

Power System Generation, Transmission and Distribution

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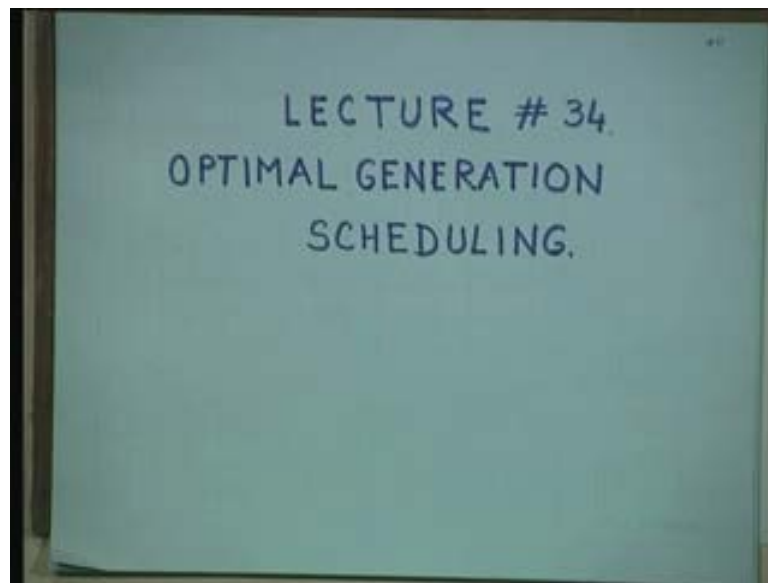
Centre for Energy Studies

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Lecture No. #34

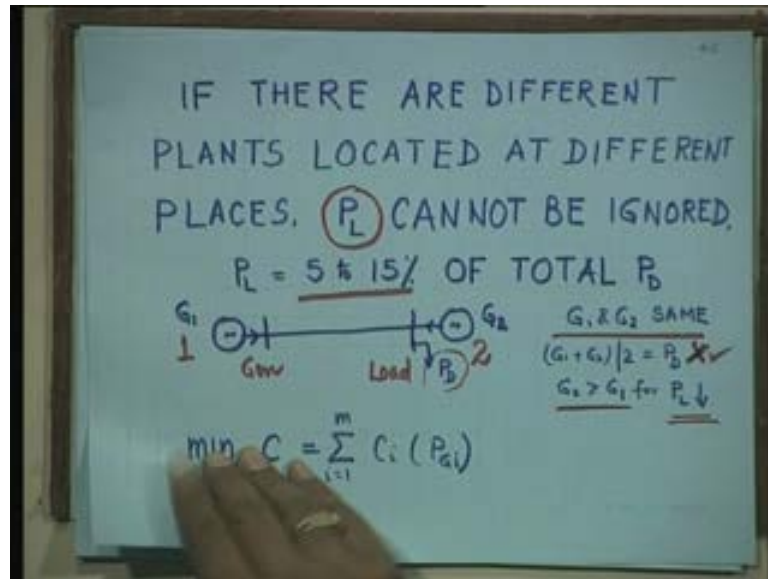
Optimal Generation Scheduling

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Today, we are starting lecture thirty-four on optimal generation scheduling, this is different than the earlier one, in the sense today we will be considering losses as well. Last lecture, of course, was unit commitment and prior to that was again generation scheduling, but without losses, and we saw in that lecture, that if losses are not there, that means, all the units are at the same station. So, no question of any loss, how they will share the load? The condition was equal incremental fuel cost, that means, $\frac{dC_i}{dP_{G_i}}$ should be equal to λ for all i , that was the condition.

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Now, today, if there are different plants located at different places: one is in Haryana, one is in Himachal, one is in U. P., one is in Delhi, then there is no way you can assume that there will be no losses, because the power will be travelling over long distances. Naturally, anything that travels over long distances is bound to be loss, even if it is water, there is a seepage loss; there is oil, there are losses; so, anything. So, P L means transmission loss, this P L stands for transmission loss. Normally, in the world, P L is 5 to 15 percent of total load to be supplied or total power, which is, both are same. Ultimately the load has to be equated to generation, that is, the power balance equation. So, these losses cannot be more than 5 to 15 percent of load or total power generated. But in India, as you know, these losses are very heavy, even ignore those thefts etcetera, even technical losses are very heavy because of bad planning, bad material, everything bad. So, you have to share the losses also, while supplying the load, so that also becomes the part of load, as if you are supplying a load the losses is gone, I mean, that is not serving any useful purpose. However, it takes 7 generation, so as far as we are concerned, it is the load.

Just to give you a technical explanation, why we have to, we cannot apply the old criterion of equal increment of your cost, very simple example. I am taking generator number 1, generator number 2, let us assume they are same. By same, I mean identical, purchased the same day, the same shop, same manufacturing concern, everything same, that means, their fuel cost characteristics are also same. That means, if there is a load at

this bus, this is a generator bus and this is a load bus, if this load is P_d, as per our earlier criterion, both the plants, or both the units will share half-half equal incremental fuel cost; so, they will share half-half. But this is a wrong solution, why it is the wrong solution? This generator has to travel all the way to supply this load, whereas this generator is right close to the load. So, even a normal IQ, I am not talking high IQ people, normal IQ people or low IQ people will understand, it will be wise, it will be prudent to supply more power from G2 rather than G1 because G2 transmission loss is 0, it is right there and hence, G2 should be more than G1 for low losses, less losses. And hence, that earlier criterion cannot be applied, when losses are to be considered; then, which criterion should be applied, next question immediately any one of you should ask. Fine, I agree with you, that you do not have to apply the old criterion, but then what is the new criterion? The new criterion to find out the objective function still remains the same, where still after the fuel cost minimization. So, the cost function is or objective function is minimization of total cost, which consists of summation of all the cost, summation is on m; m is the total number of plants.

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The whiteboard contains the following handwritten mathematical derivations:

$$\sum_{i=1}^m P_{Gi} - \sum_{i=1}^n P_{Di} - P_L = 0 \quad \text{or} \quad \sum_{i=1}^m P_{Gi} - P_D - P_L = 0$$

CONTROL VARIABLES.

$$\mathcal{L} = \bar{C} = \sum_{i=1}^m C_i(P_{Gi}) - \lambda \left[\sum_{i=1}^m P_{Gi} - P_D - P_L \right]$$

$$P_L = f(P_{G1}, \dots, P_{Gm})$$

For optimum real power dispatch

$$\frac{\partial \bar{C}}{\partial P_{Gi}} = \frac{dC_i}{dP_{Gi}} - \lambda + \lambda \frac{\partial P_L}{\partial P_{Gi}} = 0 \quad \forall i$$

If $\frac{dC_i/dP_{Gi}}{1 - \partial P_L / \partial P_{Gi}} = \lambda$ or $(IC)_i L_i = \lambda$

$$L_i = 1 / (1 - \partial P_L / \partial P_{Gi})$$

EXACT COORDINATION EQ.

This is the equality construct. Summation of total power generation at m plants minus all load at all the places n is the load buses, number of load buses, number of nodes, number of points where load is there, is not necessary to have load at all the buses. Never be under impression, that generator cannot be at all the buses because it is the costly affair, it is normally 10 to 15 percent of total buses have generations attached to them. This we

have done in our load flow chapter. minus P_L , P_L is the losses. This is an additional term, which is coming vis a vis the last to last lecture, now this can be rearranged, since load is the constant thing. Now, here I am assuming that, very big assumption, load is known with complete certainty, I am assuming, that the loads are known, that is, load for casting is perfect and whatever load has been forecasted, the same way load comes. So, load is a certain variable, is a certain thing, though in practice, it is a random variable.

All of you must have read stochastic control or control system probability theory random variables. Truly speaking, load is a random variable because even God does not know what will be load tomorrow, if it all there is any God, it varies. You may forecast it with a mean value, with a standard deviation, Gaussian distribution or any other distribution. The beauty of Gaussian or normal distribution is the first 2 moments described it completely. I do not know how, how much your background is of probability theory or the populist book, whether you have random variables or stochastic processes, etcetera. There are so many things to learn in life, it is not possible for every one of you to learn everything. So, I can lump it to a constant value P_D , this is the justification for lumping this summation to a constant value because they are all known. So, now, this has become our new load demand equation or equality constraint.

So, now, defining Lagrangian. The \bar{C} could have also been written as Lagrangian $C + \lambda P_L$, the original objective function augmented by λ times the equality constraints. Luckily, there is only 1 equality constraint; there is no other equality constraint. So, one equality constraints, that means, only 1 λ is required, that is what is Kuhn-Trucker theorem, which is given in appendix E. So, I have requested earlier also to read that, I hope you read it, otherwise you should read it. Now, this λ is called the, several names given to it in literature, it is called, it is called a Lagrange multipliers, it is called adjoint variables, it is called co-state variable; control people will know it better, but whatever it is called, it is an unknown. So, you have to it, that is the bottom line and P_L has, this is a new entry to this gate. We, just up to this point, we could get away with writing P_L , now the time has come when you have to explicitly express P_L in terms of control variables.

And what are the control variable here? Only 1, P_G , such a beautiful control problem. You would have never had practical problem, where there is only 1 control variable, this is P_G , that is, power generation at a given plant i or at i th plant, the way you want to

write. So, the process had to be a function of generations; if there are low generations, no losses. Losses are there because there is a generation, water leaks because there is water in the pipe. If you stop water as the many column is no water, where is the losses, no losses. So, losses are there because the original substance is there.

Now, this function can be evaluated and in last 50 years people have tried to evaluate in different way. People have done B.Tech projects, M.Tech projects, even PhD on finding out modeling of these losses and minimizing and still work is going on, still people do M.Tech project, PhD project on lost minimization. But the original formula given by , who is considered to be father of economic operation, he has written series of 2 books, 1958 and 1959 published by John Wiley, they are still the bible, both of them are available in low price and they are available in library, if you have time kindly go through them.

Now, for optimum real power dispatch, the Kuhn-Trucker condition, K-T. These are, mind it, these are necessary conditions, not sufficient. Those of you have done optimal control or optimization techniques will know the difference between necessary conditions and sufficient conditions. For example, to pass a course, it is necessary to attend classes, it is not sufficient. Suppose, you do not study, do not go back and work, read book or read notes, you may not pass. So, it is only necessary, not sufficient. I will, just to give a practical, real life example, to be a Brahmin it is necessary to be a Hindu, not sufficient, why? All Hindus are not Brahmins, reverse may be true, all Brahmin are Hindus, but all Hindus. It is necessary to be a Hindu to be a Brahmin, but not sufficient. So, I hope you have understood the difference between necessary conditions and sufficient conditions. Your earlier teachers in optimal control or optimization might have explained to you, what is necessary condition and what is sufficient conditions; these Kuhn-Trucker conditions are necessary conditions for optimal to exist. So, for optimal real power dispatch, you differentiate the Lagrangian to control variable. Here is, there is only 1 control variable, so I differentiate it. This any B.Sc. student will find out, this derivation or even high school, $dC_i dP_G$ minus λ plus λ times $\frac{\partial P_L}{\partial P_{Gi}}$ is equal to 0 for all i, you simplify, you get this equation $dC_i dP_{Gi} 1$ minus $\frac{\partial P_L}{\partial P_{Gi}}$ is equal to λ . Any one or all of you are intelligent, that is my assumption and I am sure my assumption is not wrong, this is the only term, which was not their earlier on, this was their earlier. So, this is the new term and this one upon this

is called L_i , penalty factor, P_f or power factor, penalty factor. I can briefly write the numerator as IC, incremental cost of I into L_i , L_i is the penalty factor, should be equal to lambda. So, modification is this term and that is why, since it is exact, now this is called exact coordination equation.

So, if the question comes, derive exact coordination equation, that means, this derivation should come, not the earlier one. All this experience of 35 years, mine, suggests 25 percent students, tell, derive the old one and not this one. So, when I ask to derive exact coordination equation, this is this equation, which you have to derive, not the earlier one without help.

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Handwritten mathematical derivation on a whiteboard:

$\lambda \rightarrow \text{Rs/MWh}$

IFC criterion.

$\partial P_L / \partial P_{Gi} \rightarrow \text{INCREMENTAL TRANSM. LOSS (ITL)}$

$P_L = P^T B P$

$= [P_{G1}, \dots, P_{Gm}] \begin{bmatrix} B_{11} & \dots & B_{1m} \\ \vdots & \ddots & \vdots \\ B_{m1} & \dots & B_{mm} \end{bmatrix} \begin{bmatrix} P_{G1} \\ \vdots \\ P_{Gm} \end{bmatrix}$

$= \sum_{i=1}^m \sum_{j=1}^m P_{Gi} B_{ij} P_{Gj}$

$P_L = B_{11} P_{G1}^2 + 2B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$

SUBSTITUTING:

$b_i + 2C_i P_{Gi} + \lambda \sum_{j=1}^m 2B_{ij} P_{Gj} = \lambda$

The unit of lambda is rupees per megawatt hour. So, incremental fuel cost criterion, this new term has come, $\partial P_L / \partial P_{Gi}$. The people in literature, experts in power systems have called it incremental transmission loss. If there is a change in a generation by ΔP_G , what is the corresponding change in loss is called incremental transmission loss, so $\Delta P_L / \Delta P_G$. Now, we come, the time has come to give you the expression for P_L , we had talked about it little ago, but we have yet advanced any expression. Now, I am, I am writing my expression in 3 different forms, the 1st form is the matrix form $P^T B P$, P is a vector of power generation. If you want, you can even write P_{G1} up to P_{Gm} , B is the loss formula coefficient matrix, which is given here in expanded form P_{G1}^2 up to P_{Gm}^2 and this is a quadratic form. Again, in linear

algebra, those of you have studied linear algebra or control system, then quadratic form, if you want to write in the index form $\sum_{i=1}^m \sum_{j=1}^m P_{Gi} B_{ij} P_{Gj}$, if it is the 2 plant system, then the answer will be $B_{11} P_{G1}^2 + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$. This will be $P L$ for 2 plant system; that is the expansion of this. If it is 4, if I expand, may be whole sheet is required to write that particular equation, so these are the 3 different ways in which you can write. The derivation of this is given in all the books; I do not think that is necessary to go through that derivation. In fact, has given chapters just on to derive this particular equation, from there you can understand how important is this equation. Even after 50 years of that 1st enunciation, people prefer this formula rather than intermediate. Many papers have come, many formulas have come, all have been discarded and this is the only formula, which is most respected, most valued and most used. Substituting various values in that exact coordination equation result, we get this $B_{ii} P_{Gi} + 2 C_{ij} P_{Gj}$, this is $d C_{ij} P_{Gj}$. If it is the quadratic equation, you get this by differentiating, you get the linear equation incremental cost curve plus lambda times. If you differentiate $\frac{\partial P L}{\partial P G}$, what you will get from quadratic form $2 \sum_{j=1}^m B_{ij} P_{Gj} = \lambda$. Any difficulty in this equation?

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$$b_i + 2c_i P_{Gi} + 2\lambda B_{ii} P_{Gi} + \lambda \sum_{\substack{j=1 \\ j \neq i}}^m 2B_{ij} P_{Gj} = \lambda$$

$$(2B_{ii} + 2\lambda B_{ii}) P_{Gi} = -\lambda \sum_{\substack{j=1 \\ j \neq i}}^m 2B_{ij} P_{Gj} - b_i + \lambda$$

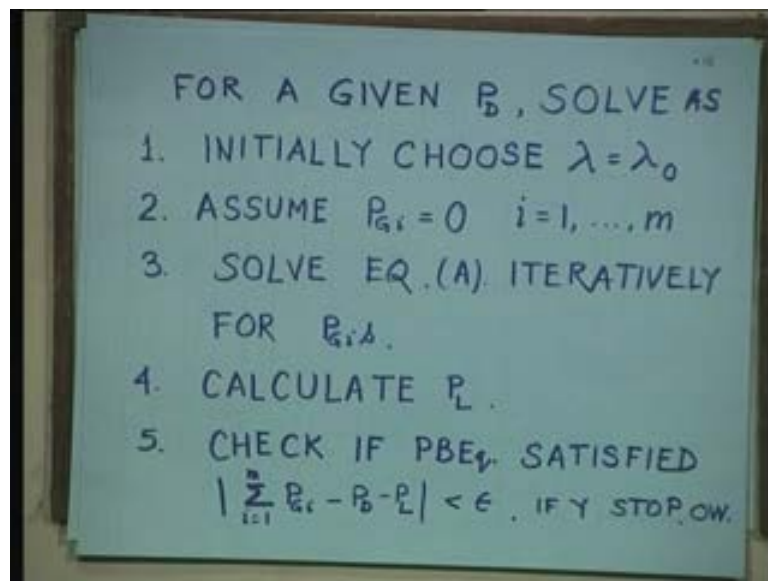
$$P_{Gi} = \frac{1 - \frac{b_i}{\lambda} - \sum_{\substack{j=1 \\ j \neq i}}^m 2B_{ij} P_{Gj}}{2\lambda B_{ii} + 2B_{ii}} \quad i=1, \dots, m, (A)$$

FOR ANY GIVEN VALUE OF λ ,
EQ. (A) CAN BE SOLVED ITERATIVELY
BY ASSUMING INITIAL VALUES OF P_{Gi}

Once you get this equation, then since I am interested in finding out the value of P_{Gi} , I separate out P_{Gi} from here. So, j not equal to i , rest terms are same. Then, I collect all the coefficient of P_{Gi} , which I normally do in deriving anything, rest of them are in right

hand side. So, $P G_i$ becomes this and this is CTM, CTM means commit to memory; this is the final derivation for $P G_i$. So, for a given value of lambda equation A can be solved iteratively by assuming initial value of $P G_i$. Why you have to assume? Because the $P G_i$ is both the places, so in order to, any iterative process, all of you know that you have to initiate iteration by assuming initial values. If you do not know anything, what you will initiate? Anybody? So, you have no knowledge what values to assume. So, what is the best value to assume? No, this is not the flat voltage start; that is the voltage. Let us assume, there are 0s in the ratio and it will converge it is like a Gauss Seidal. Once you know the value, start using them, see you are calculating $P G_4$, so up to $P G_3$ you have a values do not assume 0. There you just calculated $P G_1$, $P G_2$, $P G_3$, so use these values while calculating $P G_4$, $P G_5$, $P G_6$ etcetera. You can continue to assume 0, but not trivial values because you have updated values, that you know of course, that are, that what is Gauss Seidal.

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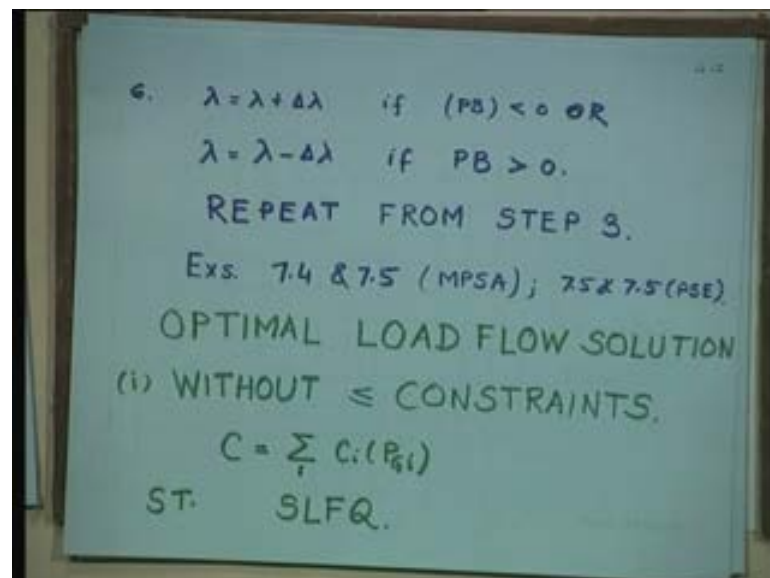
For a given value of demand, naturally you have to have a demand, then only you try to find out optimal values of various generations, solve the equation A by using the following algorithm. Initially, choose lambda is equal to lambda 0, lambda value you have to assume. Assume $P G_i$ is equal to 0, this I just now told you, assume $P G_i$ is equal to 0, for all i solve equation A iteratively; for $P G_i$'s calculate $P L$. Now, you know the formula for the $P L$. P coefficients will be given to you, if not given you can compute them, the formula is again given in the book, but as far as we are concerned, you will be

given the values of P_s as for this course is concerned, this exam is concerned, calculate P_L . Check if power-balance equation is satisfied.

I told you many times, that any optimization technique, 3 basic things are required: how do you start, how do you proceed, when do you stop. You have to stop somewhere, otherwise you go on and on and on, even in your exam you have to stop at 9 at the most, you continue up to... I have seen students even taking 5 minutes out of the exam period reading something. The best thing, the psychology say, you must stop reading 1 hour before the exam; have fresh, may be watch some TV or some walkman or chit-chat with your real friends, not the other ones who can tell you, that you are not going to get good marks today. So, demoralizing factor you should be away o.k. and then come slowly, walk towards exam hall, sit, think some good things in life and then start. So, here the power-balance equation is the thing where you can stop, see whether it is satisfied to.

The whole thing is then to supply power to a given load. So, sigma absolute value of $\sum P_{Gi} - P_d - P_L$ should be less than certain epsilon, which is again fixed by you, depends on what accuracy you want, 10 is to minus 4 point naught naught 1. Whatever, if yes, stop otherwise, otherwise continue.

(Refer Slide Time: 23:54)



How should you continue? The 6th step is very crucial; lambda should be now revised. If you recall the very 1st step, assume lambda equal to lambda 0. That means that lambda 0 is not good enough, so add to that by delta lambda. If power-balance is less than 0,

negative, it means generation is less load is more, or if it is otherwise, that is, generation is more and demand is less, then subtract λ . The new λ will be $\lambda - \Delta\lambda$, repeat from step 3. Can you give me 1 caution here? If you are caught, there is 1 danger in any iterative process; I will explain you, again by a real life example. Say, you have to go to Mehrauli, that is, Qutab Minar, you are starting from AIIMS. Now, if you, you are proceeding towards your objective, that is, Qutab Minar, see you have come up to IIT, you check, is it Qutab Minar? You, you stop your car and ask somebody, this is IIT, oh IIT, then what should I do go ahead? And suppose, he goes with that step i 's, that is, reaches DLF; you know DLF condos are there in Gurgaon? He has left Qutab Minar far behind, then I ask, suppose, again comes to IIT. What is this process? This is called oscillation; you are oscillating around the solution, which is the most dangerous thing that can happen in any iterative solution. Because then, there is no solution, then I keep on going, DLF and IIT, DLF and IIT, you never reach Mehrauli, so what should you do next time? If you have to add and subtract, do not do a $\Delta\lambda$, what will happen? You are back to the same λ . So, your program should take care of this $\Delta\lambda$ by 2, progressively reduce or increase the step size. If you keep the $\Delta\lambda$ same, one way you are adding $\Delta\lambda$, next time you are reducing $\Delta\lambda$. That means, you are still at λ , you still at IIT, you are not reaching Qutab Minar; this is the moral of the story. So, your teacher might have talked something, this should come from your general knowledge, your IQ. Example 7.4 and 7.5 from this book or 7.5 and 7.6 from other book, that power system, these are 2 good examples, you should try to solve them, they are already solved and how do you solve the solved example? By closing the book, just note down the problem and try to solve. The advantage is, you will come to know where you have gone wrong and other unsolved problem, if you do not get the answer, then you do not know where you are wrong. Then, if your friends solve independently, then it can perhaps, provided you are not making the same mistake again, there will be problem if you are making the same mistake. You may say the book is wrong and we are right, whatever answer I am getting. Sometimes, you may be right your book may be wrong, after all, Gods have not written books, human beings have written books and then, there are some proof readers and then, there are some illiterate people who are doing this job.

Now, we will come to the next article - optimal load flow solution. We have done load flow, but we have not done optimal load flow. So far whatever we have done, we are

solving the network assuming the losses, but some purist may say, this is no good, losses is after all, an approximate way of assuming by formula, de-coefficient formula. Some people call it B-constant also, but remember, they are not truly constant, they vary with operating condition. So, you may have to use the set of b coefficient and they may be reevaluated. Once there is a good change in the operating condition, I do not want to use drastic change; whenever there is a drastic change, naturally you have to change them. So, optimal load flow solution is one, where the load flow solution is optimal from some point of view, some criterion of goodness. Now, let us do with out inequality constraints to keep our life simple, we will consider inequality constraints later on, if time permits again. The objective function remains the same, C is equal to $\sum_i C_i P_i G_i$, subject to SLFE, static load flow equations, equation. We know, all load flow equations $P_i Q_i$ is equal to, you have done it in last chapter; incidentally, in exam we will have both the chapters, chapter 6 and 7. And you will get a paper from my portion of 13 marks, 1-3rd as usual, with the total paper will be of 40 marks, either there will be 1 big question of 13 marks or there will be 2 small, 6 and 7 marks. And you should prepare examples, you should prepare some derivations and some small questions as they normally ask, why do you want slack BUS; what is the significant of slack BUS; what are the conditions for the BUS to be eligible to become a slack BUS, like that, so that just to test your concept and understanding of the course.

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Handwritten notes on a piece of paper:

$$f(x,y) = \begin{bmatrix} P_i \\ Q_i \\ P_i \end{bmatrix} \text{ FOR EACH PQ BUS} \quad \text{PV BUS}$$

WHERE THE VECTOR OF DEP. VAR. IS

$$x = \begin{bmatrix} IV_i \\ \delta_i \\ \delta_i \end{bmatrix} \text{ FOR EACH PQ BUS} \quad \text{PV BUS}$$

VECTOR OF INDEPENDENT VAR IS

$$y = \begin{bmatrix} IV_i, \delta_i, \text{ Slack BUS} \\ P_i, Q_i, \text{ FOR EACH PQ BUS} \\ P_i, IV_i, \text{ PV BUS} \end{bmatrix} = \begin{bmatrix} U \\ P \end{bmatrix}$$

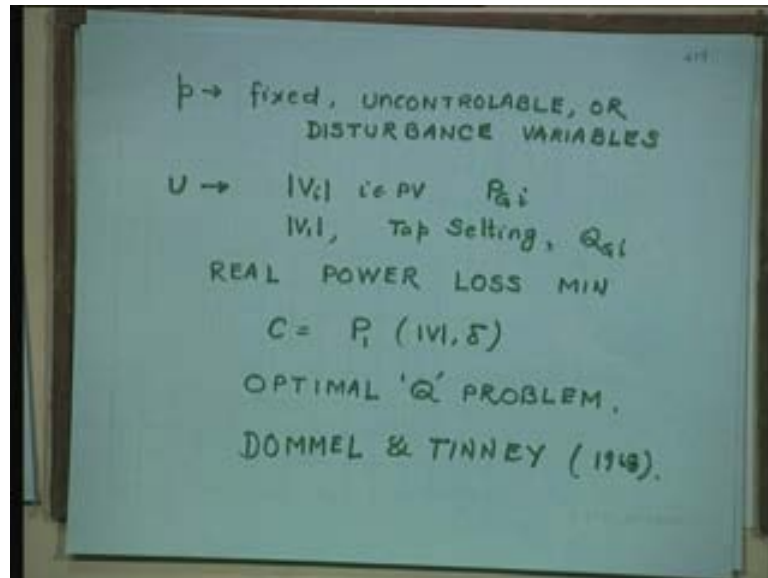
OBJ FN MUST INCLUDE P_{G_2} .

$f(x, y)$, that is a function of x and y ; what is x and what is y ? x is the state variable, y is the control variable, or at least y contains control variable; subset of y is a u . x is also called dependant variable or state variables, y is called control variable or independent variable. I think independent variable is a better term because y is only, the substitute of y is only u , the whole y is not a control variable. You may have, you can have it as a control variable, but cost, you cannot control everybody, where the vector of dependant variable is. Let us find out what is x ? So, $f(x, y)$ is $P_i Q_i$ for each PV BUS, sorry, PQ BUS and only P_i for PV BUS because Q_i is not known. So, does not pointing, I think equation for Q_i whereas for P Q BUS, both P_s and Q_s are there. So, what is x ? x is V_i magnitude and δ_i for each PQ. What is not known is dependent, they are dependent on what? What is known, everything follows from real life and δ is not known on PV BUS because voltage, so I cannot write it as that, something which is unknown, something which is dependant. Similarly, vector of independent variables is y . So, slack BUS V_1 and δ_1 are known, so they are independent. Similarly, $P_i Q_i$ are known at each PQ BUS, so they are independent variables. Similarly, P_i and V_i are known on each PV BUS, so that is again part of y . As I told you little while ago, that y , the whole y should not be considered as control variable for 2 reasons: a – there is no need; b - involves cost; anything, which you want to control, involves cost. It is like, in a locality if you are a superintendent of police, if you keep watch on only goondas that is enough. Those who are law-abiding citizens sleeping in their houses, why should you stand and go and whole night, they are sleeping; those who are on the road, those who are moving in their cars, going for bar or restaurant or whatever, you have to keep watch on them, that is all, and there will be law and order. That is why, only a portion of y should be considered as u , rest will be called P .

What is P ? P can be, truly speaking, any variable, which need not be controlled, as I said, or they are disturbances where control is not possible or tough, involves more money. So, P in literature is called either the constant, constant means you cannot control, you cannot change its value, control means you can change its behavior. Objective function must include $P G_1$, why? Suppose, you do not include $P G_1$ in this objective function C equal to $\sum C_i P G_i$, what will happen? Slack power, $P G_1$ is slack power, if you do not include that slack power, so many questions people ask, questions - why should we give slack power as a control variable? If you do not include, what will happen? The optimization process will try to allocate maximum power to the slack because you are

not paying for it, as if it is free, which will be wrong because here also, you need coal, you also need oil to produce power. So, P Gi must be included in objective function.

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So, here is the definition of P, which I have already told. P i is fixed, uncontrollable or disturbance variable, I told you all the 3 things. Some examples of control variables, out of y, which will be called u, what are they? Voltage, you can control voltage, you can vary voltage by various means, by excitation, by tabs, by SVS, by fact; you had a good dose of facts on those, last weekend this gentleman, 850 rupees. And P Gi, power generation is controllable by controlling the steam; you must have had a heavy dose of when Doctor Reddy is teaching, AGC would be far behind. So, AGC means, you have to have a, steam input can be controlled, watergate opening can be controlled. So, P Gi can be a control candidate for the control variable. Tap setting, tap setting is also a candidate for control variable. Either the, the, what you call on, load left on, load tap changing or phase shift, which can control the power, real power and these taps will change the voltage, and so the reactive power, that is, Q Gi, that is all gaining control variable. The capacitor, the value of capacitor, the location of capacitor, each changes the scenario in power system, that itself is the important topic in power system location of capacitor. Where do you want to locate facts, devices, UPFC, SVS and there are so many family members of facts, there are books written on facts, you must read them if you want to go into some power business like NTPC, Power Grid, BHEL, CA or abroad, Power Technologies and EPRI, electric power resources institution in U.S., very good, there

are so many good Indians there. Keep track of papers coming in the transactions, like to them that I am doing, they need cheap labor; pardon me I am saying that because Indians are, you know, for them, it is cheap labor. So, you can go abroad if you want to after your M. Tech.

Suppose, I am interested only in minimization of real power loss, to that it is too much in India, as I told you. So, my objective function now become not the cost, but slack power generation because it is slack power generation, which satisfies or which supplies majority of transmission losses. So, this problem is called optimal reactive power problem, reactive power minimization problem and this has been dealt in a paper by and this is a landmark paper, milestone. This paper must be read by you, 1968, IEEE transactions on power, approaches and system. Hopefully, this page number here is also 1968 if I recall properly and may be because 68 is already more than what, about 38 years, 37 years back, 2005 is round the corner. But in case you are doing a project in this, then you must read this paper. Have you finalized your projects, minor projects, major projects?

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Handwritten mathematical notes on a whiteboard:

$$\min_x C(x, u)$$

st. EQ. CONSTRAINT.
 $f(x, u, p) = 0$

$$\mathcal{L}(x, u, p) = C(x, u) + \lambda^T f(x, u, p)$$

↑
VECTOR OF LAGRANGE MULTIPLIERS

K.T. CONDS (NECESSARY CONDS)

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial C}{\partial x} + \left[\frac{\partial f}{\partial x} \right]^T \lambda = 0 \quad \rightarrow [J]$$

$$\frac{\partial \mathcal{L}}{\partial u} = \frac{\partial C}{\partial u} + \left[\frac{\partial f}{\partial u} \right]^T \lambda = 0 \quad \text{INVOLVED}$$

Again, the objective function is minimization of C of x u. Now, this is a vector subjective equality constraints; equality constraints are those 2 load flow equations as L f E f of x of u of P i removed y input u and P equal to 0, that is your equality constraint, inequality we have already ignored. We are going to consider it next time, time permitting because I

also want to do hydro-thermal, may be we will do in the power system analysis course, in that also I will have 1 hour only, Balu will take 2 hours, I will take 1 hour in the evening. Now, this lambda is no longer a variable, Lagrange variable or costate variable or adjoint variable. Now, what it is? It is a vector of Lagrange variable, as many load flow equations as many lambdas; that is why I have written lambda transpose. You cannot multiply 2 vectors just like that, they may not be compatible. So, the number of rows and number of columns should be equal, all of you know better than me the rules of vector algebra or matrix theory.

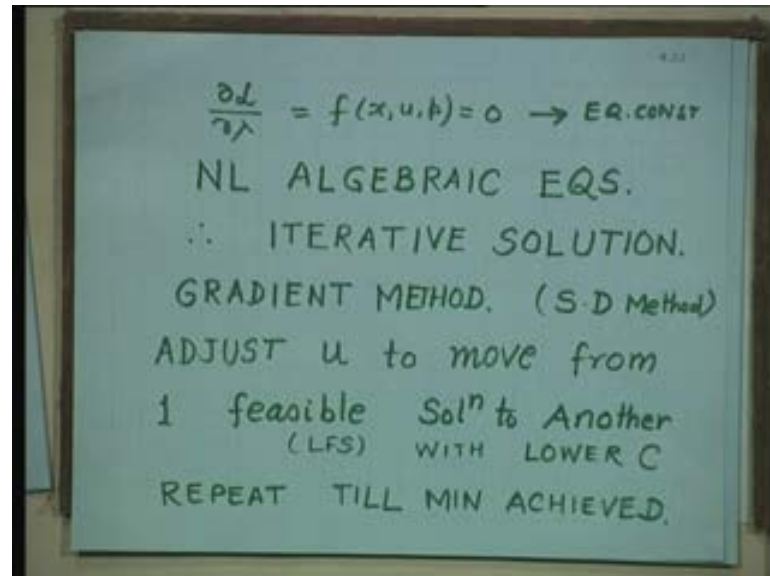
Again, condition will come into play, they are necessary conditions and what are the conditions? del Lagrangian by all 3 variables should be 0; it is no longer only control variable. delta L by delta x should be 0, delta L by delta u should be 0, delta L, L means Lagrange, I am, I do not want to say Lagrange, so I am saying L, by P should be 0. I am giving here the expanded version of delta Lagrange by delta x del C by del x plus del f by del x transpose into lambda is equal to 0, del L by del u is equal to delta C by del U Plus delta f by del u transpose into lambda is equal to 0.

The uncircled components of this equations are very involved, computation of these 2 components, the computation of this is very involved and book has, not book, paper, has spend 1 full page on each. So, in case, you are interested in, in case you get enthused to find out, why they are involved, please read that original paper provided you get in the library, because in olden days when we used to give such references, our students, who must be now in their 50s tear, they used to tear those pages. So, the 1st person who runs after the class, who is smart, lean and thin and not like me, who cannot run, he used to do this job, which is very bad. So, do not think the bad habits are come now only, they are there since time immemorial and so that journal may be there, but that paper would not be there. So, do not blame me, that paper is not there, maybe now, internet is there, downloaded it and that is the good thing. That internet has done the bad thing is, it has started producing reports: M.Tech thesis, PhD thesis by downloading. So, you do not understand what is there in the thesis because you have downloaded it, you have not worked yourself, anyway.

So, del f by del u is equally involved, but this particular fellow is nothing, but our old friend Jacobian, Jacobin you know, how to do that, because you have done it in load flow

chapter. This is $\frac{\partial L}{\partial x}$ by $\frac{\partial}{\partial x}$ is nothing, but Jacobian, that you know, this is very simple to, you will, you have already done it, $\frac{\partial C}{\partial P}$ Gi.

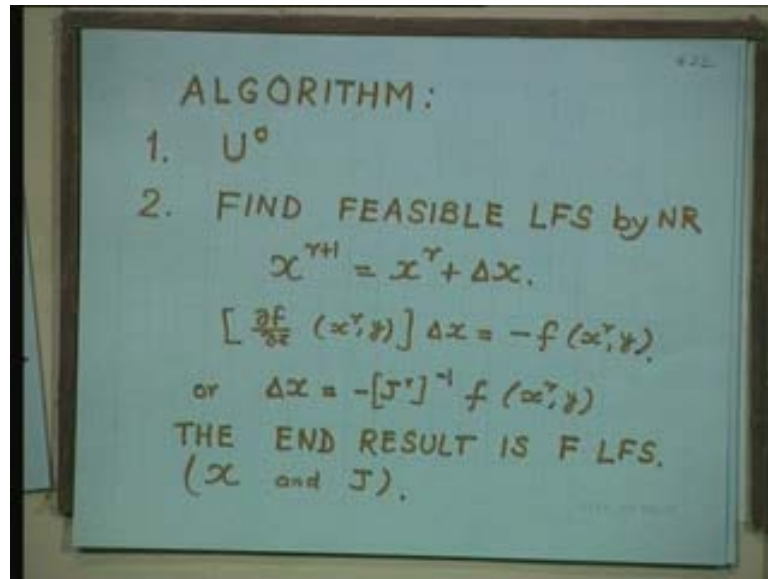
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Luckily, the 3rd equation result is same good old friend, the load flow equation $f(x, u, \lambda) = 0$ that you can immediately recognize as your equality constraints. So, does not give any extra new information, it is an old, old, old thing, non-linear. These all 3 are non-linear algebra equation, luckily there is no time involved here like stability, no differential equations. So, but again, iterative solution is required because they are non-linear, highly non-linear. We, since it is an optimization is no longer plane load flow, so we need to have an optimization technique. An optimization technique, which we use here is simplest possible called Gradient method or method. Again, we have to adjust initial u , u have to assume and then adjust u to move from one feasible solution to another.

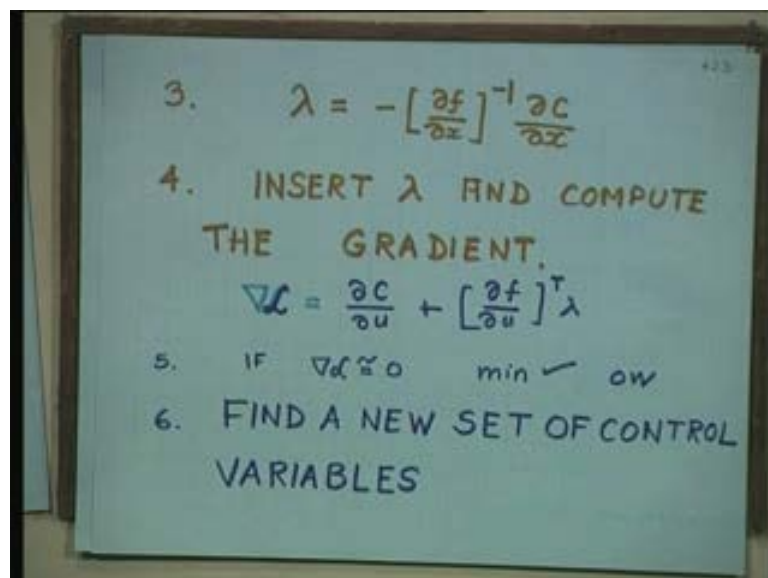
So, as long as the fellow is coming from Green Park to Hauz Khas to IIT and then Qutab Minar, he is converging. So, step size is very important in optimization. The correct direction is very important in optimization. So, but feasible solution should also be there, otherwise if you are going into infeasibility, you cannot achieve optimality. Even if you achieve optimality, serves no good. Any optimal solution, which, by which I can satisfy my load is not optimal, it can be anything, but optimal. Repeat till minimum is achieved, this is true in any optimization process.

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Let us consider the algorithm. Assume initial value of control vector; find feasible load flow solution by Newton-Rapson method. x^{r+1} is equal to $x^r + \Delta x$, this is nothing, but the load flow solution. The end result is feasible load flow solution, that way you get the value of x and the value of J . x is the dependant variable and J is the Jacobian, which you get in the load flow solution. This whole step is nothing but load flow solution. Find out lambda from one of those equations.

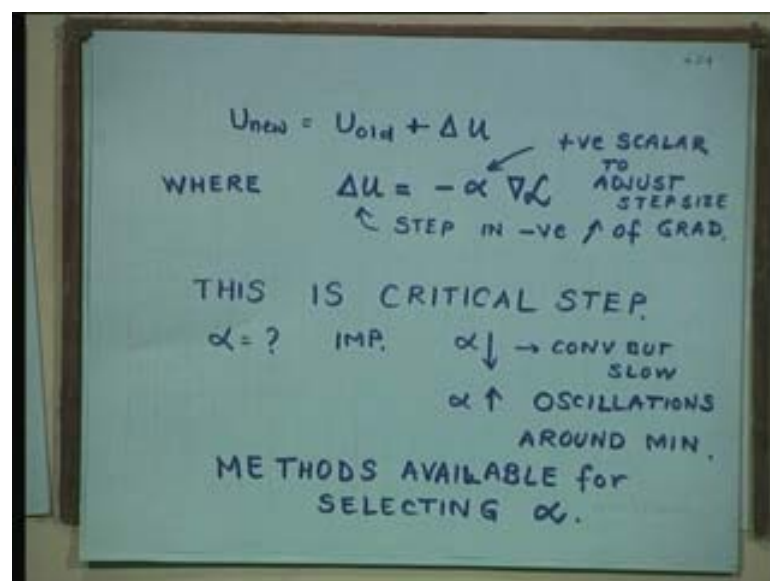
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This equation and this equation lambda converts, send this right hand side, so minus sign will come, take inverse of this, multiply it, you get lambda; is it alright? From here you get this, insert lambda in 2nd equation, 3rd equation is, has no use it gives me same good old equality constant. So, from 1st equation you find out lambda, substitute into 2nd equation to get gradient because the 2nd equation is nothing but a gradient. What is the gradient - 1st order derivative; of what – Lagrangian, respect to control variables. So, del, this is like your theory. I do not know how many of you have read Maxwell's equation, divergence is equal to 0 and all those things, so this is the del. So, del, del Lagrangian is nothing but del C by del u plus del f by del u transpose into lambda, and in this, gradient is 0 and nearly 0 within certain tolerance epsilon. Then, minimum is achieved, otherwise find a new set of control variables and repeat.

How do you find those new set of control variables? u_{new} is equal to u_{old} plus Δu ; you have to add something to the old to get the new or subtract 1 of the 2 things like we did with lambda. If you recall, where Δu is minus alpha times gradient. What is alpha? alpha is the positive scalar. Why minus? It is called technique because we are going for minima, if it is maxima then . If you have to go to Himalayas, there is an optimum path, which your Sherpas will tell you or your trainer will tell you to reach. Himalaya is an objective, how do you reach with minimum efforts and minimum time? Sometimes time is not important, then take a longer root, but with efforts, less; it all depends on what you want.

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So, Δu is the step in negative direction of gradient α ; by varying α you can vary the step. So, what is the critical step, what should be α ? Ultimately problem is postponed to deciding what should be your step size. Should you ask gentleman every 5 minutes or should you stop only once somewhere in between, but that one should not be DLF quarters, that is the real challenge in that comes from experience. If α is too low, convergence is low. Suppose, you start asking every feet, but convergence is guarantee, you will reach Qutab Minar. However, if you ask, α takes a long step size oscillation, DLF, so then what is the real thing? Well, there are again optimization techniques available to find out α . I do not think we should go in to that, but references are given in the literature and lot of people have worked to find out what should be the optimal size of this, what should be the optimum value of the α . Well, gentleman, today we are finished without inequality constraint, we will continue this problem with inequality constraints and inequality constraints are there in x as well as on u , both to be treated in different way. And then, hydro-thermal scheduling will be our last section in this particular chapter.

Thank You,