**Power System Generation, Transmission, and Distribution**
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**Lecture No. # 29**
**New Raphson (NR), Load Flow Method**

We continue with our story about the load flow method. As you know, the load flow is bread and butter for any power flow engineer. You cannot solve any power system problem without load flow being a part of it because, it gives you the pulse of the system. It tells you what are these four vital parameters in electrical engineering namely real power injected, injected reactive power, voltage magnitude and angle delta. We have just finished Gauss Seidel last time. Today, we are starting the most important method very widely used Newton Raphson load flow method.

This Newton is the same Newton that apple one you know, and it is a numerical technique as you told you is nothing to do with electrical engineering. Those of you have done a course in numerical analysis in your under graduate or anywhere, a book by like Hildebrand or Iyengar and Jain and so on, you must have read a Newton method. It is nothing, but a numerical technique to solve any non-linear algebraic equations, and as you all know by now, that your load flow equations are nothing, but non-linear algebraic equations.

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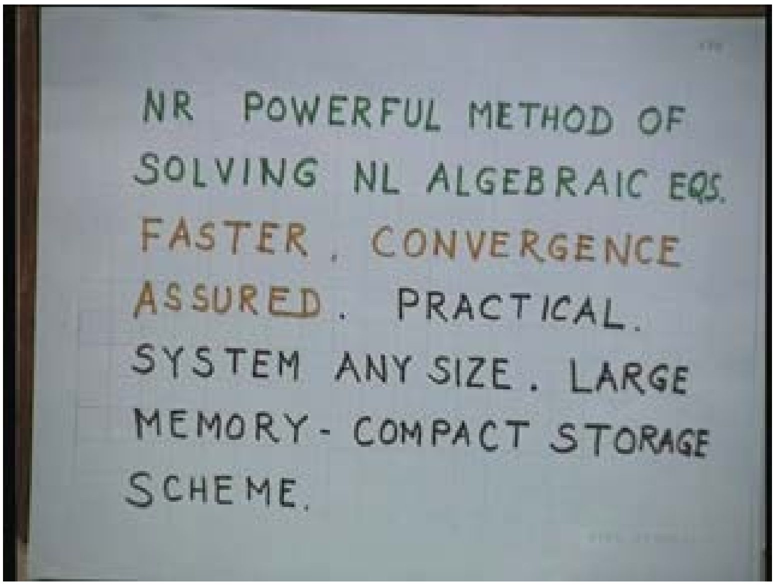

Now, NR as it is briefly called or it is abbreviated is a very powerful method of solving non-linear algebraic equations. It is very fast; convergence is very fast and not only that, convergence is assured; it is guaranteed. There is no way that method would not converge, once you use Newton Raphson. It is a very practical method, system size can be anything; let it be thousand bus; let it be whole country of size of India or former user or US whatever, it will give you solution. Only problem is as I told you every time there is plus and minus in anything that you do in life. The minus point of Newton Rapson method is you need a large memory. In olden days when memory was an issue, now it is no longer a issue. We used to use compact storage scheme which is to tell us, how to solve such a problem using Newton Rapson without crossing the memory limit. Anyway, now you do not have to worry as most powerful computers are available.

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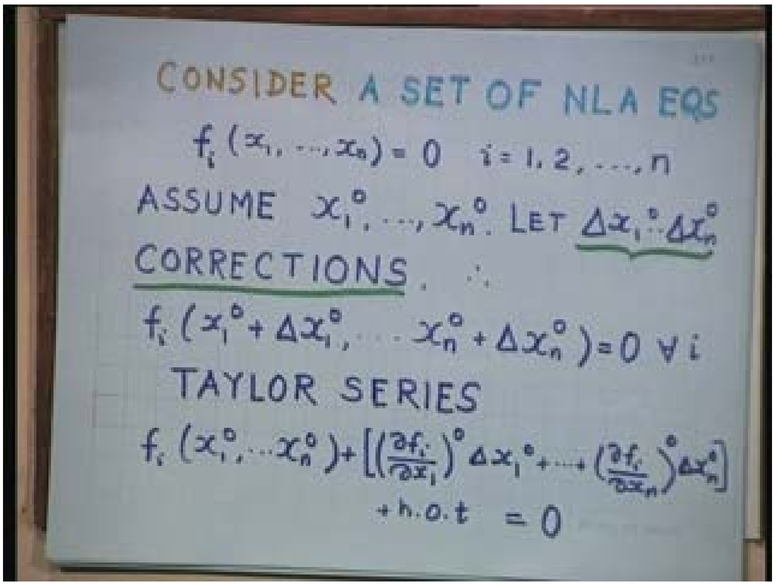


Let us consider a set of non-linear algebraic equations writing in a mathematical form f i x 1 up to x n is equal to 0 for all i; i varying from 1 to n. This is a non-linear algebraic equation. Any as I have been repeatedly telling you earlier on, when you solve any numerical technique you have iteratively that is, you have to assume a initial solution. An assumption of initial solutions more intelligently you do lesser time it will take for convergence. There were you are as an engineer, your all past experience, your knowledge will come to your rescue because, you know your system so well you know what is the starting point. In fact, if it is a daily problem, daily solution last solution could become starting point for the next solution. However, if you assume that you do not know about anything about the system, it is entirely a new system and entirely you are a new person first day in a job, you can always assume x 1 0 to x n 0 whatever values and luckily Newton Raphson method is not sensitive to the starting solution, Gauss Seidel method was.

But, naturally since you assuming initial solution, it will not give you final solution. Otherwise, there is something wrong otherwise you are knowing the solution. You are not a God, you are a human being. So whatever initial solution you assume that will not necessarily give you final solution because, it is a numerical technique. There have good ought to be in iterations, it is a iterative process. So let delta x 1 0, delta x n 0 be the corrections, which when added to x 1 0 will give you next estimate of the variable. Solve it till you get convergence criterion satisfied. Let us see, lets us proceed further.

Now these seeing are called corrections; how do you find out corrections? Unless and until you find out corrections, how do you add? So next step is, substitute of x 1 by x 1 0 delta x 1 0 and so on is equal to 0 for all i; inverted A is for all. This is nothing, but the Taylor series expression now we are going to use the first term; the second term is first derivative plus h o t is higher order terms. Now, in practice you can almost always neglect higher order terms to make your life little comfortable.

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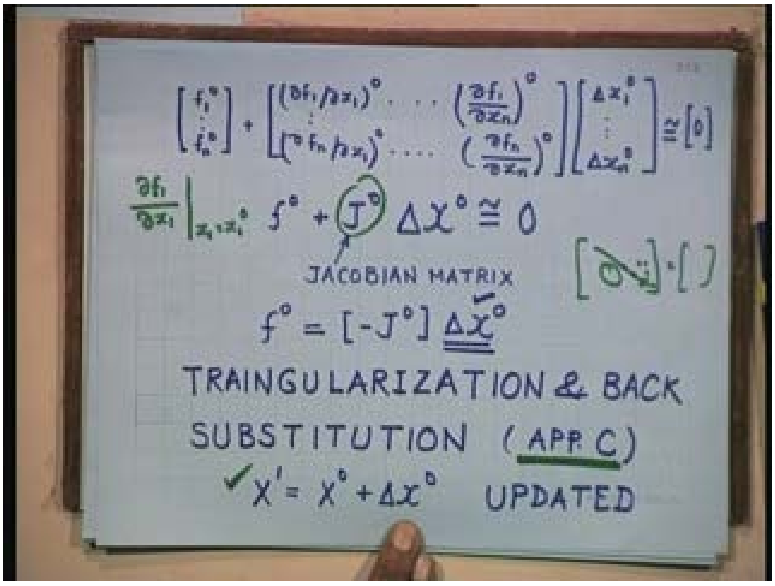


Now I can write that Taylor series expression in a form of a matrix; f 1 0 f n 0 plus del f 1 del x 1 0 and so on. This 0 means this derivatives are evaluated at x 1 0. In fact, this only means del f 1 del x 1 evaluated at x 1 is equal to x 1 0 that is all. Into simplify this I am writing this superscript 0; delta x 1 0 up to del x n is equal to nearly equal to 0. Why nearly? We have ignored those high order terms. If you want to write in a compact form rather than carrying such a big matrix equation f 0 J 0 delta x 0 is nearly equal to 0; this J 0 is called Jacobian matrix. What is Jacobian matrix? Jacobian matrix is that matrix whose each element is first order derivative. If it is become second order derivative it is called (( )) matrix, which fortunately or unfortunately we have ignored.

So now, I can write further f 0 is equal to minus J 0 delta x 0 and this delta x 0 we have to find out. So, it needs an inversion of Jacobian matrix. This is another negative point of this method. In Gauss Seidel we did not have to use any inversion; it was straight forward; most easy method to understand. Even an ordinary person can solve using Gauss Seidel. This is just one equation as we have seen. Here you got to invert this matrix in order to get corrections. However, all of you may be aware if you have done your numerical analysis course, there is a method called triangularization and back substitution. It only means, you have to create all 0 and then x n can be immediately found out; substitute x n here find out x n minus 1; substitute x n x n minus 1; find out x n minus 2 back substitution. This is called triangularization and back substitution. The details of this is given in appendix C in your text book. So you can go to appendix C, there is solved example given there; 3 by 3 matrix which you can easy solve otherwise also.

Any 3 by 3 matrix, I think even calculator can give you a solution if it is a programmable. If it is nonprogrammable, where you can solve it; after all you have to find out cofactors, determinants and things like that. Then, now we have got an updated value of x; x 1 is equal to x 0 plus delta x 0. This is an updated value.

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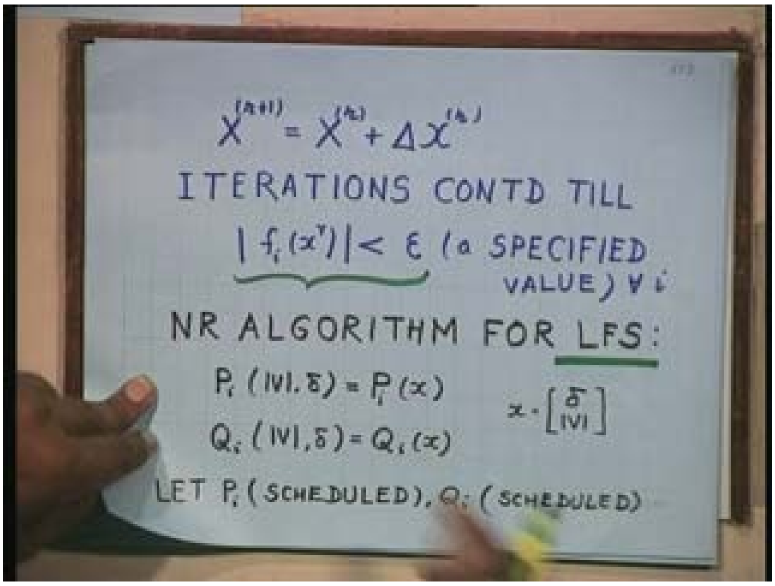


Likewise in general, you will have x r plus 1 is equal to x r plus delta x r in general. Now, how long you are going to continue with iterations, as I have been repeatedly telling in your lectures, any numerical technique not only you have to assume initial solution, you have to devise a criterion which will help you to stop. You cannot go on going to till eternity, that means qayamath. So, you have to stop it somewhere and this stopping criterion is given here, iterations continue till this become less than epsilon, where epsilon is nothing, but a specified value for all i. Again this depend on you, how accurate solution you want. Is this load flow for planning purposes? Is this load flow for real time operation or online control or it just a normal load flow? It depends on that. What accuracy you want? NR algorithm now we come to the proper. So far we have been talking about mathematics. Now, let us talk of electrical engineering or power systems for load flow studies.

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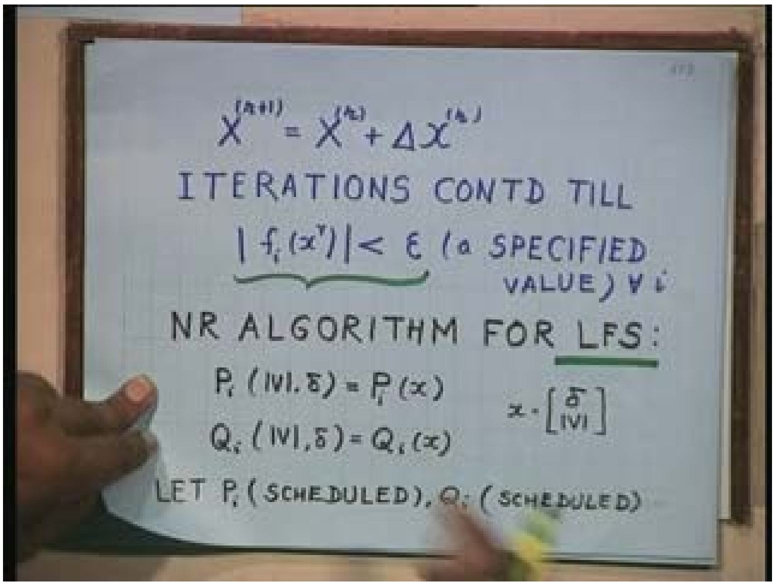


You are aware of these two equations P i and Q i these are called s l f e. Let say this P i is function of x and Q i is function of x, where x is this delta and V. Why am doing this? I am assuming that all buses are P Q buses except one; that has to be a slag bus in a load flow problem, and I also told you why there should be a slag bus. It is the slag bus which does many jobs like supplying the rest of the losses at the end of the solution. It is a reference bus because the delta are of rest of buses are referred to angle of a slag bus, which is normally assume 0. Again a slag bus is normally given number one without loss of generality you can always give it any number, if you are so fussy about it. Some books also give or some experts and bus also.

Now let P i schedule or scheduled or Q i is scheduled, they are given to you on all PQ buses. What is the definition of PQ bus? Where P and Q are given to you, they are scheduled. Bus using your initial solution, you can always calculate P i and Q i. Now, this P i and Q i which are calculated should equal to scheduled values, if you want you can write here calculated also. If it is 0, that is a ideal conditions. That means, your solution is perfect, you do not have to worry. But as you know, unless and until you know a solution, you are not God, you cannot assume a solution which is a final solution. So, there has to be delta P x and delta Q x, they are residuals mismatch factor; so many names given in the literature, and this mismatch factor or residuals; this is the Jacobian and these are the corrections. You have assume certain delta 0; you have assume certain V 0, now these has to be corrected in every iteration, updated, modified. So that, we march further towards our goal. What is our goal? Goal is to achieve a converge solution, the final solution, the correct solution. Now, guess Jacobian is known to you, because this can be evaluated at trail values at x is equal to x 0; or P is equal to P 0; or Q is equal to Q 0, and J is also sparse like your (( )) bus. You must have been told by your teacher or you must have read in your books that, why bus is a sparse matrix? Today, this class is not that sparse; it is fairly full but normally, when my only empty students are there, the class is hardly full. That is called wide bus today it is said bus because, all elements are full available. Would it wide bus is a sparse matrix and that is why, we prefer wide bus for load flow studies because, we do not have to store zeros.

Enhanced storage requirements; further wide bus has certain other qualities; namely the buildup of wide bus is easy by simple following simple rules, which you have been told by your teachers, and even modification of wide bus is easy. So building up of wide bus is easy; modification of wide bus is easy; it is sparse. That is the reason why this is a very important question normally asked in interviews, in engineering services or even IIS; why wide bus is preferred for load flow studies is the reason. Similarly, J bus is also sorry Jacobian is also equally sparse; so this J is sparse. Now for each PQ bus, you will have two rows in Jacobian; for each PV bus, you have only one row that is delta row; why? Voltage is already given to you. So, PV bus contributes only one additional row whereas, PQ contributes two rows.

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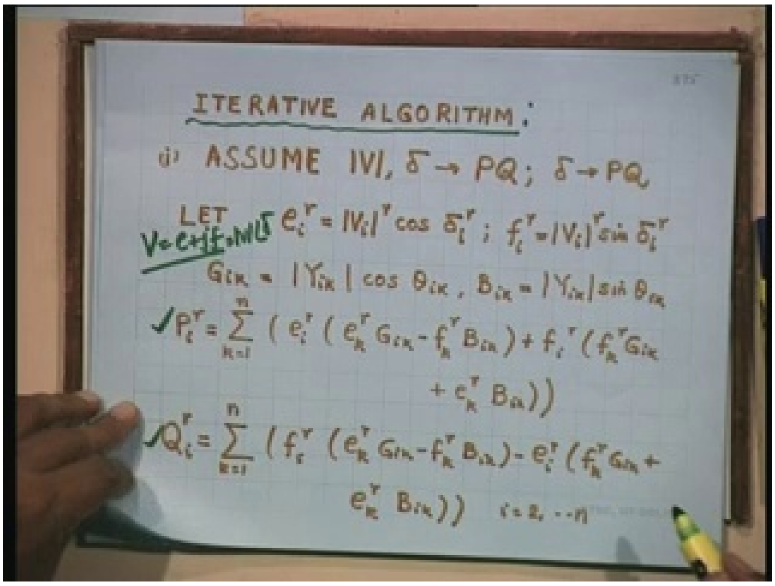

Now we come to the iterative algorithm. Assume V and delta, since your solving a power system problem, you know Joule very well that voltages in power system have got to be close to 1 per unit; 0.95 per unit to 1.45 per unit; that is the range in which voltage should vary. Forget about India, where it can be minus infinity to plus infinity as I normally say, but that should not be the case. So, in case you are not sure how to assume the initial values of V or delta, you are always welcome to use flat voltage start. The flat voltage start means 1 plus 0 or 1 angle 0, and you are likely to be close to the final solution.

Let, now certain times it is always better to compute s l f e in rectangular coordinates not in a polar coordinates. Why? Polar coordinates means you have non-linear terms sine and cosine, which takes longer time to get computed. So, it is always better to convert s l f e equations in rectangular coordinates. How do you do that? Let real e i r the real value of the voltage v i r cosine delta i r and f i r, the imaginary part of voltage is v i r sin delta i r. That means, I am assuming V is equal to e plus jf; and V is V angle delta and that is how I am writing this. G iK the conductance real part of wide bus is Y iK cosine theta iK; please remember theta iK is close to 90 degrees because, the resistance is very small. In any well power system; in any normal power system r by x ratio are ought to be very very small; x is roughly ten times. If it is not, then you are solving an ill condition system, where r becomes comparable to x. And still you can solve load flow because, certain systems are ill condition, but Newton Raphson method will always give an answer, even if it is ill condition system, which is not guaranteed with Gauss Seidel method.

So, B iK susceptance is Y iK sine theta iK; now am substituting this rectangular variables and this is final equation obtained for s l f e using rectangular coordinates. So, you see sine and cosines have vanished. So this way will get a quicker computation of Ps and Qs. You need not copy them they are given in your text book whichever text book may be using all text books have to give this. Now, as I again ask you and talk to you, we have to stop somewhere in any numeric analysis. How do you stop here? These residuals should be less than epsinon, they will not become exact 0; do not expect that to happen, even you solve x 0 is equal to x square is equal to four by numerical analysis by assuming x is equal to 2.1, you may not get two answers in two three iterations. You may have to go for some iterations, but if you are happy with 2.0001 you may get it in couple of iterations, which is as good as answer 2. So here this epsinon, how do you assume that depends on accuracy you want. it can be 10 raise to the power minus 4; it can be 10 raise to the power minus 8; it can be even 10 raise to the power minus 2.

So, it all depends on you, the designer, the analyst, the expert, the power system engineer. You may have to work in load dispatch center; you may be going to power iterate; you may be going to BHEL, NTPC, NHPC, Central City Authority or Tata Electric Companies or any private companies. You will have to solve load flow again and again because, that side I said you bad and better. Calculate finally, what is the outcome of load flow solution? The slag power, slag bus power, real as well as reactive, the losses real as well as reactive and line flows because, even in security analysis which you may be reading somewhere, you will have to see the overloads. The line flow should not be overloaded line should not be overloaded. If this is not equal to epsinon less than epsinon then, we evaluate Jacobian again; we find out the correction vector delta V r and delta delta r added to the earlier one; get new V r plus 1; get new delta r plus 1, and continue. Continue what? Till not the eternity, till this condition gets satisfied and that is the end of your load flow solution.

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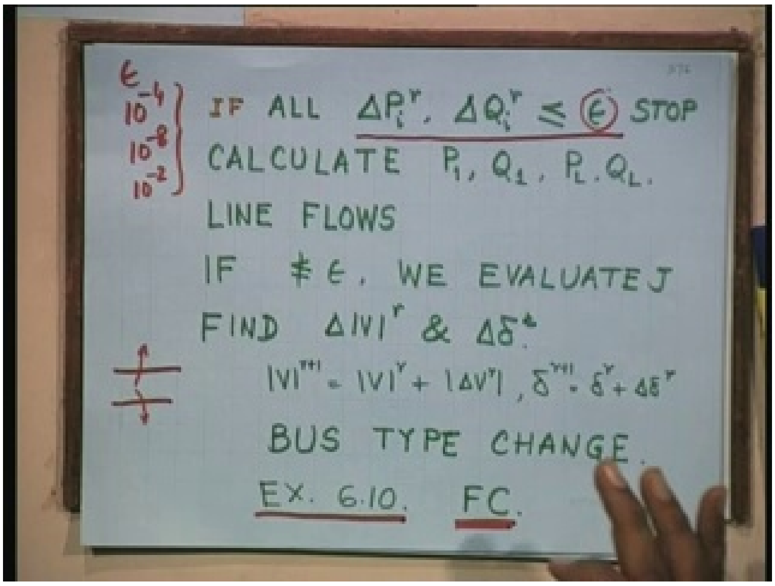


Now, as I have told you during Gauss Seidel; in case there are constraints in life there are constraints, nobody is unconstraint even Birla’s, Tata’s or Reliance or Mittal’s they have limited wealth; it is not infinite wealth. Even I have one hour class (( )) or minor is one hour; major is… There is a constraint; you have 24 hours in a day constraint. In life you have always constraint by many things. So similarly here, if voltage crosses the limit, there is a concept of change of bus. Luckily, you can either cross upper limit or lower limit not both simultaneously. Either you are inside the studio or outside the studio, you cannot be simultaneously both place; only God is omnipresent. So, if its crosses upper limit or lower limit remedy is state, you convert it into the change of tie buses PQ to PV or PV to PQ depending upon Q is crossed or V is crossed. If Q is crossed, PV become PQ bus. You fix that Q, and that becomes a PQ bus, till you are back within the range then, you can change back. So, this is the change of bus type which can take place. Finally, the final solution should be within constraint otherwise, that is not the correct solution, because feasibility is more important condition than optimality. See the solution has got to be feasible first then, only can you look for optimum. Of course, there are methods in optimization techniques where you start with optimality and go for feasibility. But, eventually you have to attain feasibility then, only you can say the solution is optimum. Kindly solve the example 6.10 of your book and read the flow chart because certain stereo type universities still ask draw the flow chart; half an hour gone you know, but depends on your teacher, and class and so on. Certain people can still ask in engineering services they can still ask. Luckily gate has become fully objective type; so they cannot ask you flow chart in gate exam. This is all about Newton Raphson.

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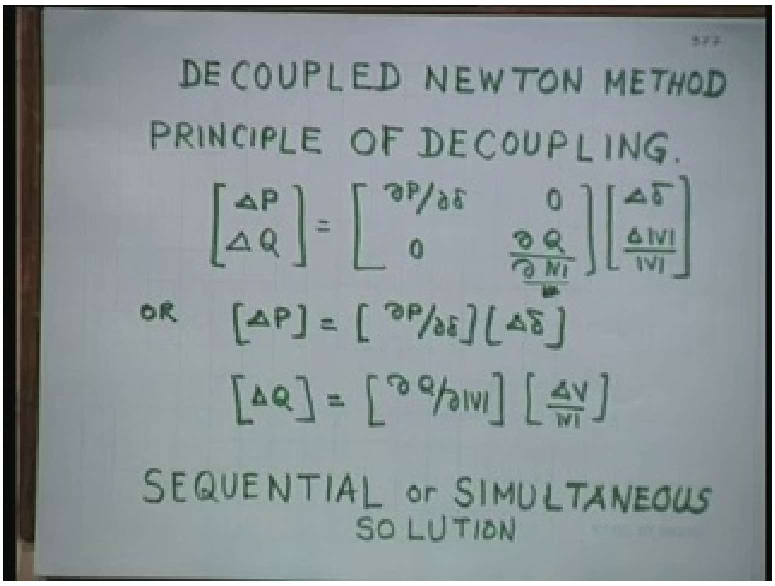

The flow chart has been given in book for polar version as well as rectangular portion. As I told you there is a plus point in polar; there is a plus point in rectangular nothing is absolute. It depends what do you want. Let me also tell you one thing polar is realistic because, direct voltage, direct angle; what is this enf then nothing practical about real part of voltage or reactive. You cannot go and catch real part and leave imaginary part in voltage, but he said numerical convenience; that is why we have gone for enf. Now next, there are so many load flow methods starting from Gauss, Gauss Seidel, Newton Raphson, approximate load flow which we have been done once, where we ignored resistance. So, it becomes a you can a close flow solution if you want to. Then there is decouple load flow which we are going to start soon; then there is a fast decouple load flow; then there is a super decouple load flow like super express and so on. Then there is a second order load flow method. And finally, we will have optimal load couple load flow then, we have stochastic load flow after all your loads are not known with complete certainty, they are random variable. When you get a phone call nobody knows it is a random event. When you get a customer in a shop is a random nobody know unless and until you have a appointment like doctors, like lawyers. There also you get 15 minutes this way, that way, the jams, the bus could not get, you could not get the parking and so on. So, that is the stochastic load flow; the very first paper in (( )) load flow came in 1973 by a polish lady (( )) and since then, stochastic load flow and hybrid load flow and so on. Other varieties of load flows; people have done master thesis, doctor thesis on load flow. There is continuation power flow that is, if you go for contingency analysis and security analysis may be for under graduate; I do not know Dr. Mishra will tell us whether it will go up to that extent or not because, you have to do full course. You do not have to do only load flow for rest of the semester. What is decoupling? The power system we have golden principle of decoupling. What is that principle? P is more friendly to delta; Q is more friendly to V; you no like you move in a group. Some of you go together to Priya cinema or to Chanakya or you for a restaurant. So, you know like togetherness, affinity.

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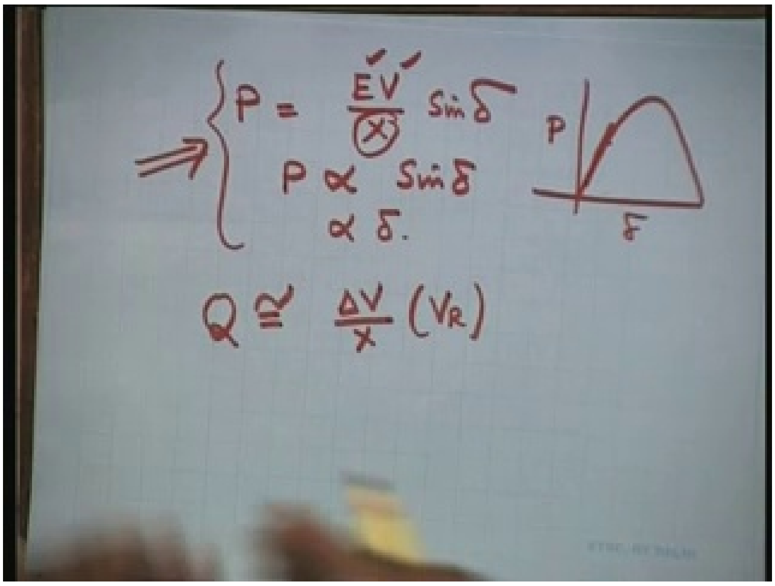


So similarly, the real power is having more affinity towards… I will write a equation which will show you that be that P is EV by X sin delta. Once transmission line is built X is fixed; of course, if we do some compensation this and that minor changes be there. But, again it is fixed having done the compensation; E and V are also fixed; they are suppose to be fixed close 1 per unit and hence, P is directly proportional to sin delta and delta as all of you know, we operate close to 30 degrees; linear portion of P delta curve power angle curve which we must done in your stability in case you have done it. For our under graduate friends, you might have not done stability, you will do it perhaps hopefully in some course. So, delta is small and all of you know from your trigonometry knowledge that sin delta is small; sin delta is close to delta; even sin 30 is 0.866 very close to 1 sorry when delta is 0 sin delta is also 0; so it is close to delta. So now, this the equation this shows that P is only depends on delta not on V. Reverse is true for Q, this is approximate. Otherwise, you can write those four big big equation; receiving end Q; sending end Q; receiving end P; sending end P and so on those for big equations, but when it is a approximated lambda delta goes away. In fact, cosine delta term is there, but delta is small cosine delta becomes 1; cosine 0 is 1. So, this is proportional to voltage and that is why my famous decoupling principle is P is close to delta; and Q is close to V. Using this what I have done, when we that this will be here. I can make this of diagonal term 0 whatever, these half diagonal terms delta P by delta V. Did you recall that?

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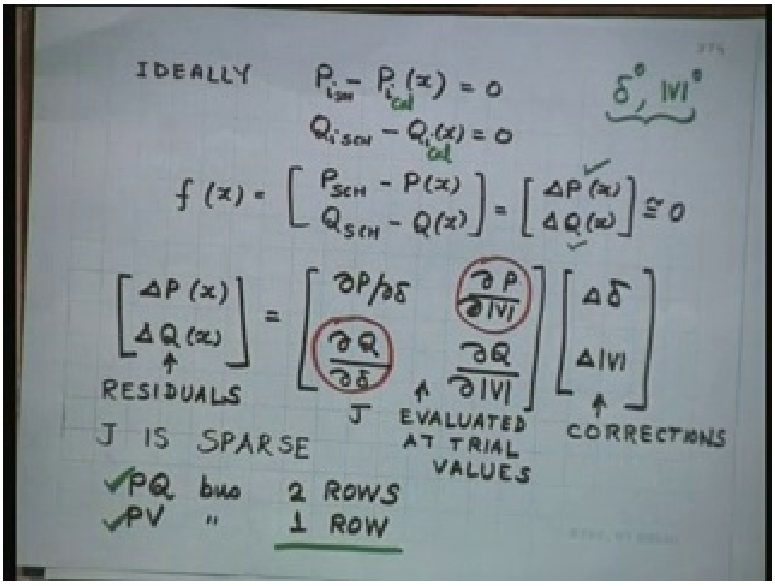


If you go back to Newton Raphson method delta P, when P is not dependent on V where the question of variation of vector V. So, I am within my right to put it 0; similarly I am within my right to put this also 0; Q is not dependent on delta at all and this has done an miracle. What is the miracle? Suddenly, half diagonal become 0 and hence, I can decouple these two equations as this. That is why this is called decoupled load flow method. Suddenly, I my dimension is become half, so a smaller computer can be used in case you have one. In fact, you can solve real problem independently of reactive power problem. If you are not interested in economic dispatch or economic power system, you are only interested in minimization of losses solve only reactive power problem, but you are interested in both then, either you can form a sequential mode or simultaneous mode. What is sequential? Solve one equation from here, solve one from here, another this like this sequentially. What is simultaneous? Solve them together as if they are one equation. Sequential is better than simultaneous; first solve all delta P so that, you may get latest deltas put them here and then you get a latest solution for (( )). As I just said sequential is better; it gives you faster convergence; it reduce memory requirement. You need to have that space only and the next problem can also be solved using the same space. So, you have reduced the memory requirement. X more iterations as compared to Newton Raphson, but reduced memory requirement, reduce computation time per iteration as compared to Newton Raphson. This question is formally asked in the examination compare various load flow methods specially, these three methods Gauss Seidel, Newton

Raphson and FDLF. We have not yet done FDLF so, let first do FDLF then we come to this comparison. Fast decouple load flow method.

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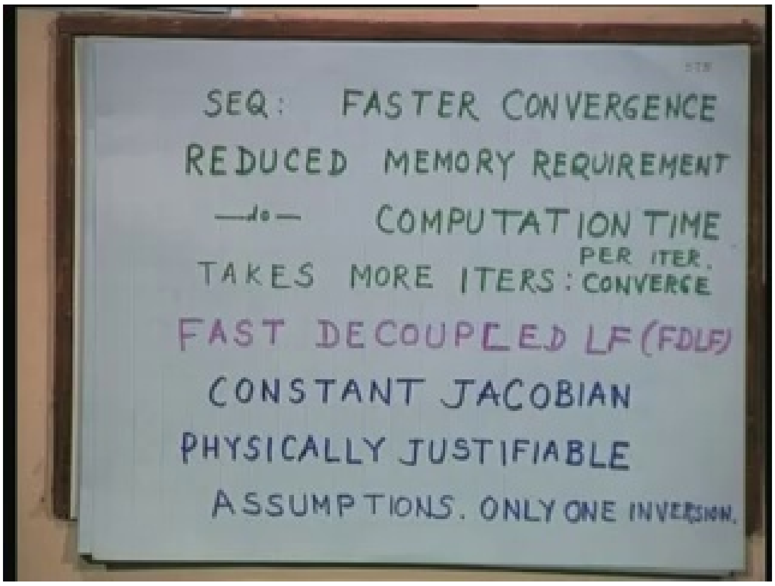


What is the concept in fast decouple load flow method. We avoid certain computation become faster. What is the biggest bottle neck or problem in Newton Raphson method or even decouple load flow method? You have to invert a matrix in each iteration that takes lot of your time. Suppose iterations are ten, normally sees Newton Raphson has a quadratic convergence, it converges fast in two to three iterations right. But, even three iterations means you have invert three times. Why do that? How do you avoid that? By making Jacobian constant; if you make it constant somehow, the constant matrix how many time you will iterate only once, because the matrix will remain the same; there is no point in iterating it again and again only a mad fellow will do it, because you are not getting new information. Matrix is same so invergence is also same and hence, if you can produce a constant Jacobian by physically justifiable assumptions. You cannot jabardasti constant; you have to prove, you have to show to the world look its possible, and there are certain conditions which make it possible those condition of course, we will look in next class and I am not going to today.

But, let me tell you to conditions are same; resistance are can be ignored as compared to reactances or deltas are small; these are the various justifiable assumptions, and G can be ignored as compared to X that is same thing as R and X. So, we will see what are those conditions and there are more conditions, and this paper came in 74 by Stuart Brand Stuart, who use to be in (( )) and that is say milestone paper; one paper can make you immortal like domalantiny newton Raphson 67 that paper came in 67. Gauss Seidel paper (( )) came in 56; that is the first load flow paper. Before that, people is to solve network by AC calculating board or DC calculating board or analog computer. Prior to that, it was only slide rule; I do not know whether you are generation has even heard the name of slide rule. I use to carry as a student everyday slide rule and we have to solve numericals using that and it use to be very approximate. Nowadays, things are so easy and only one invergence is the p any fit of this FDLF.

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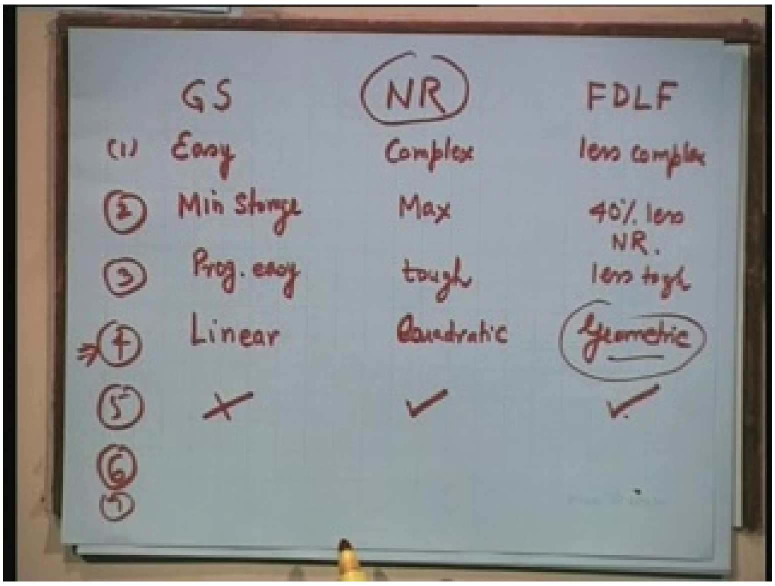


Now, let me give you brief comparison because our younger friends will not be available next time. The Gauss Seidel, Newton Raphson and FDLF; this is easy to understand; this method anybody can understand. Newton Raphson method is complex; this is slightly less complex because, only one invergence is involved; minimum storage requirement in Gauss Seidel. In fact, there is no other method which needs which can have lower then this as storage requirement. Here is the maximum storage; this is 40 percent less than NR. Third, programming is very easy; of course, nowadays mat lab and pas cad and hardly anybody has to do any programming; in our days we have to carry so many cards is to punch one mistake error, and output will be given next day. We have wait 24 hours to correct it. Programming is tough here, less tough. Convergence wise this is the worst; it has linear convergence; it is the best, it has a this what we call quadratic convergence, and this is geometric convergence. I have given one appendix on convergence also. So, if you want you can read that; if I am not mistaken it is nth appendix in this book and some appendix in that book.

So, linear quadratic geometric convergence wise NR is the best. If you want sensitivity property; what is sensitivity? If delta varies how much P varies; that is the sensitivity property delta P by delta Q. It is not available here in Gauss Seidel as there is no derivative involved. You can see remember Gauss Seidel equation there was no derivative; it was very simple equations 1 over Y i i minus P i minus JQ i upon Y i JK star in bracket sigma V i and so on; that was the Gauss Seidel equation. Here, we do have a sensitivity property here we do have sensitivity property these are required for optimal operations, optimal load flow and hence, these methods are more suitable if you want optimal load flow. System size this I am talking about the attributes; we have to make one more column, when you give this table in exam or somewhere when you write a book, you do not have to write only three columns, the fourth column is attributes which I am only telling you like here I was saying how the method is; here I was saying storage is attribute; here the programming is attribute; here the convergence is attribute; here the sensitivity problem is attribute. Now I am talking system size.

If system size is small then Gauss Seidel is wonderful, but it becomes thousand bus system then there are problems, but this NR method does not care for system size; it can be any system size whereas, FDLF again system size is not that much of a problem. The time increases linearly with system size in Gauss Seidel whereas, in NR it is hardly matters, whether it is a 3 bus, 30 bus, 300 bus or 1000 bus more or less it converges in three to four iteration; here it takes five to six iteration because, convergence is not quadratic but geometric; that is geometric is in between linear and quadratic. So, this is about system size. Seventh, type of system if it is a normal system then Gauss Seidel is no problem, but if it is a ill condition system Gauss Seidel is having a convergence may have a convergence problem. Let me also tell you some experts feel before doing Newton Raphson, it is always advisable to have one or two iterations of Gauss Seidel, but not necessary this is just one opinion. Is it (( )) proved or just an opinion?

Not yet proved.

Not yet proved. See you have an expert opinion also, but then certain people have states this, so I taught I will as well state.

It may decrease the iteration to some extent.

Yes.

If you have a starting a Gauss Seidel.

Yeah Gauss Seidel. So, this is what you should know that sometimes it helps if you have first you know Gauss Seidel one or two iterations then you go for Newton Raphson. Now, let me tell you the FDLF there is a some sacrifice in accuracy after all you have ignored those half (( )). So when the attribute becomes accuracy nothing to beat Newton Raphson because, you are not ignoring anything in Newton Raphson. You are taking the system as it is, as it stands, as it exists whereas, here for your benefit, for your own convenience, for your own comfort you are ignored no doubt you are justifiable ignoring it, but however the fact remains that it brings in certain amount of approximation and hence, certain amount of inaccuracy. So, in case somebody is very particular about accuracy where for him or her Newton Raphson is the only prescription. I think with this I thank you and we finish this. Any difficult if you have, you are most welcome to ask.