**Power System Dynamics**
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**Lecture - 01**
**Introduction to Power System Stability Problem**

Friends, during this semester we will be studying a course on power system dynamics.

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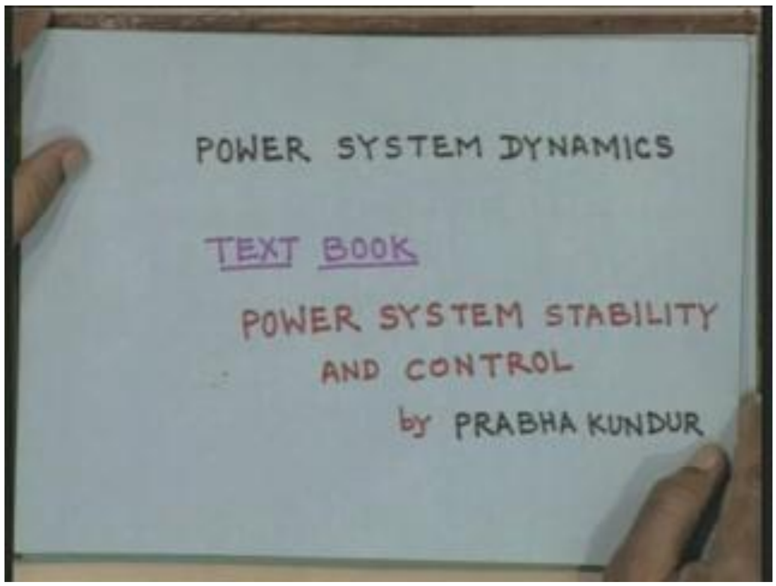


This power system dynamics is also popularly known as power system stability. The modern power systems are very widely interconnected while this interconnections result in operating economy and reliability through mutual assistance. They also contribute to the stability problem, what I am trying to emphasize here is that the power systems are widely interconnected and the stability problem has become a very important challenge to power system engineers because of this large scale interconnections.

The powers for power system stability analysis the mathematical model of the system is required to be developed. The mathematical model is a set of nonlinear differential equations and a set of algebraic equations, for this nonlinear differential equations and set of algebraic equations there is no formal solution available. These equations are to be solved using non-linear using numerical techniques, I am sorry using numerical techniques and the numerical techniques take lot of time to solve the equations.

Over the years the power system stability has posed a problem to the power system engineers. The problem is posed in two respects; the one is the modeling of the system. To get the correct assessment of power system stability a detail model of the power system need to be developed. Once the mathematical model is correctly developed, one has to obtain the solution through numerical techniques. Because of the large size of the power system, the numbers of differential equations are large in number and therefore solution through numerical techniques takes enormous time.

Historically, historically the stability problem has been attempted from 1920 onwards. Earlier we were not having digital computers and therefore computations were mainly done using hand calculations or in those days slide rules were available for calculating. Somewhere in 1950 or so the analog computers were developed and these were used for simulating the power system stability problem.

Then in late 1950’s digital computers came in and the first digital computer program for power system stability was developed in 1956. At that time the program was mainly to analyze the transient stability of the system. Over the years another development took place that is the implementation or application of high response excitation systems. The high response excitation systems were capable of improving transient stability of the system but with the application of the high response excitation systems resulted into a problem of poor damping of the system oscillations.

This problem of poor damping of system oscillations has been overcome by implementing what is commonly known as the power system stabilizers. During this semester we will try to understand, the development of mathematical model of the power system, the mathematical model includes the mathematical model for synchronous machine, excitation systems, voltage regulators, governors and loads. For having the correct assessment of the stability the models need to be accurately developed.

Earlier due to the computational difficulties, many assumptions were made and therefore the, the assessment through computation and the actual groundility there used to be lot of difference. Now, this course we will cover in about 40 hours the text for, textbook for this course is power system stability and control by Prabha Kundur. This is a very nicely written book. Over the years a number of books have been develop, written a lot of research work or research papers are available, from time to time i will refer to other books and research papers which are relevant to our study.

The first lesson on power system dynamics is, introduction to the power system stability problem. Today, we will address to these aspects, basic concepts and definitions classification to the power system stability rotor dynamics and swing equation and swing curve.

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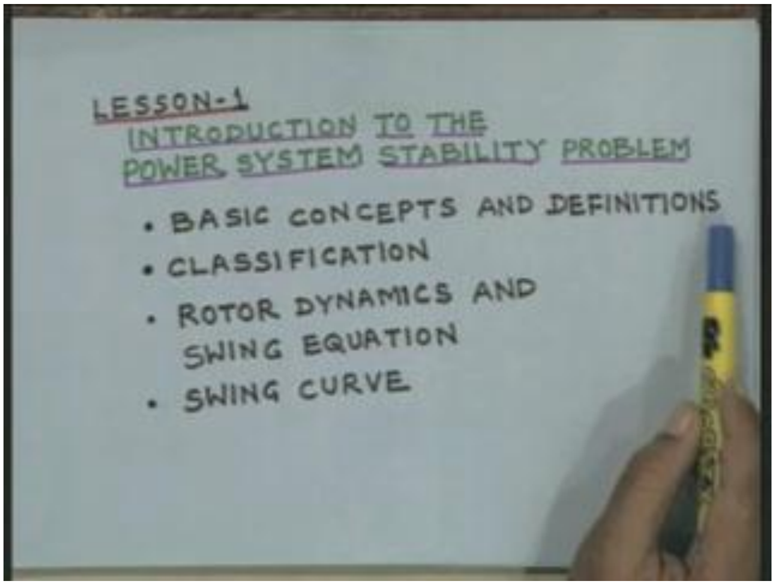


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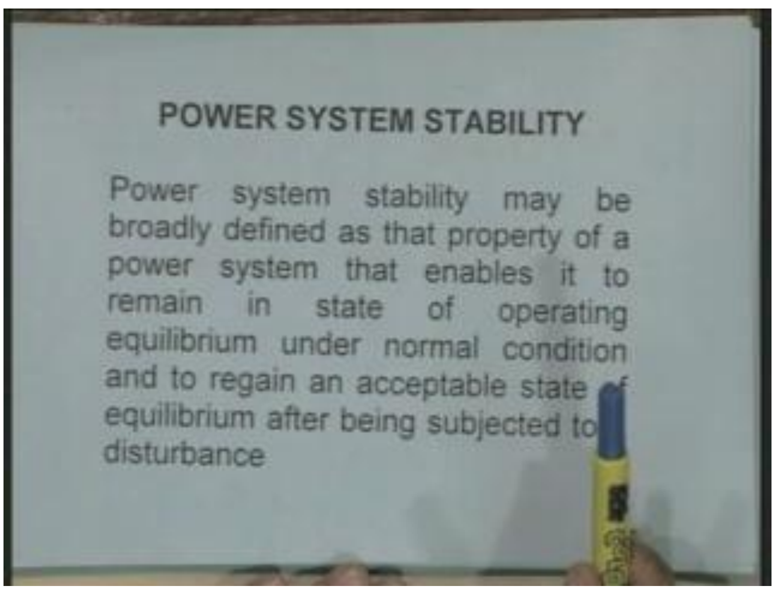


Now let us first define what is power system stability? Power system stability may be broadly defined as that property of a power system that enables it to remain in state of operating equilibrium under normal condition and to regain an acceptable state of equilibrium after being subjected to a disturbance.

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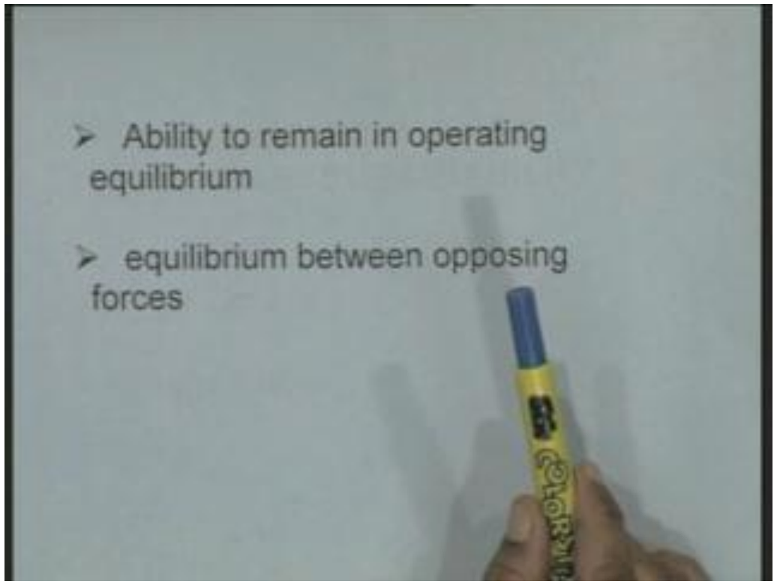


If you carefully look into this definition, you will find that one has to emphasize on ability to remain in operating equilibrium and second point we have to emphasize is the equilibrium between opposing forces. A power system is subjected to a variety of disturbances, it is never in steady state condition. Small disturbances in the form of load changes continuously come, while large perturbations in the form of faults, tripping of lines, change in large load and dropping of generators do come in the system.

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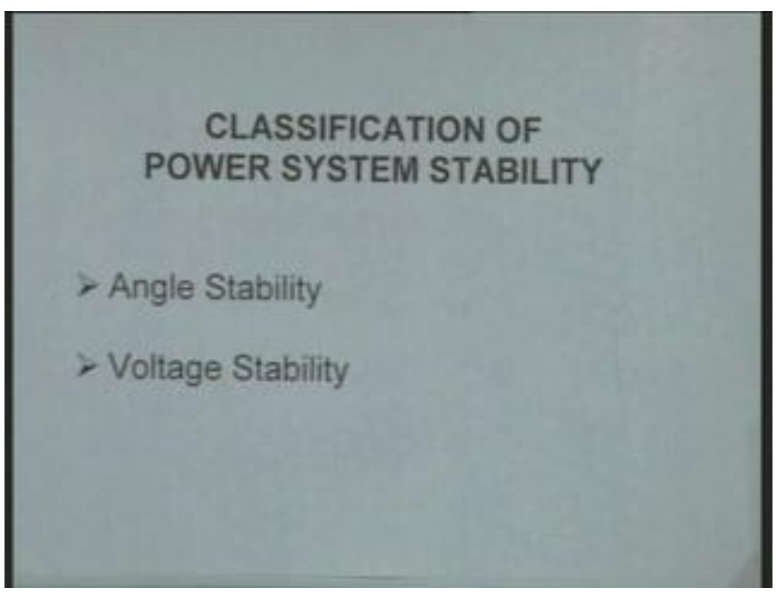

A power system is designed and operated in such a fashion. So that it can withstand certain probable contingencies. For the purpose of understanding this problem, the power system stability problem is classified into 2 broad categories, one is called voltage stability and second is called, I am sorry one is called angle stability and second is called voltage stability. Earlier years the stability means angle stability. During the last one decade the another type of stability that has come into picture is the voltage stability.

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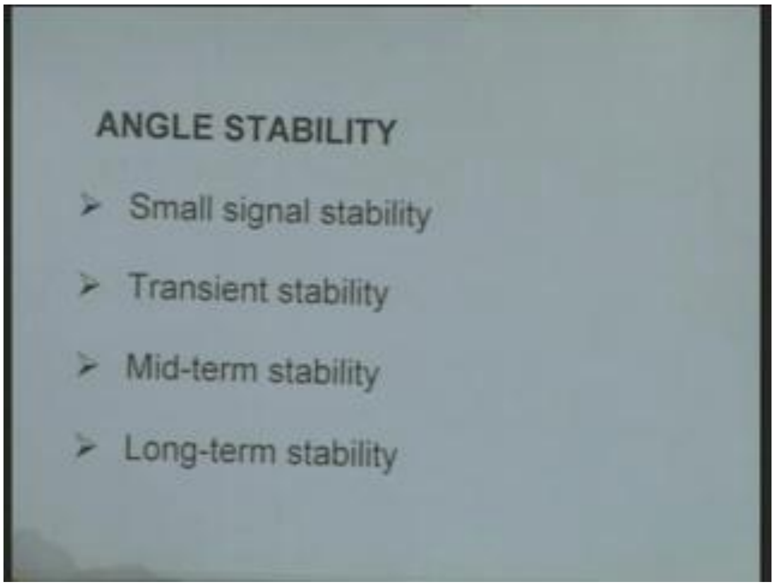


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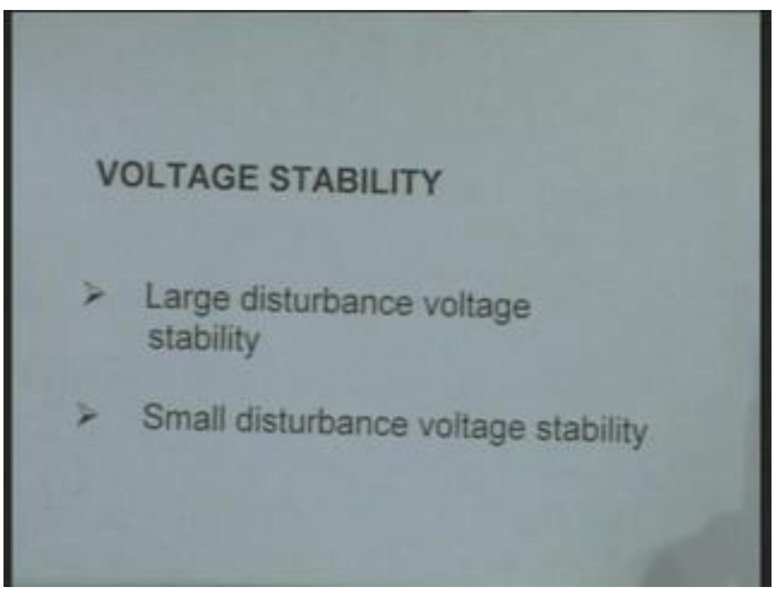

The angle stability is further classified into small signal stability, transient stability, mid-term stability and long-term stability. We will briefly define these different types of stability problems and try to understand what we mean by the different terminologies. Voltage stability is also defined into two broad categories, one is called larger disturbance voltage stability and second is small disturbance voltage stability.

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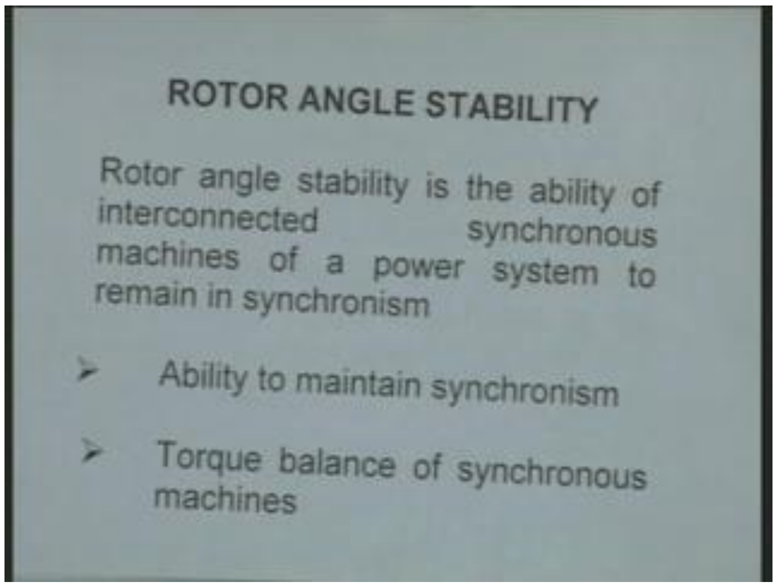


Now, let us define what is rotor angle stability? The rotor angle stability is the ability of the interconnected synchronous machines of a power system to remain in synchronism. Here the emphasis on ability to maintain synchronism. This is the primary requirement for the operation of a power system where, where all the machines of the system remain in synchronism.

Now for the system to remain in synchronism we have to study the torque balance of synchronous machines, that is synchronous machine is the primary component here, where we have to maintain equilibrium between the torque supplied by the prime over and the electromagnetic torque developed by the synchronous generator. To analyze this power system stability, we have to first understand the dynamics of the rotor and develop a mathematical equation to describe the dynamics of the rotor.

For developing the basic equation, we make use of the principle of dynamics, elementary principle of dynamics as for the rotational dynamics we all know that the accelerating torque is the product of moment of inertia and angular acceleration that is for any rotating body the accelerating torque is equal to the moment of inertia multiplied by angular acceleration and this is the fundamental law on which actually the swing equation is best. Now as you know that synchronous machine may operate as a synchronous generator or a synchronous motor.

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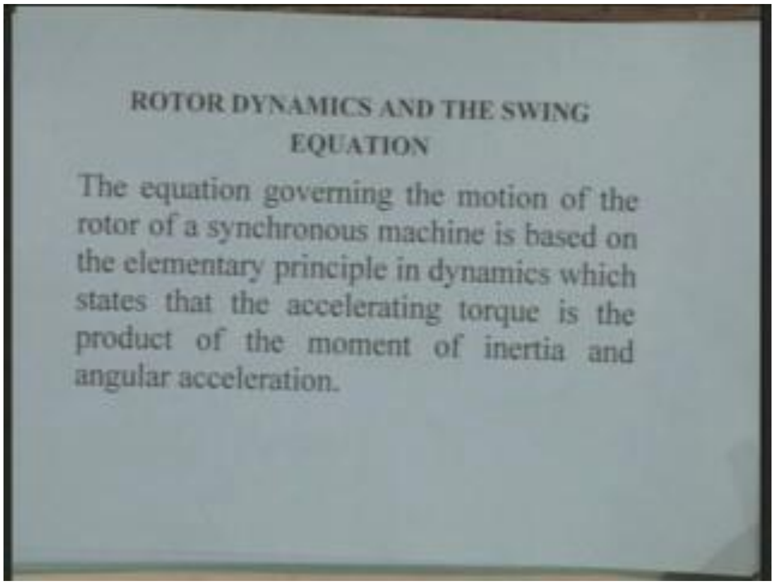


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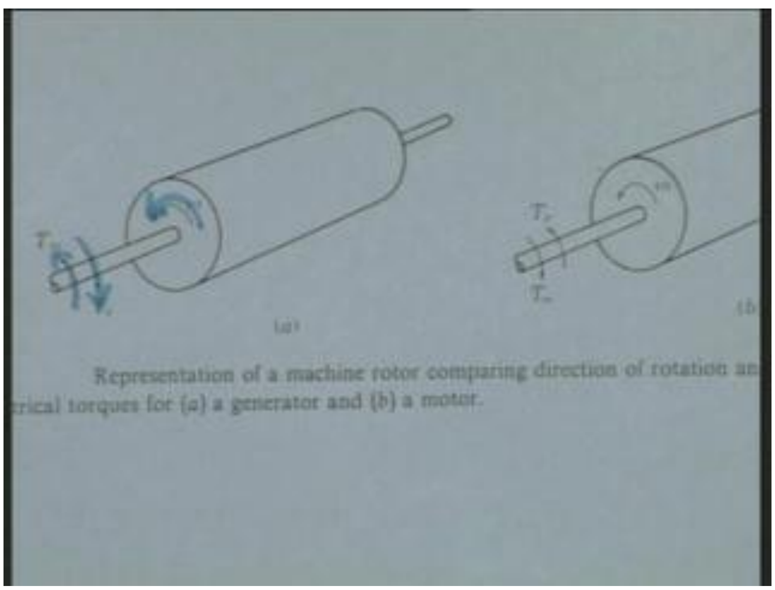


When we look at the rotor of the synchronous generator there are two torques which act on the synchronous generator rotor are, one is the mechanical torque which acts on the system and another is the electrical torque or we can call it electromagnetic torque which acts on the, these two torques operate or act on the rotor in opposite direction.

The mechanical torque is provided by the prime over and electrical torque is developed due to interaction of magnetic field and a stator currents. The rotor rotates in the direction of mechanical torque that is if you just look here, if I show the direction of rotation, it is in the same direction as the mechanical torque is applied to the system.

Under steady operating condition, these two torques are equal and the rotor of the synchronous machine rotates at synchronous speed. However, however when disturbance is occur, there exist a unequilibrium between the two torques, these two torques are not equal and this this difference is called accelerating torque.

When I look at the synchronous motor thus the driving torque is developed by the flow of electric power from the supply and it meets the load torque, therefore load torque becomes the breaking torque while the driving torque is the electrical torque and the rotor rotates in the direction of in which the electrical torque is developed.

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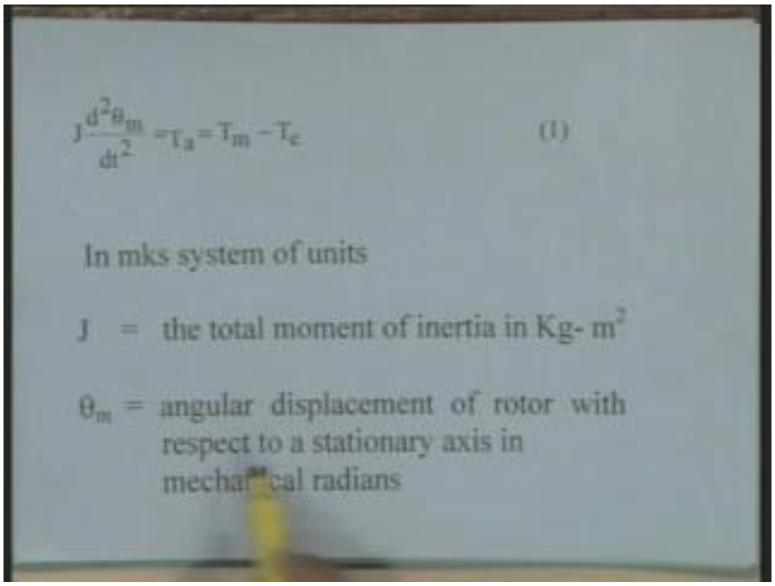


The swing equation or actually I will the swing equation is to be derived, a differential equation can be written relating the accelerating torque moment of inertia and acceleration. This can be written as J into d square theta m by dt square equal to $T_a$, where $T_a$ is difference of $T_m$ minus of $T_e$. $T_m$ is positive for generator operation and T is also positive for generator operation, while for motor operation they take negative signs. When they use mks system of units, J the total moment of inertia expressed is in kilogram meter square, theta m the angular displacement of rotor with respect to the stationary axis in mechanical radians. Now I am here emphasizing that in this equation theta m is measured with respect to a stationary axis

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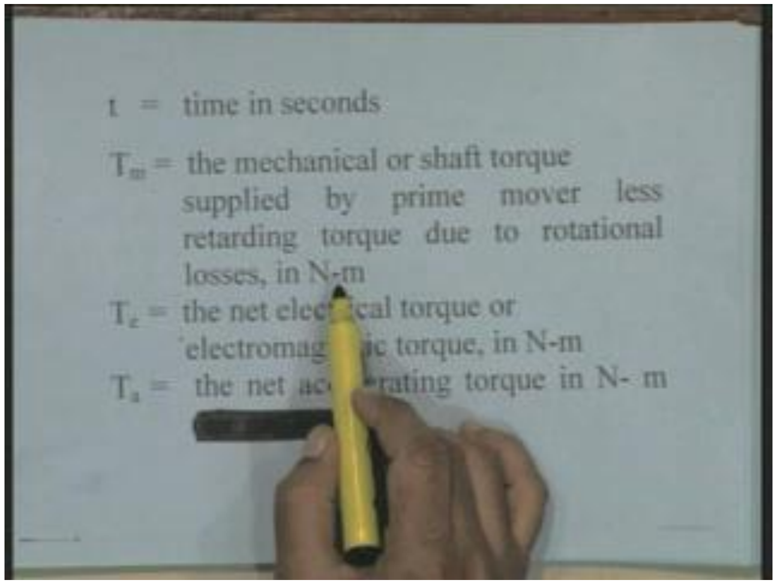


Time is measured in seconds. $T_m$ is the mechanical or shaft torque supplied by the prime over less retarding torque due to rotational losses in Newton meters, that the unit for the torque is Newton meters, this $T_m$ is torque supplied with the prime over to less rotational losses. It is the torque which is available for rotating the rotor. Similarly, $T_e$ is the net electrical torque or electromagnetic torque in Newton meters. $T_a$ is the net accelerating torque in Newton meters.

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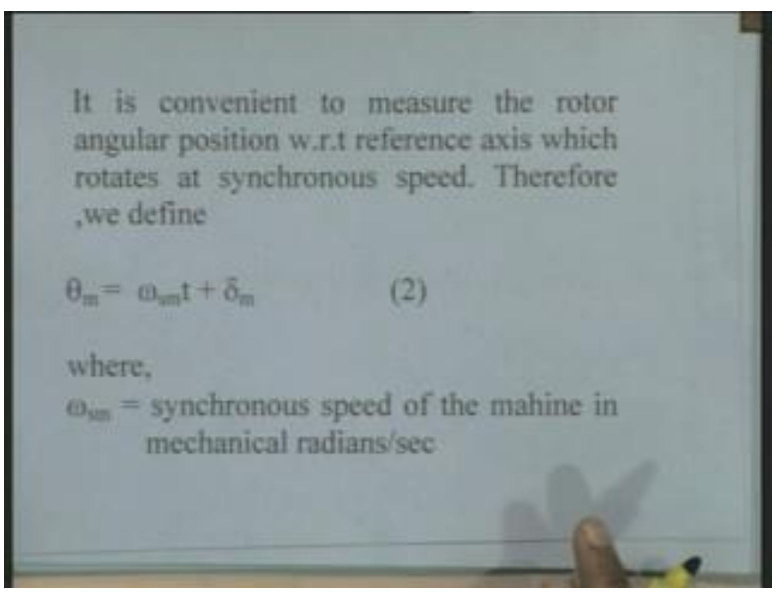

Now, if you see this equation then this theta m increases continuously even under steady state conditions because theta m is measured with respect to stationary axis, okay. Now instead of measuring the angle with respect to a stationary axis, the angle can be measured with respect to a synchronously rotating axis, therefore theta m can be defined as omega sm into t plus delta m, where omega sm is the synchronous speed of the machine. This is measured in radians per second.

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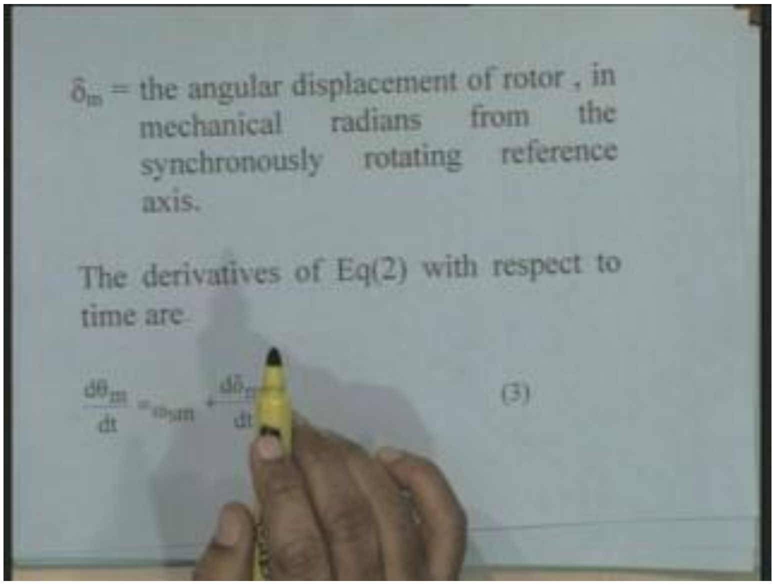


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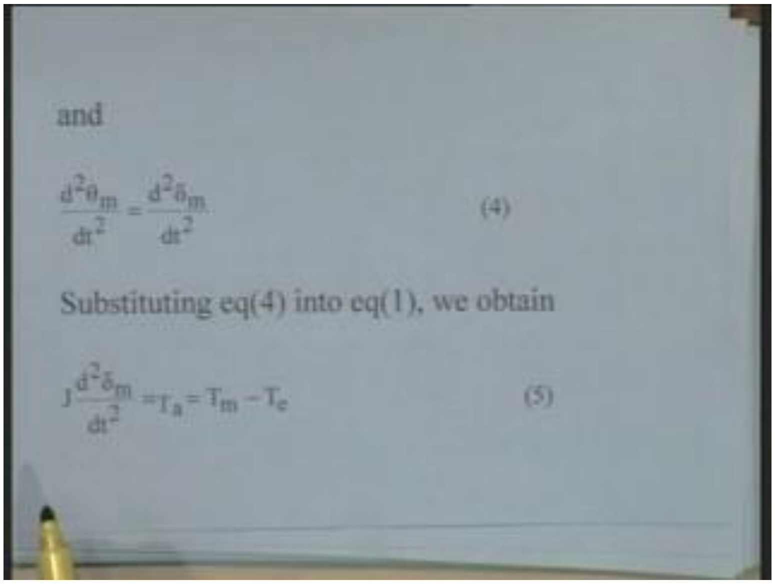

Delta m is angular displacement of the rotor in mechanical radians from the synchronously rotating reference axis. Now this equation two, when you when you take the derivatives of this equation with respect to time, the first derivative is written as d theta m by dt2 equal to omega sm plus d delta m by dt, that is we can see the rotor speed is equal to synchronous speed plus this additional torque.

Now we take the second derivative, $d^2$ theta m divided by dt square is equal to d square delta m by dt square, okay. Omega sm being constant its derivative is 0. Now we substitute the value of rotor acceleration in the equation 4, we get expression as J times d square delta m by dt square equal to $T_a$ $T_m$ minus $T_e$. Now at this point I want to emphasize that all the terms in this equation are torque terms in Newton meters. In power system studies, we are more comfortable with the terms in power, power may be watts, kilowatts, megawatts and therefore, what we do is we multiply this equation 5 by omega m that is d theta m by dt.

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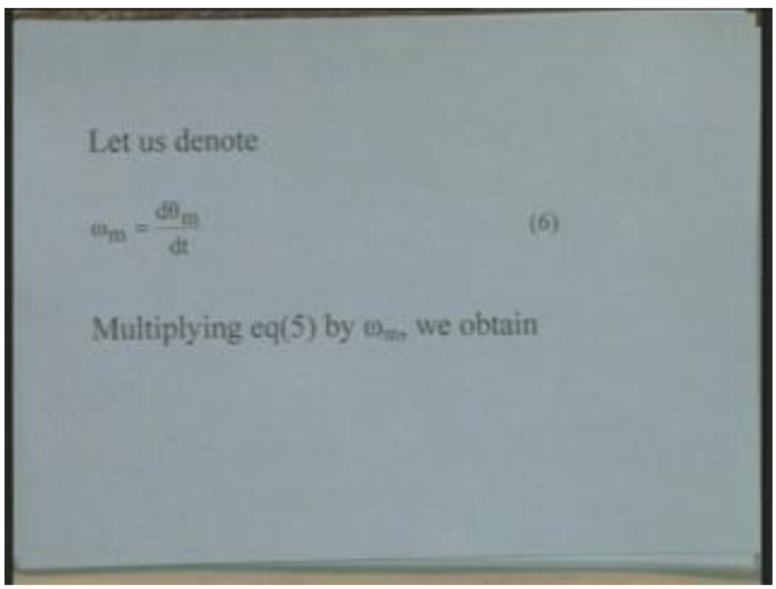


Now when we multiply this by this term omega m, our equation becomes J into omega m d square delta m by d square equal to omega m $T_m$ minus omega m $T_e$. Now here, this speed into torque, this is the power, the speed is measured in radians per second, power or torque is measured in Newton meters this product is power in watts.

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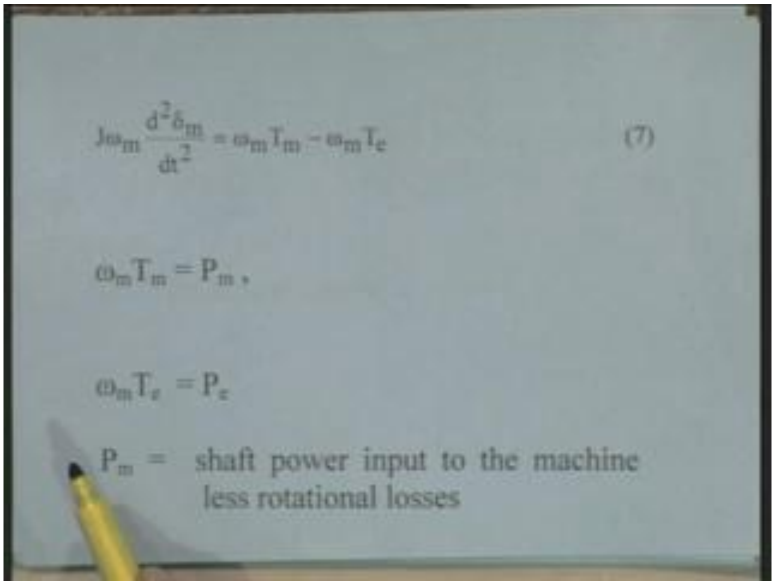


Now we will represent this term omega m $T_m$ as $P_m$, omega m $T_e$ as $P_e$, where $P_m$ is shaft power input to the machine less rotational losses. We are emphasizing all the time, the term rotational losses here. The meaning here is that this is not the torque supplied by the prime over but it is the prime over torque minus rotational losses.

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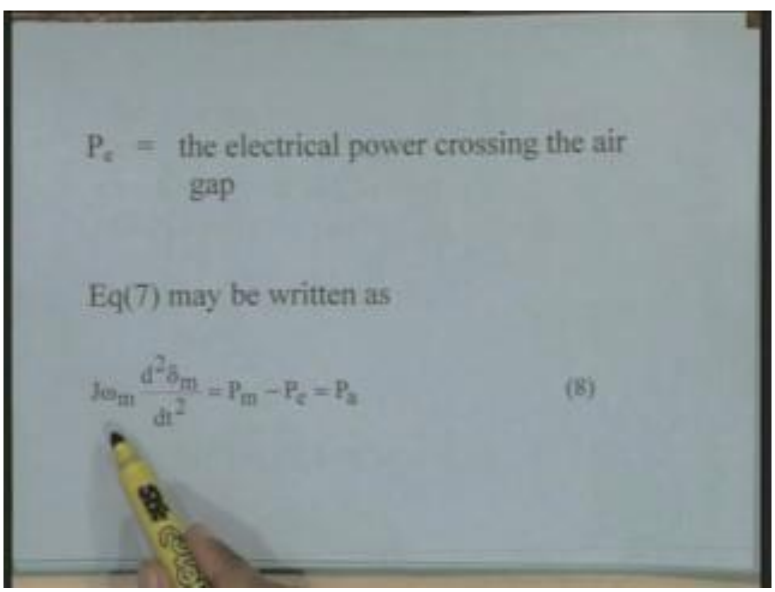


$P_e$ is the electrical power crossing the air gap also called air gap power. I mean in this equation 7, we substitute the expression for power and the equation become J into omega m d square delta m by dt square equal to $P_m$ minus $P_e$ equal to $P_a$. Now this term J into omega m, J is the moment of inertia in kilogram meter square, omega m is the speed in radians per second. Now this product is called angular momentum. Now in practical situations, the rotor speed omega m is very nearly equal to the synchronous speed.

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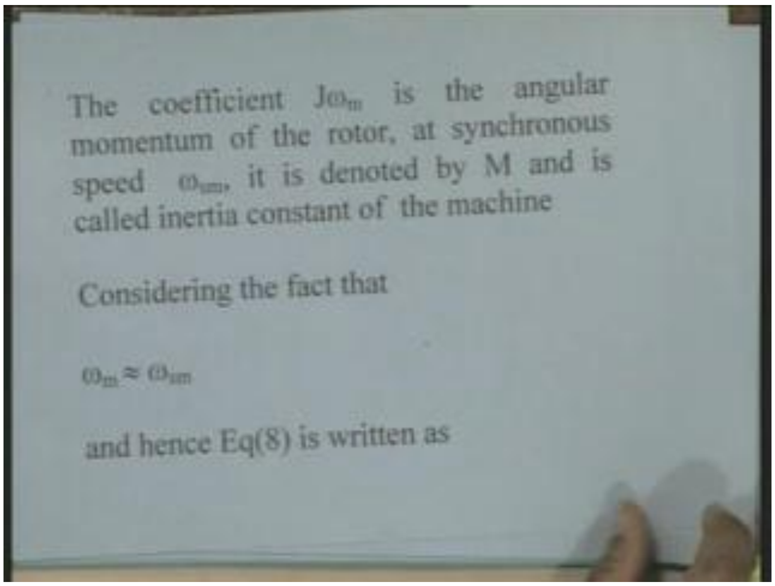


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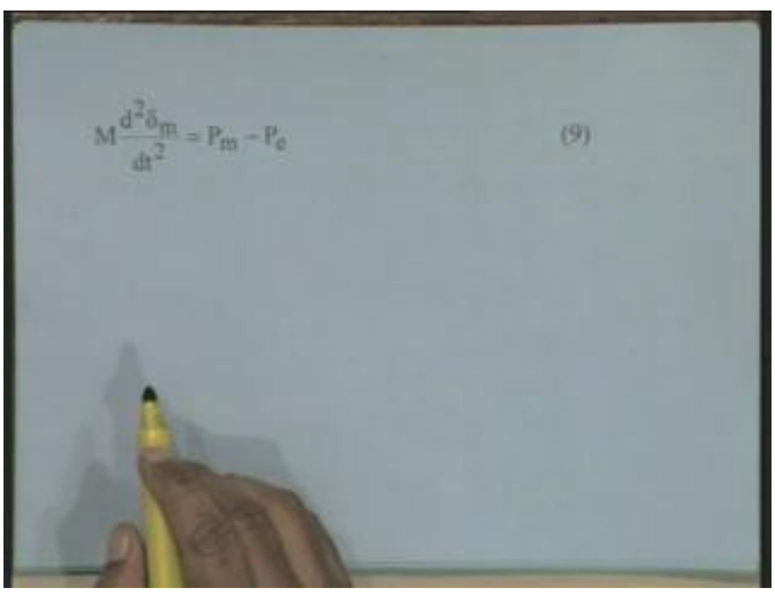


The difference is very small, the difference may become large only when machine loses synchronism right and therefore, what we do is for the purpose of simplicity, if it represents omega m in this equation by this omega sm, then this product of omega sm and moment of inertia J, it will be denoted as the inertia constant M and therefore my equation take the shape M d square delta m by dt square equal to $P_m$ minus $P_e$, where m is known as inertia constant.

Strictly speaking this coefficient should be J into omega m and since omega m is not constant therefore this coefficient term, strictly is not constant but by assuming making this assumption omega m is equal to omega sm, this coefficient become constant and it simplifies our analysis, further the error incurred by making this simplification is negligible. Okay therefore we are justified in assuming this coefficient of the acceleration term as constant and this is called inertia constant M.

Now at this stage, I would like to emphasize that this term M, varies over a wide range depending upon size of the machine, type of machine because we have two main types of synchronous generators hydro machines, hydro generators and turbo generators and these two machines have widely different value of inertia constant M and this inertia constant M will be different depending upon the size of the machine.

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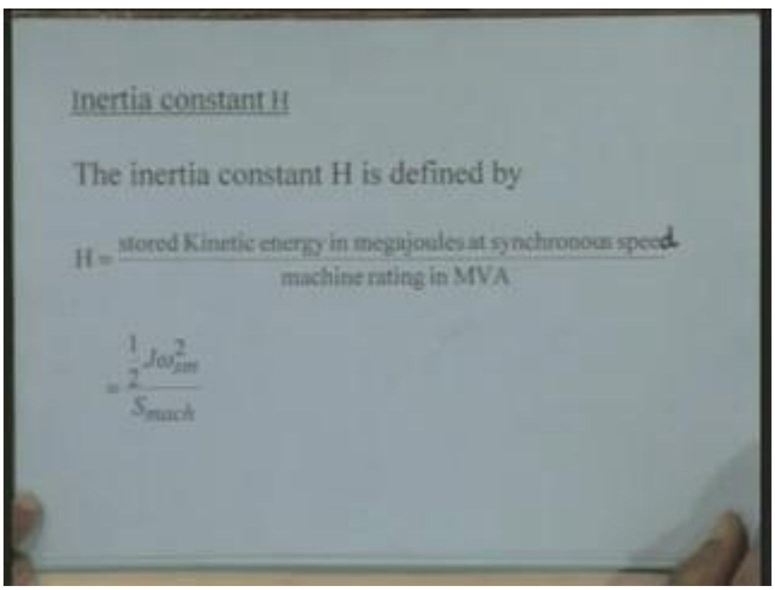


Now to overcome this problem, we define another term or another inertia constant which is denoted by the symbol H. You have to we have to very clearly understand the definition of H, the inertia constant H or the constant H is defined as the stored kinetic energy in mega joules at synchronous speed divided by machine rating in MVA. For any synchronous machine, when is when the rotor is rotating at synchronous speed, we can find out what is the kinetic energy stored in the rotor and we divide this kinetic energy stored in the rotor by machine rating. If we denote the kinetic energy stored in mega joules and machine rating in MVA, then the unit of this term becomes mega joules per MVA. We can also represent in kilo joules per kilo watt, per KVA not kilo watt, per KVA but still the machine ratings are normally in MVA, we prefer to use the rating in MVA and energy stored in mega joules, okay. Therefore, this can be written as 1 by 2J omega sm square upon the machine rating in MVA as mach.

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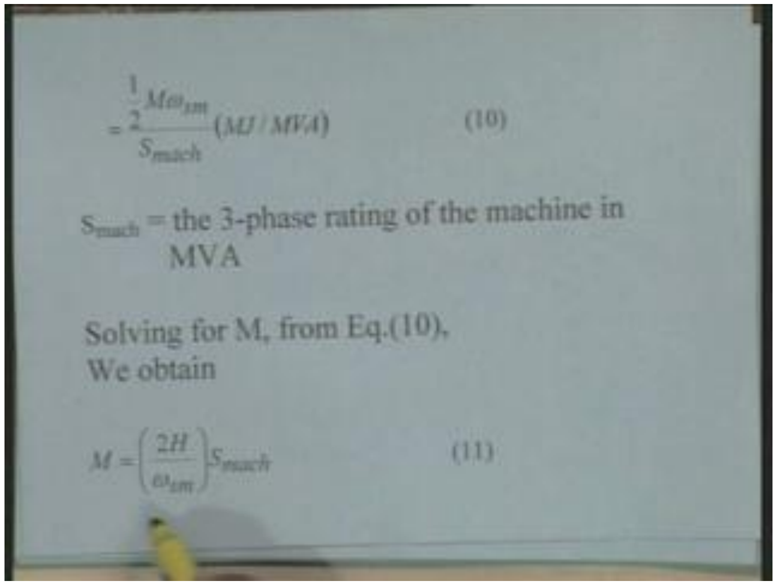


Now we will develop a relationship between the inertia constant M and constant H. So that the M which is there in our differential equation will be replaced by constant H, from this equation 10, we can write the inertia constant M as 2H upon omega sm into ah machine rating S mach.

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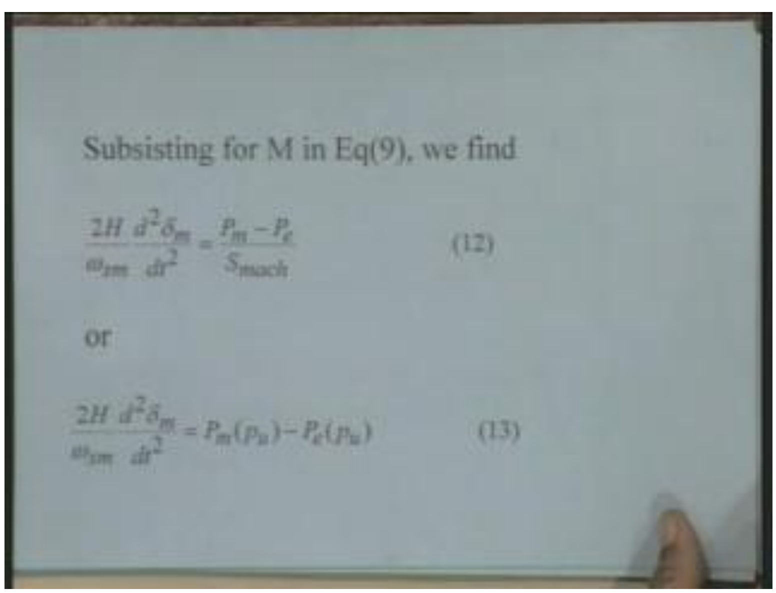

Now we substitute for inertia constant M in our equation, we find here that this becomes 2times H omega sm d square delta m upon dt square equal to $P_m$ minus $P_e$ upon of S mach. Now if we assume, MVA based for the system as the machine rating in MVA, then $P_m$ upon S mach becomes the per unit mechanical power, similarly $P_e$ divided by S mach becomes the per unit electrical power okay. Therefore, this equation can now be written in terms of power expressed in per unit, further as you all know that in power systems the per unit system of calculations are very convenient and therefore the equation is manipulated and written in terms of per unit power.

Therefore, the equation 13 here is written as 2H upon omega sm d square delta m divided dt square equal to $P_m$ in per unit minus $P_e$ in per unit. Now every time writing per unit is not very convenient, therefore what we do with that, we drop this nomenclature here but we keep in mind that power is expressed in per unit.

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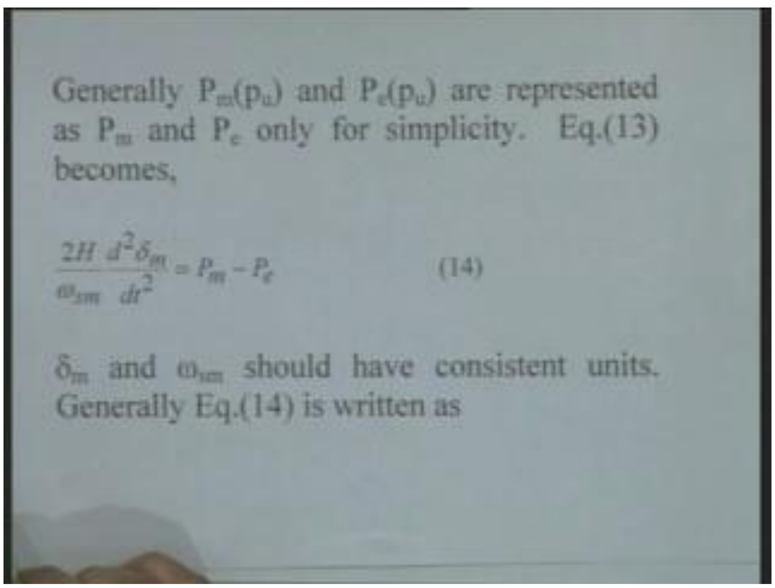


Therefore with this, we can write down equations describing the rotor dynamics of the synchronous machine as 2H upon omega sm d square delta m by dt square equal $P_m$ minus $P_e$. At this stage we have to very careful and understand very clearly that this $P_m$ and $P_e$ are expressed in per unit while expressing $P_m$ and $P_e$ in per unit, the MVA base is the machine rating in MVA. For any machine which I am writing this differential equation, if we assume the MVA base okay then the MVA base is the machine rating in MVA and therefore $P_m$ and $P_e$ are expressed in per unit in terms of MVA base of the machine.

Now here, this delta m is expressed in radians per second, I am sure radians not radians per second, this is the correction omega sm is expressed in radians per second. The units of these two terms should be consistent. We know a term known as angle in electrical radians, speed in electrical radians per second.

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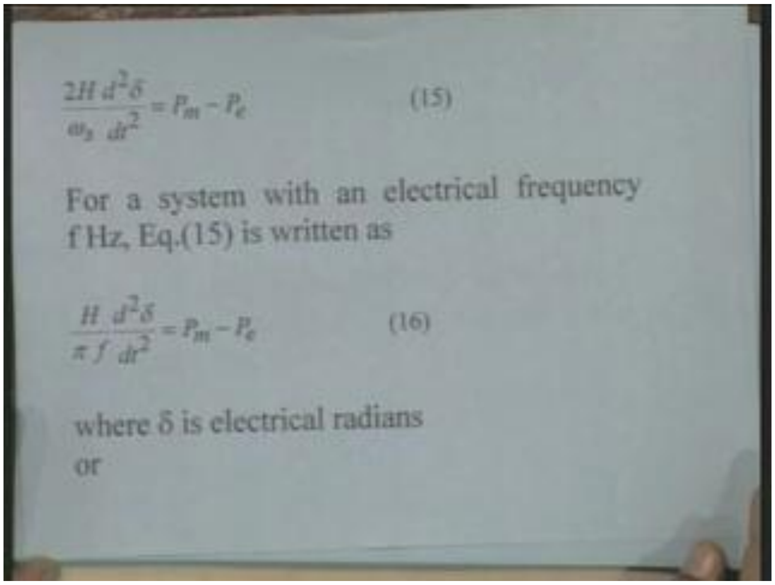


Therefore, if we use instead of the angle in mechanical radians and speed in mechanical radians per second, we can use delta in electrical radians and omega sm in electrical radians per second and therefore what we do is that keeping in mind that these 2 terms would have the consistent units, we drop the subscript m and we write the equation in the form 2 times H upon omega s equal to d square delta by dt square equal to $P_m$ minus $P_e$. In this is, this equation is known as the swing equation of the synchronous machine.

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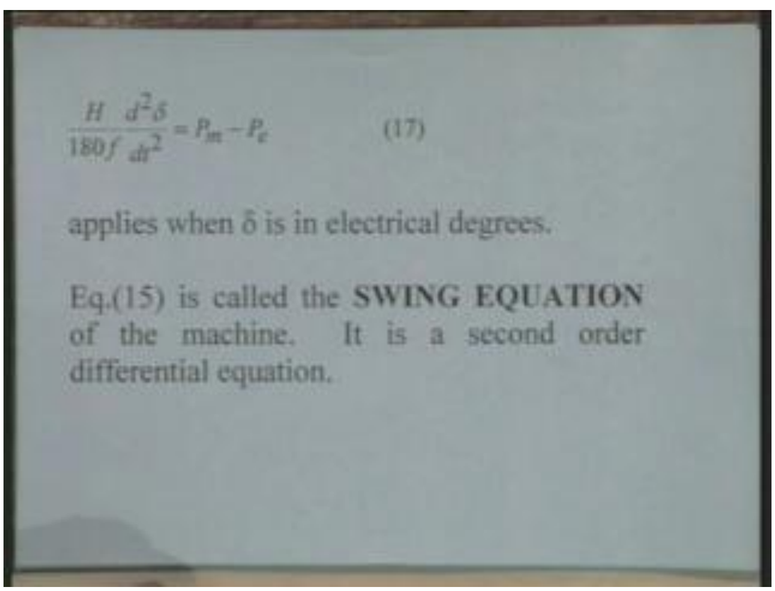

This is applicable to generator as well as to motor, only difference is when we write this equation for motor Pm becomes negative and Pe also become negative. So, that this term becomes Pe minus Pm. Now here, we can express delta in radians, omega s in radians per second. If we do in this manner, we can write the equation in the form H upon 2, H upon phi f that is omega s can be replaced by 2 phi f and you will have the equation in the form H upon phi f d square delta by dt square equal to $P_m$ minus $P_e$ okay, many time we express delta in electrical degrees and omega in electrical degrees per second.

In that case omega s will be replaced by 360 into f. So that the resulting equation will be in the form H upon 180 f d square delta by dt square equal to $P_m$ minus $P_e$. We do use swing equation either in this form or in the previous form but while using this equation, we have to very careful in expressing the delta in proper units. You can in this case delta is in the electrical degrees while in the equation 16 coefficient is H upon phi f delta is in electrical radians. As we will see here actually that, this is the basic swing equation on which or around which the stability analysis of the power system depends

Now we have 2 terms here $P_m$ minus $P_e$, this $P_e$ which is the electrical power output of the machine is, is non linearly related to or it is non-linear function of delta and therefore, this differential equation becomes a non-linear differential equation or we can say that the swing equation is a second order non-linear differential equation. Suppose, you have any system a number of machines number of synchronous machines then we have to write down swing equation of each machine while in this, we have to write down the correct expression for electrical power output.

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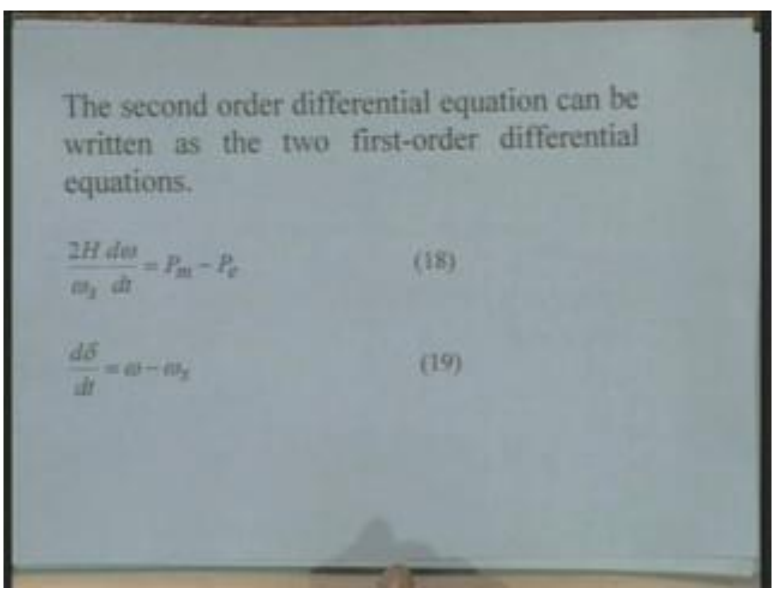


Now when this swing equation is solved, the solution of the swing equation is known as swing curve that is what we will get actually when we solve this equation, if you solve this equation you will get the delta as a function of time delta as function of time and therefore we can define this swing curve.

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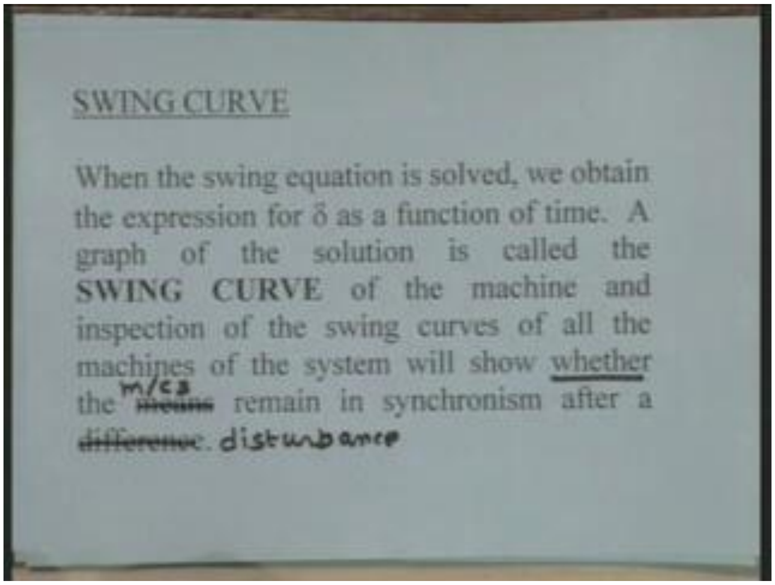


When the swing equation is solved we obtain the expression for delta as a function of time. A graph of the solution is called swing curve of the machine and inspection of the swing curves of all the machines of the system will show whether, whether the machines this is the wrong mistake, I have whether the machines remains in synchronism after a disturbance.

I am writing here whether the machines remain in synchronism after a disturbance again this is a big mistake here after a disturbance. For the purpose of solving the second order differential equation, using numerical techniques or standard practice is to represent the second order differential equation in terms of 2 first-order differential equations because when we apply the numerical techniques for solving the differential equations using digital computers, it handles first-order differential equations.

We can write down the second order differential equation as 2 first-order differential equations by defining by defining a term omega but the omega is the actual speed of the machine and therefore we can define these two differential equations in the form, 2H upon omega s, d omega by dt that is you are having the d square delta by dt square therefore omega is defined as d delta by dt okay. So that now here, we have one variable omega, then the second equation becomes d delta by dt whose expression comes out to be omega minus omega s okay.

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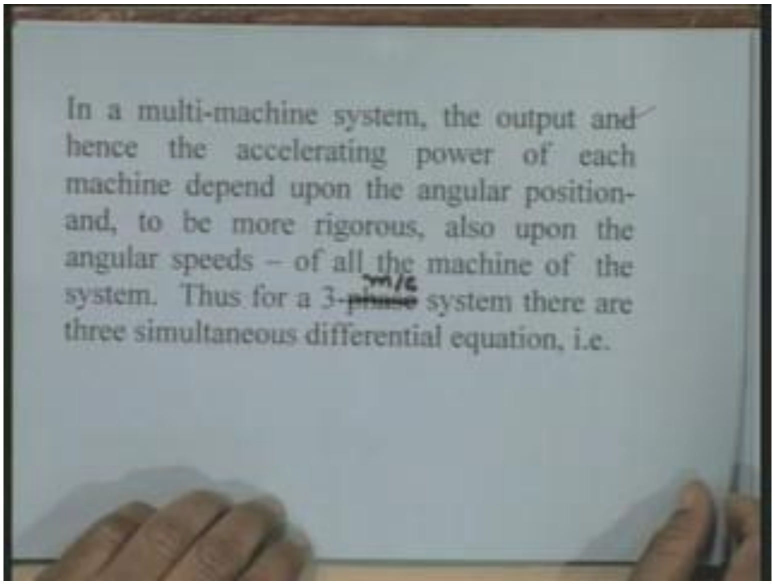


Now suppose you have a practical system where you have more than one machine that is called a multi-machine system. In a multi-machine system, the output and hence the accelerating power of each machine depend upon the angular position and, to the more rigorous also upon the angular speeds of all the machines of the system. Thus for a 3- phase system, this is written for mistake here again. Thus for a 3 machine system, for a 3 machine system, there are 3 simultaneous differential equations. The equations will look like this.

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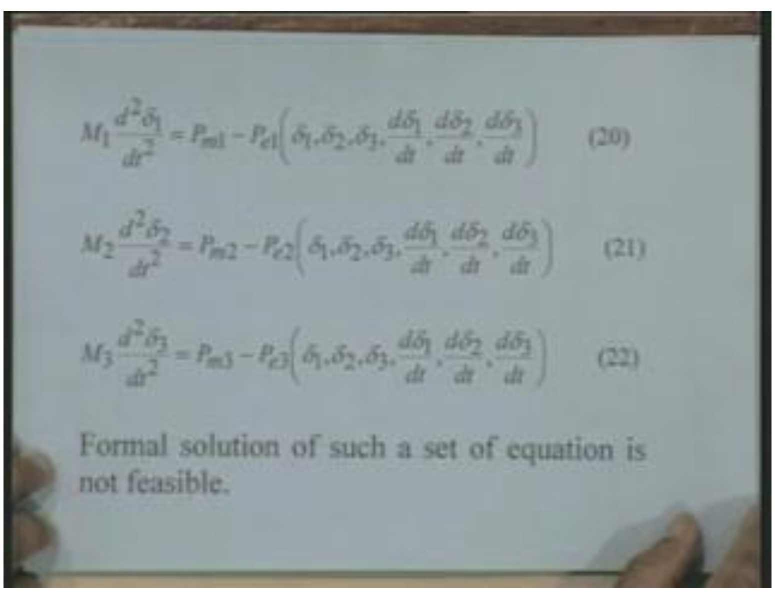

$M_1$ $d^2$ delta 1 by $dt^2$ equal to $P_{m1}$ minus $P_{e1}$ which is function of delta 1, delta 2, delta 3and it is also function of d delta 1 by dt, d delta 2 by dt, d delta 3 by dt. I will tell actually why this $P_{e1}$ is function of this derivative terms also the, for the second machine we have to put inertia constant of that machine and the electrical output of this machine is function of all these angles and these derivatives.

Similarly, for the third machine, we have $P_{e3}$ function of all these derivatives. Now since the system, we are considering here is in dynamic condition. Okay when system rotor is in dynamic condition, it develops some damping torque and this damping torque is proportional to the speed deviation. Then, let us correct it is proportional to the speed or speed deviation with respect to the synchronous speed. Now this d delta 1 by dt, d delta 2 by dt, d delta 3 by dt, these are the rotor speeds with respect to the synchronously rotating reference speed.

Now to simplify our stability analysis, we ignore this damping terms. Once we ignore this damping terms, the electrical output will exclusively be function of these angles only okay. Now I would like to tell something more about the electrical power output.

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The important characteristic that has a strong bearing on power system stability is the relationship between interchanges power and the position of rotors of the synchronous machines, this relationship is highly non-linear. I have told you here the electrical power $P_e$ right is function of rotor angels and it is non-linear. Now if I take a simple example to illustrate this, let us consider this case, we have 2 machines connected by a transmission line.

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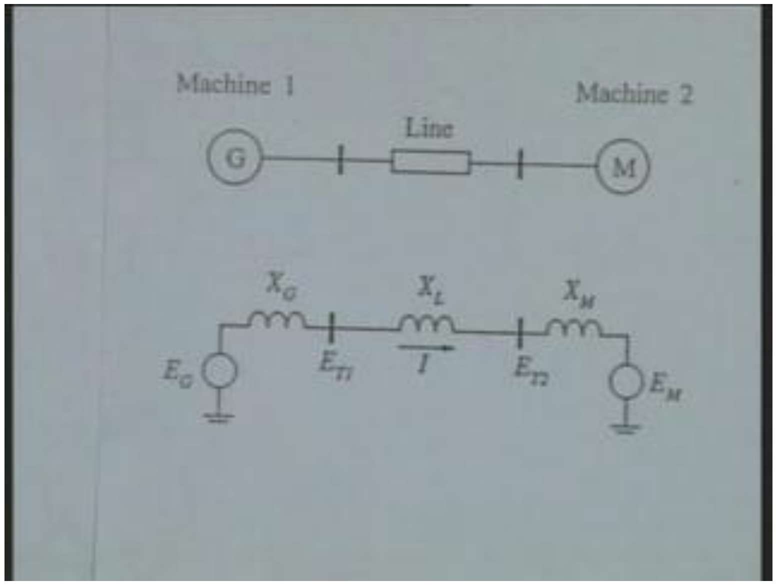


We can represent this machine 1 by a voltage source in series with a reactance. Similarly, the machine 2 can be represented by a voltage source in series with a reactance. The machine 1 is supplying power to machine 2. This can be understood as machine 1 is a synchronous generator machine 2 is asynchronous motor. For this simple system, if we develop an expression for electrical power output from machine 1, then this can be derived by writing or by first drawing a phasor diagram.

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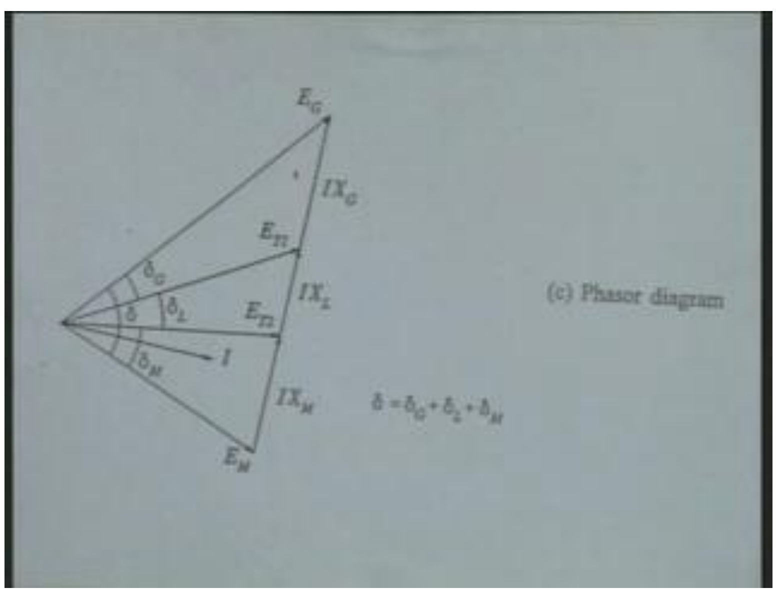

The phasor diagram can be simply drawn in this fashion. We can start with the terminal voltage of the machine, synchronous machine 2. Let I is the current, the internal voltage of the synchronous motor is represented by this phasor $E_m$, to this $E_m$ we add this voltage drops, where the voltage drops which take place in the internal reactance of the motor 9 reactance and generator reactance to get the internal voltage.

Therefore, this phasogram diagram shows that $E_m$ to this $E_m$, if we add this voltage drop I into XM, we get the voltage ET 2, to this we add the voltage drop I into XL, we get the voltage ET 1 and to this we add the voltage drop I into XG, we get the internal voltage of the synchronous generator. Delta which is the phase difference between EM and EG is called power angle and this delta is sum of all these three angles delta G delta L and delta M.

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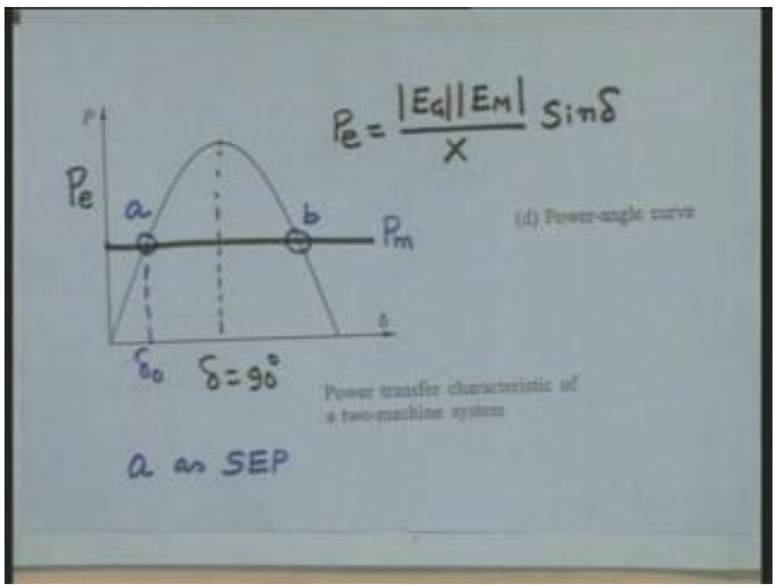


If we plot, a graph relating power angle delta and power P or electrical power $P_e$, we call this as $P_e$, then this graph comes out to be a sine curve, the expression for $P_e$ is the internal voltage of the generator, the magnitude of this voltage, internal voltage of the motor EM divided by the total reactance X into sine. Now, if we see here as the delta increases, the power output increases and becomes maximum at delta equal to 90 degrees, on the same diagram if I draw the mechanical input line, the mechanical input is not function of delta and therefore it comes out to be a line parallel to delta axis okay.

In this diagram, we will write this as $P_m$ mechanical power, now this mechanical power line and this power angle characteristic intersect at 2 points, this is 1 point I call it “a”, this is another point we call it “b”, “a” is stable equilibrium point while “b” is unstable equilibrium point that is we will denote this “a” as stable equilibrium point normally called SEP and “b” is unstable equilibrium point.

The system, if it is operating at this point “a” and if it is perturb then it develops the forces so as so that it returns back to this operating point “a”. However if the system is made to operate which is at the point b which is also the equilibrium point but if suppose some perturbation is given then the system will lose stability. It cannot come back it cannot rest develop restoring forces to come back to the positing point “b” and therefore, our stable operating point is a and we shall represent this operating angle as delta that is when the system is under steady conditions the mechanical power is equal to electrical power and operating angle delta naught.

Now I conclude here, what we have learn in this lecture. I have tried to give you the basic classification of the power system stability. We have defined the stability involved terms, we have developed swing equation of the machine and we have also defined a very important term inertia constant H. We will continue further in the next lecture.