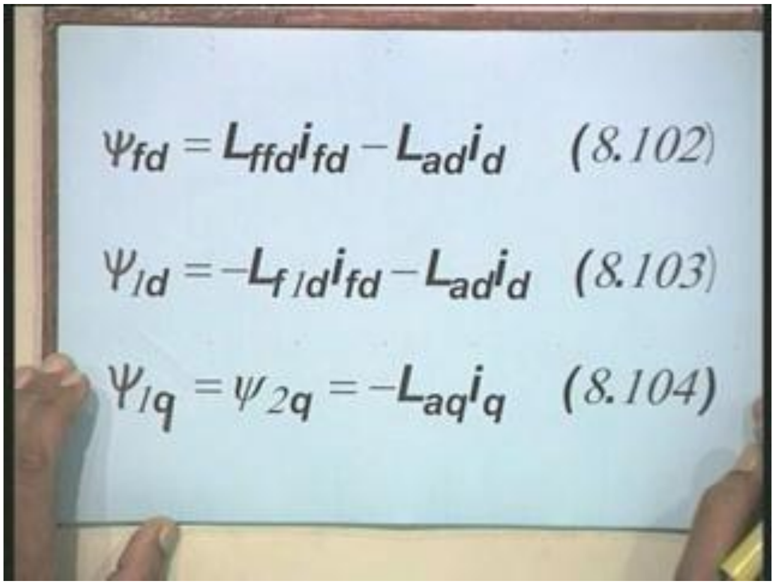
Now under a steady state operating condition the rotor speed is equal to synchronous speed therefore, we substitute here omega r equal to omega s and in a per unit system we will see that omega r is equal to omega s equal to 1. Okay now these are the 3 voltage equations these 2 voltage equations for the stator circuit this voltage equation for the field winding and these are the flux linkage equations that is psi d and psi q in per unit quantities they are all per unit terms we have not done anything except that except that in these equations the the derivative terms have been set equal to 0 right. Another thing while writing here we have also set the mutual inductances per unit mutual inductances on the d axis windings are equal right therefore, you will find here actually when you write down this psi d is $L_{ad}$ $i_{fd}$.

(Refer Slide Time: 37:34)

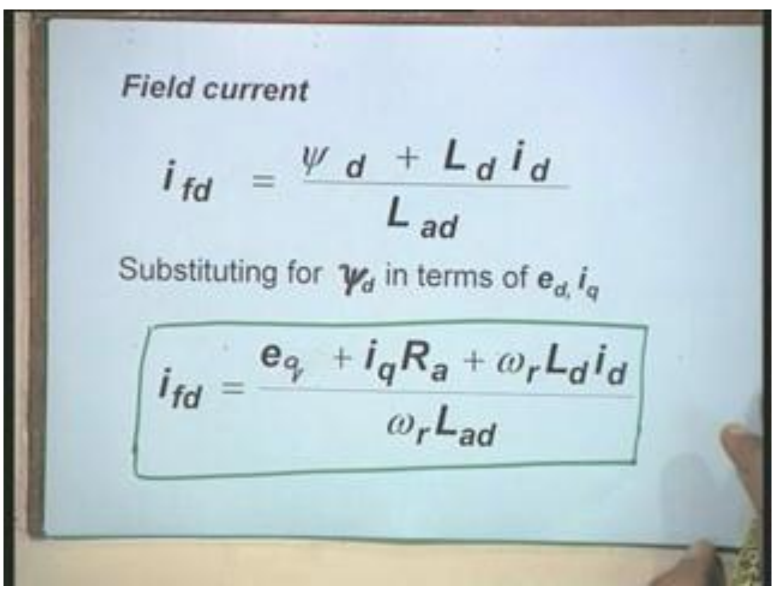



We will write down psi fd again I have put Lad because the mutual inductance is mutual inductance between the stator d axis winding and the field winding have been made equal in per unit in per unit system okay. Therefore, this simply appears as Lad okay. Further actually when we have written these equations suppose there is one amortisseur on the d axis k becomes one here, suppose you take 1 or 2 amortisseurs on the q axis one will, one equation will be for psi 1q there is a second amortisseur will be psi 2q right and they will be written in the by the same equation minus Laq iq why the same Laq will come because mutual inductances between the 2 amortisseur on the q axis, when expression per unit are equal.

(Refer Slide Time: 38:19)




(Refer Slide Time: 40:16)

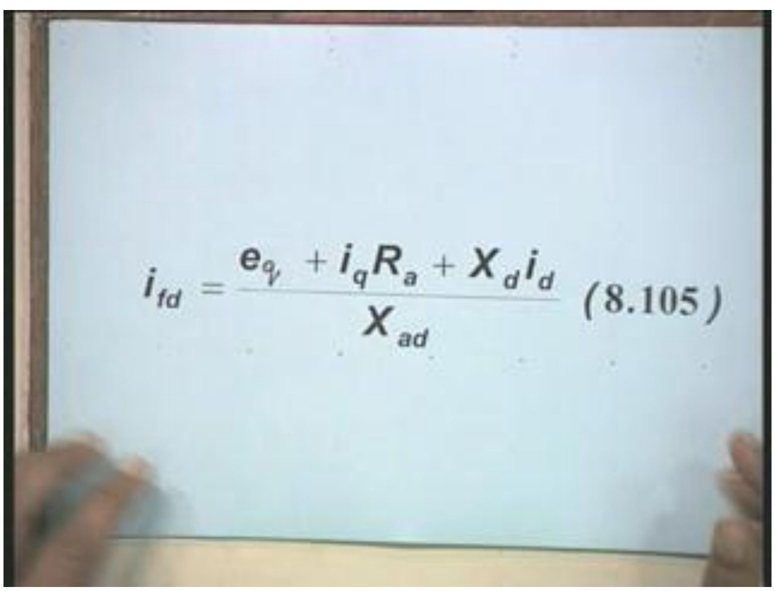



Therefore, this is okay now here we can just derive one simple expression for field current what we do is that in these equations just start with this. Suppose you take my interest is to write down what will be the value of field current required or field current in per unit required to produce the required steady state operating condition. So that using this expression you can write down the expression for ifd that is ifd is written first here as psi d plus $L_d$ $i_d$ divided by $L_{ad}$ and then this psi d that is the per unit flux linkages in the d axis will be replaced by the this equation psi d equal to minus $L_d$ $i_d$ not not here actually this psi d because we will resort back to our stator winding equations that is psi d can be written using this equation. Okay that is $e_q$ equal to omega r psi d minus $R_{aiq}$ okay and when we make these substitutions here we get very important relationship is this relationship that is the field current which is required to produce certain value of $e_q$ the $i_q$, $i_d$ with these quantities known they are all the circuit machine parameters.

Now stator per unit stator resistance per unit direct axis inductance, per unit mutual inductance between direct axis and the field winding these are all now circuit parameters or synchronous machine parameters they are all known. Okay and therefore when system is operating with some load the that load will determine $i_q$ and $i_d$, okay and to have then for a particular value of $e_q$ this is the value of $i_{fd}$ required therefore for all our studies actually as we will see further when we talk about the models of the synchronous machine for our stability studies this information is required but this is simply obtained using those equations that is the stator circuit equations flux linkage equations and making some appropriate substitutions.

Now we use this omega r is equal to omega s and this omega s into $L_d$ we will represent by $x_d$ omega s into $L_d$ will be represented by $x_d$ right. Similarly, you can see omega s into $L_{ad}$ will be represented by $x_{ad}$ reactance omega L is equal to reactance always okay.

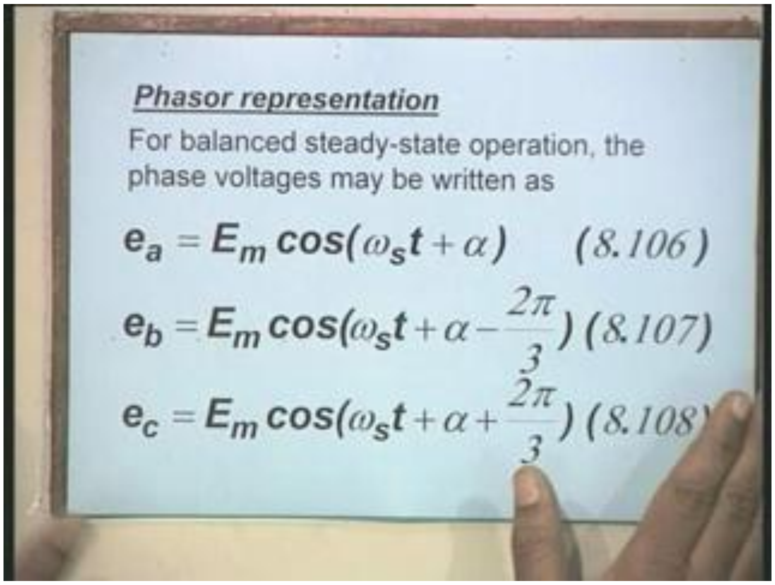
(Refer Slide Time: 42:49)




Therefore if this equation is written again exactly the same but instead of writing in terms of the per unit per unit d axis inductance and mutual inductance which is written in terms of $X_d$ and $X_{ad}$

$X_d$ is the direct axis direct axis reactance while $X_{ad}$ is the mutual reactance between between the field and the d axis winding.

(Refer Slide Time: 43:27)

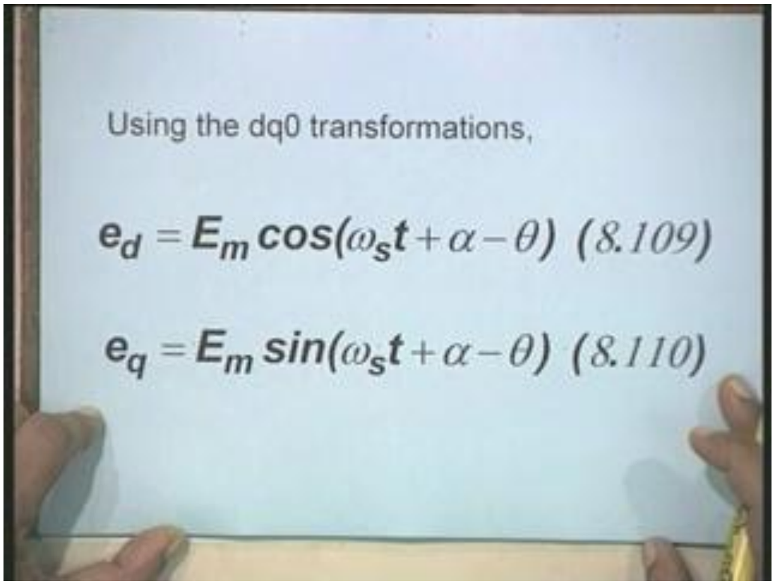



Now we will quickly see, now the phasor representation before we talk about the phasor representation let us reinstate the some of the facts. Under steady state conditions the $i_d$, $i_q$ these two currents these two currents are constant in magnitude therefore basically the current flowing in the fictitious d axis winding and the fictitious q axis winding are dc current direct currents.

Similarly, the $e_d$ and $e_q$ they came out to be constant voltages that we have already established under steady state operating conditions. Now the question arises is that the when we, when we draw a phasor diagram right all the phasor are corresponding to a sinusoidally varying quantity that is all the quantities which vary sinusoidally and having the same frequency right can be represented in the phasor diagram. Okay but here also although these quantities are constant in magnitude because of certain trigonometrical relations which exist we will be in a position to represent this d and q axis voltage and currents as phasor but with the with the clear clear understanding in our mind that d and q axis quantities are scalars.

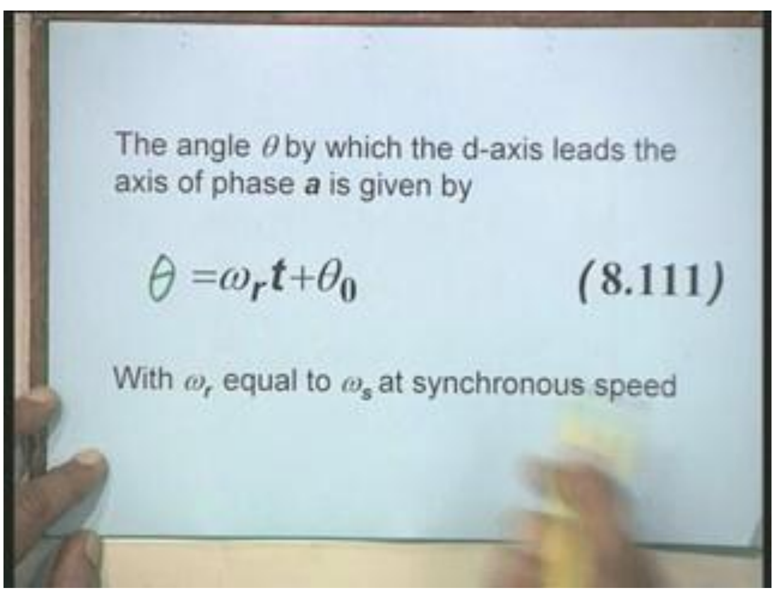
\The $e_d$, $e_q$, $i_d$, $i_q$ they are scalars right but because there exists trigonometrical relationships between the certain phasors and the d and q axis components therefore we can represent these quantities in the phasor diagram. Now to start with let us say that we have a balanced steady state operation the phase voltages are given by these equations. Okay that is $e_a$ is equal to $E_m$ cos omega s t plus alpha alpha is some arbitrary angle a reference angle or may be to start with and this $e_b$ is lagging by 2 by 3, 2 phi by 3,120 degrees $e_c$ by leading by 2 phi by 3. Now you apply dqO transformation to this equation.

(Refer Slide Time: 45:58)




When you apply the dqo transformation you will get the d axis component $e_d$ and q axis component $e_q$ as $E_m$ cos omega st plus alpha minus theta $E_m$ sin omega st plus that is in fact when you apply this transformation right to arrive at this result we have to make use of many trigonometrical identities that is when you when you when you put these equations right $e_a$ $e_b$ and $e_c$ this this column vector in that transformation matrix.

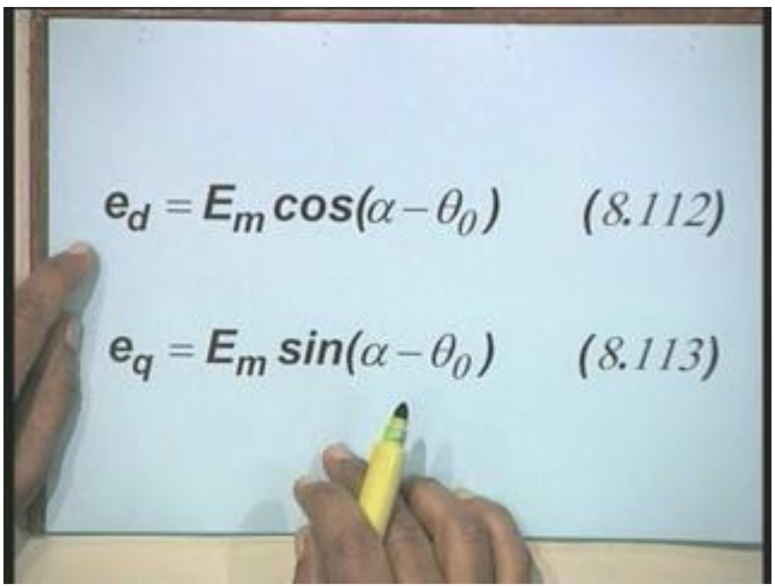
(Refer Slide Time: 47:01)



You will find actually that this trigonometrical terms will have to be multiplied with the elements of that matrix transformation matrix and finally to come to this result you may have to use number of trigonometrical identities. Okay and the final result is going to be in this form okay.

Now we can represent the angular position theta as omega r into t plus theta o and omega r is same as omega s because it is a steady state operating condition. Now when you make this substitution ed comes out to be equal to $e_m$ cos alpha minus theta o $e_q$ comes out to be equal to $e_m$ sin alpha minus theta. Okay and therefore in this equation when we find here in this equation the there is no term like omega st right and therefore $e_d$ is constant it depends upon the peak value of the stator phase voltage and the alpha and theta naught, okay that is why I said that this $e_d$ and $e_q$ come out to be scalar quantities under steady state operating conditions.

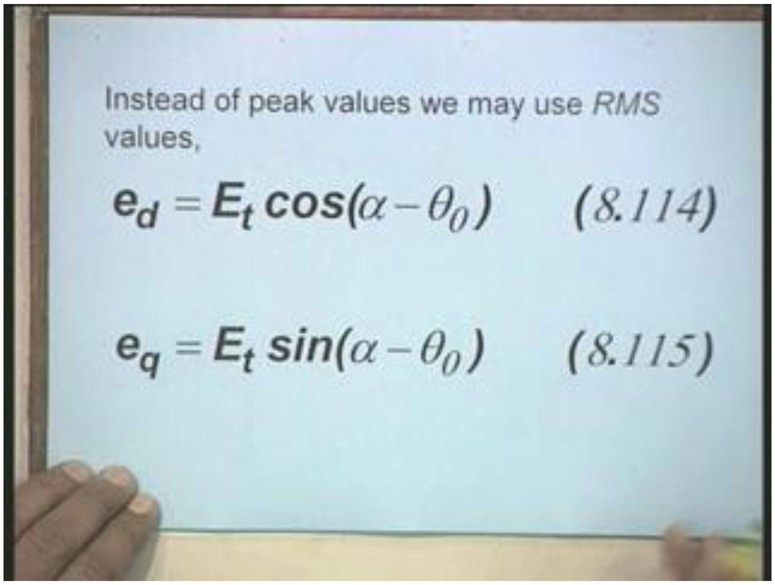
(Refer Slide Time: 47:19)



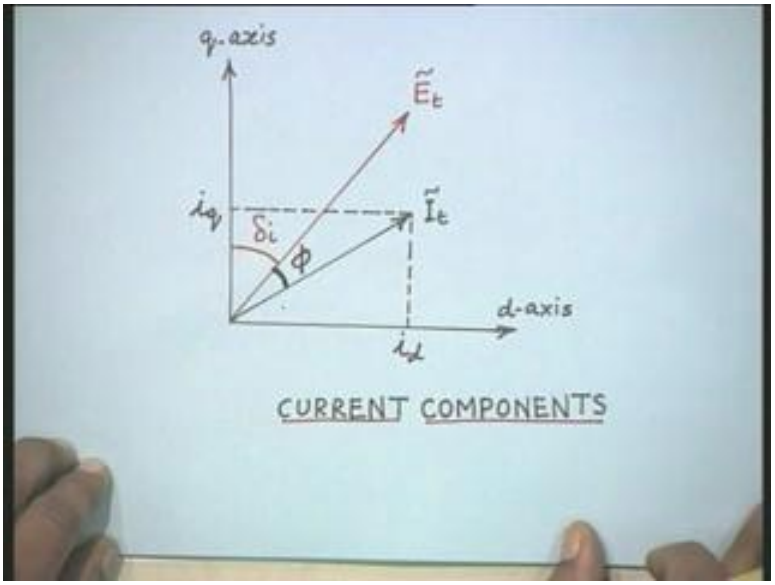

Now $e_d$ and $e_m$ the $e_d$ is d axis voltage and is in per unit. Okay now we are more more comfortable when we use the RMS value of the voltage rather than using the peak value of the voltage. Now suppose, I assume the peak value of the stator voltage as my stator base voltage then RMS value RMS value of the stator base voltage when it is expressed per unit will come out be same as the peak value. It is something like this that I have chosen the base voltage in the stator circuit and that base voltage is peak value of the per phase per unit of the phase to neutral voltage.

Okay now if I want to express ah any quantity in terms of this then they are expressed in terms of the peak value. Now if suppose I have chosen the stator base voltage as peak voltage I want to chose the the base voltage in terms of RMS value, then it is automatically going to be the base voltage which is the peak value divided by root two and therefore if you express these quantities that is $e_d$ and $e_q$ not in terms of peak values but in terms of RMS value the equation will remain same because the base values of the stator voltage, when it is expressed as a in terms of peak or where it is expressed in terms of RMS value they will have a relationship of one by root 2. Suppose I take the stator base as 100 volts then this is a peak value then RMS value, RMS base value will be equal to 100 by root 2 and therefore this equation can be written in terms of the RMS value now I put et, et as the RMS value and it remains same there is absolutely no change this is also per unit, this is also per unit okay.

(Refer Slide Time: 50:18)



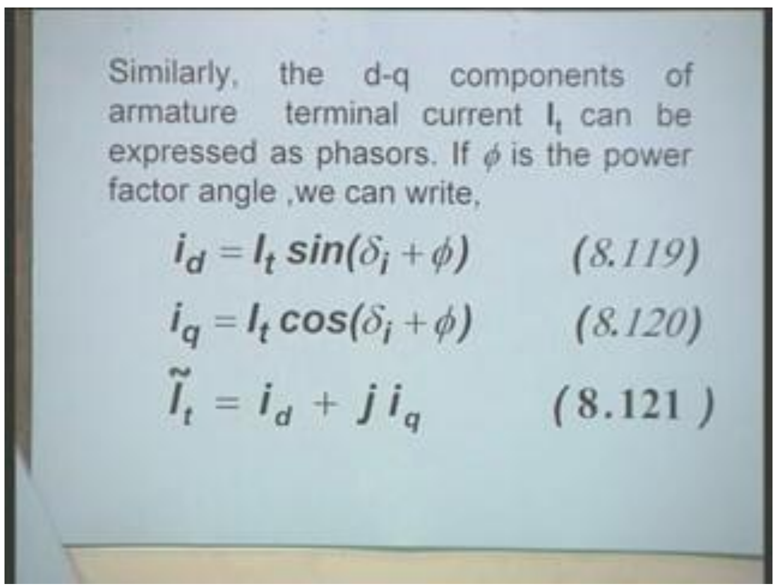

(Refer Slide Time: 50:33)



Now when we represent these quantities in the phasor diagram it will look like this. Let us say this is the terminal voltage phasor I represent this $E_t$ tilde putting this as a phasor quantity because we use bar earlier super bar earlier for representing the bar per unit quantities because that is why we are putting $E_t$ tilde and let us show that this is my d axis, this is the q axis, q axis lead d axis by 90 degrees and here the angle is alpha minus theta because you have seen here actually that the angles comes out to be alpha minus theta o right. Then, then $e_d$ is represented by Et cosine alpha minus theta o and $e_q$ is equal to $e_t$ sin alpha minus theta o therefore, this phasor diagram, phasor phasor diagram is expressing these quantities $e_d$, $e_q$ in terms of $E_t$.

Similarly, you can draw a phasor diagram showing the currents. Let us say this is my terminal voltage phasor so this is the current phasor and let us say that phase difference between the voltages and current is phi right.

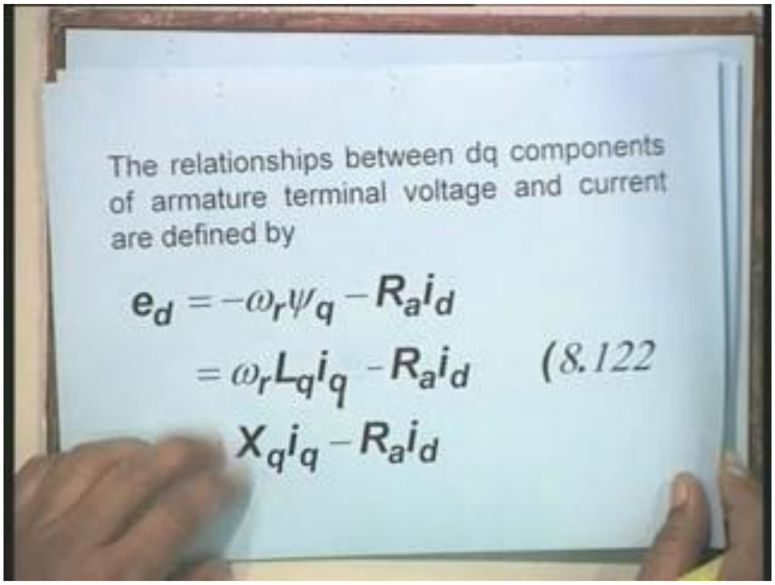
(Refer Slide Time: 52:12)



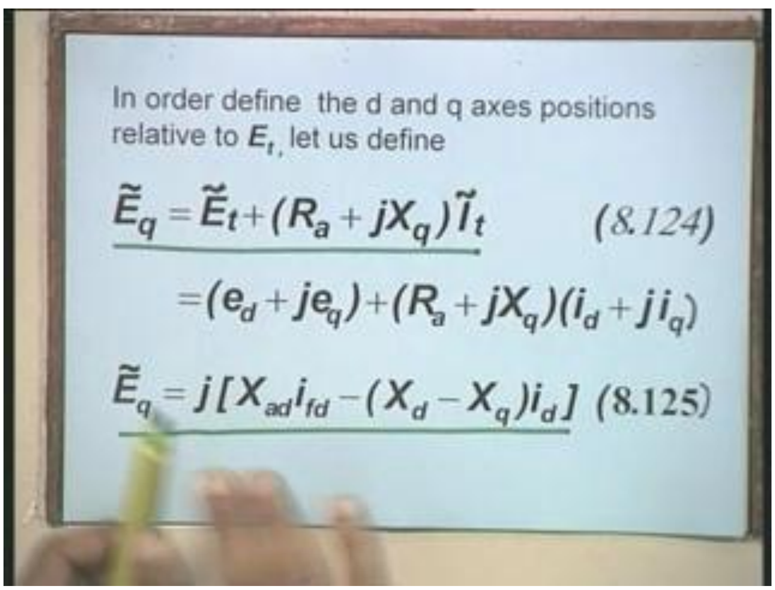

Then we can express express the d and q axis components of current as $E_t$, I am sorry current can be expressed as $i_d$ equal to $I_t$ sin delta i plus phi $i_q$ is equal to $I_t$ cos delta i plus phi. This exercise is done similarly to what we did for voltages right therefore in the phasor diagram I can simply show $I_t$ equal to $i_d$ plus j times $i_q$ okay and the diagram I have just now shown to you, okay.

Now here the angle between the q axis and the terminal voltage we will denote by delta i. Okay now one small exercise which is important exercise to be done is done is to establish the position of d and q axis with respect to the terminal voltage that is I know the terminal voltage and with this terminal voltage as phasor I have to establish what is the position of q axis what is the position that is if I establish the position of q axis I can automatically get the position of d axis, now to establish this, now the some of these are equations are all repeated that you know the $e_d$ is given by this expression, we have already established.

(Refer Slide Time: 53:31)



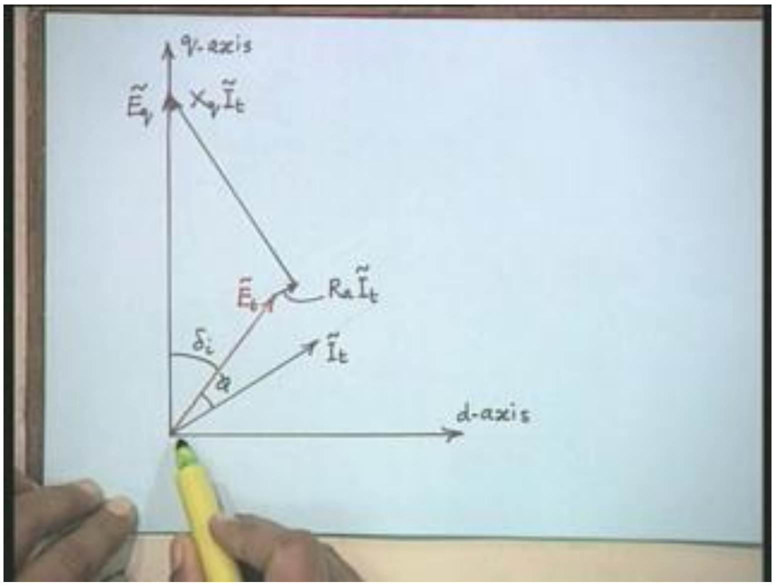

(Refer Slide Time: 53:33)




To establish the relationship what we do here is we will define a voltage $e_q$, a phasor $e_q$ as terminal voltage plus $R_a$ plus j times $X_q$ into $I_t$. This is an this relationship is a very well known to all of us right that is just start like this that my my intension is that to establish relationship between the terminal voltage and the d and q axis, we define a voltage $e_q$ as $E_t$ plus this impedance $R_a$ plus j times $X_q$ into $I_t$.

With this definition and we substitute the expression for ed $E_t$ as $e_d$ plus j times $e_q$ and $I_t$ as $i_d$ plus j times $i_q$ and we make simplifications that is you multiply and simplify this expression. You will find that $e_q$ comes out to be j times $X_{ad}$, $i_{fd}$ minus $X_d$ minus $X_q$ $i_d$ what this what does we get from this equation, what this equation conveys to us. This equation conveys that this voltage $e_q$ which I have assumed right is in quadrature with d axis that is the axis this is along q axis not part of it it is of course quadrature of d axis but it is along q axis because our reference with reference to the reference this quantity is I had by 90 degrees that is j term is existing because the other terms are all scalars here actually the terms are all scalar.

(Refer Slide Time: 55:26)




Okay and therefore the phasor diagram which I draw here showing this $E_t$ plus $R_a$ $I_t$ plus $X_q$ $I_t$ this gives a voltage $E_q$ that is with respect to this terminal voltage $E_t$ the position of $E_q$ comes out to be along q axis right and therefore we have now established a relationship between between the terminal voltage and q axis and once we establish relationship d axis position is known and all further computations can be done the moment we know the $d_q$ and other quantities. Friends I will conclude my presentation here by by summarizing what we have done in this lecture. First, thing that we have established the base volt amperes for the rotor circuit.

We have also established the how to choose the base current for the rotor circuits in order to make the mutual inductances reciprocal and also to make the mutual inductances on d axis and mutual inductances on q axis equal. Okay then we have also established the steady state relations and developed a simple phasor diagram to show that, to show that the the q axis can be obtained by obtaining the position of a phasor which is equal to terminal voltage plus plus an impedance equal to $R_a$ plus j times $X_q$ multiplied by the current that is if you compute this quantity $e_q$ equal to terminal voltage plus impedance that is $R_a$ plus j times $X_q$ multiplied by the current this gives me the position of q axis, okay. Thank you!