Power System Generation, Transmission and Distribution

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Lecture No. # 28

Load Flow Analysis (Contd.), Gauss Siedel Method

So, good morning ladies and gentleman. Let us start lecture 28, we continue with our topic of Load Flow Analysis. Today, we propose to cover Guass Siedel Method.

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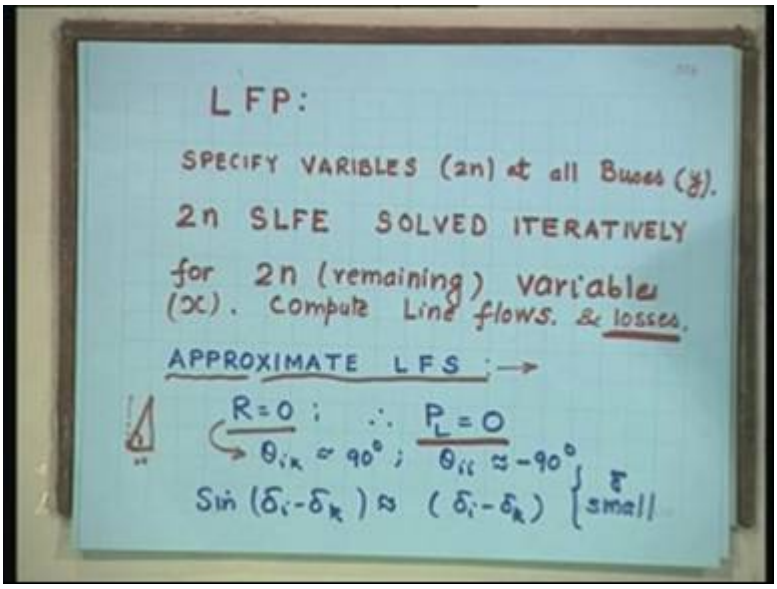

Load flow problem, what we do is, all of you know, we specify variables 2n at all the buses, we call that is a vector y and we when we solve 2n SLFE equations iteratively, because you cannot solve them in a close form manner. For remaining 2n variables are also known once the equations are solved. Finally, we compute line flows followed by losses, because these are very important for many power systems studies. So, as to see whether line flows are within limits, with the power system security is maintain, whether we are full filling the contractual obligations and losses are within limits. Now, sometimes for planning studies, as I said earlier on we need an approximate load flows, see want you know what happens in 2007 or 8 or something; planning long term midterm, short term, very long term. Then we use approximate load flow studies by approximate, we mean the R is assumed to 0 that makes losses also 0.

So, mainly we are bothered about voltage for five and line flows when losses are 0, and that this R 0 makes these both theta as nearly 90 degrees, as it is theta is close to 90 degree. I told you R is very small, x is very high. So, this angle is as it is close to 90 degree, but when these become zero then the angle becomes 90 degrees. And sine also becomes of delta equal to delta when delta is small, this we have already talked earlier also.

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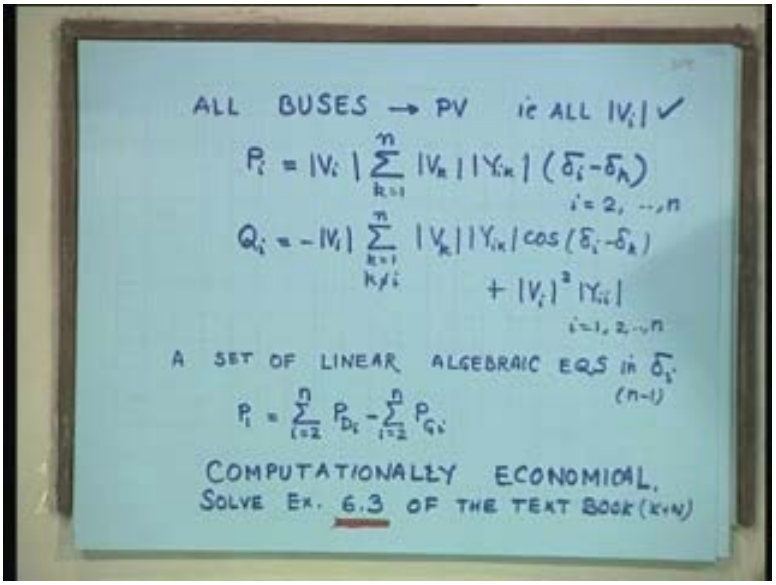

Let us assume that all buses are PV, when you solve an approximate load flow; that means, all voltages are known. The beauty of a PV bus is, P and V are known and Q and delta are to be found out, that is the minimum PV bus then our SLFE equations get reduced to P i Q i V i sigma V k Y i k delta i minus delta. k the sine delta becomes delta, as we have just seen Q i becomes minus, V i sigma V k Y i k cosine delta i minus delta k. Incidentally cosine of a small angle is large, cosine 0 becomes 1. So, you cannot apply the same theory here and cosine delta when delta is small this delta, no, that is not correct and plus V i square Y i i for all i varying from 1 to n. So, now we have reduced our SLFE two set of linear algebraic equations in delta i. Why delta? I everything as it is known all V's are known, because it is PV buses all Y is known, because it is an model of the system Y bus, and that is all what is unknown left is deltas.

So, once we solve the first equation in delta, substitute here we get P i and Q i both, and once you get P i and Q i you can find out this slack power P 1 which is unknown. So,

power is known. This is computationally very economical way of solving SLFE, because the we come in linear equations. In fact, we do not have to go for iterative solution, it gives you close form solution or use Cramers rule, whatever and you constructed we solve it. Please solve the examples 6 point 3 at the text book which talks about this method; that means, approximate method then you will realize how easy or how convenient it is to solve in an approximate load flow. Obviously, the answers will be approximate using this load flow method, but those answers are indicative of things to come, because in three years there is no guarantee what you are assuming, today will stay as it is there will be some changes. So, there is no point in solving SLFE exactly for something in future which is really not defined well defined. Now, we come to a proper method. In fact, there are four load flow methods.

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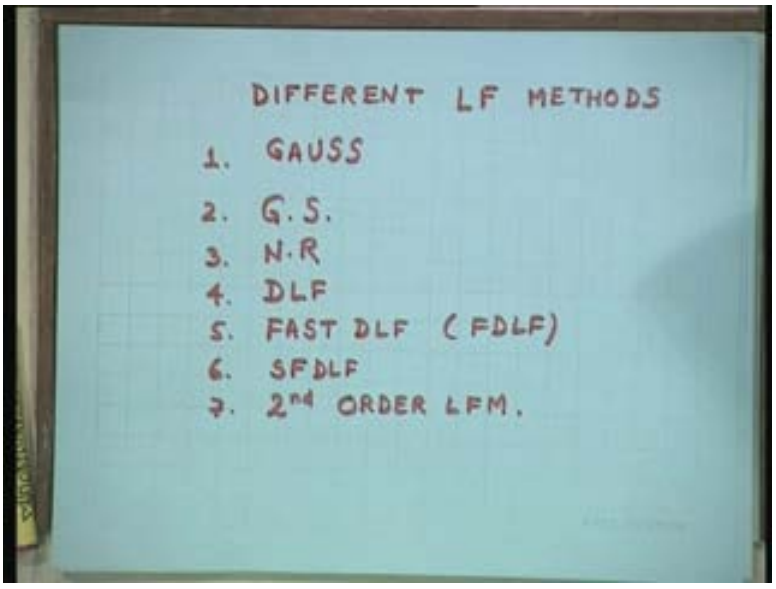

Let me list it for you, different load flow methods, you must have studied only first Gauss, Gauss Siedel, Newton Rapson, Decouple Low Flow Method, Fast Decouple Load Flow Method, which is also called FDLF Fast then Super Fast Decouple Load Flow Method then there is 2 nd Order Load Flow Method, I think we will stop here. These are the various methods which a post graduate student in load flow should know, you might have done perhaps Gauss Siedel and Newton Rapson some places may be you might have done even FDLF, but uniformly I do not think all of you must have done all these methods. So now, gauss method, we need not very much, because as I told you gauss method nobody uses nowadays because unnecessarily we are not updating the data

which you already have. It is a common sense, if you have an updated detail, if you know that current temperature at 11.25 a.m today, why should you show an T V 8 a.m temperature? Unless, until, your aim is to tell viewers whatever the morning like that is the separate issue, but when you say current temperature, you should show as much current as you have and that is why the gauss seidel method becomes the starting point of exact load flow method, you have done approximate few minutes back.

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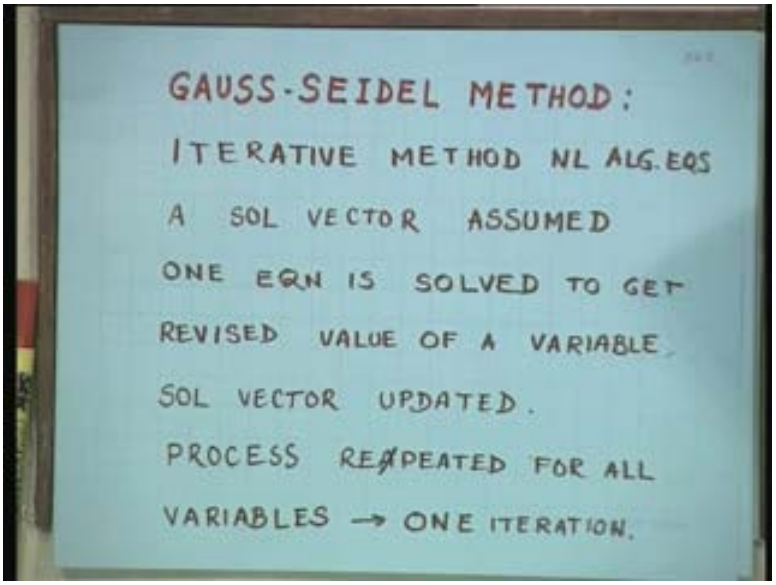

This is needless to say it goes without saying that it is an iterative method for solving any non-linear algebraic, it is nothing to do with power systems or electrical engineering or any engineering, for that matter those of you done numerical analysis course say (( )) book or any book (( )) or whatever book, you must have used in your course. If you at all done that these gauss seidel method, is there this is nothing to do with electrical engineering. This is an numerical technique which can be used to solve any set of non-linear algebraic equation.

So, we are using it for our purpose. A solution is assume, any numerical analysis there are three fundamental factors, you may be doing project work after sometime and then you may have to use certain numerical techniques which you may have to use for solving your problem. So, while selecting numerical technique you have to say so many things; the type of problem, the type accuracy you needs, suppose you say like this gauss seidel, then any numerical technique you may select to it assume the initial solution vector

starting point, because it is not known to you, a solution vector is assumed and one equation is solved to get revised value of a variable, immediately solution vector is updated. What I mean here, suppose you have started next doing it here.

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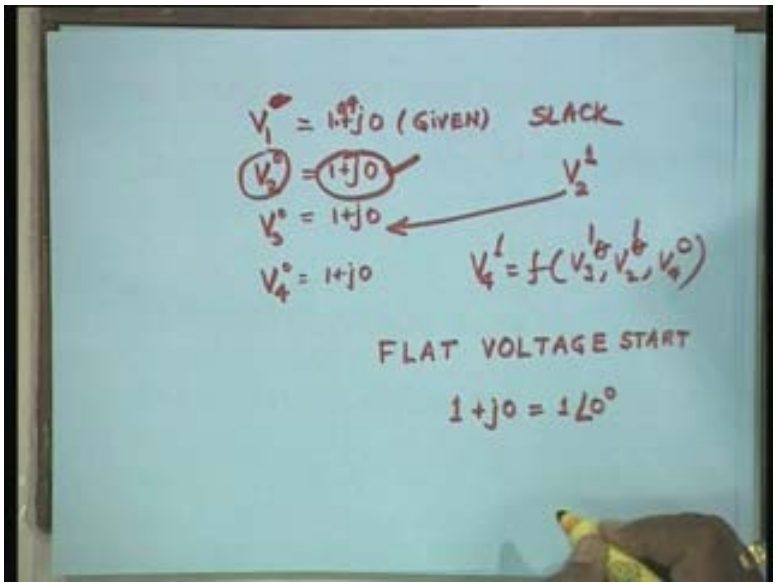

Say, you have a four bus system V 1, V 2, V 3, V 4, luckily to start a load flow solution, we know the solution vector which we should start. Why we know that voltages, at all the buses are close to one per unit and this is called flat voltage start. You must have learn this term earlier also I am not sure, but anyway I repeat it, flat voltage start where you say flat we assume that all voltages are 1 plus j 0 or 1 angle 0. So, the initial values of all voltages become 1 plus j 0 barring this slack, because there voltage is given to you so. In fact, I should not assume voltage here it is given as 1 point 04. So, there is no zero here. It is given, because one is our slack.

So, once you know V 2 0, at the end of first iterations it becomes V 2 1 and when you calculate V 3 0, use this rather using this whole value this is gauss seidel. Instead of using the earlier solution vector we immediately update it with the latest values of voltage. When we go for V 3 0, we immediately use V 2 all then we go for V 4, when you calculate V 4 1 then you should use V 3 1, V 2 1 and V 4 0. So, immediately update rather using 0 0 here, that will make it a gauss method and the difference between gauss and gauss seidel is here, we immediately update it till whatever we have already calculated.

And after updating the solution vector process is repeated for all variable, and once you have finish to all variables, we say one iteration is over. One of the important point in any numerical technique is, when you stop otherwise you can go on, on and on. See to stop some where the stop means you have to find out convergence criterion. Let us see what is of convergence criterion, we have same thing I am writing here.

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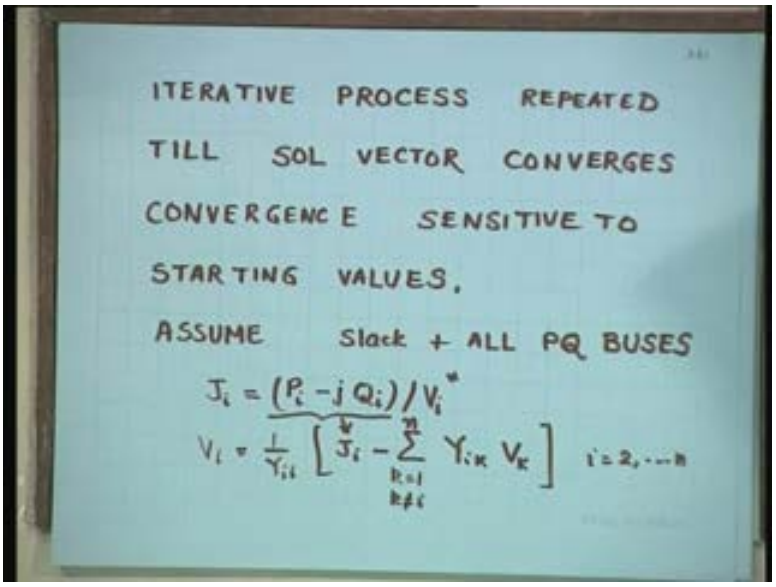

Iterative process is repeated till solution vector converges and let me tell you convergence is very sensitive to starting values is some techniques depending on your starting values, number of iterations will change. If you are not in intelligent person and you start the values of voltages as 0 plus j 0 perhaps, you may not, you will diverge because you denote know your system, you do not have a common sense to understand that voltage is 0 means it is a very bad system, it has to be close to one. And that is why we always start our gauss seidel or any method of load flow by 1 plus j 0 called flat voltage start. Assumes slack plus all PQ buses the voltages, now you know the current is equal to P i minus j Q i over V i star this equation is known to us P is equal to V i conjugate from there we have derived this. This equation also we know.

We know, V i is equal to one over i i into Y V this is nothing but ohms law. V is equal to i V and then for i we are substituting this is equation clear to you or no? This is nothing but let me tell you very simple equation j is equal to V Y now I broken these V into two parts, I have taken Y i and V i this side and rest this side and this is in the symbol j i now

what I do, I substitute in j i this equation moment I do this, because I do not want current at all moment I do this, I get with this final equation.

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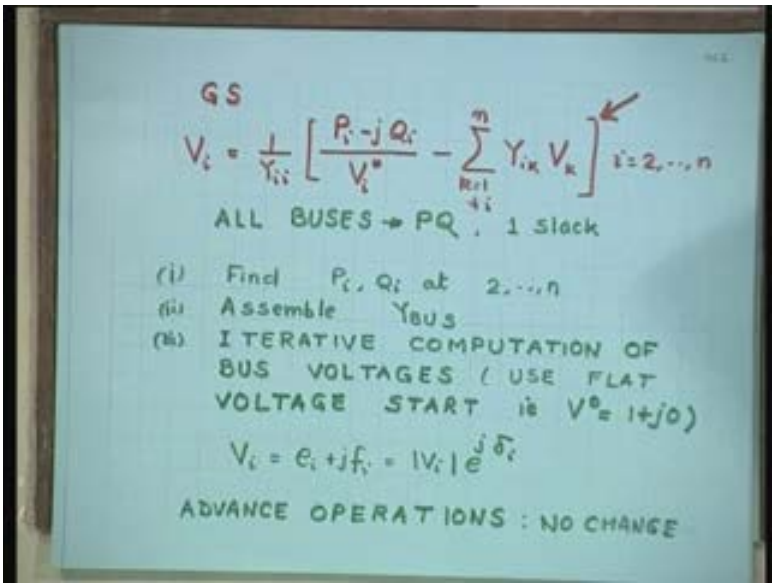

Now, you will see that there is no current anywhere, because in our load flow variables there is no current it is P Q V delta. So, we must bring our load flow equations in these four quantities. Now, V i can be found out by one y i i P i minus j Q i V i star minus k is equal to 1 to n k not equal to i because k i i as gone this way you we have already shifted it to left hand side. So, it is not equal to i Y i k V k i is equal to 2 to n, assuming all are PQ buses, if you recall for approximate solution, we assume all where PV buses it is suited as because we divide want voltage to be unknown variable, this moment voltage becomes unknown variable they do not remain linear equations they become non-linear equations z is equal to x y is an non-linear equations.

If an x and y both are variables, but here we just opposite it which is close to reality, if I if you remember, I told you that 85 percent buses are PQ, 15 percent buses are PV. Because they are not many generators, where is the money and where is the need. So, one is slack fifteen percent roughly are PV and then remaining are P Q. So, this assumption is more close or closer to reality. Now, we find out P i and Q i at 2 to n that is for n minus one equations find means, by find I means since you know load since you know generation. So, generation minus load becomes P i, in that sense fine. Otherwise, you are suppose to be specify, but before specify, you have to find out from somewhere

either you think or you get it from the old data for which system, we close the yesterday seen running power system you do not have to worry about the initial starting solution, because its already there we know after all the (( )) as been working for last so many years.

So, last night when we close the system or if you when you left system is never closed, when you left the office what was the or what is the latest on computer that you can put it here that is P i Q i if it is the theoretical exercise or a planning exercise then you have to find, because then you are assuming what is generation what is load which will common that then P G i minus P G i becomes P i. Assemble Y bus, I have already told you two methods are assembling Y bus; one was simple rules, other was by singular transformation, that is Y is equal to a transpose small y into A, where A is the bus incident matrix and small y is the primitive (( )) matrix, which is a diagonal matrix having a assemble Y bus again for a working system, you do not have to do this again and again, while unless until there is a change in Y bus; that means, there is a change in topology; that means, other a new node is added or existing node is removed or a new line is added problems will be remain same it will not change.

But in theory for a class room, we have to write to this student assemble Y bus in case it is not done earlier. Iterative computation of bus voltages use flat voltage start, I already explain you what is flat voltage start flat. Voltage start is V 0 1 plus j 0 now V i can be expressed rectangular coordinates or in polar coordinates both have plus and minus. What is the plus point here, e i plus j f i the rectangular version at times give better convergence, but this is more practical what you can major is a magnitude by voltmeter what you actually get is an angle.

So, this is more practical and this is more it is convergence twice, it is slight we get advance operations no change now this is a basic. The meaning of this last sentence is very simple, all of you must have studied computers now may be your school days or primary school or middle school or high school I do not know. The first sentence or first thing which is talk to student is whatever remains constant that mathematical operation or arithmetic operation, you should not put in a iterative loop because otherwise you are doing it again and again unnecessarily say, you have to divide 4 by 2 the answer is 2. This division should not be done 100 times if you take 100 iteration which is already known 4 divided by 2 is 2.

So, do it outside the loop, that is why some of the operations in these equations which are fixed should not be put in the loop iterative loop that is what is the meaning of this. Carry out the this operation in advance where no changes likely to take place, that will say we computer time in our days we have told division takes more time than multiplication, I do not know whether things have change in last because we were talk computer is sixties at the time.

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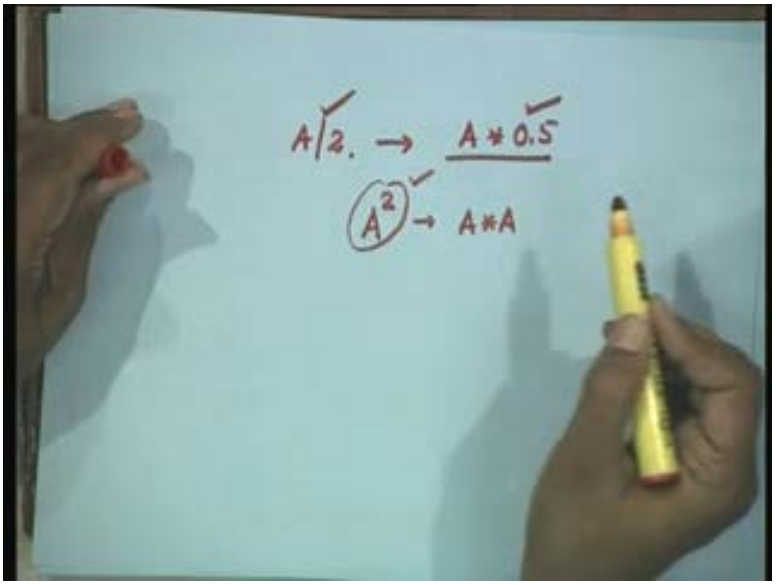

We were told, I you will correct me if there is a rechange. If you have to write A divide by two dot, make it real variable, it is better to write a in to point 5. This takes less time, than this, whether it is still true or not tell me. So, do not write such a statement where the multiplication is easier similarly, I do not know whether this is true now A square should be return as A may be this operation takes more time, I do not know this we were talked in sixties.

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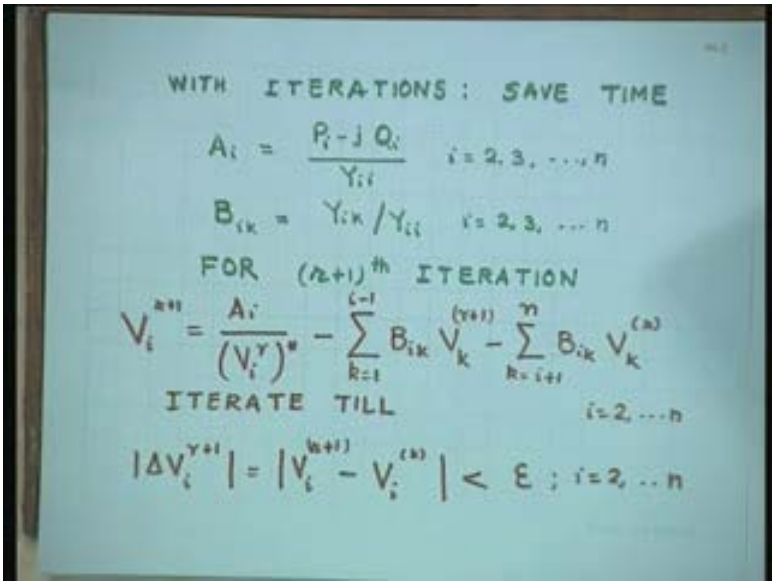

These are the two quantities which came, which are part of that equation P i minus j Q i are known to as for all PQ buses. In fact, there are specify bias they not going to change divide by y i i this y i i is now to going to change. I have already talked two minutes how Y bus remains constant, unless until there is a change in topology. So, why this particular factor should be a part of loop? So, call it A i and compute it once and for all for all buses, all PQ buses. Similarly, we call it B i k y i k divided by y i i this division takes lot of time as I told you and so why not divide it once and for all and keep it as B i k and that is why that equation now becomes this A i or V i r star minus sigma. This sigma I am breaking in to two parts, because of gauss seidel this is already A i. Similarly, this divide by this is B i that is why I am now breaking this equation into these parts, you are suppose at fourth bus; that means, first three bus is you have a latest value R plus 1 eth iteration, R eth iteration you have fourth bus because you are at the fourth bus.

Now, that is why as separated these sigma into two, this is the latest values you calculate. Here, you calculate the originally assume values which is 1 plus j 0, but here you will put the exits values which you have got and how long you will it iterate it? Till the voltage difference between immediate pass to iterations becomes less than epsilon and how do you define epsilon it depends how much accuracy you decide to have for your system. If you are very (( )) 10 is to power minus four ten is to power minus five six, if you is it know these are all rough calculations planning calculations then even 10 in to minus 2 is minus one is and so on it depends on the decide accuracy; that means, the purpose for

which you are conducting load flow as I have repeatedly set, load flow is required practically in all power system studies be it stability be it economic operation be it security or what for studies whatever these things will be more clear when you solve a numerical.

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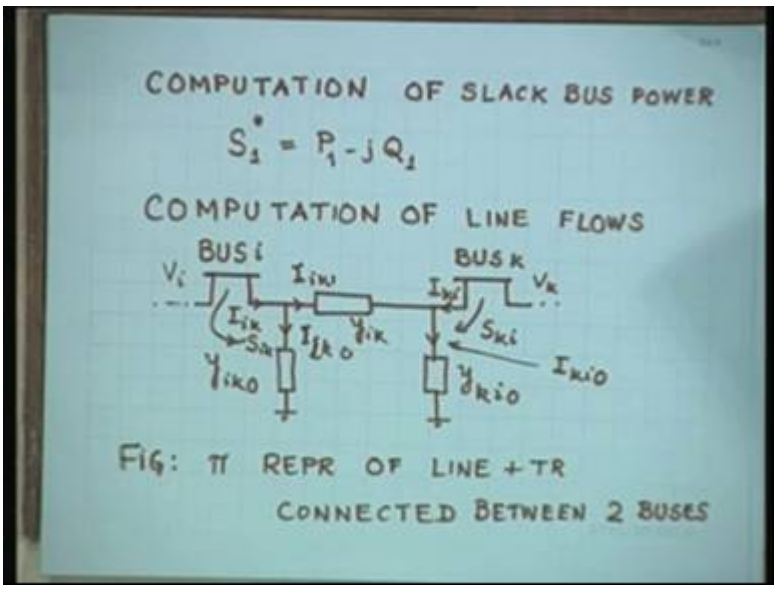

How do you compute slack bus power? Once you know all voltages you already iterations are over you are converse just substitute, S one star is equal to P one minus j Q one equal to whatever equation was two are there substitute the final conversed values of voltages you get the slack power, because that was unknown P 1 and Q 1 where not known. So, now, you know the P 1 and Q 1, now we must compute line flows I told you line flows are very important how do you compute line flows? You had to give a model of line, this is the bus i this is bus k this may be (( )) this may be (( )) they are connected by line, now this transmission line can be model by short line, but it is not a short line if it is more than hundred kilo meters it is not a short line and if it is less than hundred kilometer.

It is a short line, then you need only a series impedance, assume that you are going from, let us a Delhi to Chandigarh which is definitely not a short line, it is about both more than 5 hours journey also. So, must be two fifty kilometers or three hundred kilometers or whatever kilometers, it is definitely more than hundred kilometer. So, we represent either by pi or T, now can you tell me which model is better? pi is better or T is better

both can represent. In fact, they are call nominal pi and nominal T, both can represent medium line similarly, both can represent long line, if you talk of equivalent pi and equivalent T, where you use hyperbolic cosine and hyperbolic sine that A B C D constants this we must have done in your under graduate this has been conclusively proved that pi model is better than T.

So, we are not talking at T, I have drawn a pi model, why pi is better than T? I have already talked you perhaps that pi the shunt branches can be connected to the load combine with the load then the number of nodes are fever here then T. T has a middle node which you cannot replace you cannot ignore. So, there is a problem with T. Otherwise, T is, you find this is nothing wrong with T. So, we prepare pi model in load flow analysis. So, this is bus i, this is bus k this is V i the bus voltage this is V k bus voltage the power the complex power which is coming to the network its S i k, from here it is S k i this is the current i i k this is the part of the current i i k is one going to this series branch the i i k is zero going to the shunt branch.

Similarly, here also the current going is i k i is 0 and here i k i and this is y k i zero is the shunt branch y i k is the series branch. So, this is pi representation of line plus transformer both connected between two buses now, once you know this figure right then you can write equations and ultimately, you can find out and equation for line flows what are those equations now keep this figure in mine. I do not know how to put both of them together, but I anyway i i k apply k c L apply k c L at this point. So, this i i k is equal to sum of these two currents that is why and doing it here.

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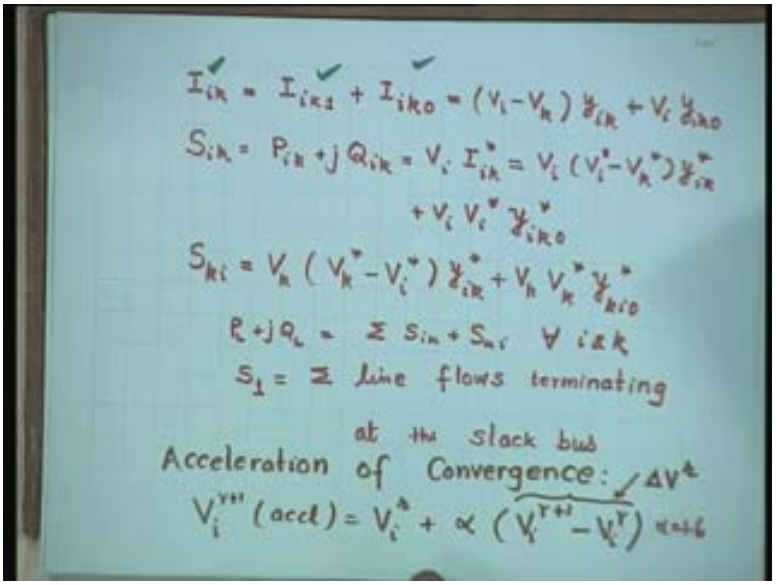

Sum of these two currents, how do you find out this current? Very simple, ohms law. V y this is V i minus V k y i k plus V i y i k 0 here, as I said we do not like current at all in a electrical engineering. So, I am replacing current into always into voltage as set y or z. So, did not that once you do the then you know the line S i k is nothing but real line flow real power flow plus reactive power flow. So, that V i i i k star we are just found out i i k substitute star means you to change here anything which has come. So, making it star wherever, there is no star. Similarly, find out S k i same way now you know the line flows S i k and S k i these are the two line flows S i k flowing from i 2 k and S k i flowing from k 2 i. Suppose, you add them what you will get ideally should give zero, but there are losses see in case you want to find out losses in these line and these two line flows, whatever you get is the loss in that line suppose you want know losses in all the lines what you do? Sum all the line flows sigma of S i k for all i and k is nothing but S L complex losses P L plus j Q L normally you are more bother about P l.

So, you can take it real part of this and you get the P l. So, P L plus j Q L is summation of or same thing it is already there for all this ulta A, inverted A is what for all i and k S one suppose, you want to know the slack power there is an alternative way of knowing slack power, if you know line flows add all the line flows immunity from slack bus suppose, this is slack bus and there are two three lines add these three line flows that is in nothing but slack bus power. So, this what is return here S 1 is sigma line flows terminating at the slack bus. Now, there are certain people who want to accelerate

convergence, see why do you think there are (( )) and Delhi roads, everybody wants to reach all destination first and that is why there are no lane system in a India. If you go any foreign country beautifully cars are coming it looks.

So, nice one after other whatever speed 100 kilo meter or 100 mails per hour the goes smoothly. First thing is, here they are not only cars, there is a car, there is a auto, there is two wheeler ,varieties of two wheeler, then there is a cycle riksha, then there is even buffalos on the road sometimes. So, all start you can people have walking in the middle of the road you know they create their own way. So, that is why it is real challenge to derive on Indian roads and therefore, there are problems because everybody wants to be ahead of everybody else. So, in power system also there are some people move on to have fast convergence we cannot wait, they no phases. So, acceleration of convergence, there is a formula V i R plus 1 eth accelerated value is V i R eth value, plus alpha times the difference between the two iterative values of the voltages, now this alpha is does the trick alpha is called acceleration factor.

Unfortunately, I cannot write everything on the slide, the phases limited plus I write it with my own hand normally people get tired, nothing other you do not attend anybody else class, the earlier class we generally must have typed it that gives more uniform beautiful lines, but I write now this alpha is called acceleration factor. Now, there is an another problem, what is the problem there are problems and problems in life, do not think life is without problem, whether your prime minister or your class there are problems and all of you know what problem is facing our problem is how to assume the value of alpha you are given me the method to accelerate the convergence fine, but how to a assume it is a million dollar position and mind it.

If you assume a wrong value instead of having you faster convergence is gives you low slower convergence sometimes divergence like if you take too much of (( )). Do not think it will do could to you, there is a limit there is a constraint now we know the word constraint. So, if you assume a bad value, a wrong value of alpha you had it instead of getting convergence you will get slower convergence or instead of getting faster convergence either you gets slower convergence or you get divergence, the normal recommended value of alpha is 1 point 6 like a every medicine water there are recommended dose and finally, is a concerned your doctor.

So, that suppose tomorrow somebody dies, we cannot report, because it return the one line small latest which nobody can write like all (( )) warning on (( )). So, small that even sixteen in whole boy with good eyes cannot read it, because there only full filing the legal requirement they actually do not want people to read that. So, similarly here it is recommended value of alpha is 1 point 6 hopefully you will get convergence. Now, we come to the reality in reality all or not PQ buses you are nicely assume all are PQ buses to make your life simpler happier always person wants part of least resistance in our Delhi life.

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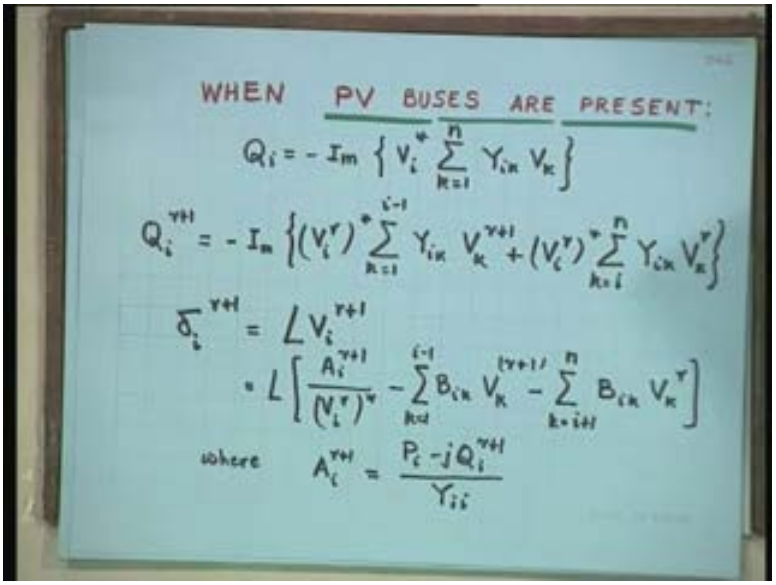

When PV buses are also present, which is the reality, PV buses are always there indeed there. So, Q i becomes unknown. So, how do you find out Q i now, minus imaginary values of V i cross sigma k is equal to one to n y i k and V k this equation we know, now again i am divided these into two parts up to i minus one eth bus, because I know the updated values upgraded values new values whatever you want to call it and these are old solution vector once you know Q i you can find out delta i also just an angle of V i R r plus 1. So, this is the old friend V i R plus one take an angle of that and you know delta i here, A i R plus one is P i minus j Q i R plus one by y P i there is no super script because P i is known whether it is PV buses or it is a PQ bus P is common to that; that means, P is always specify in a load flow studies except slack bus where P is not know till the end now, you know very well why it is not known till the end .

So, now, we have generalized to gauss seidel load flow method, we have covered we are covered when there is a approximate load flow, we also covered when there is only PQ buses we also covered when there is a PV buses also present, but mine one thing when you get this values of Q's here, you have to remember one thing, check whether they are within constraints after all Q is being supplied by other synchronous convertor or a bank of capacitors or bank of inductance you do not have infinite capacitance with you, do not even the wealthiest country in the world U.S as constrained on Q units power system. So, this Q has to be checked by if statement I do not know which language you write, I only know for I used write only if statement.

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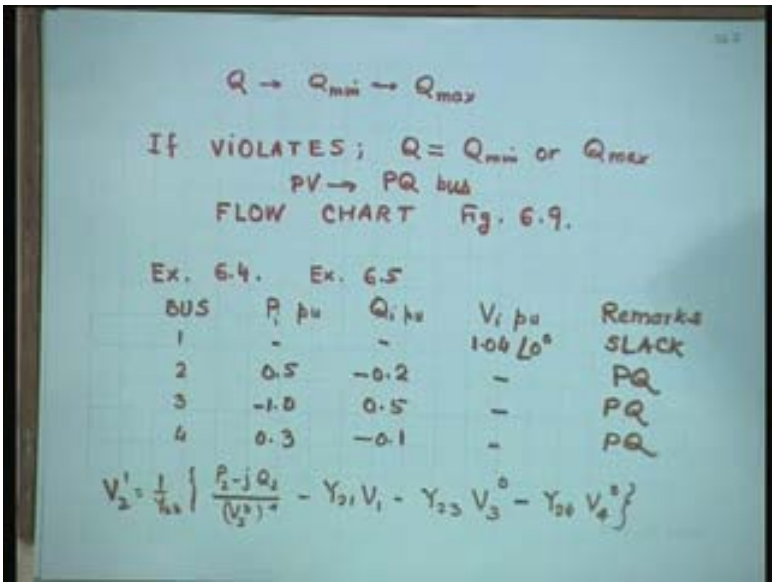

So, check if Q is between Q min and Q max, if it is have a coffee break you can go to where ever coffee or tea which you will like if not, if violates, luckily here nature also helps you this is Q max and this is Q min Q can either lie inside or outside, but outside means other either it can cross of upper limit or lower limit not both. Simultaneously, your either in this studio or your outside you cannot both, only god is omni present that to nobody knows nobody seen god this only prospection. So, here the reality is either Q crosses upper limit or Q crosses lower limit cannot cross both simultaneously it is a same time.

So, whichever violations take place make it equal to Q min or Q max and this PV, but bus gates converted in to PQ bus as long as there are violation the moment violations go

away the bus converted back in to PV bus, this is something which is very important to understand please mind it, same thing happens in PQ bus also the voltage which you are calculated if it crosses V min and V max similar thing you make to do and the bus type change from PQ to PV. Flow chart for the whole thing is given in figure 6 point 9. I just note point wasting time in here, drawing the flow chart, it would not have come in one A4 sheet, anyway and I do not know the mechanism at technology of showing your bigger chart for perhaps, I have to understand from that you still in a (( )). I mean I thought I could go whenever I want the board I can go whenever, I want here or power point something, I do not know whether it is possible of course, no work is need not I am really like do that perhaps, if technology permits that. Now, that is examples 6 point 4 kindly solve that example, examples 6 point 5 and this is the bus data given to you then and V 2 1 calculate V 2 3 and V four one and so on. And complete the problem and when we need. To next Tuesday, I assume that you will be able to solve this problem, anyway thank you very much.